DIKSHA BANSAL
V00768613

1

a) $f(x) = \ln(x+2)$, $f(-1) = \ln 1 = 0$

$f'(x) = \frac{1}{x+2}$, $f''(x) = \frac{-1}{(x+2)^2}$, $f'''(x) = \frac{2}{(x+2)^3}$

$\Rightarrow f'(-1) = \frac{1}{1} = 1$, $f''(-1) = \frac{-1}{1^2} = -1$, $f'''(-1) = 2$

$$\ln(x+2) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$$

$$= 0 + 1\cdot(x+1) + \frac{(-1)}{2!}\cdot(x+1)^2 + \frac{2}{3!}(x+1)^3$$

$$\ln(x+2) = (x+1) - \frac{(x+1)^2}{2} + \frac{(x+1)^3}{3} = P_3(x)$$

b) $$f(-1.06) = (-0.06) - \frac{(-0.06)^2}{2} + \frac{(-0.06)^3}{3}$$

$$= (-0.06) - 0.0018 - 0.000072$$

$$= -0.0618072$$

c) Part c after (d)

d) $$f^{(4)}(x) = \frac{-6}{(x+2)^4}$$

For each value of x $\exists\ \xi$ st

$$\ln(x+2) - P_3(x) = R_n = \frac{f^{(4)}(\xi)}{4!}(x+1)^4$$

$$= \frac{-\cancel{6}}{(\xi+2)^4 4\times\cancel{3!}}(x+1)^4 = \frac{-(x+1)^4}{4(\xi+2)^4}, \quad \xi \in [-1, x]$$

DIKSHA BANSAL
V00768613

$$\therefore R_3 = -\frac{(-1.06)^4}{4(\xi+2)^4} = \frac{-0.31561924}{(\xi+2)^4}, \quad \xi \in [-1.06, -1]$$

upper bound of error:

$$|R_3| = \left|\frac{-0.31561924}{(\xi+2)^4}\right| \leq \frac{0.31561924}{(-1.06+2)^4}$$

$$= 0.404251886$$

c) absolute error,

$$|E_t| = |(-0.0618754) - (-0.0618072)|$$

$$= 0.0000682$$

~~2. a~~

2.

$$g(x) = \begin{cases} \dfrac{(1+\cos x)}{(x-\pi)^2}, & x \neq \pi \\ 0.5, & x = \pi \end{cases}$$

DIKSHA BANSAL V00768613

a)

$$\cos(x) = (-1) + \frac{(x-\pi)^2}{2} - \frac{(x-\pi)^4}{24}$$

$$1+\cos x = \frac{(x-\pi)^2}{2} - \frac{(x-\pi)^4}{24}$$

$$\frac{1+\cos x}{(x-\pi)^2} = \frac{1}{2} - \frac{(x-\pi)^2}{24}$$

$$\therefore \quad g(x) = \begin{cases} \frac{1}{2} - \frac{(x-\pi)^2}{24}, & x \neq \pi \\ 0.5, & x = \pi \end{cases}$$

b) Given problem

$$g(x) = \begin{cases} \dfrac{1+\cos x}{(x-\pi)^2} & x \neq \pi \\ 0.5 & x = \pi \end{cases} \Rightarrow \text{computed sol}^n. \quad r = 0.5908$$

$x = 3.16$

perturbed problem,

$$g(3.16+\varepsilon) = \frac{1+\cos(3.16+\varepsilon)}{(3.16+\varepsilon-\pi)^2} \Rightarrow$$

exact value of $g(3.16+\varepsilon) = \dfrac{1+\cos(3.16+\varepsilon)}{(3.16+\varepsilon-\pi)^2}$ is very close to $\frac{1}{2} - \frac{1}{24}(3.16+\varepsilon-\pi)^2$ using the Taylor approx.

DIKSHA BANSAL
V00768613

If there is any value of ε s.t $\left|\frac{\varepsilon}{3.16}\right|$ is small and $g(3.16+\varepsilon) \approx 0.5908$, then the computation of $fl(g(3.16))$ is stable, else it is unstable

However

$$\frac{1}{2} - \frac{1}{24}(3.16+\varepsilon-\pi)^2 = 0.499985882 - \frac{\varepsilon^2}{24} - 0.001533945\varepsilon$$

The above expression is equal to approximately 0.4999 for all values of ε s.t $\left|\frac{\varepsilon}{3.16}\right|$ is small.

As, 0.4999 is not close to 0.5908, the computation is unstable.

c) $$g(x) = \begin{cases} \frac{1+\cos x}{(x-\pi)^2}, & x \neq \pi \\ 0.5, & x = \pi \end{cases}$$

computed sol^n, $r = 0.3871$

perturbed problem,

$$g(1.41+\varepsilon) = \frac{1+\cos(1.41+\varepsilon)}{(1.41+\varepsilon-\pi)^2},$$

exact value of $g(1.41+\varepsilon) = \frac{1+\cos(1.41+\varepsilon)}{(1.41+\varepsilon-\pi)^2}$ is very close to

$$\frac{1}{2} - \frac{1}{24}(1.41+\varepsilon-\pi)^2$$

using the above Taylor approximation

If there is any value of ε s.t. $\left|\frac{\varepsilon}{1.41}\right|$ is small and $g(1.41+\varepsilon) \approx 0.3871$,

DIKSHA BANSAL V00768613

then the computation of $fl(g(1.41))$ is stable. Else, it is unstable.

However,

$$\frac{1}{2}-\frac{1}{24}(1.41+\varepsilon-\pi)^2 = 0.37506612 + 0.144299387\varepsilon - \frac{\varepsilon^2}{24}$$
$$= h(\varepsilon)$$

If $\left|\frac{\varepsilon}{1.41}\right|$ is small, then $h(\varepsilon)$ is very close to 0.3894969 which is very close to 0.3871 for all small values of ε.

3.

a)
```
function root = Bisect(xl, xu, eps, imax, f)

i = 1;
f = f(xl);
fprintf('iteration       approximation  \n')
while(i <= imax)

    xr = (xl+xu)/2;
    fprintf('%6.0f       %18.8f \n', i, xr)
    fr = f(xr);
    if(fr == 0 || ((xu-xl)/(xu+xl)) < eps)
        root = xr;
        exit;
    end

    i = i+1;
    if(fl*fr < 0)
        xu = xr;
    else
        xl = xr;
        fl = fr;
    end
end
fprintf('failed to converge in %g iterations \n ', imax);
```

b)

```
function [vol] = height(h)
    vol = (pi*h*h*(12.3-h))/3 - 45;
end
```

root = Bisect(0, 4.1, 1e-4, 20, 'height')

c)
```
function [vel] = fall(m)
    vel = (9.81*m/13.5)*(1-e ^(-135/m);
end
```

root = bitset(1,100,10^(-4),20,'fall')