COMPUTER SCIENCE 349A

Handout Number 21

Section 18.3

The coefficients of the interpolating polynomial can be determined by solving a system of linear equations. Given a function $f(x)$ and distinct points $x_0, x_1, \ldots, x_n$, let $P(x)$ be the unique polynomial of degree $\le n$ for which $P(x_i) = f(x_i)$, $0 \le i \le n$.

Example. Consider the case $n = 2$. Let $P(x) = a_0 + a_1 x + a_2 x^2$.

Then $P(x_i) = f(x_i)$ for $i = 0, 1, 2$ implies that

$$a_0 + a_1 x_0 + a_2 x_0^2 = f(x_0)$$
$$a_0 + a_1 x_1 + a_2 x_1^2 = f(x_1)$$
$$a_0 + a_1 x_2 + a_2 x_2^2 = f(x_2)$$

or in matrix/vector notation,

$$\begin{bmatrix} 1 & x_0 & x_0^2 \\ 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} f(x_0) \\ f(x_1) \\ f(x_2) \end{bmatrix}.$$

This is a system of 3 linear equations in the 3 unknowns $\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$.

For example, if $f(x) = \sin x$ and $x_0 = 0.2$, $x_1 = 0.5$, $x_2 = 1$ (in radians), then the coefficients a_0, a_1, a_2 of the interpolating polynomial are determined by solving the following linear system:

$$\begin{bmatrix} 1 & 0.2 & 0.04 \\ 1 & 0.5 & 0.25 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0.1987 \\ 0.4794 \\ 0.8415 \end{bmatrix}$$

Relative to other methods, the disadvantages of this approach are

(i) such linear systems may be ill-conditioned

and

(ii) it's difficult and expensive to change the degree of the interpolating polynomial.