1) State the definition of a function and as composite function:

- ◯ A function is a set of rules which can be applied to a set of inputs to produce outputs.
- ◯ A composite function is function which takes another function as input.

$$f(x) = 2x - 1$$

$f(2) = ?$

$f(x) = 2x - 1$

$f(2) = 2(2) - 1$

$f(2) = 3$

$f(3) = ?$

$f(x) = 2x - 1$

$f(3) = 2(3) - 1$

$f(3) = 6 - 1$

$f(5) = 5$

$g(x) = x^2 + 2$

$g(2y+1) = ?$

$g(2y+1) = (2y+1)^2 + 2$

$= (2y+1)(2y+1) + 2$

$= 4y^2 + 2y + 2y + 1 + 2$

$= 4y^2 + 4y + 3$

$g(2y+1) = 4y^2 + 4y + 3$

$f(x) = 2x - 1$

$f(g(x)) = ?$

$f(g(x)) = f(x^2+2)$

$f(x^2+2) = 2(x^2+2) - 1$

$f(x^2+2) = 2x^2 + 4 - 1$

$f(x^2+2) = 2x^2 + 3$

$\therefore f(g(x)) = 2x^2 + 3$

$g(x) = x^2 + 2$

$g(f(x)) = ?$

$g(f(x)) = g(2x-1)$

$g(2x-1) = (2x-1)^2 + 2$

$= (2x-1)(2x-1) + 2$

$= 4x^2 - 4x + 1 + 2$

$= 4x^2 - 4x + 3$

$\therefore$

$g(f(x)) = 4x^2 - 4x + 3$

2) Calculate the inverse of these functions:

a) $f(x) = 3x/x+1$

$y = 3x/x+1$

$y(x+1) = 3x$

$yx + y - 3x = 0$

$yx - 3x = -y$

$$x(y-3) = -y$$

$$x = -y/y-3$$

$$\therefore \quad f^{-1}(x) = -x/x-3$$

b) $g(t) = t+2(t-1)$

$$y = t+2(t-1)$$

$$y = t+2t-2$$

$$y = 3t-2$$

$$y+2 = 3t$$

$$t = (y+2)/3$$

$$\therefore \quad g^{-1}(t) = \frac{t+2}{3}$$

3) State the definition of the gradient of a function.

a)The gradient of a function is the slope the function, which is the difference in y-axis divided by the difference in the x-axis, or dy/dx.

b) Sketch the graph of the following:

$y = x^2 + x + 1$

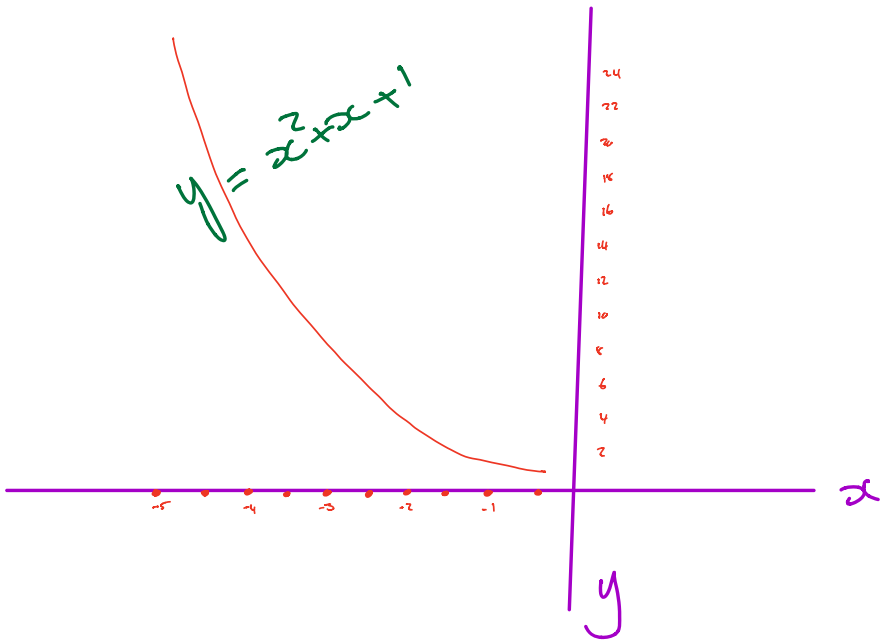
$-5 \leq x \leq 5$

i) for x $-5 \leq x < 0$

ii) for x $x \geq 0$

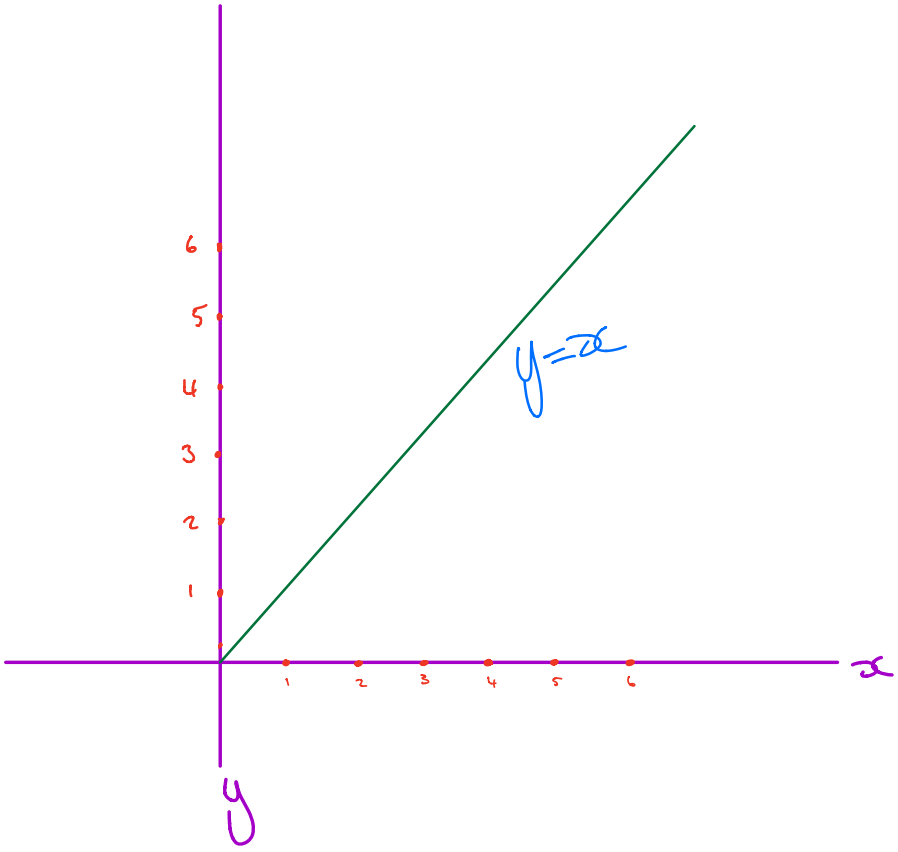
i) $y = x^2 + x + 1$ $-5 \leq x < 0$

x	$-1/2$	-1	$-3/2$	-2	$-5/2$	-3	$-7/2$	-4	$-9/2$	-5
y	$3/4$	1	$7/4$	3	$19/4$	7	$39/4$	13	$67/4$	21



ii) $y = x$ $\quad 0 \leq x \leq 5$

x	0	1/2	1	3/2	2	5/2	3	7/2	4	9/2	5
y	0	1/2	1	3/2	2	5/2	3	7/2	4	9/2	5



4) Which of the following are linear equations and which are not linear? If it is linear solve the equation.

a) $z + 8 = 0$ ✓

$z = -8$

b) $x^2 + 3x + 4 = 0$ ✗

this is not a linear equation.

c) $\sqrt{x^2+2x+1} = 9$ ✗

not linear

d) $2p + 5 = 0$ ✓

$2p = -5$

$p = -5/2$

5) Solve the following system of equations using matrix form.

$$2x + 5y = 10$$
$$6x - 4y = 24$$

$AX = B$

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

$$\begin{pmatrix} 2 & 5 \\ 6 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 24 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2(-4)-(5\times 6)}\begin{pmatrix} -4 & -5 \\ -6 & 2 \end{pmatrix}\begin{pmatrix} 10 \\ 24 \end{pmatrix}$$

$$= \frac{1}{-8-30}\begin{pmatrix} -40-120 \\ -60+48 \end{pmatrix}$$

$$= -1/38\begin{pmatrix} -160 \\ -12 \end{pmatrix}$$

$$= \begin{pmatrix} 160/38 \\ 12/38 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 80/19 \\ 6/19 \end{pmatrix}$$

Sanity check

$$2x + 5y = 10 \quad (1)$$
$$6x - 4y = 24 \quad (2)$$

① $2x + 5y = 10$

$2x = 10 - 5y$

$x = (10 - 5y)/2$ ③

sub 3 into 2

$6x - 4y = 24$

$6\left(\frac{10-5y}{2}\right) - 4y = 24$

$3(10 - 5y) - 4y = 24$

$30 - 15y - 4y = 24$

$-19y = 24 - 30$

$-19y = -6$

$y = 6/19$

sub y into 3:

$x = (10 - 5y)/2$

$x = \left(10 - 5(6/19)\right)/2$

$2x = \left(10 - 30/19\right)$

$2x = \frac{190 - 30}{19}$

$2x = 160/19$

$$x = \frac{160}{19}\left(\frac{1}{2}\right)$$

$$x = 160/38$$

$$x = 80/19$$

$x = 80/19$ ✓ $\quad$ $y = 6/19$ ✓