UNIVERSITY OF VICTORIA
MIDTERM EXAM OCTOBER 18 2017
COMPUTER SCIENCE 349A

NAME: _____ STUDENT NO. _____

INSTRUCTOR: Rich Little

. DURATION: 50 minutes

TO BE ANSWERED ON THE PAPER

STUDENTS MUST COUNT THE NUMBER OF PAGES IN THIS EXAMINATION PAPER BEFORE BEGINNING TO WRITE, AND REPORT ANY DISCREPANCY IMMEDIATELY TO ME.

PLEASE PUT YOUR NAME ON THE VERY BACK SHEET AS WELL.

THIS QUESTION PAPER HAS 4, SINGLE-SIDED PAGES. YOU MAY USE THE BACK PAGES.

NOTES: (0) CLOSED BOOK EXAM; ONLY BASIC CALCULATORS ARE ALLOWED, (1) ANSWER ALL QUESTIONS, (2) THERE ARE A TOTAL OF 30 MARKS, (3) THE BACK PAGE OF EACH QUESTION MAY BE USED FOR YOUR ANSWERS. (4) STUDENTS ARE ALLOWED ONE 8.5-by-11 INCH SHEET CONTAINING ANY INFORMATION — BOTH SIDES CAN BE USED.

Question	Possible marks	Actual marks
1	10	
2	10	
3	10	
Total	30	

1. (a) [6 points] Consider a base 5 normalized, floating-point number system. Assume that a hypothetical computer using this susytem has the following floating-point representation:

s_m	d_1	d_2	d_3	d_4	s_e	e_1	e_2

where s_m is the sign of the mantissa, s_e is the sign of the exponent (1 for negative, 0 for positive), d_i are the digits of the mantissa, and e_j are the digits of the exponent.

   i. Consider the base 5 number, given using the above representation, 02003004. What exact decicaml value does it represent?

   ii. What decimal value does 11004003 represent?

   iii. What is the smallest positive, non-zero, number that can be represented in this system? Give the answer in the above form (i.e. as 8 base-5 digits.)

   (b) [4 points] Determine the second order ($n = 2$) Taylor approximation for $f(x) = \ln(x - 1)$, expanded about $a = 2$, including the remainder term. Do not simplify the form of this polynomial; that is, do not multiply out any powers.

2. (a) [4 points] Consider the following polynomial: $f(x) = x^5 + 5x^4 - 40x^2 - 80x - 48$. Use Horner's algorithm to compute $f(-1)$ and $f'(-1)$.

   (b) [2 points] If $x_0 = -1$ is an initial approximation to a root of this polynomial, use Newton's method to determine the next approximation x_1 to the root.

   (c) [1 points] Suppose x_1, computed above, is the final approximation to a true root of $f(x)$. What is the approximate deflated polynomial $Q(x)$?

   (d) [3 points] If instead we let Newton continue until it converges to the actual root $x_t = -2$, what is the order of convergence? Justify your answer.

3. (a) [3 points] Use 4 decimal digit, idealized, chopping floating-point arithmetic, to show that $fl(g(1.011))$ gives the value 0.2727, where

$$g(x) = \frac{x^{1/3} - 1}{x - 1}.$$

(b) [3 points] The quadratic Taylor polynomial approximation for $f(x) = x^{1/3}$, expanded about $a = 1$, is

$$f(x) \approx 1 + \frac{x-1}{3} - \frac{(x-1)^2}{9}.$$

Use this to get an accurate linear approximation to $g(x)$ in part (a).

(c) [4 points]Use the approximation in (b) to show that the computation $fl(g(1.011))$ in (a) is **unstable**. Use the notation and definition of stability given in class.