1 Semantic Tableaux

a) From the open leaves of the tableau we see that there are the following models that satisfy the formula $(p \to q) \to r$: $\{\{(p \to F), (q, \to F), (r \to T)\}, \{(p \to F), (q, \to T), (r \to T)\}, \{(p \to T), (q, \to F), (r \to F)\}, \{(p \to T), (q, \to F), (r \to T)\}, \{(p \to T), (q, \to T), (r \to T)\}\}$. In the set of propositions $P = \{p, q, r\}$, the formula $(\neg p \to r) \wedge (q \to r)$ has the same models $M((p \to q) \to r) = M((\neg p \to r) \wedge (q \to r))$. Therefore, we conclude that $(p \to q) \to r$ and $(\neg p \to r) \wedge (q \to r)$ are logically equivalent.

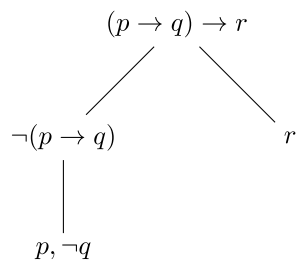

Figure 1: Left-hand side

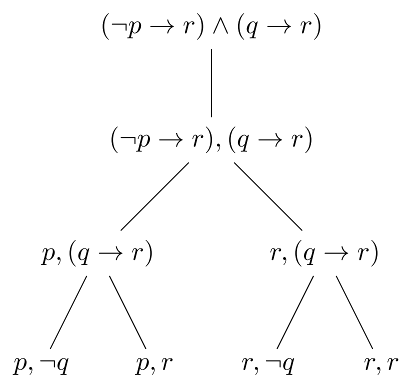

Figure 2: Right-hand side

b) To show that $(((p \to (q \wedge r)) \vee ((q \wedge r) \to p)) \to p)$ is falsifiable, we must show that there exist models for the inverse, namely $\neg(((p \to (q \wedge r)) \vee ((q \wedge r) \to p)) \to p)$. The semantic tableaux of the inverse can be found in Figure 3. Two examples of interpretations which yield false are: $\{\{(p \to F), (q \to T), (r \to T)\}, \{(p \to F), (q \to F), (r \to F)\}\}$

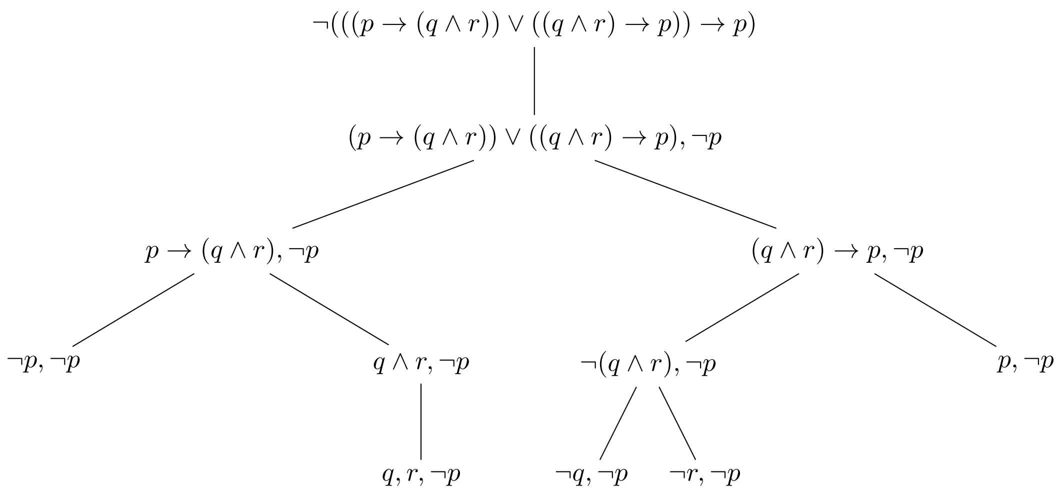

Figure 3: Semantic Tableaux for $\neg(((p \to (q \wedge r)) \vee ((q \wedge r) \to p)) \to p)$

2 Logical Equivalence

3 Gentzen

4 Hilbert