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We illustrate the usage of the master method with a few examples (with each taking the assumption that T(n) = c for n < d, for constants $c \ge 1$ and d > 1).

Example 5.7: Consider the recurrence

$$T(n) = 4T(n/2) + n.$$

In this case, $n^{\log_b a} = n^{\log_2 4} = n^2$. Thus, we are in Case 1, for f(n) is $O(n^{2-\epsilon})$ for $\epsilon = 1$. This means that T(n) is $\Theta(n^2)$ by the master method.

Example 5.8: Consider the recurrence

$$T(n) = 2T(n/2) + n\log n,$$

which is one of the recurrences given above. In this case, $n^{\log_b a} = n^{\log_2 2} = n$. Thus, we are in Case 2, with k = 1, for f(n) is $\Theta(n \log n)$. This means that T(n) is $\Theta(n \log^2 n)$ by the master method.

Example 5.9: Consider the recurrence

$$T(n) = T(n/3) + n,$$

which is the recurrence for a geometrically decreasing summation that starts with n. In this case, $n^{\log_b a} = n^{\log_3 1} = n^0 = 1$. Thus, we are in Case 3, for f(n) is $\Omega(n^{0+\epsilon})$, for $\epsilon = 1$, and af(n/b) = n/3 = (1/3)f(n). This means that T(n) is $\Theta(n)$ by the master method.

Example 5.10: Consider the recurrence

$$T(n) = 9T(n/3) + n^{2.5}$$
.

In this case, $n^{\log_b a} = n^{\log_3 9} = n^2$. Thus, we are in Case 3, since f(n) is $\Omega(n^{2+\epsilon})$ (for $\epsilon = 1/2$) and $af(n/b) = 9(n/3)^{2.5} = (1/3)^{1/2}f(n)$. This means that T(n) is $\Theta(n^{2.5})$ by the master method.

Example 5.11: Finally, consider the recurrence

$$T(n) = 2T(n^{1/2}) + \log n.$$

Unfortunately, this equation is not in a form that allows us to use the master method. We can put it into such a form, however, by introducing the variable $k = \log n$, which lets us write

$$T(n) = T(2^k) = 2T(2^{k/2}) + k.$$

Substituting into this the equation $S(k) = T(2^k)$, we get that

$$S(k) = 2S(k/2) + k.$$

Now, this recurrence equation allows us to use master method, which specifies that S(k) is $O(k \log k)$. Substituting back for T(n) implies T(n) is $O(\log n \log \log n)$.

Rather than rigorously prove Theorem 5.6, we instead discuss the justification behind the master method at a high level.