CSC 225: Spring 2018: Lab 3
Proof Techniques

January 21, 2018

1 Proof by Counterexample

Usually we use this technique to disprove some statement. It should be noted that, it is not sufficient to show the counterexample but it must be showed why it is a counterexample.

1. Disprove: All prime numbers are odd.
2. Disprove: Two circles always cross each other at exactly two points.

2 Proof by Contradiction

Suppose that we have to prove "if A then B". We assume for a contradiction that the proposition is false. Then we arrive at a contradiction, which shows that our initial assumption was not correct and the proposition is actually true.

Let's try to solve the following propositions by contradiction.

1. For all integers n, if $n^3 + 5$ is odd then n is even. `Hint: Assume both are odd.`
2. For any integer a, If $a^2 - 2a + 7$ is even, then a is odd. `Hint: Assume both are even.`
3. There is infinitely many prime numbers.
4. The number $\sqrt{2}$ is irrational. `Hint: Assume` $\sqrt{2} = \frac{a}{b}$.
5. **Ramsey's Theorem.** Prove that out of a party of 6 people, there exists a group of 3 mutual friends or a group of 3 mutual non-friends.
6. In any graph the number of odd degree vertices is even.

3 Proof by Contrapositive

If we have to prove "if A then B", we instead prove "if $\not B$ then $\not A$". Try to prove the following propositions by contrapositive.

1. If n is a positive integer such that $n mod(4)$ is 2 or 3, then n is not a perfect square.
2. If x and y are two integers whose product is odd, then both must be odd.
3. If x and y are two integers for which $x + y$ is even, then x and y have the same parity (odd or even).
4. For any integers a and b, $a + b \geq 15$ implies that $a \geq 8$ or $b \geq 8$.

4 Proof by Induction

See the attached pdf.