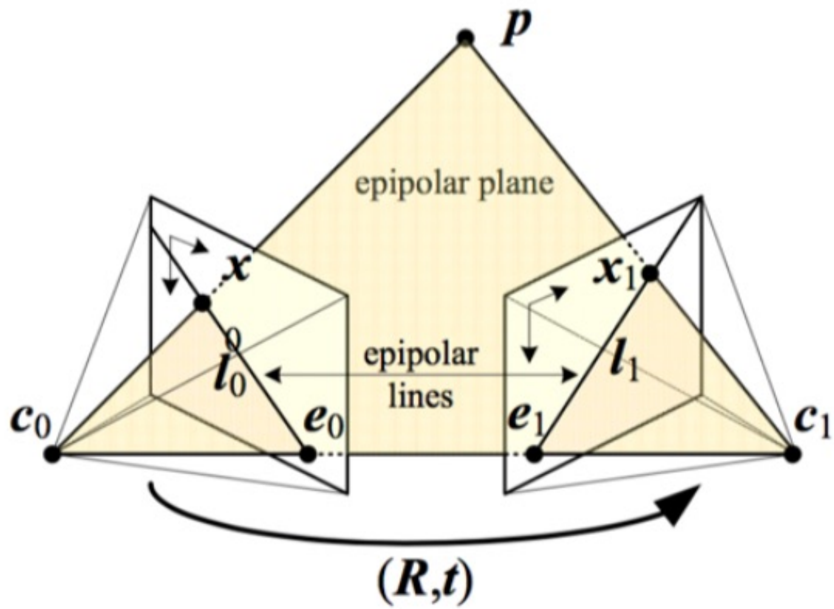

$e_0, e_1 \Rightarrow$ epipoles $\quad$ $l_0, l_2 \Rightarrow$ epipolar lines

c_0, c_1, p constructs epipolar plane.

Camera coordinate: $x = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

Image coordinate: $p = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$ (pixel coordinates)

World coordinate: $P = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$

$p \equiv KP$, K is intrinsic parameters.

$p_0 \equiv KP \quad p_1 \equiv K(RP+t) \Rightarrow x = K^{-1}p$

$\Rightarrow x_0 \equiv P, \quad x_1 \equiv RP + t \Rightarrow x_1 \equiv Rx_0 + t$

cross product t: $\quad t \times x_1 \equiv t \times (Rx_0 + t) \Rightarrow t \times x_1 \equiv t \times Rx_0$

product x_1^T:

$$x_1^T(t \times x_1) = x_1^T(t \times R x_0)$$

$\because$ $x_1^T \perp (t \times x_1)$ $\therefore$ $\underline{x_1^T\ t \times R\ x_0 = 0}$ epipolar constraint

Let $E = t \times R$, we get $x_1^T E x_0 = 0$, E is essential matrix

$x_1^T l_1 = 0 \Rightarrow x_1$ is on the line $l_1 = E x_0$

Because we set $x = K^{-1}p$:

$$x_1^T E x_0 = 0 \Rightarrow (K_1^{-1}p_1)^T E (K_0^{-1}p_0) = 0$$

$$p_1^T \cdot K_1^{-T} E K_0^{-1} p_0 = 0$$

$$p_1^T F p_0 = 0$$

F is fundamental matrix.

How to compute F? 8-points algorithm.

Let's $p_1 = [u_1, v_1, 1]^T$, $p_2 = [u_2, v_2, 1]^T$, according to $p_2^T F p_1 = 0$

$$[u_2\ v_2\ 1]\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = 0$$

$\xrightarrow{\text{transfer}}$ $Ax = b$, where $A = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}[u_1, v_1, 1]$

$x = F$, $b = 0$.

Then we can use SVD to solve.

Check F:

$$l_2 = F_{12} p_1 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} \qquad l_1 = p_2^T F_{12} = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$$

$$e_2 = \frac{1}{\sigma} \frac{x_2^T l_2}{\sqrt{a_2^2 + b_2^2}} \qquad e_1 = \frac{1}{\sigma} \cdot \frac{l_1 x_1}{\sqrt{a_1^2 + b_1^2}}$$

$$\left.\begin{aligned} score_1 &= \sum_{i=1}^{N} (th - e_1(i)) \quad e_1(i) < th \\ score_2 &= \sum_{i=1}^{N} (th - e_2(i)), \quad e_2(i) < th. \end{aligned}\right\} score = score_1 + score_2$$

How to compute E?

$$F = K_2^{-T} E K_1^{-1} \Rightarrow E = K_2^T F K_1$$

Given E, it is possible to retrieve the projective camera matrics M_1 and M_2. Assume M_1 is fixed at $[I, 0]$ M_2 can be retrieved up to a scale and four-fold rotation ambiguity. $\Rightarrow M_1 = [I|0] \quad M_2 = [R|t]$

For each point i, hope to solve for 3D coordinates $w_i = [x_i, y_i, z_i]^T$. Write $\hat{w}_i = [x_i, y_i, z_i, 1]^T$ and compute $C_1 \hat{w}_i$ and $C_2 \hat{w}_i$ to obtain the 2D homogenous coordinates projected to camera1 and camera2. C_1 and C_2 are 3×4 camera matrices

$$C_1 = K_1 M_1 = K_1 [I|0] \qquad C_2 = K_2 M_2 = K_2 [R|t]$$

For each point i:

$$A_i \hat{w}_i = 0 \quad (A_i = 4\times 4,\ \hat{w}_i \text{ is 3D in homo})$$

$$\begin{cases} x_{1i} \equiv C_1 \hat{w}_i \\ x_{2i} \equiv C_2 \hat{w}_i \end{cases}$$

use cross product:

$$x_{1i} \times (C_1 \hat{w}_i) = 0 \qquad C_1 = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}_{3\times 4}$$

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} y_1 z_2 - y_2 z_1 \\ z_1 x_2 - z_2 x_1 \\ x_1 y_2 - x_2 y_1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}_{3\times 4} \cdot \hat{w}_i \right) = 0$$

$$\begin{cases} x(c_3 \hat{w}_i) - (c_1 \hat{w}_i) = 0 \\ y(c_3 \hat{w}_i) - (c_2 \hat{w}_i) = 0 \\ x(c_2 \hat{w}_i) - y(c_1 \hat{w}_i) = 0 \end{cases}$$

$$A_i = \begin{bmatrix} x_{1i} C_{13} - C_{11} \\ y_{1i} C_{13} - C_{12} \\ x_{2i} C_{23} - C_{21} \\ y_{2i} C_{23} - C_{22} \end{bmatrix}$$

Bundle Adjustment

In BA, we will jointly optimize the reprojection error wrt the points w_i and the camera matrix C_j.

$$err = \sum_{ij} \| x_{ij} - Proj(C_j, w_i) \|^2 . \quad (C_j = K_j M_j)$$

Here we optimize the extrinsic matrix M_j.

We can use this error function and nonlinear least square optimizer to optimize the best extrinsic matrix