COMPUTER SCIENCE 349A

Handout Number 20

EXAMPLES: LAGRANGE INTERPOLATING POLYNOMIAL

1. The case $n = 1$ is called linear interpolation, and consists of constructing a straight line through 2 points $(x_0, y_0) = (x_0, f(x_0))$ and $(x_1, y_1) = (x_1, f(x_1))$.

$$L_0(x) = \frac{x - x_1}{x_0 - x_1} \text{ and } L_1(x) = \frac{x - x_0}{x_1 - x_0}$$

and thus

$$P(x) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1) \ .$$

Note that $P(x)$ is clearly a linear (first order) polynomial in the variable *x*, and that

$$P(x_0) = f(x_0) \quad \text{and} \quad P(x_1) = f(x_1),$$

which verifies that $P(x)$ is the desired interpolating polynomial.

2. A complete elliptic integral is defined by

$$K(k) = \int_0^{\pi/2} \frac{dz}{\sqrt{1 - k^2 \sin^2 z}} \ .$$

The following values can be obtained from numerical tables:

$\sin^{-1} k$	$K(k)$
65°	2.3088
66°	2.3439
67°	2.3809

Construct the quadratic (order $n = 2$) interpolating polynomial for this data, and use it to estimate the value of $K(k)$ when $\sin^{-1} k = 65.5^\circ$.

SOLUTION: with $x_0 = 65$, $x_1 = 66$ and $x_2 = 67$, we obtain

$$P(x) = \frac{(x-66)(x-67)}{(65-66)(65-67)}(2.3088) + \frac{(x-65)(x-67)}{(66-65)(66-67)}(2.3439)$$
$$+ \frac{(x-65)(x-66)}{(67-65)(67-66)}(2.3809)$$

and evaluating this at $x = 65.5$ gives the following approximation to $K(k)$ when $\sin^{-1}(k) = 65.5$:

$$P(65.5) = 2.3261 \ .$$