COMPUTER SCIENCE 349A, SPRING 2018
ASSIGNMENT #5 - SOLUTIONS - 20 MARKS

1. (a) **(2 points)** Below are the MATLAB commands and there results for setting up the system of equations,

```
>> x = [pi/4 pi/2 2*pi/3 5*pi/6]

x =

    0.7854    1.5708    2.0944    2.6180

>> f = @(x) cos(pi*x)

f =

    @(x)cos(pi*x)

>> for i=1:4
A(i,:)= [1 x(i) x(i)^2 x(i)^3];
end
>> A

A =

    1.0000    0.7854    0.6169    0.4845
    1.0000    1.5708    2.4674    3.8758
    1.0000    2.0944    4.3865    9.1870
    1.0000    2.6180    6.8539   17.9434

>> b = [f(x(1)); f(x(2)); f(x(3)); f(x(4))]

b =

   -0.7812
    0.2206
    0.9564
   -0.3623
```

NOTE: They just have to set up the system. They do not have to solve it here, they do not even need to calculate the values of the coefficients nor even use MATLAB.

(b) **(2 points)** Here is the solution to the system (c for coefficients),

```
>> c = A\b
```

```
c =

    3.7616
  -11.9028
    9.4389
   -2.0986
```

(c) **(2 points)** Here are the commands for the plot and the plot itself. Note that I define p(x) first and I also plot the interpolation points, neither of which I actually asked to do. Further note that I ask them to use fplot NOT plot, they should lose a mark if they use plot.

```
>> p = @(x) c(1) + c(2)*x + c(3)*x.^2 + c(4)*x.^3

p =

    @(x)c(1)+c(2)*x+c(3)*x.^2+c(4)*x.^3

>> fplot(f,[x(1),x(4)])
>> hold on
>> fplot(p,[x(1),x(4)])
>> plot(x,f(x),'*')
```

The figure resulting from these commands (I inserted a legend after the fact),

2. **(8 points)** Here, the cubic spline is expressed as

$$S(x) = \begin{cases} S_0(x) = a_0 + b_0 x + d_0 x^3, & \text{if } 0 \le x \le 1 \\ S_1(x) = a_1 + b_1(x-1) + c_1(x-1)^2 + d_1(x-1)^3, & \text{if } 1 \le x \le 3 \end{cases}$$

and we have $x_0 = 0, x_1 = 1, x_2 = 3, f(x_0) = 1, f(x_1) = 2, f(x_2) = -20$.

(b) $S_0(x_0) = f(x_0) \Rightarrow S_0(0) = f(0) \Rightarrow a_0 = 1$.

$S_1(x_1) = f(x_1) \Rightarrow S_1(1) = f(1) \Rightarrow a_1 = 2$.

$S_1(x_2) = f(x_2) \Rightarrow S_1(3) = f(3) \Rightarrow a_1 + 2b_1 + 4c_1 + 8d_1 = -20 \Rightarrow 2b_1 + 4c_1 + 8d_1 = -22$.

(c) $S_1(x_1) = S_0(x_1) \Rightarrow S_1(1) = S_0(1) \Rightarrow a_1 = a_0 + b_0 + d_0 \Rightarrow b_0 + d_0 = 1$.

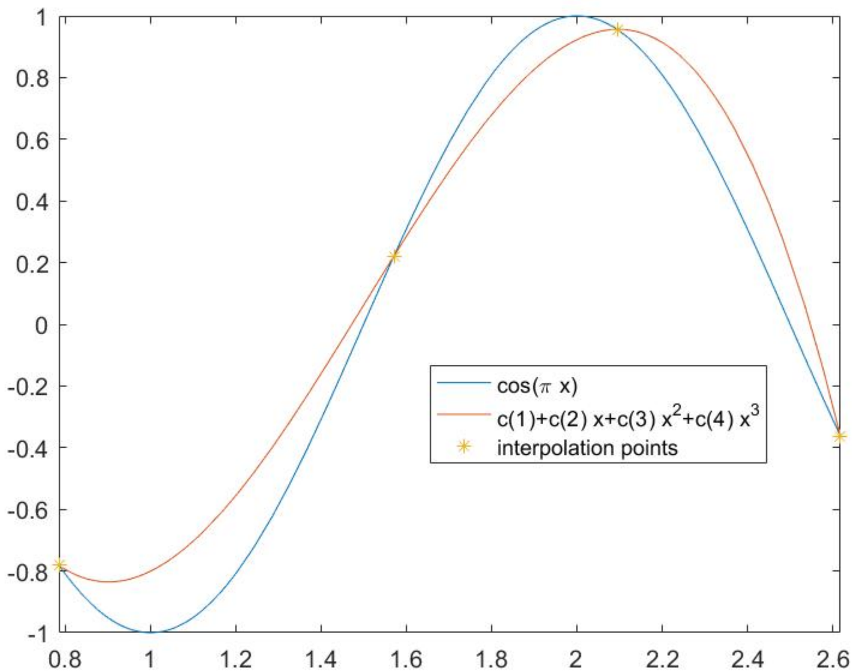
(d) $S_0'(x) = b_0 + 3d_0 x^2$ and $S_1'(x) = b_1 + 2c_1(x-1) + 3d_1(x-1)^2$

thus

$S_1'(x_1) = S_0'(x_1) \Rightarrow S_1'(1) = S_0'(1) \Rightarrow b_1 = b_0 + 3d_0 \Rightarrow b_0 + 3d_0 - b_1 = 0$.

(e) $S_0''(x) = 6d_0 x$ and $S_1''(x) = 2c_1 + 6d_1(x-1)$

thus



$S_1''(x_1) = S_0''(x_1) \Rightarrow S_1''(1) = S_0''(1) \Rightarrow 2c_1 = 6d_0 \Rightarrow 6d_0 - 2c_1 = 0.$

(f) $S_0''(x_0) = 0 \Rightarrow S_0''(0) = 0 \Rightarrow 0 = 0$, not useful.

$S_1''(x_2) = 0 \Rightarrow S_1''(3) = 0 \Rightarrow 2c_1 + 12d_1 = 0.$

So, we know that $a_0 = 1$ and $a_1 = 2$, that leaves the 5 unknowns $b_0, d_0, b_1, c_1,$, and d_1 and the 5 equations,

$$
\begin{aligned}
&2b_1 + 4c_1 + 8d_1 = -22 \\
&b_0 + d_0 = 1 \\
&b_0 + 3d_0 - b_1 = 0 \\
&6d_0 - 2c_1 = 0 \\
&2c_1 + 12d_1 = 0
\end{aligned}
$$

In matrix form, we need to solve the following 5×5 system:

$$
\begin{bmatrix} 0 & 0 & 2 & 4 & 8 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 3 & -1 & 0 & 0 \\ 0 & 6 & 0 & -2 & 0 \\ 0 & 0 & 0 & 2 & 12 \end{bmatrix} \begin{bmatrix} b_0 \\ d_0 \\ b_1 \\ c_1 \\ d_1 \end{bmatrix} = \begin{bmatrix} -22 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

I solved it in MATLAB with,

```
>> A=[0 0 2 4 8; 1 1 0 0 0; 1 3 -1 0 0; 0 6 0 -2 0; 0 0 0 2 12]

A =

     0     0     2     4     8
     1     1     0     0     0
     1     3    -1     0     0
     0     6     0    -2     0
     0     0     0     2    12

>> b=[-22; 1; 0; 0; 0]

b =

   -22
     1
     0
     0
     0

>> x=A\b

x =

     3
    -2
    -3
    -6
     1
```

So, $a_0 = 1, b_0 = 3, d_0 = -2, a_1 = 2, b_1 = -3, c_1 = -6, d_1 = 1$ and so finally the spline is,

$$S(x) = \begin{cases} S_0(x) = 1 + 3x - 2x^3, & \text{if } 0 \leq x \leq 1 \\ S_1(x) = 2 - 3(x-1) - 6(x-1)^2 + (x-1)^3, & \text{if } 1 \leq x \leq 3 \end{cases}$$

3. (a) **(3 points)** Here are the MATLAB commands,

```
>> x=[1 2 5 6 7 8 10 13 17]

x =

     1     2     5     6     7     8    10    13    17

>> y=[3 3.7 3.9 4.2 5.7 6.6 7.1 6.7 4.5]
```

```
y =

  Columns 1 through 6

    3.0000    3.7000    3.9000    4.2000    5.7000    6.6000

  Columns 7 through 9

    7.1000    6.7000    4.5000

>> pp = spline(x,[1 y -0.67])

pp =

      form: 'pp'
    breaks: [1 2 5 6 7 8 10 13 17]
     coefs: [8x4 double]
    pieces: 8
     order: 4
       dim: 1

>> [b,c]=unmkpp(pp)

b =

     1     2     5     6     7     8    10    13    17


c =

    0.0468   -0.3468    1.0000    3.0000
    0.0266   -0.2064    0.4468    3.7000
    0.3419    0.0326   -0.0745    3.9000
   -0.5745    1.0582    1.0163    4.2000
    0.1563   -0.6654    1.4091    5.7000
    0.0239   -0.1965    0.5472    6.6000
   -0.0026   -0.0529    0.0485    7.1000
    0.0057   -0.0759   -0.3381    6.7000
```

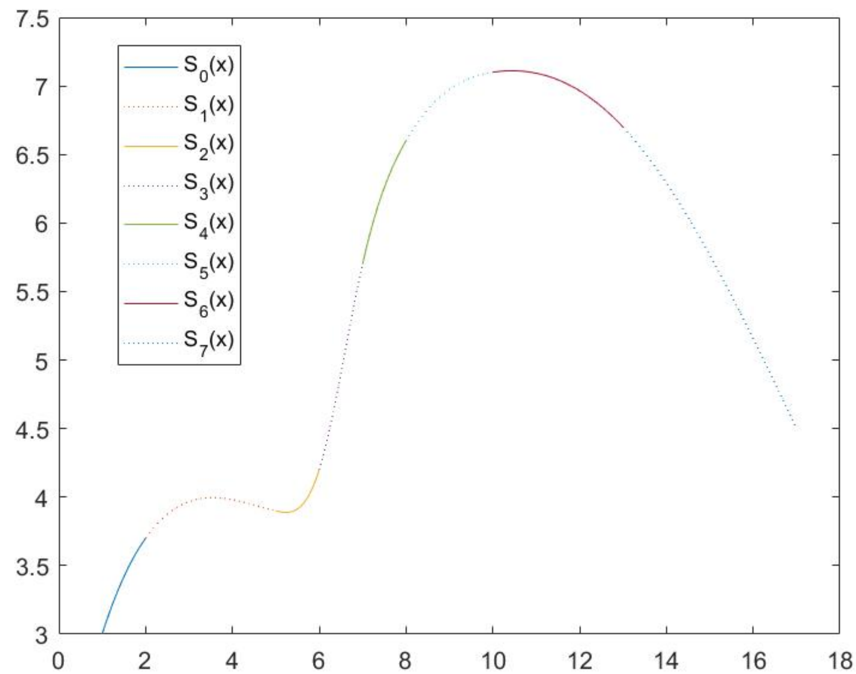
Giving the spline,

$$S(x) = \begin{cases} S_0(x) = 0.0468(x-1)^3 - 0.3468(x-1)^2 + (x-1) + 3, & \text{if } 1 \le x \le 2 \\ S_1(x) = 0.0266(x-2)^3 - 0.2064(x-2)^2 + 0.4468(x-2) + 3.7, & \text{if } 2 \le x \le 5 \\ S_2(x) = 0.3419(x-5)^3 + 0.0326(x-5)^2 - 0.0745(x-5) + 3.9, & \text{if } 5 \le x \le 6 \\ S_3(x) = -0.5745(x-6)^3 + 1.0582(x-6)^2 + 1.0163(x-6) + 4.2, & \text{if } 6 \le x \le 7 \\ S_4(x) = 0.1563(x-7)^3 - 0.6654(x-7)^2 + 1.4091(x-7) + 5.7, & \text{if } 7 \le x \le 8 \\ S_5(x) = 0.0239(x-8)^3 - 0.1965(x-8)^2 + 0.5472(x-8) + 6.6, & \text{if } 8 \le x \le 10 \\ S_6(x) = -0.0026(x-10)^3 - 0.0529(x-10)^2 + 0.0485(x-10) + 7.1, & \text{if } 10 \le x \le 13 \\ S_7(x) = 0.0057(x-13)^3 - 0.0759(x-13)^2 - 0.3381(x-13) + 6.7, & \text{if } 13 \le x \le 17 \end{cases}$$

(b) **(3 points)** Here is what I did to plot the spline. Note, I did it in a loop. Probably they will mostly use individual commands for each subinterval.

```
>> for i=1:8
X(i,:)=linspace(x(i),x(i+1),50);
Y(i,:)=c(i,1)*(X(i,:)-x(i)).^3+c(i,2)*(X(i,:)-x(i)).^2+c(i,3)*(X(i,:)-x(i))
+c(i,4);
if mod(i,2)==1
plot(X(i,:),Y(i,:),'-')
else
plot(X(i,:),Y(i,:),':')
end
hold on
end
```

The plot corresponding to this code is,


7.5
7
6.5
6
5.5
5
4.5
4
3.5
3
0
2
4
6
8
10
12
14
16
18
$S_0(x)$
$S_1(x)$
$S_2(x)$
$S_3(x)$
$S_4(x)$
$S_5(x)$
$S_6(x)$
$S_7(x)$