CSC 226 SUMMER 2017
ALGORITHMS AND DATA STRUCTURES II
MIDTERM EXAMINATION
UNIVERSITY OF VICTORIA

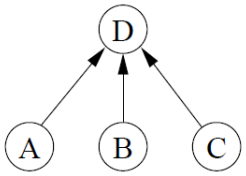
1. Student ID: _____
2. Name: _____
3. DATE: 14 JUNE 2017
   DURATION: 50 MINUTES
   INSTRUCTOR: RICH LITTLE
4. THIS QUESTION PAPER HAS SEVEN PAGES, DOUBLE-SIDED, INCLUDING THE COVER PAGE.
5. THIS QUESTION PAPER HAS FOUR QUESTIONS.
6. ALL ANSWERS TO BE WRITTEN ON THIS EXAMINATION PAPER.
7. WRITE YOUR NAME ON THE BACK OF THE EXAM.
8. THIS IS A CLOSED BOOK EXAM. CALCULATORS ARE ALLOWED.
9. READ THROUGH ALL THE QUESTIONS AND ANSWER THE EASY QUESTIONS FIRST. KEEP YOUR ANSWERS SHORT AND PRECISE.

Q1 (10)	
Q2 (10)	
Q3 (10)	
Q4 (10)	
TOTAL (40) =	

1. **(a). [2 mark]** What is the lower bound on the runtime of any comparison based sorting algorithm on n elements?

(b). [2 mark] What is the worst-case runtime of quicksort on n elements?

(c). [2 mark] Below is a tree that is the result of four Union-Find operations, with weighted-union and path compression. The first two operations are union(D,A) and union(B,C), what are the last two operations?



(d). [2 Marks] Give the definition of little-oh. That is, $f(n)$ is $o(g(n))$ if and only if… (NOTE: you cannot use the limit definition.)

(e). [2 Marks] What is the maximum number of internal nodes in a 2-3 tree with height 4 (Recall, height of a leaf is 0 and leaves are empty/null)?

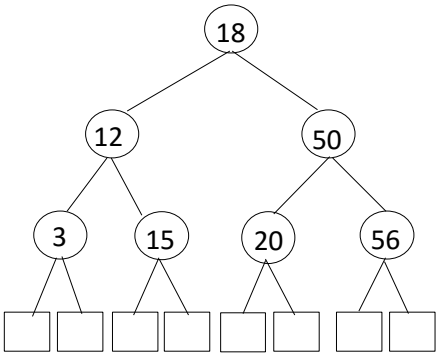
2. (a). [3 marks] Consider sequence S below. When determining the 4-th smallest element of S using the Linear Selection algorithm with subsequences of size 5, what is the first pivot that will be chosen?

$$S = \{9, 27, 4, 18, 19, 1, 21, 17, 8, 5, 111, 23, 6, 81, 2, 108, 56, 29, 67, 43, 86, 32, 71, 31, 100\}$$

(b). [3 marks] After partitioning S with the above pivot, what is the next pivot to be chosen?

(b). [4 marks] The recurrence relation for the running time of the median of medians algorithm on a sequence of n distinct elements, where you divide the sequence into groups of 3, can be expressed as: $T(n) = 2n + T(n/3) + T(2n/3)$. Using the guess and test method show that NO real number $c > 0$ exists such that $T(n) \leq cn$ for all $n > 0$. (i.e. it is not linear when using groups of 3.)

3. (a). [4 marks] Consider the following balanced binary search tree. Assume the tree is an AVL tree. Insert key 23 and then key 48 and draw the resulting AVL tree. Show your steps.



(b). [6 marks] Now assume the initial tree above is a red-black tree. Insert key 23 and then key 48 and draw the resulting red-black tree. Again, show your steps. (Note: you do not need to draw the initial tree again.)

4. (a). [5 marks] Below is the textbooks code for the union algorithm on the weighted quick-union data structure for doing union-by-size. Recall entry i in array id[] corresponds to the parent of vertex i and each entry in the sz[] array keeps track of the number of nodes in the tree rooted at node i, for each i from 0 to n-1. Rewrite the code to do union-by-height instead. Here, you will have an array ht[], instead of sz[], where each entry is the height of the tree rooted at node i.

```
public void union(int p, int q) {
    int i = find(p);
    int j = find(q);
    if (i == j) return;

    if  (sz[i] < sz[j]) {
        id[i] = j;
        sz[j] += sz[i];
    }
    else  {
        id[j] = i;
        sz[i] += sz[j];
    }
}
```

(b). [5 marks] Let $G = (V, E)$ be a connected, weighted graph with distinct edge weights. Prove that G has exactly one minimum spanning tree.

THE END