CSC 226: Summer 2018: Lab 3

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1 Walk, Trail and Path

Let $G = (V, E)$ be an undirected graph with vertex set V and edge set E. Let x, y be two (not necessarily distinct) vertices of G.

Walk: An $x - y$ walk of G is an alternating sequence of vertices and edges starting from x and ending at y. The sequence may look like this:

$$x = x_0, e_1, x_1, e_2, \ldots, e_n, x_n = y$$

The *length* of a walk is the number of edges in the walk. In the example above the length of the $x-y$ walk is n. There might be repeated vertices and/or repeated edges in a walk.

Closed and Open Walks: An $x-y$ walk is *closed* if $x = y$, otherwise it is *open*.

Trail and Circuit: A *trail* is an $x - y$ walk where no edge is repeated. A closed trail is called a *circuit*.

Path and Cycle: A *path* is an $x-y$ trail where no vertex is repeated. A closed path is called a *cycle*.

Exercise

Based on the definitions above, answer the following questions.

1. In each of the following pairs, which one is a subset of the other? For example in the pair "path, circuit", is a path always a circuit? or is a circuit always a path? or neither is true?

   - path, circuit
   - cycle, trail
   - trail, open walk

2. Draw the graph with the following edges and let's call it T_t. Try to draw it without crossing edges.

$$(a, b), (b, c), (c, a), (d, e), (e, f), (f, d), (a, d), (b, e), (c, f), (a, e), (b, f), (c, d)$$

3. How many a, c paths are there in graph T_t in Exercise 2? How many of those paths have length 4?

4. Let G be an undirected graph and let x, y $(x \neq y)$ be two distinct vetices of G. If there is an $x - y$ trail in G, prove that there is an $x - y$ path in G.

2 Subgraphs and Isomorphism

Let $G = (V, E)$ be an undirected graph with vertex set V and edge set E and let $G_1 = (V_1, E_1)$ be another undirected graph with vertex set V_1 and edge set E_1.
Subgraph: G_1 is a *subgraph* of G if $V_1 \subset V$, where $V_1 \neq \emptyset$, and $E_1 \subset E$ such that each edge in E_1 is incident to vertices in V_1.
G_1 is a *spanning subgraph* of G if $V_1 = V$ and $E_1 \subset E$.
G_1 is an *induced subgraph* of G if all the edges incident to vertices in $V_1 \subset V$ are in $E_1 \subset E$.

Complement: A *complete graph* K_n of n vertices contains an edge between each pair of vertices. The *complement graph* of $\overline{G}$ of G is the spanning subgraph of K_n that only contains all the edges that are not in G.

Isomorphism: Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two undirected graphs. G_1 and G_2 are *isomorphic* to each other if there exists a one-to-one and onto function $f : V_1 \to V_2$ such that for each $a, b \in V_1$, $(a, b) \in E_1$ if and only if $(f(a), f(b)) \in E_2$.

Exercise

Based on the definitions above, answer the following questions.

1. Draw the graph with the following edges: $(a, b), (b, c), (c, a)$. How many subgraph does it have? You don't have to draw all the subgraphs, just calculate the number. How many of those subgraphs is a spanning subgraph? How many are induced subgraphs?

2. Draw the complement of T_t from the previous exercise. How many edges are there in K_6?

3. Find all non-isomorphic (and loop-free) graphs of 3 vertices. Now find all loop-free non-isomorphic graphs of 4 vertices.

References

[1] Ralph P. Grimaldi. 2004. Discrete and Combinatorial Mathematics: An Applied Introduction (5th ed.). Addison-Wesley Longman Publishing Co., Inc., Boston, MA, USA.