# CSC349A Numerical Analysis

## Lecture 1

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# Table of Contents I

# Course Information

- Course outline: `https://heat.csc.uvic.ca/coview/course/2018011/CSC349A`
- **Optional** Textbook: Numerical Methods for Engineers (6th or 7th edition), S.C. Chapra and R.P. Canale,McGraw-Hill, ISBN: 978-0-07-340106-5
- 5-10 assignments worth 20% of final grade
- Midterm 20% and Final Exam 60%.
- Handouts and associated coursepack
- Lecture notes, and slides
- **USE CONNEX:** `https://connex.csc.uvic.ca/portal`

# Attendance

- Not compulsory - you are adults
- DO NOT email me when you will miss a lecture
- Audio/video recording is fine - just do not make public without my consent
- Informal observed pattern of attendance
- Possible to pass the course without attending any lectures but much easier if you do attend
- Lecture notes

# Large scale challenges

- 300+ students
- **USE CONNEX**
- Be precise - always CSC349A in subject line, mention assignment etc
- Less flexibility and feedback on assignments

# Assignments

- **SINGLE PDF** files
- Answers in the same order as questions
- Name, student id, and assignment number on top of every page
- Any recycling of answers from previous years automatically zero grade in assignment
- **NO LATE SUBMISSIONS** unless exceptional circumstances.
- **USE CONNEX**

# Table of Contents I

# Definitions

Common view of numerical analysis is somewhat distorted and not particularly inviting [1].

**Common definition**

Numerical analysis is the study of rounding errors.

**More correct definition**

Numerical analysis is the study of **algorithms** for the problems of **continuous** mathematics.

[1] "The definition of numerical analysis", SIAM News, L. Trefethen

# Rounding Errors

Real and complex numbers can not be represented exactly on computers therefore they are always **approximated**. This approximation introduces errors which, even though typically small, can have a significant effect in the accuracy of numerical computations especially when these computations involve lots of arithmetic operations.

Numerical analysis investigates the **stability** and **accuracy** of algorithms on these rounding errors, so to some extent the common definition is correct.

# Truncation Errors

Most continuous mathematics algorithms **cannot be solved by finite algorithms**. This is true even if we could somehow work with exact arithmetic. This is true for many problems of interest including zero finding, differential equations, and optimization.

Rapid **convergence of approximations** is the primary challenge that has been met successfully by numerical analysis.

# New Definition

A refined definition of numerical analysis:

**Numerical analysis**

**Numerical analysis** is concerned with accurate, efficient approximations of solutions to problems of continuous mathematics.

In some ways they have been the victims of their own success as everyone built on top of their work.

# Continuous mathematics applications

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- Physics engines in computer games - Angry birds
- Simulating wind turbulance - wing design
- Filter design - audio effects
- Non-linear least squares - 3D models from photographs
- Optimization - a lot of data mining, machine learning
- Matrix computations - 3D graphics for games
- Differential equations - simulation in ECE, MEC
- Study of errors - major disasters due to numerical errors

# Historical Origins of Numerical Analysis

- Computers were people before they were machines
- World War II was the catalyst for the development of computers
- Code breaking was the original discrete mathematics killer (literally) app
- Computation of ballistic tables and atomic bomb simulations were the original continuous mathematics killer (literally) apps

# Importance in science and engineering

- Powerful problem solving tools that can handle large amounts of data, non-linearities and complex geometries that **cannot** be solved by any other means
- Under the hood knowledge of packaged software
- Not all real-world problems can be solved with packaged software

# Book overview

- 8 parts and 32 chapters
- Each part begins with a Preface motivating each problem and some background mathematics
- Part 1: Modelling, Computers and Error Analysis
- Part 2: Roots of equations. Solve $f(x) = 0$ for $x$.
- Part 3: Linear algebraic equations:. Given the $a$'s and the $c$'s solve for the $x$'s.
- Part 5: Curve Fitting: Interpolation
- Part 6: Integration: $I = \int_a^b f(x)dx$ Find the area under the curve.
- Part 7: Ordinary differential equations: Given $\frac{dy}{dx} = f(x, y)$ solve for $y$ as a function of $x$.

# Table of Contents I

# A motivating example

Determine the terminal veloctiy of a free-falling body (a parachutist) near the earth's surface by using Newton's second law of motion and solving a differential equation by, (a) analytical methods, then (b) numerical methods, and (c) compare.

# Mathematical Models

These equations are typical of mathematical models of the physical world:

- Describes a natural process in mathematical terms
- It represents an idealization and simplification of reality
- It yields reproducible results and can be used for predictive purposes

# Analytical solution

It is possible to get an analytic solution to this differential equation using calculus. If the object is initially at rest i.e $v = 0$ at time $t = 0$ then:

$$v(t) = \frac{gm}{c}(1 - e^{-\frac{ct}{m}}) \tag{1}$$

This analytical solution can be used directly to solve problems. [2]

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[2] For those of you who are interested it is straightforward to show that the analytical solution satisfies the differential equation by substituting in the left hand side and differentiating and substituting in the right hand side. After a few straightforward algebraic manipulations you can show the two sides are equal. To derive the analytical solution you need to express the derivative and then take the limit of the resulting series.

# Analytical solution - Example 1.1

For example a parachutist of mass $68.1kg$ jumps out of a stationary hot air balloon. The drag coefficient is equal to $12.5kg/sec$.
Inserting the parameters into the equation we get:

| t | v |
|---|---|
| 0 | 0.0 |
| 2 | 16.40 |
| 4 | 27.77 |
| 6 | 35.68 |
| .. | .. |
| $\infty$ | 53.44 |

Thus the *terminal velocity* is $53.44m/sec$ for this example.

# Numerical solution

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Numerical approach (in constrast to analytical) for solving this mathematical model. Not exact but we can get arbitrarily close. Why numerical methods (NM) ?

- NM can be applied to functions for which we can not easily find an analytical solution through Calculus
- As any equation is to some degree an approximation of reality and therefore errors are inevitable it is possible depending on the application that the errors introduced by NM are neglible in the context of the desired application
- It's extremely simple to write a computer program to do the job for us although the number of mathematical operations involved is much larger than the analytical solution

# Numerical Solution

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- The idea is rather simple. We will approximate the derivative of the function by a "finite divided difference" effectively discretizing time.
- If we denote the discrete time steps we take as $t_i$ (we can decide what sampling rate is appropriate depending on the application) then we can approximate the derivative at time $t_i$ as follows:

$$\frac{dv}{dt} \approx \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i}$$

# Numerical iteration

- Notice that this equation gives us a way to compute the velocity value at time $t_{i+1}$ based on the previous value of the velocity.
- This approach is called Euler's method can be verbally expressed as: *New value* $=$ *old value* $+$ *slope x step size*.

# Numerical solution - Example 1.2

- For example we can use it to compute the velocity values with a step size of 2 seconds.
- At the start of the computation we use an initial velocity value $v_0 = 0$ for $t_0 = 0$ plug the numbers into the equation and get the value of $v_1$. Then we can use the value at $v_1$ to compute the value at $v_2$ and so forth.

| **t** | **v** |
|---|---|
| 0 | 0.0 |
| 2 | 19.62 |
| 4 | 32.04 |
| 6 | 39.90 |
| .. | .. |
| $\infty$ | 53.44 |

# Comparison

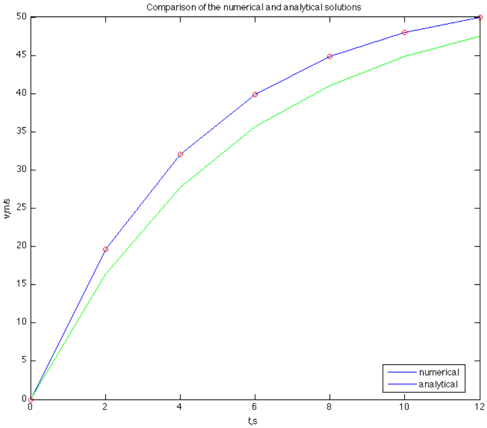


Figure: Comparison of numerical and analytical solution

# Comparison of numeric and analytic solution

Figure 1 shows a plot comparing the results of the numerical approximation using Euler's method and the exact analytical solution. In engineering there is frequently a tradeoff between accuracy and computational resources.

# Euler Method

The Euler method and more generally numerical methods allow you to control that tradeoff for example by adjusting the step size appropriately for the program at hand. There are several questions one could ask at this point that we will be exploring in the rest of the course. For example can we show that the error between the exact solution and the numerical approximate solution will always decrease with smaller step size? Can we compare how long different approaches to approximating the derivative of a function take? Are there any functions for which this would not work? Could we reduce the number of multiplication/additions needed to perform each step of the iteration?