Exercise 1

Convert the following decimal values to binary.

a) $64_{10} \Rightarrow 1000000_2$

64	32	16	8	4	2	1
1	0	0	0	0	0	0

$64 + 6(0) = \underline{\underline{64}}$

b) $129_{10} \Rightarrow 10000001$

256	128	64	32	16	8	4	2	1
0	1	0	0	0	0	0	0	1

$128 + 6(0) + 1 = \underline{\underline{129}}$

c) $255_{10} \Rightarrow 11111111_2$

256	128	64	32	16	8	4	2	1
0	1	1	1	1	1	1	1	1

$128 + 16 + 32 + 16 + 8 + 4 + 2 + 1$

$= \underline{\underline{255}}$

d) $256_{10} \Rightarrow 100000000_2$

512	256	128	64	32	16	8	4	2	1
0	1	0	0	0	0	0	0	0	0

$256 + 8(0) = \underline{\underline{256}}$

e) $99_{10} \Rightarrow 1100011$

128	64	32	16	8	4	2	1
0	1	1	0	0	0	1	1

$64 + 32 + 3(0) + 2 + 1$

$= \underline{\underline{99}}$

Exercise 2

a. What is the biggest negative number that can be represented by an 8-bit word? Give your answer in both signed magnitude and decimal.

2^{n-1}

$2^{8-1} = 2^7 = \underline{\underline{128}}$

	128	64	32	16	8	4	2	1	
Sign →	1	1	1	1	1	1	1	1	← Magnitude

$\therefore 64 + 32 + 16 + 8 + 4 + 2 + 1 = \underline{\underline{127}}$

so, $127_{10} = 1111111_2$

and

$-127_{10} = 11111111_2$

b. What is the biggest positive number that can be represented? Give your answer in both signed magnitude and decimal.

$2^{8-1} = 2^7 = \underline{\underline{128}}$

128	64	32	16	8	4	2	1
0	1	1	1	1	1	1	1

$\therefore 64 + 32 + 16 + 8 + 4 + 2 + 1 = \underline{\underline{127}}$

$127_{10} = 01111111_2$

c. Convert -68 into signed magnitude.

128	64	32	16	8	4	2	1	
0	1	0	0	0	1	0	0	$\rightarrow$ 68
1	1	0	0	0	1	0	0	$\rightarrow$ -68

$\therefore -68_{10} = 11000100_2$

d. Convert the signed magnitude value 01110101 into decimal.

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1
0	1	1	1	0	1	0	1

$\therefore 64 + 32 + 16 + 4 + 1 = \underline{\underline{117}}$

$01110101_2 = 117_{10}$

e. Convert 0 into signed magnitude.

$2^{8-1} = 2^7 = \underline{128}$

128	64	32	16	8	4	2	1
0	0	0	0	0	0	0	0

$\therefore 0_{10} = 00000000_2$

or

$0_{10} = 10000000_2$!!!!!

f. convert the signed magnitude value 10000000 into decimal.

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1
1	0	0	0	0	0	0	0

Sign

Magnitude

$\therefore$ 10000000 = +0 !!!

Exercise 3
Perform the following additions in binary. Show your work.

a) $110_{10} + 19_{10}$

110_{10}

~~128~~	64	32	16	8	4	2	1
	1	1	0	1	1	1	0

64 + 32 + 8 + 4 + 2 = 110

19_{10}

~~32~~	16	8	4	2	1
	1	0	0	1	1

$\therefore$ 16 + 2 + 1 = 19

So

```
   1 1 0 1 1 1 0
 + 0 0 1 0 0 1 1
 ---------------
 1 0 0 0 0 0 0 1
```

$\boxed{* \quad 1 + 1 = 10 \qquad * \quad 1 + 1 + 1 = 11}$

Check

1	0	0	0	0	0	0	1
128	64	32	16	8	4	2	1

$128 + 6(0) + 1 = \underline{\underline{129}}$

$$
\begin{array}{r}
110 \\
+\quad 19 \\
\hline
100 \\
20 \\
+\quad 9 \\
\hline
\underline{\underline{129}}
\end{array}
$$

$= \checkmark$

b) $63_{10} + 78_{10}$

63_{10}

78_{10}

$\cancel{64}$	32	16	8	4	2	1
	1	1	1	1	1	1

$32+16+8+4+2+1 = \underline{\underline{63}}$

$\cancel{128}$	64	32	16	8	4	2	1
	1	0	0	1	1	1	0

$64+8+4+2 = \underline{\underline{78}}$

$$
\begin{array}{rcccccccc}
 & & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
+ & & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
\hline
 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1
\end{array}
$$

Check

1 0 0 0 1 1 0 1
128 64 32 16 8 4 2 1

$128 + 8 + 4 + 1 = \underline{\underline{141}}$

$$\begin{array}{r} 63 \\ +\ 78 \\ \hline 130 \\ 11 \\ \hline \underline{\underline{141}} \end{array}$$

c) $50_{10} + 83_{10}$

50_{10}

~~64~~ 32 16 8 4 2 1
1 1 0 0 1 0

$32 + 16 + 2 = \underline{50}$

$\boxed{83_{10}}$

~~128~~ 64 32 16 8 4 2 1
1 0 1 0 0 1 1

$64 + 16 + 3 = \underline{\underline{83}}$

So

$$\begin{array}{r} 1\ 0\ 1\ 0\ 0\ 1\ 1 \\ +\ 0\ 1\ 1\ 0\ 0\ 1\ 0 \\ \hline 1\ 0\ 0\ 0\ 0\ 1\ 0\ 1 \end{array}$$

check:

1 0 0 0 0 1 0 1
128 64 32 16 8 4 2 1

$128 + 4 + 1 = \underline{\underline{133}}$ = ✓

$$\begin{array}{r} 50 \\ +\ 83 \\ \hline 130 \\ +\ \ 3 \\ \hline \underline{\underline{133}} \end{array}$$

Exercise 4

All binary values are 2’s complement

1. convert 1111111_2 to decimal.

```
  1 1 1 1 1 1 1
  0 0 0 0 0 0 0
+             1
---------------
  0 0 0 0 0 0 1
```

$-1 = 0000001$

Check

```
0 0 0 0 0 0 1
1 1 1 1 1 1 0   -> 1's com
            1   -> 2's com (add one)
-------------
1 1 1 1 1 1 1
```

-64 As the last MSB is 1

```
 1  1  1  1  1  1  1
64 32 16  8  4  2  1
```

$-64 + (32 + 16 + 8 + 4 + 2 + 1) = \underline{\underline{-1}}$

2. convert 01111111_2 to decimal.

```
0  1  1  1  1  1  1  1
  64 32 16  8  4  2  1
```

$64+32+16+8+4+2+1 = \underline{\underline{127}}$

3.) convert −63 to 2's complement.

63

64	32	16	8	4	2	1	
0	1	1	1	1	1	1	= 63

	0	1	1	1	1	1	1	
	1	0	0	0	0	0	0	→ 1's complement
+							1	→ add one
−63 →	1	0	0	0	0	0	1	

−63 = 1000001

Check:

1 0 0 0 0 0 1

	0	1	1	1	1	1	0	→ invert
+	0	0	0	0	0	0	1	→ add 1
	0	1	1	1	1	1	1	

0	1	1	1	1	1	1
64	32	16	8	4	2	1

32 + 16 + 8 + 4 + 2 + 1 = 63 ✓

4) convert 111_{10} to 2's complement.

111_{10}

~~128~~	64	32	16	8	4	2	1
	1	1	0	1	1	1	1

5) convert 10101010_2 to decimal.

	1	0	1	0	1	0	1	0	
	0	1	0	1	0	1	0	1	→ invert
+	0	0	0	0	0	0	0	1	→ Add one
	0	1	0	1	0	1	1	0	

1	0	1	0	1	1	0
64	32	16	8	4	2	1

$64 + 16 + 4 + 2 = \underline{86}$

$\therefore$ $10101010 = -86$

exercise 5
perform the following subtractions using 2’s complement arithmetic.

a) 100 − 35

~~128~~	64	32	16	8	4	2	1	
	1	1	0	0	1	0	0	→ 100
	0	1	0	0	0	1	1	→ 35

	0	1	0	0	0	1	1	
	1	0	1	1	1	0	0	→ invert
+	0	0	0	0	0	0	1	→ add one
	1	**0**	**1**	**1**	**1**	**0**	**1**	→ −35

	1	1	0	0	1	0	0
+	1	0	1	1	1	0	1
~~1~~	**1**	**0**	**0**	**0**	**0**	**0**	**1**

$$\begin{array}{r} 100 \\ +\,(-35) \\ \hline \underline{\underline{65}} \end{array}$$

check

64	32	16	8	4	2	1
1	0	0	0	0	0	1

$64 + 1 = \underline{\underline{65}}$ ✓

$$\begin{array}{r} 100 \\ -\ 35 \\ \hline 70 \\ -\ \ 5 \\ \hline \underline{\underline{65}} \end{array}$$

b) 18 − 6

16	8	4	2	1		
1	0	0	1	0	→ 18	①
0	0	1	1	0	→ 6	②

②

```
    0 0 1 1 0
    1 1 0 0 1      invert
  + 0 0 0 0 1      Add one
  -----------
③   1 1 0 1 0   →  -6
```

① + ③

```
     1 0 0 1 0
   + 1 1 0 1 0
  -------------
  ~~1~~ 0 1 1 0 0
```

check:

16	8	4	2	1
0	1	1	0	0

8 + 4 = 12

```
  18
 − 6
 ---
  10
 + 2
 ---
  12
```

✓

c) 15 − 23

16	8	4	2	1		
0	1	1	1	1	→ 15	1
1	0	1	1	1	→ 23	2

2

```
   1 0 1 1 1
   0 1 0 0 0     invert
 + 0 0 0 0 1     add one
 -----------
3  0 1 0 0 1  →  −23
```

1 + 3

```
   0 1 1 1 1
 + 0 1 0 0 1
 -----------
   1 1 0 0 0
```

check:

16	8	4	2	1
1	1	0	0	0

$-16 + 8 = -8 \rightarrow$

$$\begin{array}{r} 15 \\ -23 \\ \hline -8 \end{array}$$

✓

d) 255 − 11

	128	64	32	16	8	4	2	1	
①	1	1	1	1	1	1	1	1	⟶ 255
②	0	0	0	0	1	0	1	1	⟶ 11

②	0	0	0	0	1	0	1	1	
	1	1	1	1	0	1	0	0	invert
	0	0	0	0	0	0	0	1	add one
③	1	1	1	1	0	1	0	1	

① + ③

	1	1	1	1	1	1	1	1
+	1	1	1	1	0	1	0	1
	1	1	1	1	0	1	0	0

check:

128	64	32	16	8	4	2	1
1	1	1	1	0	1	0	0

$128 + 64 + 32 + 16 + 4 = \underline{\underline{244}}$

$$\begin{array}{r} 255 \\ -\ 11 \\ \hline 200 \\ 40 \\ +\ 4 \\ \hline \underline{\underline{244}} \end{array}$$

✓

l) $-10 - 64$

	64	32	16	8	4	2	1	
1	1	0	0	1	0	1	0	$\longrightarrow -10$
2	1	0	0	0	0	0	0	$\longrightarrow 64$

2	1	0	0	0	0	0	0	
	0	1	1	1	1	1	1	mvert
	0	0	0	0	0	0	1	Add one
3	$\boxed{1}$	$\boxed{0}$	$\boxed{0}$	$\boxed{0}$	$\boxed{0}$	$\boxed{0}$	$\boxed{0}$	$\Longrightarrow -64$