

CSC 226

Algorithms and Data Structures: II

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ECS 516

Discrete Math

The Pigeonhole Principle

If m pigeons occupy n pigeonholes and $m > n$, then at least one pigeonhole has two or more pigeons roosting in it.

Examples

1. If I draw 14 cards from a standard deck of 52, will there be a pair?
2. Given a text file with 500,000 words of size 4 or less, is it possible that the words are distinct?
3. While on a 4-week vacation I will drink at least 1 beer a day but no more than 40 total. Will there be a span of consecutive days in which I drink exactly 15 beers?

Posets

Cartesian Product

For sets A and B , the Cartesian product (or cross product) of A and B is

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Relation

For sets A and B , any subset of $A \times B$ is called a relation from A to B .

Posets

Partial Order

A relation \mathfrak{R} on a set A is called a partial order if \mathfrak{R} is

- i. Reflexive: For all $x \in A$, $(x, x) \in \mathfrak{R}$
- ii. Antisymmetric: For all $x, y \in A$, if $(x, y), (y, x) \in \mathfrak{R}$, then $x = y$.
- iii. Transitive: For all $x, y, z \in A$, if $(x, y), (y, z) \in \mathfrak{R}$, then $(x, z) \in \mathfrak{R}$.

Posets

Partially Ordered Set

Let A be a set and \mathfrak{R} a relation on A . The pair (A, \mathfrak{R}) is called a partially ordered set (or poset) if \mathfrak{R} on A is a partial order.

Hasse Diagram

If \mathfrak{R} is a partial order on A , we construct a Hasse diagram for \mathfrak{R} on A by connecting x “up” to y if and only if $(x, y) \in \mathfrak{R}$ and there are no other $z \in A$ such that $(x, z) \in \mathfrak{R}$ or $(z, y) \in \mathfrak{R}$.

Examples

4. Let $A = \{1,2,3,4\}$. Define \mathfrak{R} on A by

$$\mathfrak{R} = \{(x, y) : x, y \in A \text{ and } x|y\}$$

Draw a relation diagram and a Hasse diagram for \mathfrak{R} .

5. Consider the power set, $P(A)$, where $A = \{1,2,3\}$. Draw the Hasse diagram to illustrate the subset relation.

Posets

Total Order

If (A, \mathfrak{R}) is a poset, it is a total order if for all $x, y \in A$, either $(x, y) \in \mathfrak{R}$ or $(y, x) \in \mathfrak{R}$.