CS 520 Final: Question 3 – GoatDiscoveryBot

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Abstract

This problem is very similar to the Monty-Hall Problem which is a counter-intuitive statistics puzzle.

1 Logically, how could you (LogicalGoatDiscoveryBot) model this information? Probabilistically, how could you (ProbabilisticGoatDiscoveryBot) model this information? Hint: Consider the statements In A, In B, In C.

1. To model the information **logically**, we can write this as:

   $$(In_A \wedge \neg In_B \wedge \neg In_C) \vee (In_B \wedge \neg In_A \wedge \neg In_C) \vee (In_C \wedge \neg In_A \wedge \neg In_B)$$

   This is intuitive as when provided choices, we as humans, tend to choose either of the options. Also, without any prior information or bias, we are unlikely to rule out any option.

2. To model the information **probabilistically**, the problem can be modelled as under:

   $$P(In_A) \equiv P(In_B) \equiv P(In_C) = \frac{1}{3}$$

   Since we don't have any prior information, all the events are equally likely. Thus, the individual probabilities correspond to $\frac{1}{Number\ of\ options}$. Also, this fulfils the universal law that the sum of all the probabilities in the sample space is 1.

2 Under the logical formulation, how can you compare the value/results of actions 'Select A', 'Select B', 'Select C'? Is there an obvious choice of best action?

As discussed in Section 1, $In_A \wedge \neg In_B \wedge \neg In_C) \vee (In_B \wedge \neg In_A \wedge \neg In_C) \vee (In_C \wedge \neg In_A \wedge \neg In_B)$ Since, we don't have any further information about the choices, there **isn't any obvious choice to begin with**.

NOTE : Assuming that the initial knowledge base is set to 'True' and whenever a clue is revealed to the Bot, the knowledge base is updated by generating all possible combinations of the constraint. Implementation wise, this is similar to the Assignment 2 – Minesweeper, where we used 'combinations' method of the 'itertools' module. According to the nature of the output of selection i.e. 'True' in presence of a goat and 'False' in the absence, the value is substituted in the logical expression.

3 Under the probabilistic formulation, how can you compare the value/results of actions 'Select A', 'Select B', 'Select C'? Is there an obvious choice of best action?

Without any prior knowledge about the presence of the goat, we have no obvious best choice as each location will have an equal probability of having the goat.
The probabilities for each of the options are $\frac{1}{3}$.

4 Suppose, for argument's sake, you select location A. Before you search location A, you consult your friend, CouldBe- MoreHelpful-Bot (who knows where the goat is, but will only tell you where the goat isn't). CBMH-Bot will look at the two locations you didn't pick, and name one of them that does not have the goat. CBMHBot tells you the goat is not in location B. Given this new information: Update your logical formulation to reflect this new information.

Say the bot choses location A and the CBMHBot tells that the goat isn't in location B i.e In_B is False.
Using the above information in our equation mentioned above i.e.

$$(In_A \wedge \neg In_B \wedge \neg In_C) \vee (In_B \wedge \neg In_A \wedge \neg In_C) \vee (In_C \wedge \neg In_A \wedge \neg In_B)$$

The knowledge can be modelled in our knowledge base as:

$$(In_A \wedge True \wedge \neg In_C) \vee (False \wedge \neg In_A \wedge \neg In_C) \vee (In_C \wedge \neg In_A \wedge True)$$

i.e.

$$(In_A \wedge \neg In_C) \vee (In_C \wedge \neg In_A)$$

5 Update your probabilistic formulation to reflect this new information. *Hint: The CBMH-Bot's decision to tell you the goat is not in B depended both on which location you selected, and where the goat actually is.*

Initially, the probability of finding the goat at any location was $\frac{1}{3}$.

1. Say Location A has the goat and the location selected by Probabilistic-GoatDiscoveryBot is also A. In this case, the CBMHBot will randomly choose any location between B and C. Thus, both B and C will have an equal probability of $\frac{1}{2}$ of being selected by CBMHBot.

2. Say Location B has the goat and the location selected by Probabilistic-GoatDiscoveryBot is A. In this case, the CBMHBot has an obvious choice of C. Thus, the probability that CBMHBot selects B is 0.

3. Say Location C has the goat and the location selected by Probabilistic-GoatDiscoveryBot is A. Thus, the probability that CBMHBot selects B is 1.

With this new information, the knowledge base can be updated in the following way:

$$\begin{aligned} P(Location\ B\ is\ disclosed) = P(A) * P(Location\ B\ is\ disclosed \mid A\ )+ \\ P(B) * P(Location\ B\ is\ disclosed \mid B)+ \\ P(C) * P(Location\ B\ is\ disclosed|C) \end{aligned} \tag{1}$$

$$P(Location\ B\ is\ disclosed) = \frac{1}{3} * \frac{1}{2} + \frac{1}{3} * 0 + \frac{1}{3} * 1$$

$$P(Location\ B\ is\ disclosed) = \frac{1}{2}$$

Further using Bayes Theorem,

$$P(A|Location\ B\ is\ disclosed) = \frac{P(Location\ B\ is\ disclosed|A) * P(A)}{P(Location\ B\ is\ disclosed)}$$

$$=> P(A|Location\ B\ is\ disclosed) = \frac{\frac{1}{2} * \frac{1}{3}}{\frac{1}{2}} = \frac{1}{3}$$

$$P(B|Location\ B\ is\ disclosed) = \frac{P(Location\ B\ is\ disclosed|B) * P(B)}{P(Location\ B\ is\ disclosed)}$$

$$=> P(B|Location\ B\ is\ disclosed) = \frac{0 * \frac{1}{3}}{\frac{1}{2}} = 0$$

$$P(C|Location\ B\ is\ disclosed) = \frac{P(Location\ B\ is\ disclosed|C) * P(C)}{P(Location\ B\ is\ disclosed)}$$

$$=> P(C|Location\ B\ is\ disclosed) = \frac{1 * \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

6 At this point, you want to re-assess your earlier decision of which action to take as you now have more information than you did previously. Under the logical formulation, how can you compare the value/results of actions 'Re-Select A', 'Re-Select B', 'Re-Select C'? Is there an obvious choice of best action?

As derived in Question 4, the updated logical equation is:

$$(In_A \wedge \neg In_C) \vee (In_C \wedge \neg In_A)$$

Since the obvious choices are locations A and C, Re-select B is definitely not an option here.
Between Locations A and C, there is no obvious choice due to insufficient knowledge in the knowledge base of the LogicalGoatDiscoveryBot. Therefore, both 'Re-Select A' and 'Re-Select C' are equally good choices.

7 Under the probabilistic formulation, how can you compare the value/results of actions 'Re-Select A', 'Re-Select B', 'Re-Select C'? Is there an obvious choice of best action?

As derived in Question 5,

$$P(A|Location\ B\ is\ disclosed) = \frac{\frac{1}{2} * \frac{1}{3}}{\frac{1}{2}} = \frac{1}{3}$$

$$P(B|Location\ B\ is\ disclosed) = \frac{0 * \frac{1}{3}}{\frac{1}{2}} = 0$$

$$P(C|Location\ B\ is\ disclosed) = \frac{1 * \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

The initial choice of the ProbabilisticGoatDiscoveryBot was location A. But now with the updated knowledge base, the **obvious choice of best action is Location C** as it has a higher probability i.e. $\frac{2}{3}$.

8 Under the logical formulation, having initially selected location A, should you stick with location A or change? Justify your choice.

As derived in Question 4, the updated logical equation is:

$$(In_A \land \neg In_C) \lor (In_C \land \neg In_A)$$

Between Locations A and C, there is no obvious choice due to insufficient knowledge in the knowledge base of the LogicalGoatDiscoveryBot, there's no motivation to either stick to Location A or migrate to Location C. Either of the actions can be taken.

9 Under the probabilistic formulation, having initially selected location A, should you stick with location A or change? Justify your choice

As derived in Question 5,

$$P(A|Location\ B\ is\ disclosed) = \frac{\frac{1}{2} * \frac{1}{3}}{\frac{1}{2}} = \frac{1}{3}$$

$$P(B|Location\ B\ is\ disclosed) = \frac{0 * \frac{1}{3}}{\frac{1}{2}} = 0$$

$$P(C|Location\ B\ is\ disclosed) = \frac{1 * \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

The **obvious choice of best action is Location C** as it has a higher probability i.e. $\frac{2}{3}$. Therefore the ProbabiliticGoatDiscoveryBot will Select C.
Initially Location A was randomly selected by the ProbabiliticGoatDiscoveryBot as all the locations were equally likely to have the goat. On updating the knowledge base, changing the location to C is probabilistically advantageous.

10 Who is more successful in their mission, LogicalGoatDiscoveryBot or ProbabilisticGoatDiscoveryBot? Justify your answer.

The ProbabiliticGoatDiscoveryBot is more successful than the LogicalGoatDiscoveryBot because of the following:

1. LogicalGoatDiscoveryBot only has Boolean values which allows only True and False values with absolute certainty, doing away with the scope of likelihood.

2. Whereas, ProbabiliticGoatDiscoveryBot has room for likelihood and can thus be used to compare between different actions when there exists uncertainty in choice.

11 BONUS: You initially select location A. Suppose that you know CouldBeMoreHelpfulBot is biased, in the following way: if CBMHBot has a choice between telling you B and C, then CBMHBot tells you B with probability p, and C with probability 1 p. CBMHBot tells you that the goat is not in location B. What is the utility of sticking with your initial selection? What is the utility of switching to C? What is the rational choice, and does it depend on p?

Initially, the probability of finding the goat at any location was $\frac{1}{3}$.

1. Say Location A has the goat. Then, $P(Location\, B\, is\, disclosed \,|\, A) = p$ as the available choices will be B and C and we know that the CBMHBot is biased to choose location B with a probability 'p'.
2. Say Location B has the goat. Then, $P(Location\, B\, is\, disclosed \,|\, B) = 0$ as this is intuitive.
3. Say Location C has the goat. Then, $P(Location\, B\, is\, disclosed \,|\, C) = 1$ as it is the only option.

With this new information, the knowledge base can be updated in the following way:

$$\begin{aligned} P(Location\ B\ is\ disclosed) = P(A) * P(Location\ B\ is\ disclosed \mid A\ )+ \\ P(B) * P(Location\ B\ is\ disclosed \mid B)+ \\ P(C) * P(Location\ B\ is\ disclosed|C) \end{aligned} \tag{2}$$

$$=> P(Location\ B\ is\ disclosed) = \frac{1}{3} * p + \frac{1}{3} * 0 + \frac{1}{3} * (p + 1)$$

$$=> P(Location\ B\ is\ disclosed) = \frac{1}{3} * (2p + 1)$$

Further using Bayes Theorem,

$$P(A|Location\ B\ is\ disclosed) = \frac{P(Location\ B\ is\ disclosed|A) * P(A)}{P(Location\ B\ is\ disclosed)}$$

$$=> P(A|Location\ B\ is\ disclosed) = \frac{\frac{1}{3} * p}{\frac{1}{3} * (2p + 1)} = \frac{p}{2p + 1}$$

$$P(B|Location\ B\ is\ disclosed) = \frac{P(Location\ B\ is\ disclosed|B) * P(B)}{P(Location\ B\ is\ disclosed)}$$

$$=> P(B|Location\ B\ is\ disclosed) = \frac{0 * \frac{1}{3} * p}{\frac{1}{3} * (2p + 1)} = 0$$

$$P(C|Location\ B\ is\ disclosed) = \frac{P(Location\ B\ is\ disclosed|C) * P(C)}{P(Location\ B\ is\ disclosed)}$$

$$=> P(C|Location\ B\ is\ disclosed) = \frac{\frac{1}{3} * 1}{\frac{1}{3} * (2p + 1)} = \frac{1}{2p + 1}$$

Since $p < 1$, we can infer that

$$\frac{p}{p + 1} < \frac{1}{p + 1}$$

Thus, we can say that the utility increases if we switch to C.