COMPUTER SCIENCE 349A, SPRING 2018
ASSIGNMENT #6 - 20 MARKS

DUE THURSDAY MARCH 29, 2018 (11:30 p.m. PST)

This is a really large class and the logistics of grading assignments are challenging. Me and the markers require your help in making this process go smoothly. Please ensure that your assignments conform to the following requirements - any violation will result in getting a zero for the particular assignment.

- All assignments should be submitted electronically through the ConneX course website and shoud be **SINGLE PDF FILES**. No other formats will be accepted. Handwritten answers are ok but they will need to be scanned and merged into a single pdf file together with any code examples and associated plots.
- The assignment number, student name and student number should be clearly visible on the top of every page of your assignment submission.
- **PLEASE DO NOT COPY THE ASSIGNMENT DESCRIPTION IN YOUR SUBMISSION**
- The asnwers to the questions should be in the same order as in the assignment specification.
- Some of the questions of the assignments are recycled from previous years but typically with small changes in either the description or the numbers. Any submission that contains numbers from previous years in any questions will be immediately graded with zero.
- Any assignment related email questions should have a subject line of the form CSC349A Assignment X, where X is the number of the corresponding assignment.
- The total number of points for this assignment is 20.

1. (a) **(2 points)** Determine the Lagrange form of the interpolating polynomial $P(x)$ that interpolates a function $f(x)$ at $x = 0$, h and $4h$, where $h > 0$. (Multiply the linear factors together, but leave $P(x)$ as a sum of 3 quadratics in the variable x.)

   DELIVERABLES: All your work in constructing the polynomial. This is to be done by hand not MATLAB.

   (b) **(4 points)** Derive the quadrature formula of the form

$$a_0 f(0) + a_1 f(h) + a_2 f(4h)$$

   for approximating $I = \int_0^{4h} f(x)dx$ that results from approximating the integral I by $I \approx \int_0^{4h} P(x)dx$.

   Note: if you know only 3 function values of $f(x)$ and they are at 3 unequally-spaced points 0, h and $4h$ for any value of h, then this kind of quadrature formula can be used to approximate I.

   DELIVERABLES: All your work in deriving the quadrature formula.

   (c) **(2 points)** Suppose that you know only the following function values of $f(x)$:

x	$f(x)$
0.0	1.00000
0.1	1.11091
0.4	1.63778

   Use the quadrature formula from (b) to approximate $\int_0^{0.4} f(x)dx$.

   Note: the above data corresponds to the function $f(x) = \frac{1}{1-\sin x}$, and the exact value is $\int_0^{0.4} f(x)dx = 0.508498$. Use this information only to assess the accuracy of your computed approximation. If you do not obtain a fairly good approximation in (c), then your answer in (b) is incorrect. That is, the relative error between your answer and the true answer should be less than 1%.

   DELIVERABLES: All your work to calculate the quadrature.

2. **(6 points)** Determine constants a, b, c, and d that will produce a quadrature formula

$$\int_{-1}^{1} f(x)dx \approx af(-1) + bf(1) + cf'(-1) + df'(1)$$

   that has degree of precision $d = 3$.

   DELIVERABLES: Show all your work. This could be a combination of hand-written work and MATLAB if you use MATLAB to solve your system of equations.

3. (a) **(4 points)** Use Romberg integration to approximate

$$\int_0^{\pi/4} e^{3x} \sin 2x dx.$$

   Note that the argument for sine is in radians. Compute the first $O(h^6)$ Romberg approximation to the integral. That is, compute the Romberg table with 6 entries (3 rows and 3 columns). Do this by hand computation using a calculator.

DELIVERABLES: Show how you have computed each value in the table. Use at least 6 significant digits throughout the computations.

(b) **(2 points)** The exact value of this integral is 2.5886286 (to 8 significant digits). Show that this approxiamtion is quite accurate, even though $h = \pi/16$ is not very small, by finding the true relative error ε_t. Also, find the approxiamte relative error ε_a and compare it.

DELIVERABLES: Show all your work.