CS 520 Final: Question 1 - Decision Making　　　　　　16:198:520

> You cannot be graded on what you don't write down. This problem is to be completed individually, without any coordination with others. Complete each question to the best of your ability. If you write code to solve one of the problems, you must submit that code along with your final answers. Explain your process and algorithms. Be explicit. Answers, without evidence of the thought and process that led to them, will not earn credit.

You are taking part in the 2019 Sheep Dog Bot competition. A sheep is released into an 8x8 cell grid, and you have two sheep dog bots under your control. Your goal is to pin the sheep in the upper left corner of the field (position (0,0)):

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Example Initial State                 Sheep is Cornered
[_][_][_][_][_][_][_][D]      ->      [S][D][_][_][_][_][_][_]
[_][_][_][_][_][_][_][_]      ->      [D][_][_][_][_][_][_][_]
[_][_][_][_][_][_][_][_]      ->      [_][_][_][_][_][_][_][_]
[_][_][_][_][_][_][_][_]      ->      [_][_][_][_][_][_][_][_]
[S][_][_][_][_][_][_][_]      ->      [_][_][_][_][_][_][_][_]
[_][_][_][_][_][_][_][_]      ->      [_][_][_][_][_][_][_][_]
[_][_][_][D][_][_][_][_]      ->      [_][_][_][_][_][_][_][_]
[_][_][_][_][_][_][_][_]      ->      [_][_][_][_][_][_][_][_]
```

The rules are these:

- You see the entire field, and have complete information to use when directing your dog bots.

- Each round, you may issue commands to your dog bots in the form of where to move, and then the sheep has a chance to move in response. The commands given to each dog bot may differ.

- Valid commands to a dog bot are 'up', 'down', 'left', 'right', 'hold', and the dog bot will execute the command it is given as long as:

    - the command does not move the dog bot out of the field

    - the dog bot does not end up in the same cell as the sheep (that cell is blocked by the sheep)

    - after the dog bots execute their commands, they will not be in the same cell.

- Each dog can move at most one cell per round (you can issue commands to both dogs each round).

- After the dogs move, the sheep moves according to the following rules:

    - If the sheep has neighbors in the up/down/left/right directions, it will choose uniformly at random among the available neighbors.

    - If the sheep has no free directions to move, it will stay in place.

    - If the sheep is pinned on (0,0) and has no free directions to move, the game is over.

    - The sheep can move at most one square per round, and always moves one square unless it is cornered.

- Your final score is the number of rounds it took to corner the sheep.

Questions:

1) Compute a strategy for controlling your dog bots that attempts to minimize the number of rounds needed to corner the sheep. How did you formulate this, and how did you solve it?

2) Given the initial state in the above example, how many rounds do you need (on average) to corner the sheep? Answer as precisely as possible.

3) What is the worst possible initial state (position of dog bots, sheep), and why? Justify mathematically.

4) You are allowed to place your dog bots anywhere in the field at the start, then the sheep will be placed uniformly at random in one of the remaining unoccupied cells. Where should you place your dog bots initially, and why? Justify mathematically.

5) Do you think better strategies exist than the one you came up with? Justify.

Bonus: Answer the same questions with the following modification: the dog bots win if they can corner the sheep in any of the four corners, but when the sheep moves it is allowed to strategize about which cell to move to, and wants to keep the dog bots from cornering it as long as possible.