CSC 226: Summer 2018: Lab 1

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1 Asymptotic Notation

Let f and g be two functions that take integers as input and outputs real numbers.
Big-Oh: $f(n)$ is $O(g(n))$ if and only if there exists a *real* constant $c > 0$ and an integer $n_0 > 0$ such that $f(n) \leq c.g(n)\ \forall n \geq n_0$.
Big-Omega: $f(n)$ is $\Omega(g(n))$ if and only if there exists a *real* constant $c > 0$ and an integer $n_0 > 0$ such that $f(n) \geq c.g(n)\ \forall n \geq n_0$.
Big-Theta: $f(n)$ is $\Theta(g(n))$ if and only if $f(n)$ is $O(g(n))$ and $f(n)$ is $\Omega(g(n))$.

Based on the definitions above, prove the followings.

1. $5n^2 + 6n + 12$ is $O(n^3)$
2. $5n^2 + 6n + 12$ is $\Omega(n^2)$
3. $5n^2 + 6n + 12$ is $\Theta(n^2)$

2 Rules of Big-Oh

Prove the following theorems using the definition of Big-Oh from above.

1. **R1 (Scaling):** If $f(n)$ is $O(g(n))$ then $af(n)$ is $O(g(n))$, $a > 0$.
2. **R4 (Transitivity):** If $d(n)$ is $O(f(n))$ and $f(n)$ is $O(g(n))$, then $d(n)$ is $O(g(n))$.
3. **R7:** $\log(n^x)$ is $O(\log n)$ for any fixed $x > 0$.
4. **R6:** n^x is $O(a^n)$ for any fixed $x > 0$ and $a > 1$.

3 Permutations and Combinations

3.1 Poker Hands

If you have played poker, you probably know some or all the hands below [1]. You can choose 5 cards from 52 in $\binom{52}{5}$ ways. But how many of them would be

a *Royal Flush* or a *Four-of-a-Kind*? Let's try to calculate the numbers for all the following hands. The green ones have already been covered in the class.

1. **Royal Flush:** All five cards are of the same suit and are of the sequence 10 J Q K A.
2. **Straight Flush:** All five cards are of the same suit and are sequential in rank.
3. **Four-of-a-Kind:** Four cards are all of the same rank
4. **Full House:** A hand consisting of one pair and a three-of-a-kind of a different rank than the pair.
5. **Flush:** All five cards are of the same suit but not all sequential in rank
6. **Straight:** All five cards are sequential in rank but are not all of the same suit
7. **Three-of-a-Kind:** Three cards are all of the same rank and the other two are each of different ranks from the first three and each other
8. **Two Pair:** Two pairs of two cards of the same rank (the ranks of each pair are different in rank, obviously, to avoid a Four-of-a-Kind)
9. **One Pair:** Only two cards of the five are of the same rank with the other three cards all having different ranks from each other and from that of the pair

3.2 Some other problems

1. Six friends want to play enough games of chess and every one wants to play everyone else. How many games will they have to play?
2. There are five flavors of ice cream: banana, chocolate, lemon, strawberry and vanilla. We can have three scoops. How many variations will there be? [2]

References

[1] Jeff Duda, *Probabilities of Poker Hands with Variations.* http://www.meteor.iastate.edu/~jdduda/portfolio/492.pdf

[2] https://www.mathsisfun.com/combinatorics/combinations-permutations.html