Topics:

- …

Necessary condition of optimality

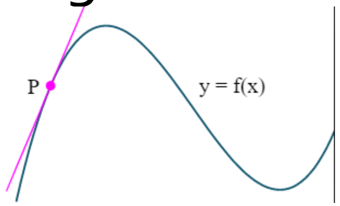
$f: \mathbb{R} \to \mathbb{R}$ is some function like $y = x^2$

If function has derivative everywhere then we can compute derivate everywhere

$f': \mathbb{R} \to \mathbb{R}$ for example above it's $y = 2x$

Value $f'(x)$ means in point "x" different things:

1. Speed with which your function moves along **y**
2. Tangent of angle for local approximation with tangent.



Necessary condition of optimality

Function $f\colon \mathbb{R} \to \mathbb{R}$ **decreases then increases**.
$f'(x)$ should in some point x^* have value
$f'(x) = 0$

Function $f\colon \mathbb{R} \to \mathbb{R}$ **increases then decreases** .
$f'(x)$ should in some point x^* have value
$f'(x) = 0$

We assume that f' has derivative everywehre

Necessary condition of optimality

For $f\colon \mathbb{R}^n \to \mathbb{R}$ the nessesary condition is that
$\nabla f = 0$

Consider constraint problem:

$$\min_x f(x)$$

$$h(x) = 0$$

It's another situation $h(x) = 0$ and directly it's not so clear how handle it. Todo it we introduce ***Lagrange Function***:

$$L(x, \lambda) = f(x) + \lambda \cdot h(x)$$

Equality constraint optimization

Simple description why

$$\min_{x,\lambda} L(x,\lambda)$$
$$L(x,\lambda) = f(x) + \lambda \cdot h(x)$$

Lead to the same solution as

$$\min_{x} f(x)$$
$$h(x) = 0$$

Proves that we do not loose anything

$\min_{x,\lambda} f(x) + \lambda \cdot h(x)$

1. If "f" and "h" are differentiable then nesessary condition for optimality is

- $\frac{\partial(f(x)+\lambda\cdot h(x))}{\partial x} = 0$
- $\frac{\partial(f(x)+\lambda\cdot h(x))}{\partial \lambda} = h(x) = 0$

2. Lagrange function achieves minimum in a point (x, λ) where both condition holds. Particulary $h(x^*) = 0$

3. And we have $f(x^*) + \lambda^* \cdot h(x^*) = f(x^*) + \lambda^* \cdot 0 = f(x^*)$

4. To make $f(x^*) + \lambda^* \cdot 0$ as small as possible we need to make $f(x^*)$ as small as possible.

5. So

$\min_{x,\lambda} f(x) + \lambda \cdot h(x)$ **solves** same optimization problem as

$\min_{x} f(x)$
$h(x) = 0$

Summary so far about nessesary condition for optimality

If there are several equality constraints. The Lagrange function has the following form:

$$L(x, \lambda) = f(x) + \sum_{i=1}^{n} \lambda_i \cdot h_i(x)$$

We know currently that nesessary optimality condition is the following

$$\boldsymbol{\nabla}_{\boldsymbol{x},\boldsymbol{\lambda}} \boldsymbol{L}(\boldsymbol{x}^*, \boldsymbol{\lambda}^*) = \mathbf{0}$$