Lemma 14.1: *In Dijkstra's algorithm, whenever a vertex u is pulled into the cloud, the label $D[u]$ is equal to $d(v,u)$, the length of a shortest path from v to u.*

Proof: Suppose that $D[t] > d(v,t)$ for some vertex t in V, and let u be the ***first*** vertex the algorithm pulled into the cloud C (that is, removed from Q), such that $D[u] > d(v,u)$. There is a shortest path P from v to u (for otherwise $d(v,u) = +\infty = D[u]$). Therefore, let us consider the moment when u is pulled into C, and let z be the first vertex of P (when going from v to u) that is not in C at this moment. Let y be the predecessor of z in path P (note that we could have $y = v$). (See Figure 14.5.) We know, by our choice of z, that y is already in C at this point. Moreover, $D[y] = d(v,y)$, since u is the ***first*** incorrect vertex. When y was pulled into C, we tested (and possibly updated) $D[z]$ so that we had at that point

$$D[z] \leq D[y] + w((y,z)) = d(v,y) + w((y,z)).$$

But since z is the next vertex on the shortest path from v to u, this implies that

$$D[z] = d(v,z).$$

But we are now at the moment when we are picking u, not z, to join C; hence,

$$D[u] \leq D[z].$$

It should be clear that a subpath of a shortest path is itself a shortest path. Hence, since z is on the shortest path from v to u,

$$d(v,z) + d(z,u) = d(v,u).$$

Moreover, $d(z,u) \geq 0$ because there are no negative-weight edges. Therefore,

$$D[u] \leq D[z] = d(v,z) \leq d(v,z) + d(z,u) = d(v,u).$$

But this contradicts the definition of u; hence, there can be no such vertex u. ■

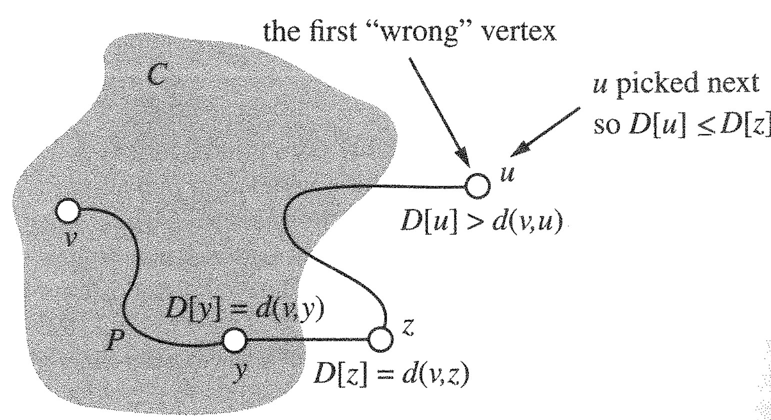

Figure 14.5: A schematic illustration for the justification of Lemma 14.1.