We illustrate the usage of the master method with a few examples (with each taking the assumption that $T(n) = c$ for $n < d$, for constants $c \geq 1$ and $d \geq 1$).

**Example 5.7:** *Consider the recurrence*

$$T(n) = 4T(n/2) + n.$$

*In this case,* $n^{\log_b a} = n^{\log_2 4} = n^2$. *Thus, we are in Case 1, for* $f(n)$ *is* $O(n^{2-\varepsilon})$ *for* $\varepsilon = 1$. *This means that* $T(n)$ *is* $\Theta(n^2)$ *by the master method.*

**Example 5.8:** *Consider the recurrence*

$$T(n) = 2T(n/2) + n\log n,$$

*which is one of the recurrences given above. In this case,* $n^{\log_b a} = n^{\log_2 2} = n$. *Thus, we are in Case 2, with* $k = 1$, *for* $f(n)$ *is* $\Theta(n \log n)$. *This means that* $T(n)$ *is* $\Theta(n \log^2 n)$ *by the master method.*

**Example 5.9:** *Consider the recurrence*

$$T(n) = T(n/3) + n,$$

*which is the recurrence for a geometrically decreasing summation that starts with* $n$. *In this case,* $n^{\log_b a} = n^{\log_3 1} = n^0 = 1$. *Thus, we are in Case 3, for* $f(n)$ *is* $\Omega(n^{0+\varepsilon})$, *for* $\varepsilon = 1$, *and* $af(n/b) = n/3 = (1/3)f(n)$. *This means that* $T(n)$ *is* $\Theta(n)$ *by the master method.*

**Example 5.10:** *Consider the recurrence*

$$T(n) = 9T(n/3) + n^{2.5}.$$

*In this case,* $n^{\log_b a} = n^{\log_3 9} = n^2$. *Thus, we are in Case 3, since* $f(n)$ *is* $\Omega(n^{2+\varepsilon})$ *(for* $\varepsilon = 1/2$*) and* $af(n/b) = 9(n/3)^{2.5} = (1/3)^{1/2} f(n)$. *This means that* $T(n)$ *is* $\Theta(n^{2.5})$ *by the master method.*

**Example 5.11:** *Finally, consider the recurrence*

$$T(n) = 2T(n^{1/2}) + \log n.$$

*Unfortunately, this equation is not in a form that allows us to use the master method. We can put it into such a form, however, by introducing the variable* $k = \log n$, *which lets us write*

$$T(n) = T(2^k) = 2T(2^{k/2}) + k.$$

*Substituting into this the equation* $S(k) = T(2^k)$, *we get that*

$$S(k) = 2S(k/2) + k.$$

*Now, this recurrence equation allows us to use master method, which specifies that* $S(k)$ *is* $O(k \log k)$. *Substituting back for* $T(n)$ *implies* $T(n)$ *is* $O(\log n \log\log n)$.

Rather than rigorously prove Theorem 5.6, we instead discuss the justification behind the master method at a high level.