

## Datasheet: Floating-Point FFT

### Features

- **FFT size:** User chosen power-of-two or non-power-of-two
- **Programmability:** Easy to change functionality to meet application requirements.
- **Dynamic Range:** Real and imaginary outputs are IEEE754 single precision
- **Scalability:** array based architecture means arbitrarily higher throughputs are obtained by increasing array size
- **Power:** array interconnects are entirely local, reducing parasitic routing capacitance to keep power dissipation low and clock speeds high
- **Implementation FPGA:** Can be used in any FPGA fabric containing embedded multipliers and memories.
- **Data I/O:** Streaming, normal order I/O with fixed-point 2's complement input words
- **Inverse FFT:** Run time selectable input

### I/O

- Can be either fixed-point or IEEE754 floating point
- Output format IEEE754

### Algorithm

The transform computation is based on a new formulation<sup>1</sup> of the discrete Fourier transform (DFT), different than any other FFT implementation, which decomposes it into structured sets of small, matrix-based DFTs. In particular the locality, simplicity and regularity of the processing core keeps interconnect delays lower than cell delays, leading to clock speeds that can approach the FPGA fabric limitations, e.g., "worst case" Fmax speeds determined by embedded elements. Short critical-path lengths with less delay in cells than interconnects also lower power dissipation. Additionally, a novel "base-4" algorithm reduces the number of cycles needed per FFT to less than the transform size value  $N$ .

### Architecture

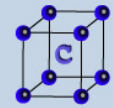
The architecture consists of two small arrays of pipelined, fine-grained, locally connected, simple processing elements, each containing a complex adder, a few registers and multiplexors. By cycling data through this array in a programmable way any size transform can be supported as well as desired variations from the standard DFT calculation.

### Precision

Unlike traditional pipelined FFTs, all additions are performed at full precision so that the round-off errors occur only in the twiddle multiplication steps. Consequently, the resulting precision is high. In the table below are accuracy measurements for 256/1024-pt streaming floating-point single-precision FFTs calculated based on 100 blocks of random 24-bit real and imaginary input

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<sup>1</sup> J. Greg Nash, "Computationally Efficient Systolic Array for Computing the Discrete Fourier Transform, IEEE Trans. Signal Processing, Vol. 53, No.12, December 2005, pp.4640-4641.



data (random phase) obtained from Matlab simulations (Altera's model is generated by IP v15). The comparison reference is a double precision Matlab calculation.

The "mean absolute error" numbers are obtained by subtracting each reference output from each circuit output, taking the magnitude of this and then dividing by the magnitude of the reference value for that output point. The "maximum absolute error" is the largest of these errors computed over all 100 blocks of input data. (In terms of the signal-to-noise-quantization-ratio both Centar designs in the table below have values >150db.)

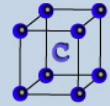
	<b>Altera (256pt)</b>	<b>Centar (256pt)</b>	<b>Altera (1024pt)</b>	<b>Centar (1024pt)</b>
<b>Mean Absolute Error</b>	8.60e-08	3.11e-08	8.82e-08	4.1637e-08
<b>Std Deviation Absolute Error</b>	2.34e-07	3.74e-08	1.77e-07	9.0471e-08
<b>Maximum Absolute Error</b>	2.80e-05	2.41e-06	1.47e-05	6.1269e-06

## Performance and Resources

Here example performance and resource usage data is provided on a 256/1024-point, streaming FFT. The circuits were compiled using Altera's software tools (Quartus II v15) using a Stratix IV EP4SE230F29C2 FPGA (40nm technology) device. The TimeQuest static timing analyzer was used to determine maximum clock frequencies (Fmax) at 1.1V and 85C (worst case settings). The same Quartus settings were used for both the Centar and Altera designs (IP v15).

	<b>Altera</b>	<b>Centar v1</b>	<b>Centar v2</b>	<b>Altera</b>	<b>Centar</b>
<b>Transform Size</b>	256 points			1024 points	
<b>ALMs</b>	10545	6537	7424	12883	6662
<b>ALUTs</b>	16957	9657	11411	20826	9935
<b>Registers</b>	15027	10265	12387	17967	10509
<b>M9Ks</b>	57	62	30	90	62
<b>Multipliers (18-bits)</b>	48	129	129	64	129
<b>Fmax (data rate, MHz)</b>	308	367	366	298	311

In the table above the adaptive logic module (ALM) is the basic unit of a Stratix IV FPGA (one 8-input LUT, two registers plus other logic). Comparison with Xilinx Virtex 6 devices (also 40nm) can be made by noting that two M9K memories are equivalent to a Xilinx BRAM and that an ALM is equivalent to between 1.2 and 1.8 LEs. These numbers come from benchmark studies which show 1 ALM=1.2 LEs (Xilinx white paper WP284 v1.0, December 19, 2007) and 1 ALM=1.8 LEs (Altera white paper), respectively. These papers actually compare Stratix III and



Virtex 5 FPGAs; however, the Stratix IV/Virtex 6 architectures are essentially the same so the comparisons should still be valid.

## Device Family Support

Altera Device Families Supported

- Stratix
- Aria
- Cyclone
- Hardcopy

## Xilinx Device Families Supported

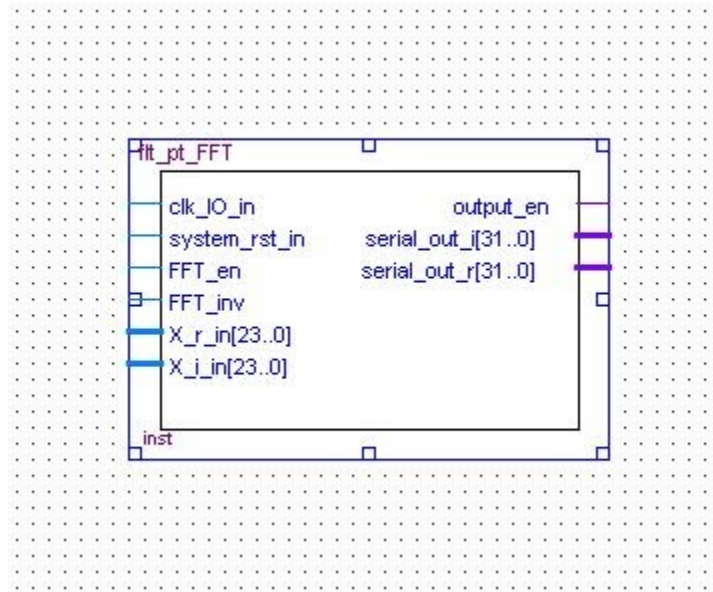
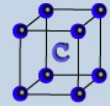
- Virtex 4-7
- Spartan
- Artix-7

## Deliverables

- Netlist (e.g., for Altera FPGAs a \*.qxp file for synthesis or a \*.vo file for simulation)
- Synthesis constraints (e.g., for Altera FPGA's an \*.sdc file)
- Modelsim Testbench (\*.vo file for DFT circuit plus verilog testbench for control). Matlab verification utilities also available.
- Altera Stratix III FPGA board development kit testbench
- Matlab behavioral bit-accurate model (p-code)
- Documentation

## Pin-outs

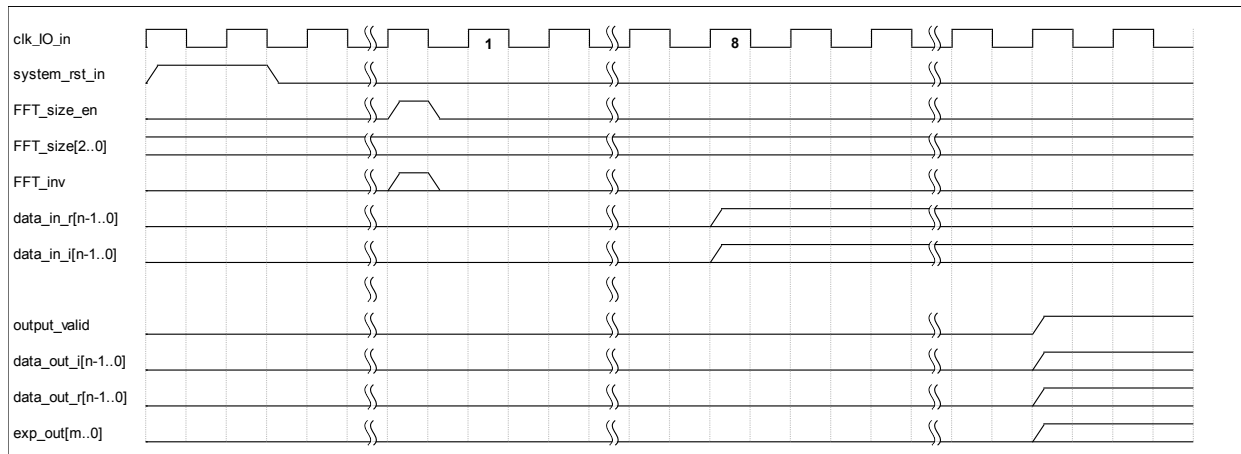
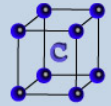
A symbol list corresponding to the pin-outs shown below are provided in the accompanying table. Depending upon the desired interfaces, some signals could change.



Name	Signal	Description
clk_IO_in	clock	Can be either a low frequency board oscillator clock output in which case circuit clock is derived from a PLL or actual data I/O clock
system_rst_in	control	Resets circuit; active high
FFT_en	control	Registers FFT_inv signal; active high
FFT_inv	control	High->forward; low->inverse
data_in_r/i	24-bit signed	Real and imaginary inputs (can be IEEE754)
output_en	control	High during data output
data_out_r/i	32-bit signed	Real and imaginary outputs IEEE754

### Timing Diagram

A variety of timing possibilities exist depending upon the desired interface. That shown here is applicable to a streaming, normal order input/output scheme.



At any time after `system_rst_in` goes low, `FFT_en` is used to latch the direction of the transform (`FFT_inv=0/1` for forward/inverse). Following this, the circuit expects to see continuous data appearing on the 8th subsequent cycle. Valid data out occurs when `output_valid` (`output_en`) goes high. For streaming operation output is continuous from this point on.