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20U433 division-4

Assignment 1

Batch - S16

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1) $(D^2-4D+3)y = x^3 \cdot e^{2x}$

$\rightarrow$ $D^2-4D+3=0$ (Auxilary equation)

$(D-1)(D-3)=0$

$D = 1, 3$

C.F is $K = c_1 e^x + c_2 e^{3x}$

for particular integral

$$y_p = \frac{1}{D^2-4D+3}(x^3 e^{2x})$$

replace $D \rightarrow D+2$

$$y_p = e^{2x}\frac{1}{(D+2)^2-4(D+2)+3}x^3$$

$$y_p = \frac{e^{2x}}{D^2+\cancel{4D}+4-\cancel{4D}-8+3}x^3$$

$$y_p = \frac{e^{2x}}{D^2-1}x^3$$

$$y_p = e^{2x}(-1)(1-D^2)^{-1}x^3$$

$$y_p = e^{2x}(-1)(1-D^2)^{-1}x^3$$

$$y_p = -e^{2x}(1+D^2+D^4+D^4\ldots)x^3$$

$$y_p = -e^{2x}(x^3+6x+0)$$

$$y_p = -e^{2x}(x^3+6x+0)$$

$$y_p = -e^{2x}(x^3+6x)$$

$$y = y_c + y_p$$

$$\boxed{y = 4e^x + c_2 e^{3x} - e^{2x}(x^3+6x)}$$

$$\frac{d^2y}{dx^2} - \frac{3dy}{dx} - 12y = x^2 + \sin x$$

$$(D^2 - 3D + 2)y = x^2 + \sin x$$

$$D^2 - 3D + 2 = 0 \quad (A \cdot E)$$

$$(D-1)(D-2) = 0$$

$$D = 1, 2$$

CF is ; $y_c = C_1 e^x + C_2 e^{2x}$

P I is $y_p = \frac{1}{D^2 - 3D + 2}(x^2 + \sin x)$

$$= \frac{x^2}{D^2 - 3D + 2} + \frac{\sin x}{D^2 - 3D + 2}$$

$$= \frac{1}{2\left(1 + \frac{D^2 - 3D}{2}\right)} + \frac{\sin x}{-1 - 3D + 2}$$

$$= \frac{1}{2}\left(1 - \left(\frac{D^2 - 3D}{2}\right) + \left(\frac{D^2 - 3D}{2}\right)^2 - \cdots\right)x^2$$

$$- + \frac{\sin x}{1 - 3D}$$

$$y_p = \frac{1}{2}\left(x^2 - \frac{D^2x^2 + 3Dx^2}{2} + \frac{D^4x^2 - 6D^3x^2 +}{4}\right.$$

$$\left. 3D^2x^2 + 0\right) + \frac{\sin x(1+3D)}{1 - 9D^2}$$

$$y_p = \frac{1}{2}\left(x^2 - \frac{2 + 6x}{2} + \frac{\cancel{18}\,9}{\cancel{4}\,2}\right) + \frac{\sin x\,(1+3D)}{1 - 9(-1)}$$

$$y_p = \frac{1}{2}\left(x^2 - \frac{2 + 6x + 9}{2}\right) + \frac{1}{10}(\sin x + 3D\sin x)$$

$$y_p = \frac{1}{2}\left(x^2 + 3x + \frac{7}{2}\right) + \frac{1}{10}(\sin x + 3\cos x)$$

$$y = C_1 e^x + C_2 e^{2x} + \frac{1}{2}\left(x^2 + 3x + \frac{7}{2}\right) + \frac{1}{10}(\sin x + 3\cos x)$$

3) $(D^2-2D+2)y = e^x + \sin x$ (By VOP)

$D^2-2D+2=0$

$D^2 - D(1-i) - D(1+i) + (i-1)(i+1) = 0$

$(D-(1-i)) - (1+i)(D-(1-i)) = 0$

$(D-(1-i)(D-(1-i)) = 0$

$D = (1-i), (1+i)$

So C.E is

$y_c = e^x(c_1 \cos x + c_2 \sin x)$

Comparing with $C.F = c_1 v_1 + c_2 v_2$

$y_1 = e^x \cos x$, $v_2 = e^x \sin x$

$y_1 = e^x \sin x + e^x \cos x$; $y_2 = e^x \cos x + e^x \sin x$

$$W = \begin{vmatrix} y_1 & y_2 \\ v_1' & y_2' \end{vmatrix}$$

$$W = \begin{vmatrix} e^x \cos x & e^x \sin x \\ -e^x \sin x + e^x \cos x & e^x \cos x + e^x \sin x \end{vmatrix}$$

$W = e^{2x}\cos^2 x + e^{2x}\cos x \cancel{\sin x} + e^{2x}\sin^2 x -$
$e^{2x} \sin x \cos x$.

$W = e^{2x}$

$U = \int -\frac{y_2 x}{W}\, dx$

$U = \int -\frac{\cancel{e^x} \sin x \cdot e^x \tan x}{\cancel{e^{2x}}}\, dx$

$U = -\int \frac{\sin^2 x}{\cos x}\, dx = -\int \frac{1-\cos^2 x}{\cos x}\, dx$

$= -\int \sec x - \cos x \, dx$

$U = -(\log(\sec + \tan x) + \sin x)$

$$v = \int \frac{y_1 x}{W} dx$$

$$v = \int \frac{e^x \cos x \, e^x \tan x}{e^{2x}} dx$$

$$v = \int \sin x \, dx$$

$$v = -\cos x$$

$$P.I = u v_1 + v v_2$$

$$y_p = e^x \cos x[-\log(\sec x + \tan x) + \sin x]$$

$$+ e^x \sin x(-\cos x)$$

$$y_p = e^x \cos x \log(\sec x \tan x) + e^x \cancel{\sin x} \cos x$$
$$- e^x \sin x \cos x$$

$$y_p = -e^x \cos x \log(\sec x + \tan x)$$

Complete solution

$$y = y_c + y_p$$

$$y = e^x(c_1 \cos x + c_2 \sin x) - e^x \cos x \log(\sec x + \tan x)$$

4) $x^3 \frac{d^2y}{dx^2} + 3x^2 \frac{dy}{dx} + xy = \cos(\log x)$

→ divide by x for standard form

$$x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = \frac{\cos(\log x)}{x}$$

by putting $x = e^z$, $z = \log x$

$$D(D-1)y + 3D + y = \frac{\cos z}{e^z}$$

$$(D^2 + 2D + 1)y = e^{-z} \cos z$$

$$yp = \frac{1}{D^2+2D+1} e^{-z} \cos z$$

$D \to D-1$

$$yp = \frac{1}{(D-1)^2+2(D-1)+1} e^{-z} \cos z$$

$$yp = e^{-z} \frac{1}{D^2} \cos z$$

$$yp = e^{-z} \iint \cos z \, dz$$

$$yp = -e^{-z} \cos z$$

$$yp = -e^{-z} \cos(\log x)$$

$yc \quad D^2+2D+1=0 \quad (A \cdot E)$

$D = -1, -1$

$$yc = (c_1 z + c_2) e^{-z}$$

$$yc = (c_1 \log x + c_2) e^{-\log x}$$

Complete Sol^n

$$y = yc + yp$$

$$y = (c_1 \log x + c_2) + ^{-\log x} - e^{-z} \cos \log x$$

5) $[3x+2]^2 D^2 + 3(3x+2)D - 36] y = 3x^2+4x+1$

$\to$ $3x+2 = e^z$; $z = \log(3x+2)$

$$(3x+2)\frac{dy}{dx} = 3Dy$$

$$(3x+2)^2 \frac{d^2y}{dx^2} = 9D(D-1)y$$

Eq^n becomes

$$9D(D-1)y + 3 \cdot 3Dy - 36y = 3\left(\frac{e^z-2}{3}\right)^2 + 4\left(\frac{e^z-3}{3}\right)+1$$

$$9[D^2 - \cancel{D} + \cancel{D} - 4]y = 3\left(\frac{e^{2z}-4e^z+4}{\cancel{9}\,3}\right) + \frac{4e^z-8+1}{3}$$

$$9\,[D^2-4]y = \frac{e^{2z} - 4e^{z} + 4 + 4e^{z} - 8 + 3}{3}$$

$$9\,[D^2-4]y = \frac{e^{2z}-1}{3}$$

$$(D^2-4)y = \frac{1}{27}(e^{2z}-1)$$

$$D^2-4=0 \quad (A \cdot E)$$

$$D^2-4=0 \quad (A \cdot E)$$

$$D \pm 2$$

$$y_c = c_1 e^{2z} + c_2 e^{-2z}$$

$$y_p = \frac{1}{(D^2-4)}(e^{2z}-1) \times \frac{1}{27}$$

$$= \frac{1}{27}\left(\frac{e^{2z}}{D^2-4} - \frac{e^{0z}}{D^2-4}\right)$$

$$= \frac{1}{27}\left(z\frac{e^{2z}}{2D} + \frac{1}{4}\right) = \frac{1}{27}\left(\frac{ze^{2z}+1}{4}\right)$$

$$y_p = \frac{1}{108}(2e^{2z}+1) = \frac{1}{108}(\log(3x+2)\cdot(3x+2)^2+$$

Complete sol^n $y = y_c + y_p$

$$y = c_1(3x+2)^2 + c_2(3x+2)^{-2} + \frac{1}{108}((3x+2)^2 \cdot \log(3x+2)+$$

6) $x\,\dfrac{dx}{y^2 z} = \dfrac{dy}{dz} = \dfrac{dz}{y^2}$

$\longrightarrow$ by Combinat^n

$$\frac{x\,dx}{y^2 z} = \frac{dy}{xz}$$

$$x^2\,dx = y^2\,dy$$

Integrating; $\int x^2\,dx = \int y^2\,dy$

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$$\frac{x^3}{3} = \frac{y^3}{3} + c_1$$

$$\boxed{x^3 - y^3 = c_1}$$

Again by Commbinations

$$\frac{x\,dx}{y^2 z} = \frac{dz}{y^2}$$

$$x\,dx = z\,dz$$

Integrating

$$\int x\,dx = \int z\,dz$$

$$\frac{x^2}{2} = \frac{z^2}{2} + c_2$$

$$\boxed{x^2 - z^2 = c_2}$$

equatⁿ ① & ② together Constitute the Solution of the System

4) $(D^4 - 1)y = \cos x \cosh x$

→ $D^4 - 1 = 0$ (A.E)

$(D^2+1)(D^2-1) = 0$

$D^2 = -1 \quad D^2 = 1$

$D = \pm i \quad D = \pm 1$

Two roots are real and two roots are Complex

$C.F = c_1 e^{x} + c_2 e^{x} + c_3 \cos x + c_4 \sin x$

$P.I = y_p = \frac{1}{D^4 - 1} \cos x \cosh x$

$$y_p = \frac{1}{D^4-1} \cos x \left(\frac{e^x + e^{-x}}{2}\right) \left(\cosh x = \frac{e^x + e^{-x}}{2}\right)$$

$$y_p = \frac{1}{2}\left(\frac{1}{D^4-1} e^{x}\cos x + \frac{1}{D^4-1} e^{-x}\cos x\right)$$

$D \to D+1$; $D \to D-1$

$$y_p = \frac{1}{2}\left(e^{x}\frac{1}{(D+1)^4-1}\cos x + e^{-x}\frac{1}{((D-1)^4-1)}\cos x\right)$$

$$y_p = \frac{1}{2}\left(e^{x}\frac{1}{D^4+4D^3+6D^2+4D+1-1}\cos x + e^{-x}\frac{1}{D^4-4D^3+6D^2-4D+1-1}\cos x\right)$$

Replace $D^2 = -1$

$$y_p = \frac{1}{2}\left(e^{x}\frac{1}{1-4\cancel{D}-6+4\cancel{D}}\cos x + e^{-x}\frac{1}{1+4D-6-4D}\cos x\right)$$

$$y_p = \frac{1}{2}\left(\frac{e^{x}\cos x}{-5} + \frac{e^{-x}\cos x}{-5}\right) = \frac{-1}{5}\cos x\left(\frac{e^{x}+e^{-x}}{2}\right)$$

$$y_p = -\frac{1}{5}\cos x\cosh x$$

Complete Solution $y = y_c + y_p$

$$\boxed{y = c_1 e^{x} + c_2 e^{-x} + c_3\cos x + c_4\sin x - \frac{1}{3}\cos x\cosh x}$$

8) $(D^2+3D+2)y = e^{e^{x}} + \cos e^{x}$

$D^2+3D+2 = 0$

$(D+2)(D+1) = 0$

$D = -2, -1$

C.F is $y_c = c_1 e^{-2x} + c_2 e^{-x}$

P.I $y_p = \frac{1}{D^2+3D+2}\left(e^{e^{x}} + \cos e^{x}\right)$

$$y_p = \frac{1}{(D+2)(D+1)}\left(e^{e^{x}} + \cos e^{x}\right)$$

$y_p = \frac{1}{D+2} e^{-x} \int e^{x} (e^{e^x} + \cos e^x)\, dx$

$e^x = t$

$e^x dx = dt$

$y_p = \frac{1}{D+2} e^{-x} \int (e^t + \cos t)\, dt$

$y_p = \frac{1}{D+2} e^{-x} (e^t + \sin e^x)$

$y_p = e^{-2x} \int e^{2x} e^{-x} (e^{e^x} + \sin e^x)\, dx$

$y_p = e^{-2x} \int e^{x} (e^{e^x} + \sin e^x)\, dx$

$y_p = e^{-2x} \int (e^t + \sin t)\, dt$

$y_p = e^{-2x} \int e^x (e^{e^x} + \sin e^x)\, dx$

$e^x = t$

$y_p = e^{-2x} (e^t + (-\cos t))$ $\quad e^x dx = dt$

$y_p = e^{-2x} (e^{e^x} - \cos e^x)$

Complete Solution is

$y = y_c + y_p$

$$\boxed{y = c_1 e^{-2x} + c_2 e^{x} + e^{-2x} (e^{e^x} - \cos e^x)}$$

9) $(D^2 - 6D + 9)y = \frac{e^{3x}}{x^2}$ by VOP

$D^2 - 6D + 9 = 0$ A.E

$(D-3)(D-3) = 0$; $D = 3, 3$

C.F ; $y_c = c_1 x e^{3x} + c_2 e^{3x}$

Comparing y_c with $y_c = c_1 y_1 + c_2 y_2$

$y_1 = x e^{3x}$, $y_2 = e^{3x}$

$y_1' = 3xe^{3x} + e^{3x}$; $y_2' = 3e^{3x}$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} x e^{3x} & e^{3x} \\ 3x e^{3x} + e^{3x} & 3e^{3x} \end{vmatrix}$$

$$= 3\cancel{x} e^{6x} - 3\cancel{x} e^{6x} - e^{6x}$$

$$\boxed{W = -e^{6x}}$$

$$U = \int \frac{-y_2 X}{W} dx$$

$$= \int \frac{e^{3x}}{-e^{6x}} \times \frac{e^{3x}}{x^2} dx$$

$$U = \frac{-1}{x}$$

$$V = \int \frac{y_1 X}{W} dx$$

$$= \int \frac{xe^{3x}}{-e^{6x}} \times \frac{e^{3x}}{x^2} dx = \int -\frac{1}{x} dx$$

$$\boxed{V = -\log x}$$

$P.I = y_p = Uy_1 + Vy_2$

$$y_p = -\frac{1}{x} \times e^{3x} + (-\log x) e^{3x}$$

$$y_p = -e^{3x}(1 + \log x)$$

$y = y_c + y_p$

$$\boxed{y = c_1 x e^{3x} + c_2 e^{3x} - e^{3x}(1 + \log x)}$$