

1) State the definition of a function and as composite function:

- ☐ A function is a set of rules which can be applied to a set of inputs to produce outputs.
- ☐ A composite function is function which takes another function as input.

$$f(x) = 2x - 1$$

$$f(2) = ?$$

$$f(x) = 2x - 1$$

$$f(2) = 2(2) - 1$$

$$\underline{\underline{f(2) = 3}}$$

$$f(3) = ?$$

$$f(x) = 2x - 1$$

$$f(3) = 2(3) - 1$$

$$f(3) = 6 - 1$$

$$\underline{\underline{f(3) = 5}}$$

$$g(x) = x^2 + 2$$

$$g(2y+1) = ?$$

$$g(2y+1) = (2y+1)^2 + 2$$

$$= (2y+1)(2y+1) + 2$$

$$= 4y^2 + 2y + 2y + 1 + 2$$

$$= 4y^2 + 4y + 3$$

$$\underline{\underline{g(2y+1) = 4y^2 + 4y + 3}}$$

$$f(x) = 2x - 1$$

$$f(g(x)) = ?$$

$$f(g(x)) = f(x^2 + 2)$$

$$f(x^2 + 2) = 2(x^2 + 2) - 1$$

$$f(x^2 + 2) = 2x^2 + 4 - 1$$

$$f(x^2 + 2) = 2x^2 + 3$$

$$\therefore \underline{\underline{f(g(x)) = 2x^2 + 3}}$$

$$g(x) = x^2 + 2$$

$$g(f(x)) = ?$$

$$g(f(x)) = g(2x - 1)$$

$$g(2x - 1) = (2x - 1)^2 + 2$$

$$= (2x - 1)(2x - 1) + 2$$

$$= 4x^2 - 4x + 1 + 2$$

$$= 4x^2 - 4x + 3$$

\therefore

$$\underline{\underline{g(f(x)) = 4x^2 - 4x + 3}}$$

2) Calculate the inverse of these functions:

$$a) f(x) = 3x / x + 1$$

$$y = 3x / x + 1$$

$$y(x + 1) = 3x$$

$$yx + y - 3x = 0$$

$$yx - 3x = -y$$

$$x(y-3) = -y$$

$$x = -y / y-3$$

$$\therefore \underline{\underline{f^{-1}(x) = -x / x-3}}$$

$$b) \quad g(t) = t + 2(t-1)$$

$$y = t + 2(t-1)$$

$$y = t + 2t - 2$$

$$y = 3t - 2$$

$$y + 2 = 3t$$

$$t = (y+2) / 3$$

$$\therefore \underline{\underline{g^{-1}(t) = \frac{t+2}{3}}}$$

3) State the definition of the gradient of a function.

a) The gradient of a function is the slope of the function, which is the difference in y-axis divided by the difference in the x-axis, or dy/dx .

b) Sketch the graph of the following:

$$y = x^2 + x + 1$$

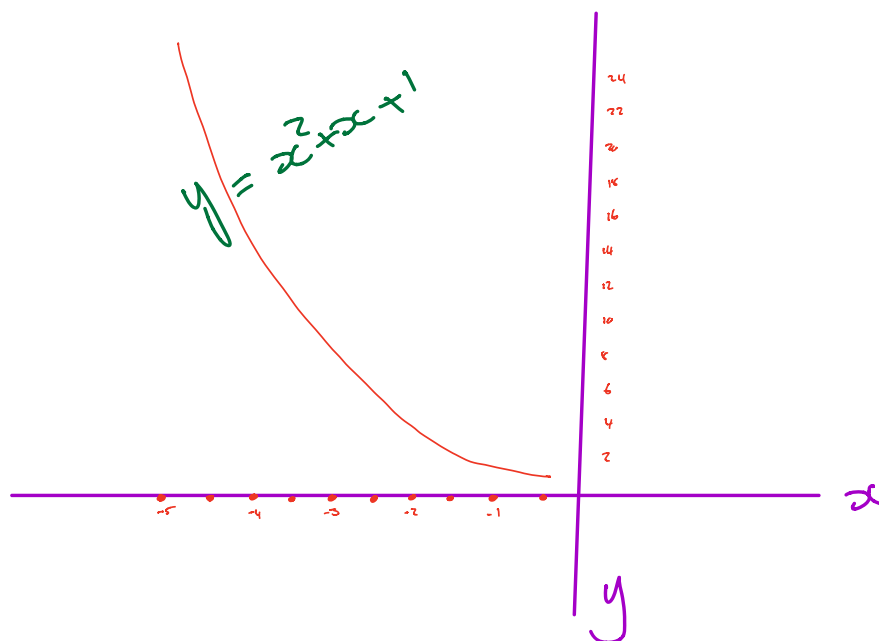
$$-5 \leq x \leq 5$$

i) for x $-5 \leq x < 0$

ii) for x $x \geq 0$

i) $y = x^2 + x + 1$ $-5 \leq x < 0$

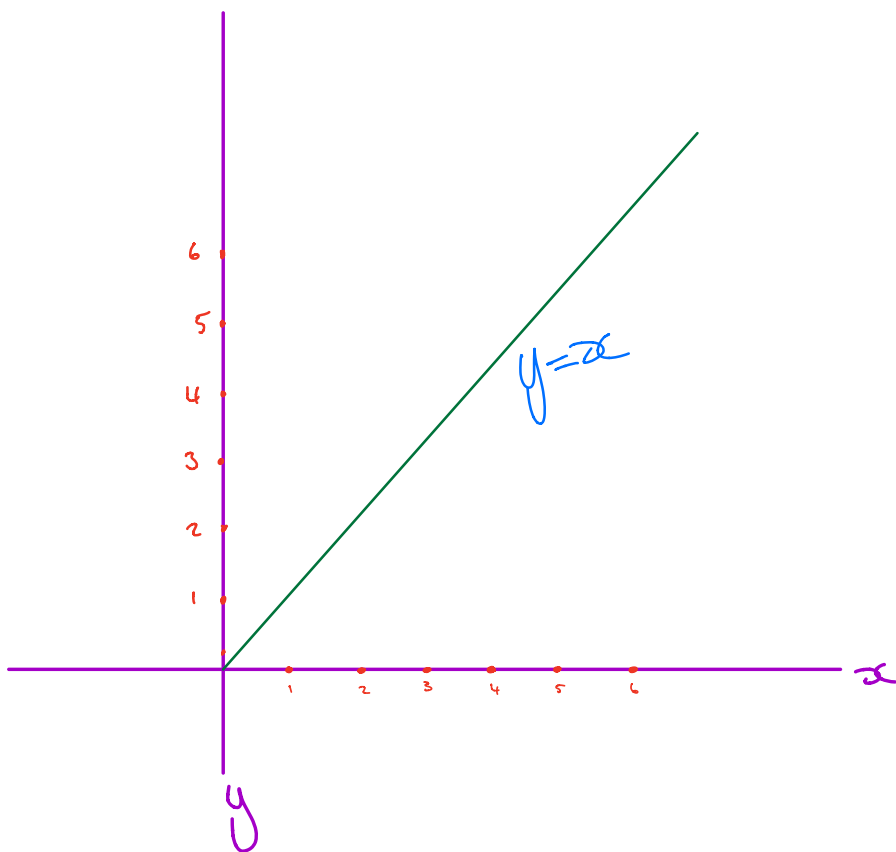
x	$-1/2$	-1	$-3/2$	-2	$-5/2$	-3	$-7/2$	-4	$-9/2$	-5
y	$3/4$	1	$7/4$	3	$19/4$	7	$39/4$	13	$67/4$	21



ii) $y = x$

$0 \leq x \leq 5$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5
y	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5



4) Which of the following are linear equations and which are not linear? If it is linear solve the equation.

a) $x + 8 = 0$ ✓
 $x = -8$

b) $x^2 + 3x + 4 = 0$ ✗
 this is not a linear equation.

c) $\sqrt{x^2 + 2x + 1} = 9$ ✗
 not linear

d) $2p + 5 = 0$ ✓
 $2p = -5$
 $p = -\frac{5}{2}$

5) Solve the following system of equations using matrix form.

$$2x + 5y = 10$$

$$6x - 4y = 24$$

$$\begin{pmatrix} 2 & 5 \\ 6 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 24 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2(-4) - (5 \times 6)} \begin{pmatrix} -4 & -5 \\ -6 & 2 \end{pmatrix} \begin{pmatrix} 10 \\ 24 \end{pmatrix}$$

$$= \frac{1}{-8 - 30} \begin{pmatrix} -40 - 120 \\ -60 + 48 \end{pmatrix}$$

$$= -1/38 \begin{pmatrix} -160 \\ -12 \end{pmatrix}$$

$$= \begin{pmatrix} 160/38 \\ 12/38 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \underline{\underline{\begin{pmatrix} 80/19 \\ 6/19 \end{pmatrix}}}$$

Sanity check

$$2x + 5y = 10 \quad \textcircled{1}$$

$$6x - 4y = 24 \quad \textcircled{2}$$

$$\begin{aligned} Ax &= B \\ A^{-1}Ax &= A^{-1}B \\ Ix &= A^{-1}B \\ x &= A^{-1}B \end{aligned}$$

$$\textcircled{1} \quad 2x + 5y = 10$$

$$2x = 10 - 5y$$

$$x = (10 - 5y)/2 \quad \textcircled{2}$$

sub 2 into 1

$$6x - 4y = 24$$

$$6\left(\frac{10 - 5y}{2}\right) - 4y = 24$$

$$3(10 - 5y) - 4y = 24$$

$$30 - 15y - 4y = 24$$

$$-19y = 24 - 30$$

$$-19y = -6$$

$$\underline{\underline{y = 6/19}}$$

sub y into 2:

$$x = (10 - 5y)/2$$

$$x = (10 - 5(6/19))/2$$

$$2x = (10 - 30/19)$$

$$2x = \frac{190 - 30}{19}$$

$$2x = 160/19$$

$$x = \frac{160}{19} \left(\frac{1}{2} \right)$$

$$x = 160/38$$

$$\underline{\underline{x = 80/19}}$$

$$\underline{\underline{x = 80/19}} \quad \checkmark$$

$$\underline{\underline{y = 6/19}} \quad \checkmark$$