CSC 225 SPRING 2014
ALGORITHMS AND DATA STRUCTURES I
MIDTERM EXAMINATION
UNIVERSITY OF VICTORIA

1. Student ID: ________________________
2. Name: ___________________________
3. DATE: 21 FEBRUARY 2014
   DURATION: 50 MINUTES
   INSTRUCTOR: V. SRINIVASAN
4. THIS QUESTION PAPER HAS FIVE PAGES (INCLUDING THE COVER PAGE).
5. THIS QUESTION PAPER HAS FOUR QUESTIONS.
6. ALL ANSWERS TO BE WRITTEN ON THIS EXAMINATION PAPER.
7. THIS IS A CLOSED BOOK EXAM. NO CALCULATORS OR ANY OTHER ITEMS ARE ALLOWED.
8. KEEP YOUR ANSWERS SHORT AND PRECISE.

Q1 (4)	
Q2 (4)	
Q3 (4)	
Q4 (3)	
TOTAL (15) =	

1. The two parts of Question 1 test the basics of **asymptotic analysis**. All logarithms in this question are to the base 2.

   (a) [2 Marks] Order the following functions by increasing growth rates. Justify whenever needed.
   $n^{3/2}$, $2n(\log n)^2$, $\sqrt{\log n}$, $n^{0.5}$, $2^{\sqrt{n}}$, 100.

   (b) [2 Marks] Compute the running time of the following algorithm in terms of n using O-notation. Show your calculations.

   Algorithm Loop(n)
   $s \leftarrow 0$
   for $i \leftarrow 1$ to $2n$ do
       for $j \leftarrow 1$ to i do
           $s \leftarrow s + i$

2. Question 2 checks if we can solve **recurrence equations**.

   [4 Marks] Solve the following recurrence equation to get a closed-formula for $T(n)$. You can assume the n is a power of two.

$$\begin{aligned} T(n) &= 1 \text{ if } n = 1 \\ &= 2T\left(\frac{n}{2}\right) + \log n \text{ if } n \geq 2 \end{aligned}$$

3. Question 3 is about **Sorting Algorithms**. We assume that the output should be sorted in ascending order from smallest to largest.

   (a) [2 Marks] Suppose that your Quick-Sort algorithm uses a pivot rule that picks the element in the "middle". That is, for an array $A[0, 1, \ldots, n-1]$ of size n, it uses the element in $A[n/2]$ as pivot if n is even and the element in $A[(n-1)/2]$ as pivot if n is odd. Illustrate how this algorithm works using a quick-sort tree on the input:

   $$[7\ \ 6\ \ 5\ \ 4\ \ 3\ \ 2\ \ 1]$$

   (b) [2 Marks] What is the running time of this version of quick-sort on sequences of size n that are already sorted from largest to smallest? Explain why by writing down the recurrence equation for the running time of your algorithm on such sequences.

4. Question 4 is based on **Proof by Induction**.

   We say that a tree is binary if every internal node has exactly two children. **Using induction on the size of the tree**, prove the following claim: In any binary tree, the number of external nodes (also called as leaves) is one more than the number of internal nodes. (**Hints**: For the base case, use a tree of size 1. For the induction step, use the fact that the left and right subtrees of the root node are smaller in size.) [3 Marks]

END OF EXAM