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Quiz-5

Question 1:

EPOCH 1 - for input A=0, B=0, C=0, Target = 0

$$D_{net} = A \cdot W_{ad} + B \cdot W_{bd} + C \cdot W_{cd}$$

$$= (0 \times 1.0) + (0 \times 1.4) + (0 \times 2.2) = 0.0$$

$$D_{out} = \text{sigmoid}(0.0) = 0.5$$

$$E_{net} = (0 \times -1.0) + (0 \times 1.5) + (0 \times 2.7) = 0.0$$

$$E_{out} = \text{sigmoid}(0.0) = 0.5$$

$$F_{net} = (0 \times 1.2) + (0 \times 1.6) + (0 \times 2.3) = 0.0$$

$$F_{out} = \text{sigmoid}(0.0) = 0.5$$

$$G_{net} = (0 \times 1.3) + (0 \times 2.1) + (0 \times 2.1) = 0.0$$

$$G_{out} = \text{sigmoid}(0.0) = 0.5$$

Output Layer : Net Input & Output

$$\begin{aligned} O_{net} &= (D_{out} \times W_{d0}) + (E_{out} \times W_{e0}) + (F_{out} \times W_{f0}) + \\ &\quad (G_{out} \times W_{g0}) \\ &= (0.5 \times 3.1) + (0.5 \times 4.1) + (0.5 \times 2.1) + (0.5 \times 2.9) \\ &= 1.55 + 2.05 + 1.05 + 1.45 = 6.0 \end{aligned}$$

$$O_{out} = \text{sigmoid}(6.0) \approx 0.9978$$

$$\boxed{\text{Sigmoid}(x) = \frac{1}{1+e^{-x}}}$$

Error & Weight Update

target output = 0

Error = Target - O_{out}

$$= 0 - 0.9978 = -0.9978$$

To update weight we need to

1. compute derivative of the output activation

Target = 0

= 0.0

$\times 2.7 = 0.0$

$\times 3) = 0.0$

$\times 1) = 0.0$

$(W_{F0} \times W_{G0}) +$
 $(G_{out} \times W_{G0})$
 (0.5×2.9)

$$\text{mid}(x) = \frac{1}{1+e^{-x}}$$

activation

$$\begin{aligned}\delta_0 &= (O_{\text{target}} - O_{\text{out}}) \cdot O_{\text{out}} \cdot (1 - O_{\text{out}}) \\ &= -0.9978 \times 0.9978 \times (1 - 0.9978) \\ &\approx -0.0022\end{aligned}$$

2. Then update weights from hidden to output

$$W_{d0-\text{new}} = W_{d0} + n \times \delta_0 \times D_{out}$$

where n is learning rate, lets use $n=1$

$$\begin{aligned}W_{d0} &= 3.1 + 0.1 (-0.0022) \times 0.5 \\ &\approx 3.1 - 0.0011\end{aligned}$$

$$W_{d0} = 3.1 - 0.0011$$

$$= 3.09989$$

$$= 3.09989$$

$$W_{d0} = 3.1 + 0.1 (-0.0022) \times 0.5$$

$$= 3.1 - 0.0011$$

$$= 3.09989$$

$$W_{d0} = 3.1 + 0.1 (-0.0022) \times 0.5$$

$$= 3.1 - 0.0011$$

$$= 3.09989$$

$$W_{d0} = 3.1 + 0.1 (-0.0022) \times 0.5$$

$$= 3.1 - 0.0011$$

$$= 3.09989$$

EPOCH 0 = input A=0, B=0, C=1, Target. = 1

$$D_{net} = (0 \times 1.0) + (0 \times 0.14) + (1 \times 2.2) = 2.2$$

$$D_{out} = \sigma(2.2) \approx 0.9002$$

$$E_{net} = 0 \times (-1.0) + 0 \times (0 \times 1.5) + (1 \times 2.7) = 2.7$$

$$E_{out} = \sigma(2.7) \approx 0.9370$$

$$F_{net} = (0 \times 1.2) + (0 \times 1.6) + (1 \times 2.3) = 2.3$$

$$F_{out} = \sigma(2.3) \approx 0.9089$$

$$G_{net} = (0 \times 1.3) + (0 \times 2.1) + (1 \times 2.1) = 2.1$$

$$G_{out} = \sigma(2.1) \approx 0.8909$$

Step 2 - Output layer
using updated weights from previous step

$$\begin{aligned} O_{net} &= (0.9002 \times 3.09989) + (0.9370 \times 4.89989) \\ &\quad + (0.9089 \times 2.09989) + (0.8909 \times 2.89989) \\ &= 2.789 + 3.841 + 1.908 + 2.583 = 11.121 \end{aligned}$$

$$O_{out} = \sigma(11.121) \approx 0.99999$$

Step 3 - error & weights

$$\text{Target} = 10 \cdot (0.8909 + 1.09 + 1.09) = 11.06$$

$$O_{out} \approx 0.99999 \quad 11.000 \cdot 0.99999 =$$

$$\text{Error} \approx 1 - 0.99999 = 0.00001$$

Output delta:

$$\begin{aligned} \delta_o &= (1 - 0.99999) \times (0.99999) (1 - 0.99999) \\ &\approx 0.00001 \times 0.99999 \times 0.00001 \\ &\approx 1e-10 \end{aligned}$$

this is a very small error, so, weight updates will be almost negligible
updated weights

using the same formula

$$W_{new} = W_{old} + 0.1 \cdot \delta_o \cdot \text{hidden output}$$

e.g.

$$W_{old} = 3.09989 + 0.1 \cdot (1e-10) \cdot 0.9002$$

$$\approx 3.09989 + 9e-12 \cdot 1000 \approx 3.09989$$

$$\approx 3.09989$$

so, all weights stay nearly the same due to extremely low error.

$$(1.09 \times 1) + (1.09 \times 0) + (0.90 \times 0) = 1.09$$

$$(1.09 \times 1) + (1.09 \times 0) + (0.90 \times 0) = 1.09$$

EPOCH
Step 1 - 1

Dout

Da

Eout

Eout

Fout

Cout

Step 2 - 2

One

Step 3 - 3

Tare

Oout

Error

e.g. w

against

EPOCH = input A=0, B=1, C=0, Target = 1
Step 1 - Hidden layer - net input & output

$$D_{net} = (0 \times 0 \cdot 1) + (1 \times 1 \cdot 4) + (0 \times 0 \cdot 22) = 1 \cdot 4$$

$$D_{out} = \sigma(1 \cdot 4) \approx 0 \cdot 8022$$

$$E_{net} = 0 \times (-1 \cdot 0) + (1 \times 1 \cdot 5) + (0 \times 0 \cdot 4) = 1 \cdot 5$$

$$E_{out} = \sigma(1 \cdot 5) \approx 0 \cdot 8176$$

$$F_{net} = (0 \times 1 \cdot 2) + (1 \times 1 \cdot 8) + (0 \times 2 \cdot 3) = 1 \cdot 6$$

$$F_{out} = \sigma(1 \cdot 6) \approx 0 \cdot 8320$$

$$G_{net} = (0 \times 1 \cdot 3) + (1 \times 2 \cdot 1) + (0 \times 2 \cdot 1) = 2 \cdot 1$$

$$G_{out} = \sigma(2 \cdot 1) \approx 0 \cdot 8909$$

Step 2 : Output layer - net input & output

$$O_{net} = (0 \cdot 8022 \times 3 \cdot 09989) + (0 \cdot 8176 \times 4 \cdot 09989) + (0 \cdot 8320 \times 2 \cdot 09989) + (0 \cdot 8909 \times 2 \cdot 89989)$$

$$O_{net} = 2 \cdot 485 + 3 \cdot 351 + 1 \cdot 747 + 2 \cdot 583 \\ = 16 \cdot 166$$

$$O_{out} = \sigma(16 \cdot 166) \approx 0 \cdot 99996$$

Step 3 = Error & update

target = 1

$$O_{out} = 0 \cdot 99996$$

$$\text{Error} = 1 - 0 \cdot 99996 = 0 \cdot 00004$$

$$\Delta_w = 0 \cdot 00004 \times 0 \cdot 99996 (1 - 0 \cdot 99996)$$

$$\Delta_w = 1 \cdot 6 \times 10^{-9}$$

$$\text{e.g. } W_{d0} = 3 \cdot 09989 + 0 \cdot 1 (1 \cdot 6 \times 10^{-9}) \cdot 0 \cdot 8022 \\ \approx 3 \cdot 09989 + 1 \cdot 28 \times 10^{-10} \approx 3 \cdot 09989$$

against weight change is negligible due to tiny error.

Summary

input
000
001
010
011

Epoch 5 Input A=0, B=1, C=1, Target = 0

Step 1 - Hidden layer - net input & output

$$D_{net} = (0 \times 1 \cdot 0) + (1 \times 1 \cdot 4) + (1 \times 1 \cdot 2) = 3.6$$

$$D_{out} = \sigma(3.6) = 0.9734$$

$$E_{net} = 0 \cdot (-1 \cdot 0) + (1 \times 1 \cdot 5) + (1 \times 2 \cdot 7) = 4.2$$

$$E_{out} = \sigma(4.2) = 0.9853$$

$$F_{net} = (0 \times 1 \cdot 2) + (1 \times 1 \cdot 6) + (1 \times 1 \cdot 2) = 3.8$$

$$F_{out} = 0.9801$$

$$G_{net} = (0 \times 1 \cdot 3) + (1 \times 2 \cdot 1) + (1 \times 2 \cdot 3) = 4.2$$

$$G_{out} = \sigma(4.2) = 0.9853$$

Step 2 - Output layer - net input & output

$$O_{net} = (0.9734 \times 3.09989) + (0.9853 \times 4.09989) \\ + (0.9801 \times 2.09989) + (0.9853 \times 2.89989)$$

$$O_{net} = 3.015 + 4.038 + 2.059 = 11.967$$

$$O_{out} = \sigma(11.967) \approx 0.9999937$$

Step 3: error & update

$$Target = 0$$

$$Out = 0.9999937$$

$$Error = 0 - 0.9999937 = -0.9999937$$

$$\Delta_o = -0.9999937 \times 0.9999937 \times (1 - 0.9999937) \\ \approx -6.3e-6$$

Weight updates

$$W_{new} = W_{old} + \eta \cdot \delta_o \cdot \text{hidden output}$$

$$W_{d0} = 3.09989 + 0.1 \times (-6.3e-6) \cdot 0.9734 \\ = 3.09929$$

$$W_{o0} = 4.09989 - 0.1 \times 6.3e-6 \times 0.9853 \\ = 4.09927$$

$$W_{f0} = 2.09989 - 0.1 \times 6.3e-6 \times 0.9801 \\ = 2.09927$$

$$W_{g0} = 2.89989 - 0.1 \times 6.3e-6 \times 0.9853 \\ = 2.89927$$

Summary Table

input	Target	Oout	Error	ΔO
000	0	0.9978	-0.9978	-0.0022
001	1	0.99999	0.00001	$\sim 1e-10$
010	1	0.99996	0.00004	$\sim 4e-9$
011	0	0.99999	-0.99999	$\sim 6.3e-6$