Correlation-function peak detector

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Abstract: Accurate detection of the peak position of a correlation function requires a large number of correlation-function points to be determined. To achieve an acceptable dynamic performance, a parallel implementation of the function is required. In the paper techniques are described for achieving high-resolution peak detection with a limited amount of parallel circuitry.

1 Introduction

Bendat and Piersol [1] have shown that an unbiased estimate of the mathematically-defined (i.e. averaged over infinite time) cross-correlation function is given by

$$R_{yx}(\tau) = \frac{1}{T}\int_0^T y(t)x(t-\tau)\,dt$$

where $y(t)$ and $x(t)$ will, for the purposes of this paper, be considered to be the output and input, respectively, of a system, and T is the finite integration period. If the system is linear then input and output are also related by the convolution integral

$$y(t) = \int_0^\infty h(t-\tau)x(\tau)\,d\tau$$

where $h(t)$ is the system impulse response. These two integrals can be related to give what is commonly referred to as the Wiener-Lee relation

$$R_{yx}(\tau) = \int_0^\infty h(t-\tau)R_{xx}(\tau)\,d\tau$$

where $R_{xx}(\tau)$ is the input autocorrelation function

$$R_{xx}(\tau) = \frac{1}{T}\int_0^T x(t)x(t-\tau)\,dt$$

The Wiener-Lee relation provides the mathematical basis for correlation-based measurement systems. For example, if the input conditions are arranged so that $R_{xx}(\tau)$ approximates to an impulse of strength K then $R_{yx}(\tau) = Kh(\tau)$ and hence the system transfer function can be directly evaluated from the cross-correlation function relating input and output of the system. Wide-bandwidth analogue noise has an autocorrelation-function shape that approximates the impulse-function shape. However it is statistically more reliable to achieve this approximation by using a pseudorandom noise signal as an input signal or as a perturbation superimposed on an existing signal level [2].

An alternative simplification of the Wiener-Lee relation is obtained when the system transfer function is a pure transport lag D. In this case the system will not distort the amplitude of the input signal in any way. In this case $h(t)$ becomes a unit impulse at time delay D and therefore

$$R_{yx}(\tau) = R_{xx}(\tau - D)$$

Hence it is possible to determine the transport lag by measuring the time-delay position of the peak of the cross-correlation function.

Measurement systems based on transit-time determination by correlation techniques have been extensively investigated [3]. The correlation flowmeter [4] is the best known example of this type of instrument. Flow noise created by turbulence conditions and conveyed particles is detected by two sensors separated by distance L along the flow stream. The two flow-noise signals are converted to an electrical signal by, for example, measuring the modulation resulting from the interaction of an ultrasonic beam with the flowing medium. These two signals are cross-correlated to give a function having a peak position delayed from the origin by the time delay D. Flow velocity is then obtained from

$$\text{Velocity} = (1 + C)\frac{L}{D}$$

where C is a small and constant correction factor.

The hardware cost of implementing the correlation function is greatly reduced if the polarity form of the function is used:

$$R_{yxp} = \frac{1}{T}\int_0^T \left\{\frac{y(t)}{|y(t)|}\right\}\left\{\frac{x(t-\tau)}{|x(t-\tau)|}\right\} dt$$

where, for example, $y(t)/|y(t)| = 1$ if $y(t)$ is positive and -1 if $y(t)$ is negative. In this case maximum positive correlation is obtained when $R_{yxp} = 1$, zero correlation when $R_{yxp} = 0$ and maximum negative correlation when $R_{yxp} = -1$. Digital-circuit realisation is further simplified if the sign function $y(t)/|y(t)|$ is forced to be $+1$ if $y(t)$ is positive and 0 when $y(t)$ is negative. In this case $R_{yxp} = +1$ for maximum positive correlation, $R_{yxp} = +\frac{1}{2}$ for zero correlation and $R_{yxp} = 0$ for maximum negative correlation.

As a result of using the polarity function, Van Vleck [5] has shown that a distortion of the amplitude of the function is obtained which for Gaussian noise signal is described by

$$R_{yxp}(\tau) = \frac{2}{\pi}\arcsin\left\{\frac{R_{yx}(\tau)}{R_{xx}(0)R_{yy}(0)^{1/2}}\right\}$$

Fortunately the positions of the peaks of the function are not distorted. A further comparison of polarity correlation with the direct-correlation function shows that the integration time must be increased by approximately 2.5 times to achieve the same variance.

Many correlator circuits have been proposed and successfully implemented [6]. However, it was impossible to produce a low-cost circuit to measure the function and reliably detect the peak position until large-scale integrated circuits

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became readily available. For routine industrial use of correlation based measurement systems it is essential that the most-significant peak position is detected reliably with no likelihood of the circuit locking on to spurious lower-significance peaks.

The overloading-counter correlator [7] provides one hardware solution to the problem of reliably detecting the position of the most-significant peak. Consider a parallel-polarity correlator using an array of binary counters to count coincidence conditions at each time delay of interest. The equivalent time-delay position of the first counter to overload (i.e. to output a carry from its most significant bit position) gives the position of the most-significant peak of the function. This technique has been successfully implemented as an LSI digital circuit. Each silicon chip implements 12 points of the correlation function and a number of chips are connected in series to cover a specified time-delay range. Use of the circuit in a wide variety of experimental and industrial prototype systems has demonstrated the reliability of correlation flowmeters based on this peak-detection method.

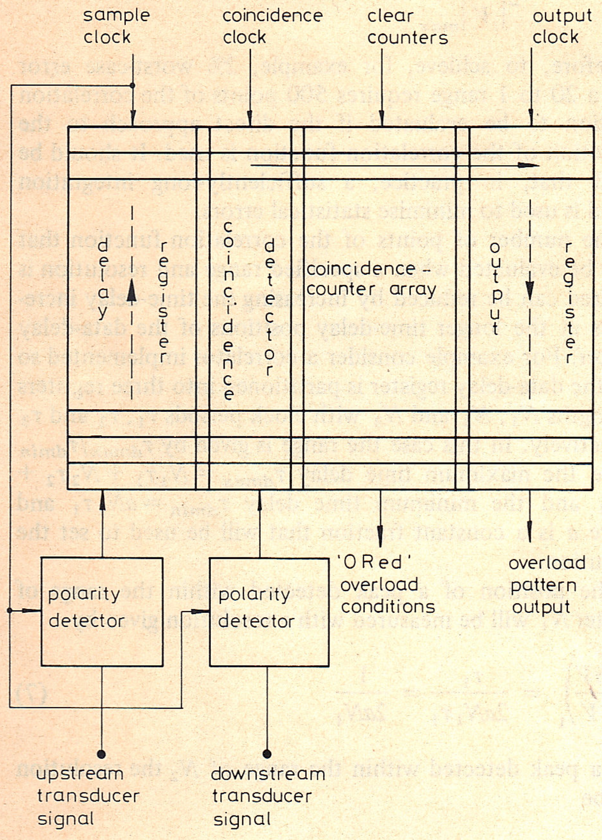

Fig. 1 *Block diagram of overloading-counter correlator*

The resolution with which the peak position can be detected is determined by the quantisation of the time-delay axis. A large number of shift-register stages and associated counters will be required to meet, for example, a 1% resolution specification over a 20 to 1 flow range. In this paper, two methods will be described for overcoming this difficulty. The first method involves the use of an increasing sample clock period as the time delay increases. The second involves the use of the pattern of overload conditions that will continue to evolve after the peak overload pattern has been detected.

3 Overloading counter correlator

A block diagram showing the significant features of an overloading-counter polarity correlator is shown in Fig. 1. Three related clock signals are used: the sample clock shifts the sampled, and polarity-detected, upstream signal U along the delay register, a coincidence clock is 'ANDed' with the coincidence signal formed by the binary operation $UV + \bar{U}\bar{V}$ (where V is the downstream signal) to produce a pulse train that is counted by coincidence counters and an output clock shifts overload patterns out of the output register. Overload patterns are stored by a parallel-load operation selected by the mode control. Starting from the counter-cleared state the array of counters sum coincidences between the two signals U and V until one counter overloads. This is indicated by the 'ORed' overload output, and the external control logic then puts the output register into its parallel-load state and at the same time inhibits the coincidence-count clock, thereby freezing the peak overload pattern. The peak overload pattern is synchronously stored in the output register by an output-register clock pulse. The counters are then cleared, the output register is put into its serial-shift mode and the integration sequence is repeated. When the overload pattern is serially shifted from the output register (smallest time delay first) the time-delay position of the most significant peak of the function is found by counting output clock pulses until a logic 1, corresponding to the peak overload pattern, is detected. At the end of the shift operation the counter therefore contains a binary number proportional to the time-delay position of the function peak value.

The number N stored in a coincidence counter after T count clock pulses can be related to the polarity correlation coefficient R_{yxp}, as follows. When the waveforms to be correlated are completely coincident then the output from the coincidence gate is always at logic 1, hence count clock pulses are continuously counted and therefore $N = T$ corresponds to $R_{yxp} = 1$. If the input waveforms are never coincident (i.e. $R_{yxp} = 0$), then the output from the coincidence gate is a square waveform spending as much time at logic 1 as at logic 0 and therefore $N = T/2$ corresponds to $R_{yxp} = 0$. For negatively correlated waveforms (i.e. when $R_{yxp} = -1$) the output from the coincidence gate is always at logic 0 and therefore $N = 0$.

These specific results can be summarised by writing

$$R_{yxp} = 2\frac{N}{T} - 1 \qquad (1)$$

The contents of coincidence counters in the LSIC correlator are not available, hence the time at which an overload occurs, relative to the start of the integration cycle, must be used to determine the correlation significance of an overload pattern. In this case a timing counter, operating synchronously with the coincidence counters, is used to monitor the time at which an overload pattern occurs. If τ is the sample clock period and T the number stored in the timing counter at time t_1 then $t_1 = \tau N$ and the polarity-correlation coefficient corresponding to an overload pattern obtained at time t_1 is given by

$$R_{yxp} = 2\frac{\tau N}{t_1} - 1 \qquad (2)$$

However, overloads cannot occur until $t_1 = \tau N$, since this time corresponds to the unity correlation coefficient, therefore it is convenient to write

$$t_1 = t + \tau N$$

where t is a time, derivable from the contents of the timing counter, that is nonlinearly related to correlation significance by

$$R_{yxp} = \frac{2\tau N}{t + \tau N} - 1$$

or

$$R_{yxp} = \frac{2N}{\alpha + N} - 1 \tag{3}$$

where $\alpha = t/\tau$ represents the additional count clock pulses counted by the timing counter when an overload pattern corresponding to R_{yxp} less than one is monitored.

Note that the input signal must be oversampled to give adequate resolution to the detection of the peak position and consequently the coincidence signals will consist of groups of 1 s and 0 s. It is therefore not necessary to count coincidences at the data sample rate F_s. Indeed, large reductions in the capacity of the integrating counters to give the required variance can be obtained by reducing the count clock frequency F_c to the point where redundant information is not counted.

The output clock effectively samples the evolving pattern of overloads. The sample time is determined by the number of shift-register stages used (i.e. this corresponds to the number of points of the function monitored) and the output clock period τ_o. An additional output-register clock pulse must be allowed for the pattern of overloads to be parallel loaded into the register.

Let P = number of output-register clock pulses required to inspect the contents of the register and load the next pattern. Then the time to inspect N_{op} overload patterns

$$= PN_{op}\tau_o$$

The number of count clock pulses in this time

$$= PN_{op}\tau_o f_c$$

Hence the way in which the correlation function is amplitude sampled by this circuit when the overload patterns are allowed to evolve beyond the peak overload condition is described by

$$R_{yxp} = \frac{2N}{PN_{op}\tau_o f_c + N + \alpha} - 1 \tag{4}$$

where α is as defined in eqn. 3. Despite the nonlinear relationship between R_{yxp} and N_{op}, techniques have been developed that enable the evolving pattern of overloads to be used in a low-cost correlation-function display [8].

It is found in practice, after careful transducer design, that the measured cross-correlation function has a symmetrical shape. In this case the worst error in the detection of the peak position caused by the quantisation of the time-delay axis is given by $\tau_s/2$ where τ_s is the period of the data-delay register clock.

The worst-case velocity error, δV, is given by

$$\delta V = \frac{L}{n\tau_s} - \frac{L}{n\tau_s \pm \dfrac{\tau}{2}}$$

or

$$\frac{\delta V}{V} \simeq \pm\frac{1}{2n} \quad \text{if} \quad n \geqslant 10 \tag{5}$$

where $V = L/n\tau_s$ is the indicated velocity.

Clearly, the velocity error will have its maximum value when n has its minimum value n_{min}. But n_{min} is related to the maximum delay n_{max} by the range R_e, of the instrument i.e.:

$$n_{max} = R_e n_{min}$$

Hence

$$\left|\frac{\delta V}{V}\right|_{max} = \frac{R_e}{2n_{max}}$$

or

$$n_{max} = \frac{R_e}{2\left|\dfrac{\delta V}{V}\right|_{max}} \tag{6}$$

Therefore, to achieve, for example, 2% worst-case error with a 20 to 1 range requires 500 points of the correlation function to be evaluated if the direct approach to the evaluation of the correlation function is used. It should be noted that, in practice, a sufficiently-long integration period is used to minimise statistical errors.

The number of points of the correlation function that must be evaluated when a specified range and resolution is required can be reduced by increasing the time-delay increments at the longer time-delay positions of the data-delay register. For example consider a correlator implemented so that the data-delay register is partitioned into three registers of lengths N_1, N_2 and N_3 with clock periods τ_1, τ_2 and τ_3 respectively. In this case the range is given by τ_{dmax}/τ_{dmin} where the maximum time delay $\tau_{dmax} = N_1\tau_1 + N_2\tau_2 + N_3\tau_3$ and the minimum time delay $\tau_{dmin} = aN_1\tau_1$ and where a is a constant fraction that will be used to set the resolution.

The position of a peak detected within the range of register N_1 will be measured with a resolution given by

$$\left(\frac{\delta V}{V}\right)_1 = \frac{\tau_1}{2aN_1\tau_1} = \frac{1}{2aN_1} \tag{7}$$

For a peak detected within the range of N_2 the resolution will be

$$\left(\frac{\delta V}{V}\right)_2 = \frac{\tau_2}{2(N_1\tau_1 + \tau_2)} \tag{8}$$

For a peak detected within the range of N_3 the resolution will be

$$\left(\frac{\delta V}{V}\right)_3 = \frac{\tau_3}{2(N_1\tau_1 + N_2\tau_2 + \tau_3)} \tag{9}$$

Let, for example, $\tau_2 = 2\tau_1$ and $\tau_3 = 4\tau_1$ and $N_2 = 2N_1$ and $N_3 = 2N_1$. Then the range $= 13/a = 26/1$ when $a = \frac{1}{2}$ and $\delta V/V = 1/50$ (i.e. 2% resolution) if $N_1 = 50$. Substitution into eqns. 8 and 9 will show that

$$\left(\frac{\delta V}{V}\right)_2, \left(\frac{\delta V}{V}\right)_3 < \left(\frac{\delta V}{V}\right)_1$$

The total number of stages required to achieve this worst-case resolution of 2% is given by $N_1 + 2N_1 + 2N_1 = 5N_1$, = 250. To achieve the same resolution with a single sample-frequency system will require 650 points of the correlation function. Clearly correlator implementations using the multiple-frequency-sampling method will require significantly less hardware than a directly-implemented correlator.

4 Interpolated-moment peak detector

The resolution of a correlation-based measurement system using the overloading-counter peak-detection method can be significantly improved by making use of the overload patterns that will be output after the peak overload pattern has been detected. Fig. 2 shows an idealised pattern of over-

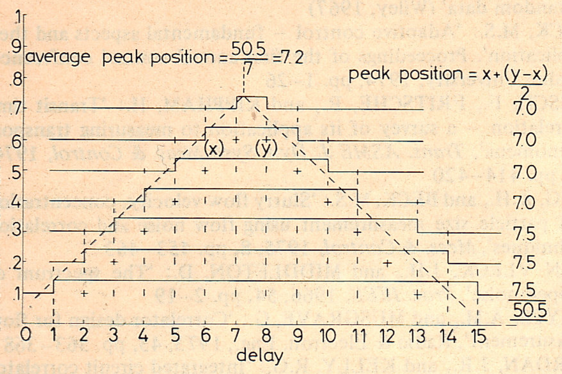

Fig. 2 *Interpolated moment peak detector*

loads. Note that in practice a symmetrical, approximately Gaussian-shaped, correlation function is obtained. Therefore a symmetrical triangular-function shape provides a reasonable first-order model of experimentally-obtained correlation functions.

Since the function is symmetrically shaped any of the overload patterns derived from the output register shortly after the first (i.e. the peak) pattern has been output can be used to define the peak position. Let X and Y be the time-delay positions of the edge of the overload pattern with $Y > X$. Each overload pattern allows the peak position D to be estimated by using

$$D = X + \frac{Y - X}{2} \tag{10}$$

The statistical quality of the estimate is improved by averaging successive values of D over the first N_{op} overload patterns:

$$D = \frac{\sum_{i=1}^{i=N_{op}} \left(X_i + \frac{Y_i - X_i}{2} \right)}{N_{op}} \tag{11}$$

It will be convenient to choose N_{op} to be a power of two since division by N_{op} can then be implemented with a simple operation. An interpolated measure of the peak position is obtained as can be seen from the idealised example shown in Fig. 2.

The peak-detection algorithm, eqn. 11, has been implemented by an up-down counter and a small amount of additional logic. Output-register shift pulses are counted at full rate until the rising edge of the overload pattern is detected, and causes the count rate to be halved. The count continues until the falling edge of the pattern stops the count. This procedure is repeated for the first N_{op} overload patterns with the counter acting as an accumulator and therefore implementing the numerator of eqn. 11. When this operation is completed the counter contents are transferred to a buffer store and the correlator system is reset to start the next integration cycle. Division by N_{op} is achieved by a simple wiring shift of the connection between the buffer store and a DAC generating an analogue output signal.

The statistical quality of the correlation function is reduced at lower significance magnitudes below the peak value. It is important, therefore, to ensure that only a small part of the top of the function is used to interpolate the peak position. The top fraction of the correlation function used by this peak-detection method can be evaluated by re-writing eqn. 4.

Let R_{yxp1} be the peak magnitude of R_{yxp}, then

$$R_{yxp1} = \frac{2}{1 + \alpha_1} - 1 \tag{12}$$

where

$$\alpha_1 = \frac{\alpha}{N}$$

Let R_{yxp2} be the value of R_{yxp} after N_{op} overload patterns beyond the peak overload pattern, then

$$R_{yxp2} = \frac{2}{1 + \alpha_1 + \alpha_2} - 1 \tag{13}$$

where

$$\alpha_2 = \frac{PN_{op}\tau_o f_c}{N}$$

Then $\alpha_1 = (1 - R_{yxp1})/(1 + R_{yxp1})$ can be calculated given R_{yxp1} and R_{yxp2} evaluated by using eqn. 13.

An experimental system has been constructed having a time-delay range increased by the multiple-frequency-sampling method to give a P value of 240. For this system the integrating-counter capacity N was 3.2×10^3, the output clock period was 1 μs, the count clock frequency was 26 KHz and the number of overload patterns used N_{op} was 16. Using these values it will be found that $\alpha_2 = 0.12$. A psuedo-random noise signal was used to test the system so that $R_{yxp1} = 1$, hence $\alpha_1 = 0$ and therefore $R_{yxp2} =$

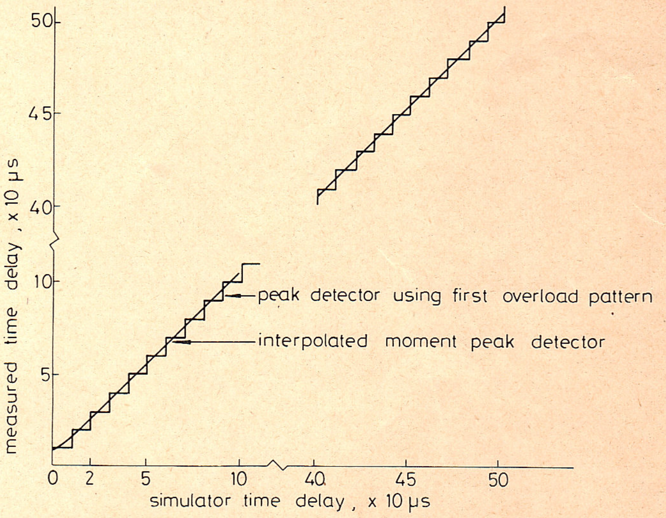

Fig. 3 *Improved response of interpolated moment peak detector*

0.79 (i.e. the top 21% of the function was being used). Fig. 3 shows the performance of the interpolated moment peak detector obtained by measuring the time-delay setting of a simulated-noise generator. The undesirable stepped response of a peak detector only using the first (i.e. peak) overload pattern is also shown in this Figure. An improved response is clearly obtained. An extensive programme of experiments on a prototype ultrasonic-correlation flowmeter using the interpolated moment peak detector has shown that a similarly improved performance can be expected in practical industrial-measurement situations.

5 Concluding comments

It is clearly essential for any measurement system implementing statistical functions to be able to operate without fear of malfunction caused by the fluctuations introduced, for example, by a finite averaging time or nonstationary-signal conditions. To ensure that, for example, peak-tracking systems always track the most-significant peak position additional circuits must be used to provide a peak-search facility. The overloading counter correlation and peak-interpolation technique enables a very reliable and accurate detection of the position of the most-significant peak of a correlation function.

The design and fabrication of the LSIC overloading counter correlator was supported by the National Research and Development Corporation. After successful trials in prototype correlation flowmeters the chip design has been licensed for use in correlation flowmeters and is not available for general-purpose use. A low-cost implementation of the overloading counter correlator using commercially-available components has however been described by Thorn [9]. By using the simplifying methods derived by Kam *et al.* [10] a polarity correlator based on a sequentially up-dated RAM circuit has been constructed to cover the lower-bandwidth applications. This system has been successfully used in a variety of measurement situations and has confirmed the reliability of the overloading-counter peak-detection method.

6 Acknowledgments

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7 References

1 BENDAT, J.S., and PIERSOL, A.G.: 'Measurement and analysis of random data' (Wiley, 1967)

2 BECK, M.S.: 'Adaptive control – fundamental aspects and their application'. Proceedings of the first annual control conference, Purdue University, 1974, pp. 1–26

3 MESCH, F., FRITSCHE, R., and KIPPHAM, H.: 'Transit time correlation – a survey of its application to measuring transport phenomena', *Trans. ASME J. Dyn. Syst. Meas. & Control,* 1974, **96**, pp. 414–420

4 ONG, K.H., and BECK, M.S.: 'Slurry flow velocity, concentration and particle size measurement using flow noise and correlation techniques', *Meas. & Control,* 1975, **8**, pp. 453–463

5 VAN VLECK, J.H., and MIDDLETON, D.: 'The spectrum of clipped noise', *Proc. IEEE,* 1966, **54**, pp. 2–19

6 HAYES, A.M., and MUSGRAVE, G.: 'Correlator design for flow measurement', Radio & Electron. Eng., 1973, **43**, pp. 363–368

7 JORDAN, J.R., and KELLY, R.G.: 'Integrated circuit correlator for flow measurement', *J. Inst. Meas. & Control,* 1976, **9**, pp. 267–270

8 MANOOK, B.A.: M.Phil. thesis, University of Edinburgh, 1976

9 THORN, R.: Ph.D. thesis, University of Bradford, 1980

10 KAM, Z., SHORE, H.B., and FEHER, G.: 'Simple schemes for measuring autocorrelation functions', *Rev. Sci. Instrum.,* 1975, **46**, pp. 269–277