UNIVERSITY OF WOLVERHAMPTON

FACULTY OF SCIENCE & ENGINEERING

SCHOOL OF MATHEMATICS & COMPUTER SCIENCE

Coursework (Replacing Examination)

Module Code	:	**4MM013**
Module Title	:	**Computational Mathematics**
Date	:	**Semester II 2020/2021**
Examiner(s)	:	**Dr. Nurdan Cabukoglu & Dr. Desmond Case**
Time Allowed	:	**2 weeks**

Answer All Questions

Q1. **(a)** Given the function $f(x) = 2x^3 + 4$ find
(i) $f(7)$, **(ii)** $f(-0.5)$.

(3 marks)

(b) Calculate $f(4x + 5)$ when $f(x) = x^2$. Write down the corresponding expression for $f(4x + 2) - f(x)$.

(4 marks)

(c) If $f(x) = 4x + 2$ and $g(x) = x + 3$ find $(gof)(x)$.

(3 marks)

(d) Consider the function given by $g(x) = x^2 + 5, -2 \leqslant x \leqslant 3$.
(i) State the domain of the function.
(ii) Plot the graph of the function.
(iii) Deduce the range of the function from the graph. (State the input and output table.)

(5 marks)

Q2. **(a)** Find the inverse of each of the following functions:
(i) $f(x) = 7 + 5x$, **(ii)** $f(x) = 3x$. **(5 marks)**.

Q3. **(a)** For the matrices $A = \begin{bmatrix} 1 & 5 & -1 \\ 3 & 0 & 4 \\ 1 & 1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 3 \\ 1 & -2 & 1 \\ 2 & 3 & -2 \end{bmatrix}$ find, where possible,
(i) A^T, **(ii)** AB.

(5 marks)

(b) Use expansion along the first row to find $A = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{vmatrix}$.

(5 marks)

Questions continue on next page

Q4. **(a)** Find the inverse of the following matrices using determinants if possible:

(i) $A = \begin{bmatrix} -1 & -1 \\ -2 & -2 \end{bmatrix}$,

(ii) $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

(5 marks)

(b) **(i)** Using Cramer's rule obtain the solution to the following set of equations:

$$\begin{aligned} 2x + y &= 8 \\ 3x - 4y &= 5. \end{aligned}$$

(5 marks)

Q5. **(a)** Solve the following system of equations using inverse matrix method ($AX = B$ and $X = A^{-1}B$):

$$\begin{aligned} 2x + 3y &= 3 \\ 5x + 4y &= 10. \end{aligned}$$

(5 marks)

(b) Diffie-Hellman key exchange is based on the difficulty of solving the Discrete Logarithm Problem. For instance, Alice and Bob agree on a prime number $p = 7$ and a number (base) $g = 4$. This is the public aspect of the system. Using this algorithm, find the $k_{private}$.

(5 marks)

Q6. **(a)** Solve the following using Gauss elimination method:

$$\begin{aligned} 2x_1 + x_2 - x_3 &= 0 \\ x_1 + x_3 &= 4 \\ x_1 + 2x_2 + x_3 &= 0 \end{aligned}$$

(5 marks)

(b) Find the inverse of the matrix from (a) using the same method and steps.

(5 marks)

Questions continue on next page

Q7. **(a)** **(i)** Write out all the terms of the series $\sum_{p=1}^{6} \frac{3^{p-1}}{p!(17-p)}$.

(ii) Write the simple formula for the n^{th} Fibonacci number for $n \geqslant 2$. Write the first 10 element of this sequence (including F_{10}.)

(5 marks)

(b) Is the following harmonic sequence convergent? Explain why.

$$a_n = \frac{n+2}{n(n-1)}.$$

(5 marks)

Python Questions

Question 1:

a) Write and test a function that counts and returns the number of bits (binary digits) set to 1 in a positive integer:

```
def countBits(n):
    'Returns the count of bits set to 1 in a positive integer'
    ...
```

For example, countBits(12) should return 2, because 12 = 0...011002.
Hints:
1. Check the digits from right to left.
2. n % 2 returns the rightmost (least significant) digit in the binary representation of n. (a % b is read “a modulo b” and means the remainder when a is divided by b.)
3. n //= 2 divides n by 2 and truncates the result to an integer, in effect shifting the binary digits to the right by one and getting rid of the least significant digit.

(8 marks).

Question 2:

a) Let $\vec{x} = (x_1, x_2, ..., x_n)$ and $\vec{y} = (y_1, y_2, ..., y_n)$ be two n-dimensional vectors. Their *dot product*, denoted $\vec{x} \cdot \vec{y}$, is defined as $\vec{x} \cdot \vec{y} = x_1 y_1 + x_2 y_2 + x_3 y_3 + ... + x_n y_n$. Write a Python function (call it *dotProduct*) that will return the dot product of two given vectors (assume the lists of the same length).

(8 marks).

Question 3:

a) Consider a function:

$$f(n) = \begin{cases} 1, \text{ if } n = 1 \\ 2 \cdot f(n-1) + 1, \text{ if } n > 1 \end{cases}$$

Encode the definition into a recursive Python function (for readability name your function my*func*). **(6 marks).**

b) Using the function given in part a) calculate the result of $f(5)$?

(2 marks).

Question 4:

a) Credit card numbers have 16 digits. The checksum is calculated as follows. Each digit in an odd position (first, third, etc.) is multiplied by 2. If the result is a two-digit number, its two digits are added together; otherwise it is left alone. The result is added to the sum. Each digit in an even position (second, fourth, etc.) is added to the sum as is. The resulting sum must be 0 modulo 10. For example, 4111111111111111 is a valid credit card number: its checksum is

30. 4111111111111178 is another valid number: its checksum is 40. Write and test a Python function that checks whether a given string of 16 digits represents a valid credit card number. Come up with a few other valid numbers and use a few invalid numbers for testing.

(8 marks).