

## HW 3: Numerical solutions of ODE models

Due April 19, 2024.

1. In Chapter 9 of Kot (“Elements of Mathematical Ecology”), a predator-prey model with type IV functional response is presented:

$$\begin{aligned}\frac{dN}{dT} &= rN \left(1 - \frac{N}{K}\right) - \phi(N)P \\ \frac{dP}{dT} &= b\phi(N)P - mP\end{aligned}$$

where

$$\phi(N) = \frac{cN}{\frac{N^2}{i} + N + a}$$

and  $N$  represents the prey,  $P$  the predator. Substituting in  $\phi(N)$  and nondimensionalizing yields the following equations

$$\begin{aligned}\frac{dx}{dt} &= x \left(1 - \frac{x}{\gamma}\right) - \frac{xy}{\frac{x^2}{\alpha} + x + 1} \\ \frac{dy}{dt} &= \frac{\beta\delta xy}{\frac{x^2}{\alpha} + x + 1} - \delta y\end{aligned}$$

with  $x = \frac{N}{a}$ ,  $y = \frac{c}{ra}P$ , and  $t = rT$ .

Numerically solve this system of equations with the following parameters:

$\alpha = 5.2$ ,  $\beta = 2.0$ ,  $\gamma = 4.1$ ,  $\delta = 2.5$ ,  $x(0) = 1.3$ ,  $y(0) = 1.6$ .

To visualize your results, create a figure with a two-by-two subplot grid. In the top two plots, plot both solutions,  $x(t)$  and  $y(t)$ , in separate plots. In the bottom left subplot, plot the phase plane for this system using a quiver plot. In the bottom right subplot, plot the  $(x, y)$  solution over a chosen time period with  $x$  on the x-axis and  $y$  on the y-axis. You should see the solution spiral out to a steady oscillation (an orbit).

2. In Chapter 10 of Kot, a double mass-action chemostat model is presented including rate equations for substrate, heterotroph, and holozoic predator

$$\begin{aligned}\frac{dS}{dT} &= D(S_i - S) - \frac{\mu_1}{Y_1}SH \\ \frac{dH}{dT} &= \mu_1SH - DH - \frac{\mu_2}{Y_2}HP \\ \frac{dP}{dT} &= \mu_2HP - DP.\end{aligned}$$

Nondimensionalizing yields the following equations:

$$\begin{aligned}\frac{dx}{dt} &= 1 - x - Axy \\ \frac{dy}{dt} &= Axy - y - Byz \\ \frac{dz}{dt} &= Byz - z\end{aligned}$$

with  $x = \frac{S}{S_i}$ ,  $y = \frac{H}{Y_1 S_i}$ ,  $z = \frac{P}{Y_1 Y_2 S_i}$ ,  $t = DT$ .

Numerically solve this system of equations with the following parameters,

$A = 4.0$ ,  $B = 8.0$ . Choose your own initial conditions (strictly positive, not too big... say less than 5) and plot each of  $x$ ,  $y$ , and  $z$  against time in their own subplot (time on the x-axis, each variable on the y-axis) so that you can see damped oscillations. Use a one-column (three rows) subplot for this. Then, in a separate plot, create a 3D line plot of the  $(x, y, z)$  solution over the chosen time period with each variable on its respective axis.