

# DESIGNER'S NOTEBOOK



## Extending Fixed-Point Dynamic Ranges

Contributed by Alex Tessarolo

### Design Problem

How can you extend the fixed-point math dynamic range beyond the range of a Q15 number with a minimum of instructions?

### Solution

In many advanced control problems such as state estimators, Kalman filters and some high Q filters, the dynamic range/accuracy of the coefficient can sometimes be beyond the range of a Q15 number while the data value can be typically represented as a Q15 number.

Aside from trying to dynamically scale the coefficients to extract as much accuracy as possible or trying to use floating point math, there is a technique that can perform 32-bit  $\times$  16-bit math at an effective 4 cycles per Tap and potentially 2 cycles per Tap for larger than 6th order systems (+ some fixed overhead of about 8-13 cycles).

The trick is to re-scale the numbers and represent the problem as an integer value + a fractional value. For example:

$$Y = 2.391456 * X_0 - 0.0235045 * X_1 + 0.000329758 * X_2 - 34.3392345 * X_3$$

In the above equation, the filter Coefficients have a dynamic range exceeding a 16-bit Q15 number. If we re-scale the problem as follows:

$$Y = [1224.425472 * X_0 - 12.034304 * X_1 + 0.168836096 * X_2 - 17581.68806 * X_3] / 512$$

And then allocate the following coefficient values:

$$Y = [(A0i + A0f) * X_0 + (A1i + A1f) * X_1 + (A2i + A2f) * X_2 + (A3i + A3f) * X_3] / 512$$

where:

$$\begin{aligned} A0i &= 1224 = 04C8h \\ A0f &= 0.425472 = 3676h (= 0.425476074) \\ A1i &= -12 = FFF4h \\ A1f &= -0.034304 = FB9Ch (= -0.034301758) \\ A2i &= 0 = 0000h \\ A2f &= 0.168836096 = 159Ch (= 0.168823242) \\ A3i &= -17581 = BB53h \\ A3f &= -0.68806 = A7EEh (= -0.688049316) \end{aligned}$$

The problem then reduces to calculating the following:

$$Y = (A0i \cdot X0 + A1i \cdot X1 + A2i \cdot X2 + A3i \cdot X3) + (A0f \cdot X0 + A1f \cdot X1 + A2f \cdot X2 + A3f \cdot X3)$$

This is like calculating two filter banks. The above problem is coded in the example below:

$$sY = 1k0 \cdot sX0 + \dots + 1k3 \cdot sX3$$

movd Y, Round  
madd.q Y, Y, k, XL, #0  
madd.q Y, Y, k, XU, #0  
madd.q Y, Y, k, XL, #0  
madd.q Y, Y, k, XU, #0

```
; Assume:      X0,X1,X2,X3 = Q15 (-1 range 0.999053955)
;              Y = Q10 (-32 range +31.99902344)
; Ymin-max = 2.391456 + 0.0235045 + 0.000329758 + 34.3392345
;              = +/- 36.75452476
;              Sat      = 06000h
;              Round    = 08000h
```

```
SETC OVM ; Enable saturation.
SETC SXM ; Enable sign extension.
SPM 3 ; Set shift mode = -6
LT A0f
MPY X0 ; P = A0f*X0
LTP A1f ; ACC = A0f*X0
MPY X1 ; P = A1f*X1
LTA A2f ; ACC = ACC + A1f*X1
MPY X2 ; P = A2f*X2
LTA A3f ; ACC = ACC + A2f*X2
MPY X3 ; P = A3f*X3
LTA A0i ; ACC = ACC + A3f*X3
SPM 0
SACH Temp,6 ; On C5X replace by BSAR 9
LAC Temp,1 ; ACC = ACC/512
; instruction.
MPY X0 ; P = A0i*X0
LTA A1i ; ACC = ACC + A0i*X0
MPY X1 ; P = A1i*X1
LTA A2i ; ACC = ACC + A1i*X1
MPY X2 ; P = A2i*X2
LTA A3i ; ACC = ACC + A2i*X2
MPY X3 ; P = A3i*X3
APAC ; ACC = ACC + A3i*X3
ADDS Round ; Round result.
ADDS Sat ; Saturate Y to Q10 value
SUBH Sat
SUBH Sat
ADDS Sat
SACH Y,1 ; Y = Q10 number.
```

; Cycles = 13 + 4n cycles (n = number of taps).

; Note: If saturation is not required, Cycles = 8 + 4n cycles

Figure 1.

If the number of taps is greater than 6, then a RPT loop can be used for each bank and the effective cycles/tap can be approximately 2.

The above technique is almost equivalent to a floating-point notation with a 4-bit exponent and a 16-bit mantissa.