Diksha Bansal
V00768613

CSC 349A
ASSIGNMENT 5

1. a)

1. a) $p(x) = C_0 + C_1 x + C_2 x^2 + C_3 x^3$

$f(x) = \cos(\pi x)$

$x = \left[\dfrac{\pi}{4}, \dfrac{\pi}{2}, \dfrac{2\pi}{3}, \dfrac{5\pi}{6}\right]$

$f(x_0) = C_0 + C_1 x_0 + C_2 x_0^2 + C_3 x_0^3$

$f(x_1) = C_0 + C_1 x_1 + C_2 x_1^2 + C_3 x_1^3$

$f(x_2) = C_0 + C_1 x_2 + C_2 x_2^2 + C_3 x_2^3$

$f(x_3) = C_0 + C_1 x_3 + C_2 x_3^2 + C_3 x_3^3$

$$\begin{bmatrix} 1 & x_0 & x_0^2 & x_0^3 \\ 1 & x_1 & x_1^2 & x_1^3 \\ 1 & x_2 & x_2^2 & x_2^3 \\ 1 & x_3 & x_3^2 & x_3^3 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} f(x_0) \\ f(x_1) \\ f(x_2) \\ f(x_3) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & \pi/4 & (\pi/4)^2 & (\pi/4)^3 \\ 1 & \pi/2 & (\pi/2)^2 & (\pi/2)^3 \\ 1 & 2\pi/3 & (2\pi/3)^2 & (2\pi/3)^3 \\ 1 & 5\pi/6 & (5\pi/6)^2 & (5\pi/6)^3 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} \cos(\pi^2/4) \\ \cos(\pi^2/2) \\ \cos(2\pi^2/3) \\ \cos(5\pi^2/6) \end{bmatrix}$$

b)
```
>> a_1 = pi/4
a_1 =
    0.7854

>> a_2 = a_1^2
a_2 =
    0.6169

>> a_3 = a_1^3
a_3 =
```

    0.4845

>> b_1 = pi/2
b_1 =
   1.5708

>> b_2 = b_1^2
b_2 =
   2.4674

>> b_3 = b_1^3
b_3 =
   3.8758

>> c_1 = 2*pi/3
c_1 =
   2.0944

>> c_2 = c_1^2
c_2 =
   4.3865

>> c_3 = c_1^3
c_3 =
   9.1870

>> d_1 = 5*pi/6
d_1 =
   2.6180

>> d_2 = d_1^2
d_2 =
   6.8539

>> d_3 = d_1^3
d_3 =
   17.9434

>> A = [1 a_1 a_2 a_3; 1 b_1 b_2 b_3; 1 c_1 c_2 c_3; 1 d_1 d_2 d_3]

A =

   1.0000   0.7854   0.6169   0.4845
   1.0000   1.5708   2.4674   3.8758
   1.0000   2.0944   4.3865   9.1870
   1.0000   2.6180   6.8539   17.9434

>> B = [cos(a_2); cos(b_2); cos(c_2); cos(d_2)]

B =

   0.8157
  -0.7812
  -0.3202

0.8415

>> A\B

ans =
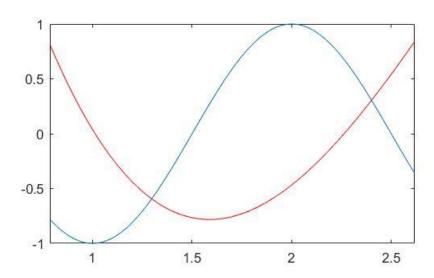
  6.4957
-10.4697
  4.5287
-0.5174

c) fplot(@(x) cos(pi*x), [pi/4 5*pi/6], 'b')
  >> hold on
  >> fplot(@(x) 6.4957 - 10.4697.*(x) + 4.5287.*(x.^2) - 0.5174.*(x.^3), [pi/4 5*pi/6], 'r')
  >> hold off

2.

$$S(x) = \begin{cases} S_0(x) = a_0 + b_0 x + d_0 x^3 & 0 \le x \le 1 \\ S_1(x) = a_1 + b_1(x-1) + c_1(x-1)^2 + d_1(x-1)^3, & 1 \le x \le 3 \end{cases}$$

	x_i	$f(x_i)$
x_0	0	1
x_1	1	2
x_2	3	-20

$\Rightarrow S(x)$ is a cubic spline interpolant if : $B=$

a) $S_j(x_j) = f(x_j)$, $0 \leq j \leq 1$ and $S_1(x_2) = f(x_2)$ $\underline{n=2}$

b) $S_{j+1}(x_{j+1}) = S_j(x_{j+1})$, $j = 0 \Rightarrow S_1(x_1) = S_0(x_1)$

c) $S'_{j+1}(x_{j+1}) = S'_j(x_{j+1})$ $j = 0 \Rightarrow S'_1(x_1) = S'_0(x_1)$

d) $S''_{j+1}(x_{j+1}) = S''_j(x_{j+1})$ $j = 0 \Rightarrow S''_1(x_1) = S''_0(x_1)$

e) $S'(x_0) = f'(x_0)$ and $S'(x_2) = f'(x_2)$

$\underline{OR}$

$\quad S''(x_0) = S''(x_2) = 0$

$\underline{j=0}$
$S_0(x_0) = f(x_0)$
$\Rightarrow a_0 + b_0(0) + d_0(0)^3 = f(0)$
$\Rightarrow$ $\boxed{a_0 = 1}$

$j=1$:
$S_1(x_1) = f(x_1)$
$\Rightarrow S_1(1) = f(x_1) \Rightarrow a_1 + b_1(1-1) + c_1(1-1)^2 + d_1(1-1)^3 = 2$
$\quad\quad\quad\quad\quad \Rightarrow \boxed{a_1 = 2}$

$\quad S_1(x_2) = f(x_2)$
$\Rightarrow S_1(3) = f(x_2) \Rightarrow a_1 + b_1(3-1) + c_1(3-1)^2 + d_1(3-1)^3 = -20$
$\quad\quad \Rightarrow 2 + 2b_1 + 4c_1 + 8d_1 = -20$
$\quad\quad \Rightarrow b_1 + 2c_1 + 4d_1 = -11$ _____ (i)

$$S_1(x_1) = S_0(x_1)$$

$$\Rightarrow S_1(1) = S_0(1)$$

$$\Rightarrow a_1 + b_1(1-1) + c_1(1-1)^2 + d_1(1-1)^3 = a_0 + b_0(1) + d_0(1)^3$$

$$\Rightarrow \quad a_1 = a_0 + b_0 + d_0$$

$$\Rightarrow \quad 2 = 1 + b_0 + d_0 \Rightarrow b_0 + d_0 = 1 \quad \text{———(ii)}$$

$$S_1'(x) = b_1 + 2c_1(x-1) + 3d_1(x-1)^2$$

$$S_0'(x) = b_0 + 3d_0 x^2$$

$$S_1'(x_1) = S_0'(x_1)$$

$$\Rightarrow S_1'(1) = S_0'(1)$$

$$\Rightarrow \quad b_1 = b_0 + 3d_0 \Rightarrow b_1 - b_0 - 3d_0 = 0 \quad \text{———(iii)}$$

$$S_1''(x_1) = S_0''(x_1)$$

$$S_1''(x) = 2c_1 + 6d_1(x-1)$$

$$S_0''(x) = 6d_0 x$$

$$S_1''(1) = S_0''(1)$$

$$\Rightarrow 2c_1 + 6d_1(0) = 6d_0(1) \Rightarrow c_1 = 3d_0 \quad \text{———(iv)}$$

$$S''(x_0) = 0 \qquad\qquad S''(x_2) = 0$$

$$\Rightarrow S''(0) = 0 \quad , \qquad \Rightarrow S''(3) = 0$$

$$\Rightarrow \quad 0 = 0 \qquad\qquad \Rightarrow 2c_1 + 6d_1(2) = 0$$

$$\Rightarrow c_1 + 6d_1 = 0 \quad \text{———(v)}$$

$\therefore$ The five equations are:

$$0.b_0 + 0.d_0 + b_1 + 2c_1 + 4d_1 = -11$$
$$b_0 + d_0 + 0.b_1 + 0.c_1 + 0.d_1 = 1$$
$$-b_0 - 3d_0 + b_1 + 0.c_1 + 0.d_1 = 0$$
$$0.b_0 - 3d_0 + 0.b_1 - c_1 + 0.d_1 = 0$$
$$0.b_0 + 0.d_0 + 0.b_1 + c_1 + 6d_1 = 0$$

$$\begin{bmatrix} 0 & 0 & 1 & 2 & 4 \\ 1 & 1 & 0 & 0 & 0 \\ -1 & -3 & 1 & 0 & 0 \\ 0 & 3 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix} \begin{bmatrix} b_0 \\ d_0 \\ b_1 \\ c_1 \\ d_1 \end{bmatrix} = \begin{bmatrix} -11 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} b_0 \\ d_0 \\ b_1 \\ c_1 \\ d_1 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -3 \\ -6 \\ 1 \end{bmatrix}, \quad a_0 = 1, \quad a_1 = 2$$

$$\therefore S(x) = \begin{cases} S_0(x) = 1 + 3x - 2x^3 & , \quad 0 \leq x \leq 1 \\ S_1(x) = 2 - 3(x-1) - 6(x-1)^2 + (x-1)^3 & , \quad 1 \leq x \leq 3 \end{cases}$$

>> C = [0 0 1 2 4; 1 1 0 0 0; -1 -3 1 0 0; 0 3 0 -1 0; 0 0 0 1 6]

C =

```
   0    0    1    2    4
   1    1    0    0    0
  -1   -3    1    0    0
   0    3    0   -1    0
   0    0    0    1    6
```

>> D = [-11; 1; 0; 0; 0]

D =

```
  -11
    1
    0
    0
    0
```

>> C\D

ans =

```
    3
   -2
   -3
   -6
    1
```

3. a) >> X = [1 2 5 6 7 8 10 13 17]
    X =

```
              1    2    5    6    7    8    10    13    17
```

   >> Y= [1 3 3.7 3.9 4.2 5.7 6.6 7.1 6.7 4.5 -0.67]
    Y =

  1.0000   3.0000   3.7000   3.9000   4.2000   5.7000   6.6000   7.1000   6.7000   4.5000   -
    0.6700

>> pp = spline(X, Y)

pp =

  struct with fields:

    form: 'pp'
   breaks: [1 2 5 6 7 8 10 13 17]

```
   coefs: [8×4 double]
  pieces: 8
   order: 4
     dim: 1
```

```
>> format short
>> [b,c] = unmkpp(pp)
```

b =

```
   1   2   5   6   7   8   10   13   17
```

c =

```
  0.0468  -0.3468   1.0000   3.0000
  0.0266  -0.2064   0.4468   3.7000
  0.3419   0.0326  -0.0745   3.9000
 -0.5745   1.0582   1.0163   4.2000
  0.1563  -0.6654   1.4091   5.7000
  0.0239  -0.1965   0.5472   6.6000
 -0.0026  -0.0529   0.0485   7.1000
  0.0057  -0.0759  -0.3381   6.7000
```

$$b = \begin{bmatrix} 1 & 2 & 5 & 6 & 7 & 8 & 10 & 13 & 17 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.0468 & -0.3468 & 1.0000 & 3.0000 \\ 0.0266 & -0.2064 & 0.4468 & 3.7000 \\ 0.3419 & 0.0326 & -0.0745 & 3.9000 \\ -0.5745 & 1.0582 & 1.0163 & 4.2000 \\ 0.1563 & -0.6654 & 1.4091 & 5.7000 \\ 0.0239 & -0.1965 & 0.5472 & 6.6000 \\ -0.0026 & -0.0529 & 0.0485 & 7.1000 \\ 0.0057 & -0.0759 & -0.3381 & 6.7000 \end{bmatrix}$$

$S_0(x) = 0.0468(x-1)^3 - 0.3468(x-1)^2 + 1.0000(x-1) + 3.0000$

$S_1(x) = 0.0266(x-2)^3 - 0.2064(x-2)^2 + 0.4468(x-2) + 3.7000$

$S_2(x) = 0.3419(x-5)^3 + 0.0326(x-5)^2 - 0.0745(x-5) + 3.9000$

$S_3(x) = -0.5745(x-6)^3 + 1.0582(x-6)^2 + 1.0163(x-6) + 4.2000$

$S_4(x) = 0.1563(x-7)^3 - 0.6654(x-7)^2 + 1.4091(x-7) + 5.7000$

$S_5(x) = 0.0239(x-8)^3 - 0.1965(x-8)^2 + 0.5472(x-8) + 6.6000$

$S_6(x) = -0.0026(x-10)^3 - 0.0529(x-10)^2 + 0.0485(x-10) + 7.1000$

$S_7(x) = 0.0057(x-13)^3 - 0.0759(x-13)^2 - 0.3381(x-13) + 6.7000$

b) X1 = linspace(1,2,50);
>> Y1 = 0.0468*(X1-1).^3-0.3468*(X1-1).^2+1.0000*(X1-1)+3.0000;
>> plot(X1,Y1,'-')
>> hold on
>> X2 = linspace(2,5,50);
>> Y2 = 0.0266*(X2-2).^3-0.2064*(X2-2).^2+0.4468*(X2-2)+3.7000;
>> plot(X2,Y2,':')
>> hold on
>> X3 = linspace(5,6,50);
>> Y3 = 0.3419*(X3-5).^3+0.0326*(X3-5).^2-0.0745*(X3-5)+3.9000;
>> plot(X3,Y3,'-')
>> hold on
>> X4 = linspace(6,7,50);
>> Y4 = -0.5745*(X4-6).^3+1.0582*(X4-6).^2+1.0163*(X4-6)+4.2000;
>> plot(X4,Y4,':')

```
>> hold on
>> X5 = linspace(7,8,50);
>> Y5 = 0.1563*(X5-7).^3-0.6654*(X5-7).^2+1.4091*(X5-7)+5.7000;
>> plot(X5,Y5,'-')
>> hold on
>> X6 = linspace(8,10,50);
>> Y6 = 0.0239*(X6-8).^3-0.1965*(X6-8).^2+0.5472*(X6-8)+6.6000;
>> plot(X6,Y6,':')
>> hold on
>> X7 = linspace(10,13,50);
>> Y7 = -0.0026*(X7-10).^3-0.0529*(X7-10).^2+0.0485*(X7-10)+7.1000;
>> plot(X7,Y7,'-')
>> hold on
>> X8 = linspace(13,17,50);
>> Y8 = 0.0057*(X8-13).^3-0.0759*(X8-13).^2-0.3381*(X8-13)+6.7000;
>> plot(X8,Y8,':')
>>
```