CSC 225 A01 FALL 2015 (CRN: 10789)
ALGORITHMS AND DATA STRUCTURES I
MIDTERM EXAMINATION
UNIVERSITY OF VICTORIA

1. Student ID: _____

2. Name: _____

3. DATE: 20 OCTOBER 2015
   DURATION: 45 MINUTES
   INSTRUCTOR: V. SRINIVASAN

4. THIS QUESTION PAPER HAS SIX PAGES INCLUDING THE COVER PAGE.

5. THIS QUESTION PAPER HAS FOUR QUESTIONS.

6. ALL ANSWERS ARE TO BE WRITTEN ON THIS EXAMINATION PAPER.

7. THIS IS A CLOSED BOOK EXAM. NO CALCULATORS OR ANY OTHER AIDS ARE ALLOWED.

8. READ THROUGH ALL THE QUESTIONS AND ANSWER THE EASY QUESTIONS FIRST. KEEP YOUR ANSWERS SHORT AND PRECISE.

Q1 (5)	
Q2 (3)	
Q3 (4)	
Q4 (3)	
TOTAL (15) =	

1. **Asymptotic Notation**

(i) Indicate for each pair of expressions (A,B) in the table below, whether A is O or Ω of B. Your answer should be in the form of a "yes" or "no" in each box. [3 Marks]

A	B	O	Ω
n^3	$(\log n)^6$		
$2n^3 + 4n^2$	$4n^3$		
$n!$	2^n		
n^5	n^4		
10000	5		
n^5	$(1.1)^n$		

(ii) Prove that $2n^3 + 3n^2$ is $O(n^3)$. Show your calculations and write down the values of the constants c and n_0 clearly. [2 Marks]

2. **Solving Recurrence Equations**

   (a) Solve the following recurrence equation to get a closed-formula for $T(n)$. [3 Marks]

$$
\begin{aligned}
T(n) &= 2 \text{ if } n = 1 \\
     &= T(n-1) + 2n \text{ if } n \geq 2
\end{aligned}
$$

3. **Sorting Algorithms**

(a) What are the minimum and the maximum number of nodes in a heap of height 3? As done in the class, you must assume that the heap only stores keys in the internal nodes and that all the leaf nodes are empty. [1 Mark]

(c) Suppose you are given an input sequence $S = [10\ 8\ 23\ 18\ 4\ 32]$. Show how a heapsort algorithm runs on input S by constructing the heap at the end of each Insertion step and each RemoveMin step of the algorithm. [3 Marks]

4. **Proof by Induction**

The Stacking Game. You begin with a stack of n boxes. Then you make a sequence of moves. In each move, you divide one stack of boxes into two nonempty stacks. The game ends when you have n stacks, each containing a single box. You earn points for each move; in particular, if you divide one stack of height $a + b$ into two stacks with heights a and b, then you score ab points for that move. Your overall score is the sum of the points that you earn for each move. An example of this game for $n = 8$ is shown below. On each line, the underlined stack is divided in the next step. Is there a strategy that you can use to maximize your total score? Prove the claim on the next page to answer this question. [3 Marks]

Stack Heights	Score in Points
<u>8</u>	
<u>5</u> 3	15
3 2 <u>3</u>	6
3 <u>2</u> 2 1	2
<u>3</u> 2 1 1 1	1
2 1 <u>2</u> 1 1 1	2
<u>2</u> 1 1 1 1 1 1	1
1 1 1 1 1 1 1 1	1
Total Score	28

Claim: Prove, by induction on n, that every way of unstacking n blocks gives a total score of $\frac{n(n-1)}{2}$ points for any $n \geq 1$. Show the base case, the induction hypothesis and calculations inside the induction step clearly.

END OF EXAM