COMPUTER SCIENCE 349A, SPRING 2018
ASSIGNMENT #2 - 20 MARKS - SOLUTIONS

Question #1 - 6 marks.

(a) First, I will calculate $fl(g(x,y))$.

$$\begin{aligned}
fl(x^2) &= fl(152.2756) = 152.2 \text{ or } 0.1522 \times 10^3. \\
fl(y^2) &= fl(0.91298025) = 0.9129 \\
fl(x^2+y^2) &= fl(152.2 + 0.9129) = fl(153.1129) = 153.1 \\
fl(\sqrt{x^2+y^2}) &= fl(\sqrt{153.1}) = fl(12.37335...) = 12.37 \\
fl(\sqrt{x^2+y^2} - x) &= fl(12.37 + 12.34) = 24.71 \\
fl(g(x,y)) &= fl((\sqrt{x^2+y^2} - x)/y) = fl(24.71/0.9555) = fl(25.86080586) = 25.86
\end{aligned}$$

The relative error associated with this calculation is

$$|\varepsilon_t| = \frac{|25.868066 - 25.86|}{|25.868066|} = 0.000311813 \approx 0.03\%$$

(b)

data	$f(x,y)$ more accurate	$g(x,y)$ more accurate	both accurate
$x = 0.1234, y = 12.34$			X
$x = 12.34, y = -0.9123$	X		
$x = -0.1234, y = -0.005678$		X	

Question #2 - 6 Marks.

(a)

$$\begin{aligned}
f(x) &= \sqrt{x-1} - \sqrt{x} \\
&= \sqrt{x-1} - \sqrt{x}\left(\frac{\sqrt{x-1}+\sqrt{x}}{\sqrt{x-1}+\sqrt{x}}\right) \\
&= \frac{-1}{\sqrt{x-1}+\sqrt{x}}
\end{aligned}$$

Now, when x is large and positive we are no lnger subtracting two values that are close in magnitude.

(b)

$$\begin{aligned} g(x) &= \ln x - \ln y \\ &= \ln \frac{x}{y} \end{aligned}$$

Here, when x and y are near to equal there is no subtraction.

(c)

$$\begin{aligned} h(x) &= \frac{x}{x+1} - 1 \\ &= \frac{x}{x+1} - 1 - \frac{x+1}{x+1} \\ &= \frac{x-(x+1)}{x+1} \\ &= \frac{-1}{x+1} \end{aligned}$$

Now, when x is large we are no longer subtracting 1 from a number close to 1.

Note: They should support there conclusions. At the very least they should say that none function includes the subtraction of close values anymore.

Question #3 - 8 Marks

(a) $3_{(10)} + (1/9)_{(10)} = 1 \times 3^1 + 1 \times 3^{-2} = 10.01_{(3)}$. In normalized form $10.01_{(3)} = 0.1001_{(3)} \times 3^{2_{(10)}} = 0.1001_{(3)} \times 3^{02_{(3)}}$. So our computer representation is

00100102

(b) $-29_{(10)} = -(1 \times 3^3 + 2 \times 3^0) = -1002_{(3)}$. Now, $-1002_{(3)} = -0.1002_{(3)} \times 3^{4_{(10)}} = -0.1002_{(3)} \times 3^{11_{(3)}}$. So our computer representation is

10100211

(c) The smallest positive non-zero number that can be represented in this system has to be positive but the exponent should be negative to make it as small as possible. So the first two trits are 0 and 1. The smallest number in terms of the mantissa is .1000 (notice that .0001 is smaller but is not normalized). Finally, as the exponent is negative the larger it is, the smaller the number so the two digts are 2 and 2. Thus,

01100022

In ternary, this value is $0.1000_{(3)} \times 3^{-22_{(3)}} = 0.1000_{(3)} \times 3^{-8_{(10)}}$. Which, in decimal, is $1 \times 3^{-1} \times 3^{-8} = 0.000050805_{(10)}$

(d) The spacing between floating point numbers in a system of base b is given by b^{t-k} where $b = 3$, $t = 3$ (since $27_{(10)} = 100_{(3)}$ and $9_{(10)} = 10_{(3)}$). Therefore the spacing will be:

$$3^{3-4} = \frac{1}{3}$$

You can also observe this:

$$\begin{aligned} 100.0_{(3)} &= 9, \\ 100.1_{(3)} &= 9_{(10)} + \frac{1}{3}_{(10)} \\ 100.2_{(3)} &= 9_{(10)} + \frac{2}{3}_{(10)} \\ 101.0_{(3)} &= 10_{(10)} \end{aligned}$$