COMPUTER SCIENCE 349A, SPRING 2018
ASSIGNMENT #4 - SOLUTIONS - 20 MARKS

Question #1 Answers

(a) **(3 points)**

$$f(x) = x^m - R$$
$$f'(x) = mx^{m-1}$$

Newton's method is

$$\begin{aligned}
x_{i+1} &= x_i - \frac{f(x_i)}{f'(x_i)} \\
&= x_i - \frac{x_i^m - R}{mx_i^{m-1}} \\
&= \frac{mx_i^m - x_i^m + R}{mx_i^{m-1}} \\
&= \frac{(m-1)x_i^m + R}{mx_i^{m-1}} \\
&= \left((m-1)x_i + \frac{R}{x_i^{m-1}} \right) / m
\end{aligned}$$

(b) **(3 points)**

```
function root = mth_root ( m, R, x0, eps, imax );
   i = 1;
   fprintf( 'iteration   approximation\n' );
   while i <= imax
      root = ((m-1)*x0+R/x0^(m-1))/m;
      fprintf( '%6.0f %18.10f\n', i, root );
      if abs( 1-x0/root ) < eps
         return
      end
      i = i+1;
      x0 = root;
   end
   fprintf( 'failed to converge in %g iterations\n', imax );
end
```

(c) **(2 points)**

```
>>  mth_root( 5, 65.25, 1, 1e-10, 20 )
iteration   approximation
     1       13.8500000000
     2       11.0803546594
     3        8.8651494842
     4        7.0942324256
     5        5.6805380948
     6        4.5569634056
     7        3.6758334715
     8        3.0121472143
     9        2.5682456827
    10        2.3545571677
    11        2.3082394421
    12        2.3063048664
    13        2.3063016155
    14        2.3063016155

ans =

    2.3063
```

Question #2 Answer.

(a) **(4 points)**
First note that:

$$\begin{aligned} p(x) &= a_1 + a_2(x+y_1) + a_3(x+y_1)(x+y_2) + a_4(x+y_1)(x+y_2)(x+y_3) + \cdots \\ &\quad \cdots + a_{n+1}(x+y_1)(x+y_2)(x+y_3)\cdots(x+y_n) \\ &= a_1 + (x+y_1)(a_2 + (x+y_2)(a_3 + (x+y_3)(\cdots(a_n + a_{n+1}(x+y_n))\cdots))) \end{aligned}$$

Therefore, applying Horner's algorithm to this it looks like:

$$\begin{aligned} b_{n+1} &= a_{n+1} \\ b_n &= a_n + b_{n+1}(x+y_n) \\ &\vdots \\ b_i &= a_i + b_{i+1}(x+y_i) \\ &\vdots \\ b_1 &= a_1 + b_2(x+y_1) = p(x) \end{aligned}$$

Thus, the MATLAB function is:

```
function p = PolyEval( n, a, y, x )
    b(n+1) = a(n+1);
```

```
    for i = n:-1:1
        b(i) = a(i) + b(i+1)*(x+y(i));
    end
    p = b(1);
end
```

(b) **(2 points)**

```
>> a = [-1 3.3 0 -2.2 5 -1.6];
>> y = [-1 1 -1 1 -1];
>> PolyEval(5,a,y,1.53)

ans =

    6.6509
```

Question #3 Answer

(a) **(4 points)** First we swap E_1 and E_3 to get

$$\left(\begin{array}{rrrr|r} 2 & -2 & 4 & -2 & 0 \\ 2 & 0 & -2 & 2 & 0 \\ 4 & 2 & 6 & -8 & 1 \\ 0 & -2 & 2 & -2 & 0 \end{array}\right) \quad \rightarrow \quad \left(\begin{array}{rrrr|r} 4 & 2 & 6 & -8 & 1 \\ 2 & 0 & -2 & 2 & 0 \\ 2 & -2 & 4 & -2 & 0 \\ 0 & -2 & 2 & -2 & 0 \end{array}\right)$$

Next we pivot about a_{11}, so $m_{21} = 1/2$, $m_{31} = 1/2$, and $m_{41} = 0$. Thus,

$$\begin{aligned} a_{22} &= 0 - (1/2)(2) = 1 \\ a_{23} &= -2 - (1/2)(6) = -5 \\ a_{24} &= 2 - (1/2)(-8) = 6 \\ b_2 &= 0 - (1/2)(1) = -1/2 \\ a_{32} &= -2 - (1/2)(2) = -3 \\ a_{33} &= 4 - (1/2)(6) = 1 \\ a_{34} &= -2 - (1/2)(-8) = 2 \\ b_3 &= 0 - (1/2)(1) = -1/2 \end{aligned}$$

Then swap rows 2 and 3.

$$\left(\begin{array}{rrrr|r} 4 & 2 & 6 & -8 & 1 \\ 0 & -1 & -5 & 6 & -1/2 \\ 0 & -3 & 1 & 2 & -1/2 \\ 0 & -2 & 2 & -2 & 0 \end{array}\right) \quad \rightarrow \quad \left(\begin{array}{rrrr|r} 4 & 2 & 6 & -8 & 1 \\ 0 & -3 & 1 & 2 & -1/2 \\ 0 & -1 & -5 & 6 & -1/2 \\ 0 & -2 & 2 & -2 & 0 \end{array}\right)$$

We now pivot about a_{22}, so $m_{32} = 1/3$ and $m_{42} = 2/3$. Thus,

$$
\begin{aligned}
a_{33} &= -5 - (1/3)(1) = -16/3 \\
a_{34} &= 6 - (1/3)(2) = 16/3 \\
b_3 &= -1/2 - (1/3)(-1/2) = -1/3 \\
a_{43} &= 2 - (2/3)(1) = 4/3 \\
a_{44} &= -2 - (2/3)(2) = -10/3 \\
b_4 &= 0 - (2/3)(-1/2) = 1/3
\end{aligned}
$$

$$
\left(\begin{array}{rrrr|r}
4 & 2 & 6 & -8 & 1 \\
0 & -3 & 1 & 2 & -1/2 \\
0 & 0 & -16/3 & 16/3 & -1/3 \\
0 & 0 & 4/3 & -10/3 & 1/3
\end{array}\right)
$$

Finally, we pivot about a_{33}, so $m_{43} = -1/4$ and

$$
\begin{aligned}
a_{44} &= -10/3 - (-1/4)(16/3) = -2 \\
b_4 &= 1/3 - (-1/4)(-1/3) = 1/4
\end{aligned}
$$

$$
\left(\begin{array}{rrrr|r}
4 & 2 & 6 & -8 & 1 \\
0 & -3 & 1 & 2 & -1/2 \\
0 & 0 & -16/3 & 16/3 & -1/3 \\
0 & 0 & 0 & -2 & 1/4
\end{array}\right)
$$

We now back-substitute:

$$
\begin{aligned}
x_4 &= (1/4)/(-2) = -1/8 \\
x_3 &= \frac{-1/3 - (16/3)(-1/8)}{-16/3} = -1/16 \\
x_2 &= \frac{-1/2 - 1(-1/16) - (2)(-1/8)}{-3} = 1/16 \\
x_1 &= \frac{1 - (2)(1/16) - 6(-1/16) - 8 * (-1/8)}{4} = 1/16
\end{aligned}
$$

(b) **(2 points)** The number of row interchanges that were performed was 2, which is an even number. Therefore the determinant is simply the product of the diagonal elements:

$$
(4)\,(-3)\left(\frac{-16}{3}\right)(-2) = -128
$$