CSC 226: Summer 2018: Lab 9

July 17, 2018

1 Maximum Bipartite Matching

Problem: We have 6 workers and 8 task to assign to them under the condition that a person can be assigned at most one task. But not every worker qualify for every job. Figure 1 shows the qualification of the workers as a graph. We have a set of vertices $a, \ldots, f$ representing the workers and another set of vertices $1, \ldots, 8$ representing the tasks. An edge between a worker and a task means that that worker qualify for that task, and an absence of an edge means that a person does not qualify for that task. For example, Worker a qualifies for Tasks $1, 4, 5$. Now, how can we assign tasks to the workers so that maximum number of tasks are accomplished?

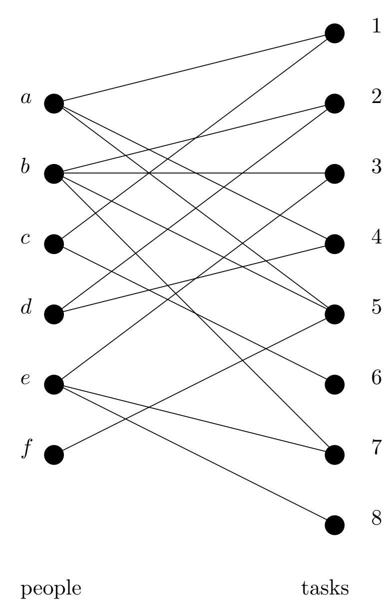


Figure 1: A bipartitte graph representing qualifications of a set of workers for a set of tasks.

Reduce the problem to a network flow problem as shown in Figure 2. Direct the original edges to the tasks and assign capacity 1 to each of the edges. Add a vertex s and draw edges of capacity 1 from s to each of $a, \ldots, f$ (all the people vertices). Add a vertex t and draw edges of capacity 1 from each of $1, \ldots, 8$ (all the task vertices) to t. Now we have to find the maximum flow in this network.

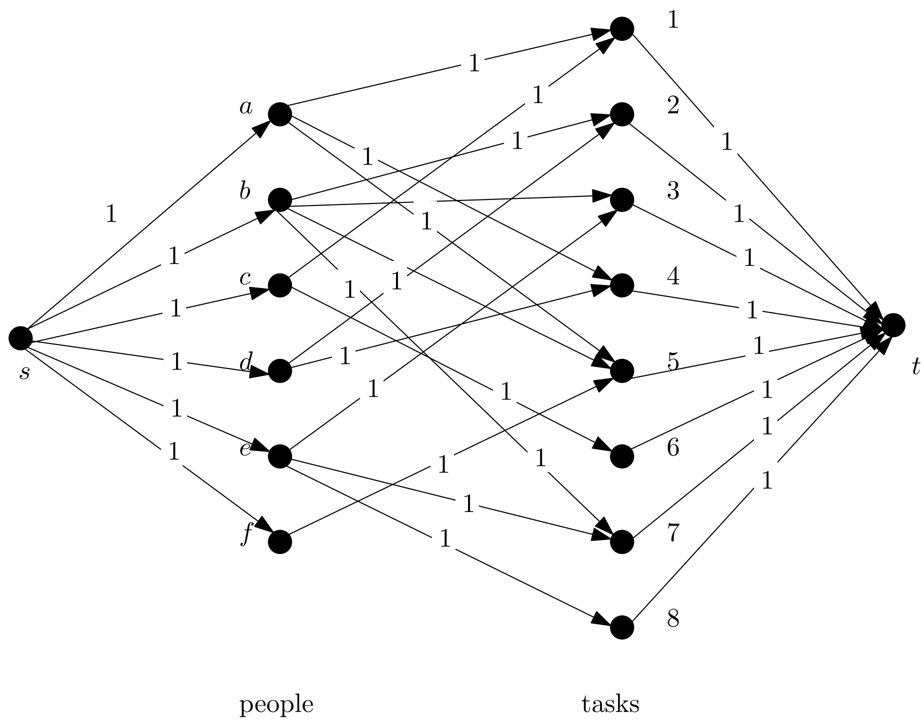
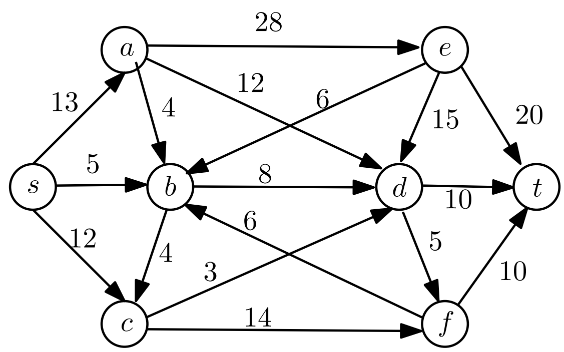
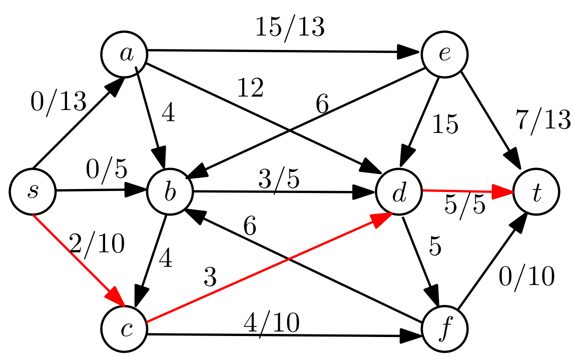


Figure 2: Maximum bipartite matching problem reduced to network flow problem.

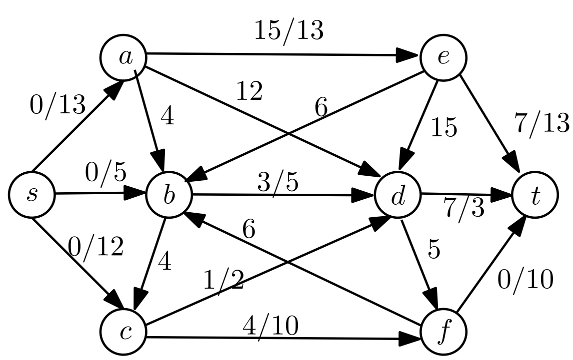
2 Exercise



Create a flow of 5 through s, b, d, t
a flow of 13 through s, a, e, t and
a flow of 10 through s, c, f, t.

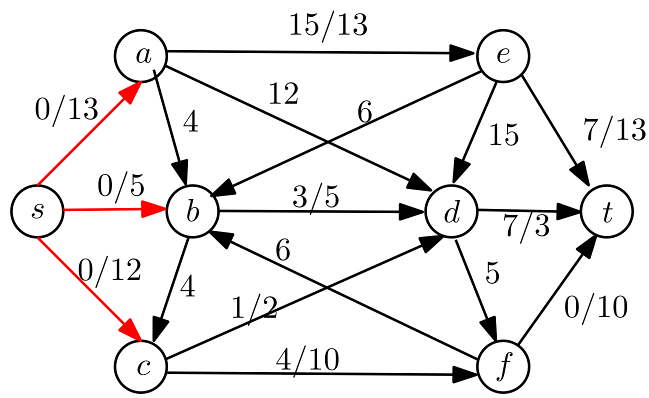


The capacity of the augmenting path is 2



No more augmenting paths.
So we found the maximum flow which is 30

Finding Min-cut: Look at the edges with full capacity used (or forward flow 0), and find all the cuts. Choose the minimum one to be the min-cut.


a
b
c
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s
t
15/13
12
6
4
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5
1/2
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