# $i2^k \mod n$

## Ian McLoughlin

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## 2

 $\begin{array}{ccc} 0 & 0 \\ 1 & 0 \end{array}$ 

## 3

#### 4

## 5

## 7

#### 8

## 9

#### 

9 8 7 

#### 

| 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
|----|----|----|----|----|----|----|----|----|----|----|----|
| 1  | 2  | 4  | 8  | 3  | 6  | 12 | 11 | 9  | 5  | 10 | 7  |
| 2  | 4  | 8  | 3  | 6  | 12 | 11 | 9  | 5  | 10 | 7  | 1  |
| 3  | 6  | 12 | 11 | 9  | 5  | 10 | 7  | 1  | 2  | 4  | 8  |
| 4  | 8  | 3  | 6  | 12 | 11 | 9  | 5  | 10 | 7  | 1  | 2  |
| 5  | 10 | 7  | 1  | 2  | 4  | 8  | 3  | 6  | 12 | 11 | 9  |
| 6  | 12 | 11 | 9  | 5  | 10 | 7  | 1  | 2  | 4  | 8  | 3  |
| 7  | 1  | 2  | 4  | 8  | 3  | 6  | 12 | 11 | 9  | 5  | 10 |
| 8  | 3  | 6  | 12 | 11 | 9  | 5  | 10 | 7  | 1  | 2  | 4  |
| 9  | 5  | 10 | 7  | 1  | 2  | 4  | 8  | 3  | 6  | 12 | 11 |
| 10 | 7  | 1  | 2  | 4  | 8  | 3  | 6  | 12 | 11 | 9  | 5  |
| 11 | 9  | 5  | 10 | 7  | 1  | 2  | 4  | 8  | 3  | 6  | 12 |
| 12 | 11 | 9  | 5  | 10 | 7  | 1  | 2  | 4  | 8  | 3  | 6  |

| 0  | 0  | 0  | 0  |
|----|----|----|----|
| 1  | 2  | 4  | 8  |
| 2  | 4  | 8  | 2  |
| 3  | 6  | 12 | 10 |
| 4  | 8  | 2  | 4  |
| 5  | 10 | 6  | 12 |
| 6  | 12 | 10 | 6  |
| 7  | 0  | 0  | 0  |
| 8  | 2  | 4  | 8  |
| 9  | 4  | 8  | 2  |
| 10 | 6  | 12 | 10 |
| 11 | 8  | 2  | 4  |
| 12 | 10 | 6  | 12 |
| 13 | 12 | 10 | 6  |

| 0  | 0  | 0  | 0  |
|----|----|----|----|
| 1  | 2  | 4  | 8  |
| 2  | 4  | 8  | 1  |
| 3  | 6  | 12 | 9  |
| 4  | 8  | 1  | 2  |
| 5  | 10 | 5  | 10 |
| 6  | 12 | 9  | 3  |
| 7  | 14 | 13 | 11 |
| 8  | 1  | 2  | 4  |
| 9  | 3  | 6  | 12 |
| 10 | 5  | 10 | 5  |
| 11 | 7  | 14 | 13 |
| 12 | 9  | 3  | 6  |
| 13 | 11 | 7  | 14 |
| 14 | 13 | 11 | 7  |

| 0  | 0  | 0  | 0 | 0 |
|----|----|----|---|---|
| 1  | 2  | 4  | 8 | 0 |
| 2  | 4  | 8  | 0 | 0 |
| 3  | 6  | 12 | 8 | 0 |
| 4  | 8  | 0  | 0 | 0 |
| 5  | 10 | 4  | 8 | 0 |
| 6  | 12 | 8  | 0 | 0 |
| 7  | 14 | 12 | 8 | 0 |
| 8  | 0  | 0  | 0 | 0 |
| 9  | 2  | 4  | 8 | 0 |
| 10 | 4  | 8  | 0 | 0 |
| 11 | 6  | 12 | 8 | 0 |
| 12 | 8  | 0  | 0 | 0 |
| 13 | 10 | 4  | 8 | 0 |
| 14 | 12 | 8  | 0 | 0 |
| 15 | 14 | 12 | 8 | 0 |
|    |    |    |   |   |

| 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
|----|----|----|----|----|----|----|----|
| 1  | 2  | 4  | 8  | 16 | 15 | 13 | 9  |
| 2  | 4  | 8  | 16 | 15 | 13 | 9  | 1  |
| 3  | 6  | 12 | 7  | 14 | 11 | 5  | 10 |
| 4  | 8  | 16 | 15 | 13 | 9  | 1  | 2  |
| 5  | 10 | 3  | 6  | 12 | 7  | 14 | 11 |
| 6  | 12 | 7  | 14 | 11 | 5  | 10 | 3  |
| 7  | 14 | 11 | 5  | 10 | 3  | 6  | 12 |
| 8  | 16 | 15 | 13 | 9  | 1  | 2  | 4  |
| 9  | 1  | 2  | 4  | 8  | 16 | 15 | 13 |
| 10 | 3  | 6  | 12 | 7  | 14 | 11 | 5  |
| 11 | 5  | 10 | 3  | 6  | 12 | 7  | 14 |
| 12 | 7  | 14 | 11 | 5  | 10 | 3  | 6  |
| 13 | 9  | 1  | 2  | 4  | 8  | 16 | 15 |
| 14 | 11 | 5  | 10 | 3  | 6  | 12 | 7  |
| 15 | 13 | 9  | 1  | 2  | 4  | 8  | 16 |
| 16 | 15 | 13 | 9  | 1  | 2  | 4  | 8  |

| 0  | 0  | 0  | 0  | 0  | 0  | 0  |
|----|----|----|----|----|----|----|
| 10 | 14 | 16 | 8  | 4  | 2  | 1  |
| 2  | 10 | 14 | 16 | 8  | 4  | 2  |
| 12 | 6  | 12 | 6  | 12 | 6  | 3  |
| 4  | 2  | 10 | 14 | 16 | 8  | 4  |
| 14 | 16 | 8  | 4  | 2  | 10 | 5  |
| 6  | 12 | 6  | 12 | 6  | 12 | 6  |
| 16 | 8  | 4  | 2  | 10 | 14 | 7  |
| 8  | 4  | 2  | 10 | 14 | 16 | 8  |
| 0  | 0  | 0  | 0  | 0  | 0  | 9  |
| 10 | 14 | 16 | 8  | 4  | 2  | 10 |
| 2  | 10 | 14 | 16 | 8  | 4  | 11 |
| 12 | 6  | 12 | 6  | 12 | 6  | 12 |
| 4  | 2  | 10 | 14 | 16 | 8  | 13 |
| 14 | 16 | 8  | 4  | 2  | 10 | 14 |
| 6  | 12 | 6  | 12 | 6  | 12 | 15 |
| 16 | 8  | 4  | 2  | 10 | 14 | 16 |
| 8  | 4  | 2  | 10 | 14 | 16 | 17 |
|    |    |    |    |    |    |    |

| 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1  | 2  | 4  | 8  | 16 | 13 | 7  | 14 | 9  | 18 | 17 | 15 | 11 | 3  | 6  | 12 | 5  | 10 |
| 2  | 4  | 8  | 16 | 13 | 7  | 14 | 9  | 18 | 17 | 15 | 11 | 3  | 6  | 12 | 5  | 10 | 1  |
| 3  | 6  | 12 | 5  | 10 | 1  | 2  | 4  | 8  | 16 | 13 | 7  | 14 | 9  | 18 | 17 | 15 | 11 |
| 4  | 8  | 16 | 13 | 7  | 14 | 9  | 18 | 17 | 15 | 11 | 3  | 6  | 12 | 5  | 10 | 1  | 2  |
| 5  | 10 | 1  | 2  | 4  | 8  | 16 | 13 | 7  | 14 | 9  | 18 | 17 | 15 | 11 | 3  | 6  | 12 |
| 6  | 12 | 5  | 10 | 1  | 2  | 4  | 8  | 16 | 13 | 7  | 14 | 9  | 18 | 17 | 15 | 11 | 3  |
| 7  | 14 | 9  | 18 | 17 | 15 | 11 | 3  | 6  | 12 | 5  | 10 | 1  | 2  | 4  | 8  | 16 | 13 |
| 8  | 16 | 13 | 7  | 14 | 9  | 18 | 17 | 15 | 11 | 3  | 6  | 12 | 5  | 10 | 1  | 2  | 4  |
| 9  | 18 | 17 | 15 | 11 | 3  | 6  | 12 | 5  | 10 | 1  | 2  | 4  | 8  | 16 | 13 | 7  | 14 |
| 10 | 1  | 2  | 4  | 8  | 16 | 13 | 7  | 14 | 9  | 18 | 17 | 15 | 11 | 3  | 6  | 12 | 5  |
| 11 | 3  | 6  | 12 | 5  | 10 | 1  | 2  | 4  | 8  | 16 | 13 | 7  | 14 | 9  | 18 | 17 | 15 |
| 12 | 5  | 10 | 1  | 2  | 4  | 8  | 16 | 13 | 7  | 14 | 9  | 18 | 17 | 15 | 11 | 3  | 6  |
| 13 | 7  | 14 | 9  | 18 | 17 | 15 | 11 | 3  | 6  | 12 | 5  | 10 | 1  | 2  | 4  | 8  | 16 |
| 14 | 9  | 18 | 17 | 15 | 11 | 3  | 6  | 12 | 5  | 10 | 1  | 2  | 4  | 8  | 16 | 13 | 7  |
| 15 | 11 | 3  | 6  | 12 | 5  | 10 | 1  | 2  | 4  | 8  | 16 | 13 | 7  | 14 | 9  | 18 | 17 |
| 16 | 13 | 7  | 14 | 9  | 18 | 17 | 15 | 11 | 3  | 6  | 12 | 5  | 10 | 1  | 2  | 4  | 8  |
| 17 | 15 | 11 | 3  | 6  | 12 | 5  | 10 | 1  | 2  | 4  | 8  | 16 | 13 | 7  | 14 | 9  | 18 |
| 18 | 17 | 15 | 11 | 3  | 6  | 12 | 5  | 10 | 1  | 2  | 4  | 8  | 16 | 13 | 7  | 14 | 9  |

| 0  | 0  | 0  | 0  | 0  | 0  |
|----|----|----|----|----|----|
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| 2  | 4  | 8  | 16 | 12 | 4  |
| 3  | 6  | 12 | 4  | 8  | 16 |
| 4  | 8  | 16 | 12 | 4  | 8  |
| 5  | 10 | 0  | 0  | 0  | 0  |
| 6  | 12 | 4  | 8  | 16 | 12 |
| 7  | 14 | 8  | 16 | 12 | 4  |
| 8  | 16 | 12 | 4  | 8  | 16 |
| 9  | 18 | 16 | 12 | 4  | 8  |
| 10 | 0  | 0  | 0  | 0  | 0  |
| 11 | 2  | 4  | 8  | 16 | 12 |
| 12 | 4  | 8  | 16 | 12 | 4  |
| 13 | 6  | 12 | 4  | 8  | 16 |
| 14 | 8  | 16 | 12 | 4  | 8  |
| 15 | 10 | 0  | 0  | 0  | 0  |
| 16 | 12 | 4  | 8  | 16 | 12 |
| 17 | 14 | 8  | 16 | 12 | 4  |
| 18 | 16 | 12 | 4  | 8  | 16 |
| 19 | 18 | 16 | 12 | 4  | 8  |
|    |    |    |    |    |    |

| 0  | 0  | 0  | 0  | 0  | 0  |
|----|----|----|----|----|----|
| 1  | 2  | 4  | 8  | 16 | 11 |
| 2  | 4  | 8  | 16 | 11 | 1  |
| 3  | 6  | 12 | 3  | 6  | 12 |
| 4  | 8  | 16 | 11 | 1  | 2  |
| 5  | 10 | 20 | 19 | 17 | 13 |
| 6  | 12 | 3  | 6  | 12 | 3  |
| 7  | 14 | 7  | 14 | 7  | 14 |
| 8  | 16 | 11 | 1  | 2  | 4  |
| 9  | 18 | 15 | 9  | 18 | 15 |
| 10 | 20 | 19 | 17 | 13 | 5  |
| 11 | 1  | 2  | 4  | 8  | 16 |
| 12 | 3  | 6  | 12 | 3  | 6  |
| 13 | 5  | 10 | 20 | 19 | 17 |
| 14 | 7  | 14 | 7  | 14 | 7  |
| 15 | 9  | 18 | 15 | 9  | 18 |
| 16 | 11 | 1  | 2  | 4  | 8  |
| 17 | 13 | 5  | 10 | 20 | 19 |
| 18 | 15 | 9  | 18 | 15 | 9  |
| 19 | 17 | 13 | 5  | 10 | 20 |
| 20 | 19 | 17 | 13 | 5  | 10 |

| 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
|----|----|----|----|----|----|----|----|----|----|----|
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| 2  | 4  | 8  | 16 | 10 | 20 | 18 | 14 | 6  | 12 | 2  |
| 3  | 6  | 12 | 2  | 4  | 8  | 16 | 10 | 20 | 18 | 14 |
| 4  | 8  | 16 | 10 | 20 | 18 | 14 | 6  | 12 | 2  | 4  |
| 5  | 10 | 20 | 18 | 14 | 6  | 12 | 2  | 4  | 8  | 16 |
| 6  | 12 | 2  | 4  | 8  | 16 | 10 | 20 | 18 | 14 | 6  |
| 7  | 14 | 6  | 12 | 2  | 4  | 8  | 16 | 10 | 20 | 18 |
| 8  | 16 | 10 | 20 | 18 | 14 | 6  | 12 | 2  | 4  | 8  |
| 9  | 18 | 14 | 6  | 12 | 2  | 4  | 8  | 16 | 10 | 20 |
| 10 | 20 | 18 | 14 | 6  | 12 | 2  | 4  | 8  | 16 | 10 |
| 11 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 12 | 2  | 4  | 8  | 16 | 10 | 20 | 18 | 14 | 6  | 12 |
| 13 | 4  | 8  | 16 | 10 | 20 | 18 | 14 | 6  | 12 | 2  |
| 14 | 6  | 12 | 2  | 4  | 8  | 16 | 10 | 20 | 18 | 14 |
| 15 | 8  | 16 | 10 | 20 | 18 | 14 | 6  | 12 | 2  | 4  |
| 16 | 10 | 20 | 18 | 14 | 6  | 12 | 2  | 4  | 8  | 16 |
| 17 | 12 | 2  | 4  | 8  | 16 | 10 | 20 | 18 | 14 | 6  |
| 18 | 14 | 6  | 12 | 2  | 4  | 8  | 16 | 10 | 20 | 18 |
| 19 | 16 | 10 | 20 | 18 | 14 | 6  | 12 | 2  | 4  | 8  |
| 20 | 18 | 14 | 6  | 12 | 2  | 4  | 8  | 16 | 10 | 20 |
| 21 | 20 | 18 | 14 | 6  | 12 | 2  | 4  | 8  | 16 | 10 |

| 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
|----|----|----|----|----|----|----|----|----|----|----|
| 1  | 2  | 4  | 8  | 16 | 9  | 18 | 13 | 3  | 6  | 12 |
| 2  | 4  | 8  | 16 | 9  | 18 | 13 | 3  | 6  | 12 | 1  |
| 3  | 6  | 12 | 1  | 2  | 4  | 8  | 16 | 9  | 18 | 13 |
| 4  | 8  | 16 | 9  | 18 | 13 | 3  | 6  | 12 | 1  | 2  |
| 5  | 10 | 20 | 17 | 11 | 22 | 21 | 19 | 15 | 7  | 14 |
| 6  | 12 | 1  | 2  | 4  | 8  | 16 | 9  | 18 | 13 | 3  |
| 7  | 14 | 5  | 10 | 20 | 17 | 11 | 22 | 21 | 19 | 15 |
| 8  | 16 | 9  | 18 | 13 | 3  | 6  | 12 | 1  | 2  | 4  |
| 9  | 18 | 13 | 3  | 6  | 12 | 1  | 2  | 4  | 8  | 16 |
| 10 | 20 | 17 | 11 | 22 | 21 | 19 | 15 | 7  | 14 | 5  |
| 11 | 22 | 21 | 19 | 15 | 7  | 14 | 5  | 10 | 20 | 17 |
| 12 | 1  | 2  | 4  | 8  | 16 | 9  | 18 | 13 | 3  | 6  |
| 13 | 3  | 6  | 12 | 1  | 2  | 4  | 8  | 16 | 9  | 18 |
| 14 | 5  | 10 | 20 | 17 | 11 | 22 | 21 | 19 | 15 | 7  |
| 15 | 7  | 14 | 5  | 10 | 20 | 17 | 11 | 22 | 21 | 19 |
| 16 | 9  | 18 | 13 | 3  | 6  | 12 | 1  | 2  | 4  | 8  |
| 17 | 11 | 22 | 21 | 19 | 15 | 7  | 14 | 5  | 10 | 20 |
| 18 | 13 | 3  | 6  | 12 | 1  | 2  | 4  | 8  | 16 | 9  |
| 19 | 15 | 7  | 14 | 5  | 10 | 20 | 17 | 11 | 22 | 21 |
| 20 | 17 | 11 | 22 | 21 | 19 | 15 | 7  | 14 | 5  | 10 |
| 21 | 19 | 15 | 7  | 14 | 5  | 10 | 20 | 17 | 11 | 22 |
| 22 | 21 | 19 | 15 | 7  | 14 | 5  | 10 | 20 | 17 | 11 |

| 0  | 0  | 0  | 0  | 0  |
|----|----|----|----|----|
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| 2  | 4  | 8  | 16 | 8  |
| 3  | 6  | 12 | 0  | 0  |
| 4  | 8  | 16 | 8  | 16 |
| 5  | 10 | 20 | 16 | 8  |
| 6  | 12 | 0  | 0  | 0  |
| 7  | 14 | 4  | 8  | 16 |
| 8  | 16 | 8  | 16 | 8  |
| 9  | 18 | 12 | 0  | 0  |
| 10 | 20 | 16 | 8  | 16 |
| 11 | 22 | 20 | 16 | 8  |
| 12 | 0  | 0  | 0  | 0  |
| 13 | 2  | 4  | 8  | 16 |
| 14 | 4  | 8  | 16 | 8  |
| 15 | 6  | 12 | 0  | 0  |
| 16 | 8  | 16 | 8  | 16 |
| 17 | 10 | 20 | 16 | 8  |
| 18 | 12 | 0  | 0  | 0  |
| 19 | 14 | 4  | 8  | 16 |
| 20 | 16 | 8  | 16 | 8  |
| 21 | 18 | 12 | 0  | 0  |
| 22 | 20 | 16 | 8  | 16 |
| 23 | 22 | 20 | 16 | 8  |
|    |    |    |    |    |