

1. The order would be $O(n^2)$. Since in the worst case scenario, the inversions would be opposite to each other.

2.

```
public long merge(int[] a, int[] aux, int lo, int mid, int hi) {
    long inversions = 0;

    // copy to aux[]
    for (int k = lo; k <= hi; k++) {
        aux[k] = a[k];
    }

    // merge back to a[]
    int i = lo, j = mid+1;
    for (int k = lo; k <= hi; k++) {
        if (i > mid)
            a[k] = aux[j++];
        else if (j > hi)
            a[k] = aux[i++];
        else if (aux[j] < aux[i]) {
            a[k] = aux[j++]; inversions += (mid - i + 1);
        }
        else
            a[k] = aux[i++];
    }
    return inversions;
}
```

```
public long count(int[] a, int[] b, int[] aux, int lo, int hi) {
    long inversions = 0;
    if (hi <= lo) return 0;
    int mid = lo + (hi - lo) / 2;
    inversions += count(a, b, aux, lo, mid);
    inversions += count(a, b, aux, mid+1, hi);
    inversions += merge(b, aux, lo, mid, hi);
    assert inversions == brute(a, lo, hi);
    return inversions;
}
```

3.

```
public static long distance(int[] a, int[] b) {
    if (a.length != b.length) {
        throw new IllegalArgumentException("Array dimensions disagree");
    }
    int n = a.length;

    int[] ainv = new int[n];
    for (int i = 0; i < n; i++)
        ainv[a[i]] = i;

    Integer[] bnew = new Integer[n];
    for (int i = 0; i < n; i++)
        bnew[i] = ainv[b[i]];

    return Inversions.count(bnew);
}
```

4. To sort S , do a radix sort on the n elements, viewing them as pairs (i, j) such that i and j are integers in the range $[0, n - 1]$.
5. We will assume that the priority queue can be considered a min heap (though it is not necessarily so) where each node stores a distinct number in S , called its key. And, each node's key is always greater than its parents.

Therefore, to *insert()* we will need to perform a series of comparisons to ensure the new node is placed appropriately within the 'heap'.

To *removeMin()* we can use an $O(\log(n))$ operation of pulling off our 'heap's root and bubbling as necessary.

Therefore we will attempt to prove that, in a comparison based implementation following from the above, that *insert()* requires $> O(\log(\log(n)))$ time.