COMPUTER SCIENCE 349A

Handout Number 19

Section 18.2 POLYNOMIAL INTERPOLATION

Let $y = f(x)$ be any given function. For any value of $n \geq 0$ and any given values $x_0, x_1, \ldots, x_n$, let $y_i = f(x_i)$. The polynomial interpolation problem is to determine a polynomial $P(x)$ of degree less than or equal to n for which

$$P(x_i) = y_i \quad \text{for } i = 0, 1, \ldots, n.$$

The set of $n+1$ data points (x_i, y_i) may be the only functional values known (that is, $f(x)$ is a discrete function, which could occur for example with experimental data) or $f(x)$ may be a known continuous function and the $n+1$ data points (x_i, y_i) are a finite sample of values with $y_i = f(x_i)$.

If z is some value between 2 of the given values x_i and if $P(z)$ is computed as an approximation to $f(z)$, then this approximation to $f(z)$ is said to be determined by polynomial interpolation.

On the other hand, if z lies outside of the interval containing all of the values x_i and if $P(z)$ is computed as an approximation to $f(z)$, then this approximation to $f(z)$ is said to be determined by polynomial extrapolation.

Note that an interpolating polynomial and the Taylor polynomial both determine polynomial approximations to $f(x)$. However, in general they are very different approximations to $f(x)$. Note that an interpolating polynomial uses the information

$$y_0 = f(x_0),\ y_1 = f(x_1), \ldots,\ y_n = f(x_n)$$

to determine the polynomial approximation, whereas the Taylor polynomial uses the information

$$f(x_0),\ f'(x_0), \ldots,\ f^{(n)}(x_0)$$

to determine the polynomial approximation.