

Arrays

→ An array is a collection of similar data elements.

Ex; to store names of all students in a class.

→ Array is a list of finite number n of homogeneous data elements such that the elements of the array are stored in successive memory locations.

→ Array is basically used to store relatively permanent collections of data, whereas if the size of the structure and data in the structure are constantly changing, then the array may not be as useful as the linked list.

→ Elements of the array are referenced respectively by an index set consisting of n consecutive nos.

→ ~~The~~ The length or no. of elements of an array can be obtained by the formula:-

$$\| \text{Length} = \underset{\substack{\uparrow \\ \text{Upper} \\ \text{bound}}}{UB} - \underset{\substack{\uparrow \\ \text{Lower} \\ \text{bound}}}{LB} + 1 \|$$



When $LB = 1$, then $UB = \text{Length or size}$.

→ The elements of the array are denoted as: -

$A[i]$ Subscript or index

Subscripted variable

Describes the position of any element in an array.

→ The array declaration must involve three items of information:

- 1) Name of the array
 - 2) Data type of the array
 - 3) Index set of the array
- Eg., $\text{Int } A[10]$.

→ One-Dimensional Array:-

Since, each element in the array is referenced by a single subscript, thus it is 1-D Array or linear arrays.

Eg., $a[10]$

[0]	10
[1]	20
[2]	30
[3]	40
⋮	⋮
[9]	100

Index used to find the element

Value stored in memory.

Array.

⇒ Traversing in Array

LA, lower Bound = LB, Upper bound = UB, S $\xrightarrow{\text{Process}}$ each element

Algo:- i) Initialize Counter : $I = LB$
ii) loop (while $I \leq UB$)
iii) Apply S to $LA[I]$
iv) $I = I + 1$
v) End loop
vi) Exit

Insertat loc(A, n, loc, ele)

// A is an array with n no of element and ele is element to be inserted at location loc

1. $I = n$

2. Repeat steps 3 to 4

while ($I \geq \text{loc}$)

3. Set $A[I+1] = A[I]$

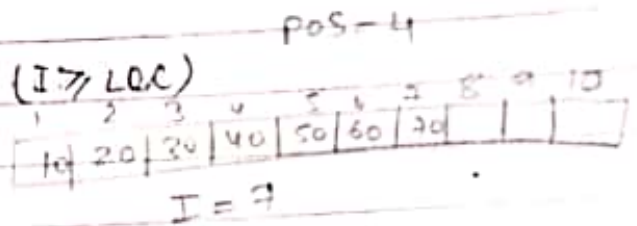
4. Set $I = I - 1$

[End of loop]

5. Set $A[\text{loc}] = \text{ele}$

6. Set $N = N + 1$

7. Exit.



Algorithm to delete an element at given location

Delete at LOC(A, n, loc)

// A is an array with n no of element and loc is location

10	20	30	40	55	2	8
[1]	[2]	[3]	[4]	[5]	[6]	[7]

1. Set $I = \text{loc}$

2. Repeat steps 3 to 4 while ($I < N$)

3. Set $A[I] = A[I+1]$

4. $I = I + 1$

[End of loop]

5. Set $N = N - 1$

6. Exit.

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Representation of linear Array in memory -

marks[] = {99, 67, 78, 56, 88, 90, 34, 85}

marks[4] = ? BA = 1000

$$\text{marks}[4] = 1000 + 2(4 - 0)$$

$$= 1008$$

for 1-D array. $A[k] = \text{Base}(A) + w(k - \text{lower Bound})$

Memory Rep
 Consider an Array A. Base address = 2000 and its index starts from 1932. No. of words required are 4. Find the address of 1965.
 $A[1965] = \text{Base add.} + w(K - \text{lower bound})$
 $= 2000 + 4(1965 - 1932)$
 $= 2000 + 4 \times 33$
 $= 2000 + 132 = 2132$

2D Array Representation in memory

	1	2	3	4
1	A[1,1]	A[1,2]	A[1,3]	A[1,4]
2	A[2,1]	A[2,2]	A[2,3]	A[2,4]
3	A[3,1]	A[3,2]	A[3,3]	A[3,4]

There are two ways of representing 2-D array
 1. Row Major Order Implementation:

A[1,1]	A[1,2]	A[1,3]	A[1,4]	A[2,1]	A[2,2]	A[2,3]	A[2,4]	A[3,1]	A[3,2]	A[3,3]	A[3,4]
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$$A[J, K] = \text{Base address of A} + w \left(\underset{\substack{\downarrow \\ \text{length of column}}}{N} (J - \text{Lower Bound}) + \underset{\substack{\downarrow \\ \text{for row}}}{(K - \text{LB})} \right)$$

2. Column Major Order Implementation:

A[1,1]
A[2,1]
A[3,1]
A[1,2]
A[2,2]
A[3,2]
⋮
A[3,4]

By default LB = 1

$$A[J, K] = \text{Base address (A)} + w \left[\underset{\substack{\downarrow \\ \text{length of row}}}{M} (K - \text{LB}) + \underset{\substack{\downarrow \\ \text{for col.}}}{(J - \text{LB})} \right]$$

eg A[2...6, 3...10]

Suppose a 2D Array A is declared as $A[-2:2, 2:6]$

- words per cell = 4, Base address = 200
- Find out length of each dimension and no. of elements in array
- Find the location of $A[1,2]$

$$\begin{aligned} \text{a) length of row} &= 2 - (-2) + 1 = 5 \\ \text{length of column} &= 6 - 2 + 1 = 5 \\ \text{No. of elements} &= 5 \times 5 \Rightarrow 25 \end{aligned}$$

b) Row Major -

$$\begin{aligned} A[1,2] &= 200 + 4 \left[5(1 - (-2)) + (2 - (2)) \right] \\ &= 200 + 4[15] \\ &= 200 + 60 = 260 \end{aligned}$$

Column Major -

$$\begin{aligned} A[1,2] &= 200 + 4 \left[5(2 - 2) + (1 - (-2)) \right] \\ &= 200 + 4[3] \Rightarrow 212 \end{aligned}$$

Consider a 2D Array $A[25][4]$ B.A. = 200 and it requires 4 words memory cell. Find location of $A[12][3]$ if the array is stored row major

$$\begin{aligned} A[12][3] &= 200 + 4 \left[4(12 - 1) + (3 - 1) \right] \quad (\text{Assume array index from 1}) \\ &= 200 + 4[4 \times 11 + 2] \\ &= 200 + 4[46] = 200 + 184 = 384 \end{aligned}$$

Array index = 0 then $A[12][3] = 404$

General Multi Dimensional Array :

An n dimensional $(m_1 \times m_2 \dots m_n)$ Array A is a collection of elements of $(m_1, m_2, m_3 \dots m_n)$ data elements in which each element is specified by a list of n integers such as k_1, k_2, \dots, k_n called subscript with property eg. $A[2][3][3]$

$$1 \leq k_3 \leq m_3 \quad 1 \leq k_1 \leq m_1 \quad 1 \leq k_2 \leq m_2$$

$$\begin{array}{l} 5-1+1 \\ \text{Length of Row} = 5 \end{array} \quad \begin{array}{l} 4-1+1 \\ \text{Length of Col} = 4 \end{array}$$

Q. Calculate the address of $X[4,3]$ in a 2D-array $X(1 \dots 5, 1 \dots 4)$ stored in a row-major order in the main memory. Assume the base address to be 1000 and that each element requires 4 words of storage.

$$\begin{aligned} X[4,3] &= 4 \left(\overset{I}{[4-1]} \underset{\substack{\downarrow \\ N}}{4} + \overset{J}{[3-1]} \right) + 1000 \\ &= 1056 \end{aligned}$$

Multidimensional Array

$$E_i = K_i - \text{Lower Bound} \quad L_i = UB - LB + 1$$

Column Major Order: L

$$\text{Loc}(A[K_1, K_2, \dots, K_n]) \Rightarrow$$

$$\Rightarrow \text{Base}(A) + w \left[(((\dots (E_n L_{n-1} + E_{n-1}) L_{n-2} + \dots + E_3) L_2 + E_2) L_1 + E_1] \right]$$

$$\text{for } n=1 \Rightarrow \text{Base}(A) + w[E_1]$$

$$n=2 \Rightarrow \text{Base}(A) + w[E_2 L_1 + E_1]$$

$$n=3 \Rightarrow \text{Base}(A) + w[(E_3 L_2 + E_2) L_1 + E_1]$$

$$n=4 \Rightarrow \text{Base}(A) + w[((E_4 L_3 + E_3) L_2 + E_2) L_1 + E_1]$$

Row Major Order

$$\text{Loc}(A[K_1, K_2, \dots, K_n]) \Rightarrow \text{Base}(A) + w \left[(\dots ((E_1 L_2 + E_2) L_3 + E_3) L_4 + \dots + E_{n-1}) L_n + E_n \right]$$

1D Address Calculation Numerical

1. $A[-15 \dots 64]$ is stored in computer memory whose base address is 459. word size is 2 byte
- How many no. of element are there.
 - Total memory size (size of array \times word size)
 - Find memory location of $A[10]$
 - Which element is located in memory address. 589.

a. size of array $= 64 - (-15) + 1$ $UB - LB + 1$
 $= 80$

b) $80 \times 2 = 160$ byte

c) $A[10] = B.A + w(K - LB)$
 $= 459 + 2(10 - (-15))$
 $= 459 + 2(25)$
 $= 459 + 50$
 $= 509$

d) $589 = 459 + 2(K - (-15))$
 $589 = 459 + 2(K + 15)$
 $589 = 459 + 2K + 30$
 $589 - 489 = 2K$
 $100 = 2K$
 $K = 50$

$$\begin{array}{r} 459 \\ + 30 \\ \hline 489 \end{array}$$

Multi-dimensional Array Numerical

1. Suppose A is a 3D array given as : $A(1:9, -4:1, 5:10)$
The array stores in memory in Row-major order and base address = 400 and $w = 2$ words per memory cell. Then find out the address of Element $A(5, -1, 8)$ row-major wise.

Sol:

Row-Major:

$$\text{Loc } A[i][j][k] = \text{Base}(A) + w[(E_1 L_2 + E_2) L_3 + E_3]$$

$$L_1 = 9 - 1 + 1 = 9$$

$$E_1 = 5 - 1 = 4$$

$$L_2 = 1 - (-4) + 1 = 6$$

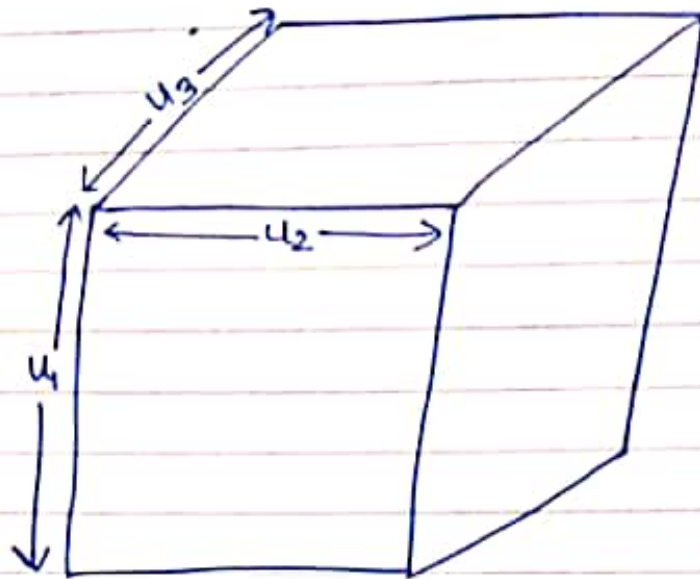
$$E_2 = -1 - (-4) = 3$$

$$L_3 = 10 - 5 + 1 = 6$$

$$E_3 = 8 - 5 = 3$$

$$\begin{aligned} \text{Loc } A[5][-1][8] &= 400 + 2[(4 \times 6 + 3)6 + 3] \\ &= 730 \end{aligned}$$

Index Formula Computation 3-D Array Row Major Order



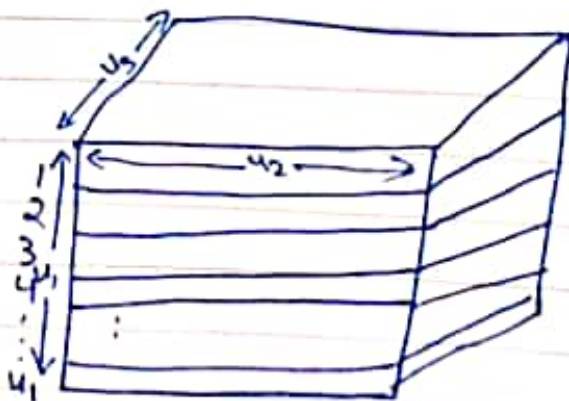
Let us say there is first dimension representing u_1 . Second dimension u_2 . Third dimension u_3 . We can imagine it as a cuboid.

Array is:

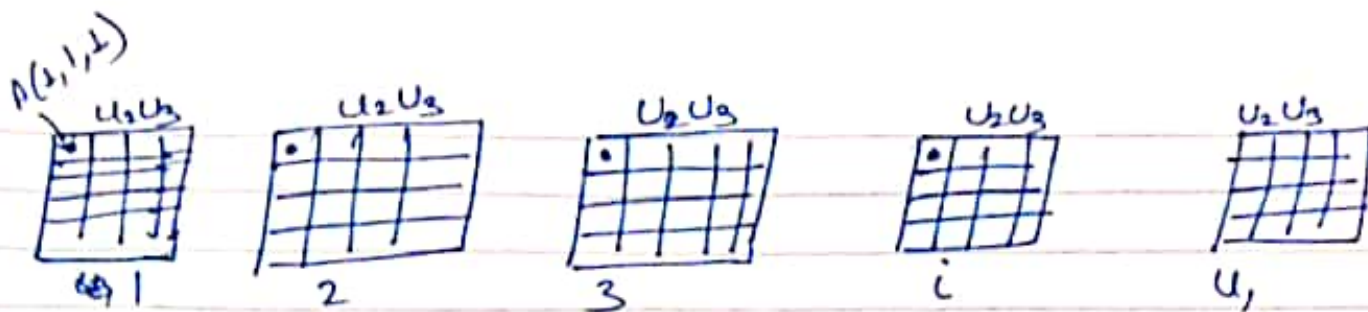
$$A[L_1:U_1, L_2:U_2, L_3:U_3]$$

For the assumptions let's say the first index is 1. So, $A[1:U_1, 1:U_2, 1:U_3]$ and every element is requiring one byte for the storage.

Now, if I say this is a 3D array and we break it in various pieces or the slices.



So, the total no. of slices can be cut upto u_1 .



Every slice is of size $u_2 \times u_3$

So, if I know the address of 1st element that is,

$A[1,1,1] = \alpha$, then what will be address of

$A[2,1,1] = \alpha + u_2 \times u_3$ (As we can only come to the storage of this element after storing 1st 2D slice)

Similarly, $A[3,1,1] = \alpha + 2u_2 u_3$

Similarly, $A[i,1,1] = \alpha + (i-1)u_2 u_3$

Now let us expand i array:

So, we already know the address of 1st element, what will be the address of k^{th} element of j^{th} row. It will be address of 1st row 1st element + u_3

$$A[i,2,1] = \alpha + (i-1)u_2 u_3 + u_3$$

$$A[i,3,1] = \alpha + (i-1)u_2 u_3 + 2u_3$$

Similarly, address of 1st element of j^{th} row

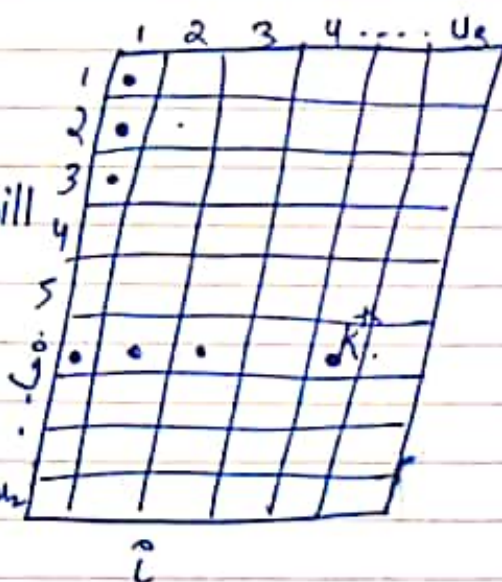
$$A[i,j,1] = \alpha + (i-1)u_2 u_3 + (j-1)u_3$$

$$\text{Similarly } A[i,j,2] = \alpha + (i-1)u_2 u_3 + (j-1)u_3 + 1$$

$$A[i,j,3] = \alpha + (i-1)u_2 u_3 + (j-1)u_3 + 2$$

Similarly for k^{th} element of j^{th} row

$$A[i,j,k] = \alpha + (i-1)u_2 u_3 + (j-1)u_3 + (k-1) \quad \text{--- (1) this is a formula with assumptions.}$$



Now, if I try to remove assumptions, then I is

$$A[i, j, k] = \alpha + (i-1)u_2u_3 + (j-1)u_3 + (k-1)$$

on 1st dimension side, j for 2nd dimension & k for 3rd dimension and every element is acquiring n byte for storage.

So, removing the assumptions:

$$A[i, j, k] = \alpha + [(i-1)u_2u_3 + (j-1)u_3 + (k-1)]n$$

i will be replaced by $i - L_1 + 1$, j by $j - L_2 + 1$ and k by $k - L_3 + 1$.

$$A[i, j, k] = \alpha + [(i - L_1)(u_2 - L_2 + 1)(u_3 - L_3 + 1) + (j - L_2)(u_3 - L_3 + 1) + (k - L_3)] \times n \quad (2)$$

So, then replacing $i - L_1 = E_1$

$$j - L_2 = E_2$$

$$k - L_3 = E_3$$

$$\alpha = B.A$$

$$u_2 - L_2 + 1 = L_2$$

$$u_3 - L_3 + 1 = L_3$$

$$wn = L_4$$

$$= B.A + [((E_1 L_2 + E_2) L_3 + E_3) L_4 + E_4] \omega$$