A CALIBRATION SYSTEM FOR BARCODE STAVES

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Abstract

This article describes a staff calibration range constructed on the Darling Scarp in Perth for the calibration of barcode staves by DLI and the formulae used for the calibration of the range with precise equipment.

1. Introduction

With the increasing use of digital levelling systems by the surveying industry, there is a requirement to determine the accuracy of barcode staves. Surveyors are well aware of the necessity to maintain and regularly calibrate the equipment they use. The Department of Land Information of Western Australia (DLI) provides a calibration service for conventional staves but to date there had not been a convenient and cost effective method of calibrating barcode staves in Western Australia.

2. Staff Calibration Range

DLI has recently developed a test range and software for this purpose which can be freely accessed by all surveyors. The range has been established on the Land Surveyors Licensing Board's examination site at Boya and consists of two observing pillars and a series of 21 stainless steel pins glued into a solid granite outcrop in a semi arc around the observing pillars.

The two observing pillars were first constructed beside a large piece of sloping granite which had the required 4 metres of height difference between top and bottom. The highest pillar is set at a comfortable observing height and the lowest a metre lower and closer to the rock and range. Because of the known problem of large systematic errors with Leica digital levels when observing at sighting distances of 7.5 m and multiples thereof *(1)*, the pins were set 10 metres from the high pillar and 8.8 metres from the low one.

The pins were glued into drilled holes in the granite and the pillars were concreted deep into the ground to ensure their stability. Initially the pillars were old tower heads welded to galvanized pipe and footings but these were found to be too unstable. To strengthen the pillars, cardboard tube formwork was used to concrete around the pipes above ground level.

Observations were then made to determine the best values for the true height differences between the pins. For this purpose, Leica's precise digital level NA3003 was used chosen along with a Leica invar barcode staff. This instrument is capable of making repeated measurements and recording the mean and standard deviations. In addition, it can be set to reject measurements above a set tolerance. For observing the pins, the instrument was configured to observe 10 readings and store the mean and standard deviations with the rejection standard criteria set at 0.2 mm. To ensure that there was no systematic error in the level or staff, a second NA3003 level was borrowed from Main Roads WA together with a second Leica invar barcode staff. An internal collimation test was done on both levels and the level and staff bubbles checked and adjusted before commencing. The instruments were shaded by an umbrella and temperature observations were taken throughout the observing period.

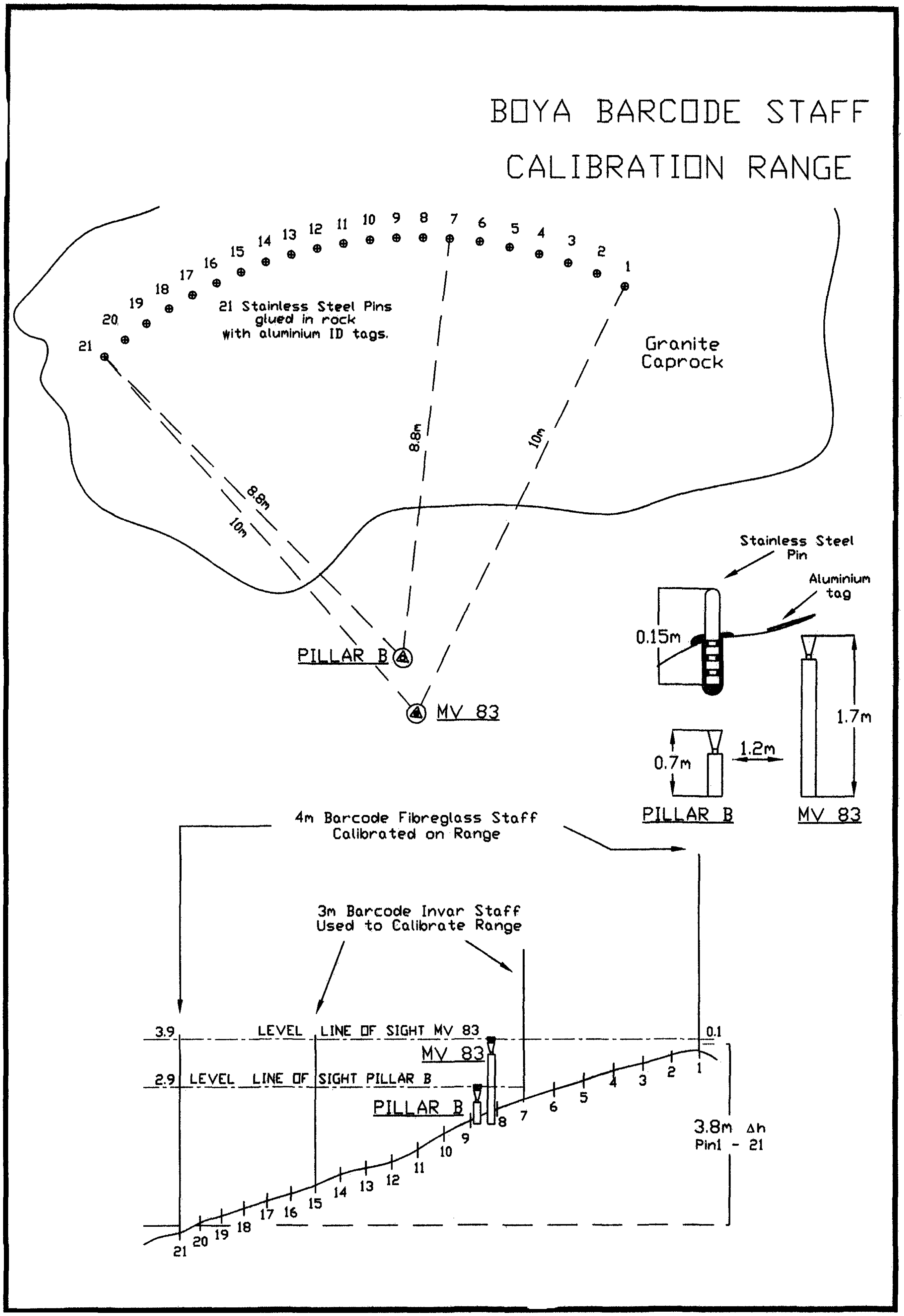

Figure 1. The Boya Barcode Staff Calibration Range

Figure 2. Calibrating the range with a supported invar staff and shaded instrument

With the 3 metre invar staff, it was possible to observe from Pin 1 to 15 from the high pillar and from Pin 7 to Pin 21 from the lower pillar. Using the two levels and two staves in all combinations, four full sets of readings were taken on the pins. The staff was supported with an adjustable legs and each full set of readings took about an hour.

The first time the range was calibrated, we found that the first two readings with a new staff showed much higher than expected residuals. We concluded that this had occurred because the staff had not yet stabilized to the ambient temperature and was still settling in the vertical position. The observations were repeated after allowing the staff to stabilize on Pin 1 for 5 minutes before commencing the range calibration.

All observed values were then input into *Staff*, a purpose written program developed by DLI to calibrate the range and constants from barcode calibrations which also caters for conventional calibrations carried out at the DLI laboratory.

The results obtained proved that there was no systematic bias in the instruments or staves. The maximum residual between the four sets of measurements on any individual pin was 0.21 mm with the majority between 0.02 mm and 0.06 mm, which was very impressive. The mean of the four sets of observations was then accepted as the "fixed" values for the pin heights of the range.

DLI's production fibreglass barcode staves were then calibrated using the range and the software "Staff". The values obtained are used by DLI as the calibration constant of the staves to be multiplied by height differences obtained in geodetic levelling. DLI strongly recommends that all surveyors undertaking geodetic and other high order levelling in WA with barcode staves use this range to obtain constants for their staves.
The software *Staff*, details of the range, procedures for use and booking forms are available from the DLIs web site at:

http://www.dli.wa.gov.au/corporate.nsf/web/Barcoded+Staff+Calibrations

Figure 3. The stainless steel pins glued in granite

3. Mathematical model for the calibration of the Barcode Calibration Range

The following mathematical model fits the observations, which are required for the calibrations of the range. There is a requirement to carry out more observations than are absolutely necessary to define the mathematical model in order to carry out a least squares adjustment for the height differences between the pins of the range. The primary purpose of the least squares adjustment is to ensure that all observations made are used to give the most probable values for the height differences between these pins.

$$\Delta H_{ij} = (M_{jk} - M_{ik}) \qquad (1)$$

Where ΔH_{ij} = Height difference between Pins i and j,
M_{ik} = Staff reading at Pin i with a digital level,
M_{jk} = Staff reading at Pin j with a digital level,
i and j are two adjoining pins, and
k = Observation set number. Each set contains several observations taken with the same level, staff and observing pillar. Different data sets contain different combinations of levels, staves and observing pillars.

4. Observation equations for each height interval between adjoining pins

The observation equations are derived from the mathematical model where the residual is the difference between the observation and the most probable height difference between the pins. One observation equation is used for each observation. Initially the most probable height difference is estimated from a previous calibration.

$$v_k = \Delta H_{ij} - [M_{jk} - M_{ik}] \qquad (2)$$

Where v = residual of the observation sets between Pins i and j in metres.

For example, to determine the height interval ΔH between adjoining Pins i and j using 8 observations sets, the following equations can be used.

$\Delta H_{ij} - [M_{j1} - M_{i1}] = v_1$
$\Delta H_{ij} - [M_{j2} - M_{i2}] = v_2$
$\Delta H_{ij} - [M_{j3} - M_{i3}] = v_3$
$\Delta H_{ij} - [M_{j4} - M_{i4}] = v_4$
$\Delta H_{ij} - [M_{j5} - M_{i5}] = v_5$
$\Delta H_{ij} - [M_{j6} - M_{i6}] = v_6$
$\Delta H_{ij} - [M_{j7} - M_{i7}] = v_7$
$\Delta H_{ij} - [M_{j8} - M_{i8}] = v_8$

Where v_k = residual of observation set k in metres.

These equations can be expressed in a matrix form.

$$\mathbf{Ax + w = v} \qquad (3)$$

Where $\mathbf{A} = \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{vmatrix}$ $\quad \mathbf{X} = \begin{vmatrix} \Delta Hij \end{vmatrix}$ $\quad \mathbf{w} = \begin{vmatrix} M_{j1} - M_{i1} \\ M_{j2} - M_{i2} \\ M_{j3} - M_{i3} \\ M_{j4} - M_{i4} \\ M_{j5} - M_{i5} \\ M_{j6} - M_{i6} \\ M_{j7} - M_{i7} \\ M_{j8} - M_{i8} \end{vmatrix}$

5. Least squares solution

A least squares adjustment computes the most probable values for the height differences between the pins. Any changes to the observations should be as small as possible. A traditional least squares adjustment minimises the sum of the squares of the weighted residuals. The observation matrix (3) is combined with the weight matrix (5) to form the following standard least squares equation (4).

To determine the best estimate for ΔHij use equation $\mathbf{x = (A^TPA)^{-1} A^TPw}$ (4)

Where P is the diagonal weight matrix.

$$P = \begin{vmatrix} 1/\sigma^2_1 & & & & & & & \\ & 1/\sigma^2_2 & & & & & & \\ & & 1/\sigma^2_3 & & & & & \\ & & & 1/\sigma^2_4 & & & & \\ & & & & 1/\sigma^2_5 & & & \\ & & & & & 1/\sigma^2_6 & & \\ & & & & & & 1/\sigma^2_7 & \\ & & & & & & & 1/\sigma^2_8 \end{vmatrix} \qquad (5)$$

And σ is the a priori standard deviation of each observation set between Pins i and j in metres.

$$\sigma = \sqrt{(\sigma^2_{Mi} + \sigma^2_{Mj} + \sigma^2_{Tij})} \qquad (6)$$

σ_{Mi} = standard deviation of the observations to Pin i.
σ_{Mj} = standard deviation of the observations to Pin j.
σ_{Tij} = standard deviation of the temperature correction between Pin i and j.

The standard deviations σ_{Mi} and σ_{Mj} can generally be obtained from the level. At the Boya calibration range, each set of observations between adjoining pins consists of 10 measurements. The level generally computes and displays the standard deviation of these measurements. There were a total of eight observation sets for each interval between adjoining pins.

$A^T PA$ can also be expressed as $\sum_{k=1}^{k=n} \frac{1}{\sigma^2_k}$ (7)

Where n = number of observation sets.

The uncertainty of the calibrated height difference is the a posteriori standard deviation of the adjusted height difference is and can be expressed as

$$\sigma_{Hij} = \sqrt{\left(A^T PA\right)^{-1}} \qquad (8)$$

Example 1: Estimating the calibrated height difference and its uncertainty at the 95% confidence level.

Observations between pins 7 and 8

$$\text{Where } A = \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{vmatrix} \qquad X = \begin{vmatrix} \Delta H \end{vmatrix} \qquad W = \begin{vmatrix} 0.24695 \\ 0.24687 \\ 0.24693 \\ 0.24693 \\ 0.24686 \\ 0.24683 \\ 0.24697 \\ 0.24690 \end{vmatrix}$$

The a priori standard deviations of each observation set is computed using equation (6).

$$\sigma_k = \sqrt{(\sigma_{Mi}^2 + \sigma_{Mj}^2 + \sigma_{Tij}^2)}$$

σ_1 = 0.00011 metres
σ_2= 0.00007 metres
σ_3= 0.00011 metres
σ_4= 0.00014 metres
σ_5= 0.00007 metres
σ_6= 0.00011 metres
σ_7= 0.00011 metres
σ_8= 0.00011 metres

$$P = \begin{vmatrix} 82644628 & & & & & & & \\ & 204081633 & & & & & & \\ & & 82644628 & & & & & \\ & & & 51020408 & & & & \\ & & & & 204081633 & & & \\ & & & & & 82644628 & & \\ & & & & & & 82644628 & \\ & & & & & & & 82644628 \end{vmatrix}$$

$$A^TPA = \sum_{k=1}^{k=n} \frac{1}{\sigma_k^2} = 649266014$$

Where n = number of observation sets.

Adjusted height difference between Pins 8 and 9 = ΔH_{ij} = $\mathbf{x = -(A^TPA)^{-1} A^TPw}$ = 0.246893 metres.

A total of 80 observations have been carried out for each height difference (8 observation sets, each containing 10 observations). The Degrees of Freedom = Number of observations –1 = 79.

The 95% confidence level is obtained by multiplying the standard deviation σ_{Hij} by a coverage factor (k). The Student's t distribution is used to determine the Coverage Factor at the 95% confidence level with 79 degrees of freedom. A Coverage Factor (k) of 1.99 can be derived from the Student's t-Distribution table.

A posteriori standard deviation of the adjusted height difference

$$\sigma_H = \sqrt{(A^T PA)^{-1}} = 0.00003925\text{metres}.$$

Uncertainty at 95% confidence level = 1.99 x 0.00003925 = 0.000078 metres = 0.08 mm.

6. Detecting gross and systematic errors

Gross and systematic errors in the observations are detected by analysing the residuals and standardised residuals. If the observations contain only random errors and the correct assumptions for the input weights have been made then the standardised residuals should be small. Any observation containing a very large standardised residual should be investigated for possible rejection. If observations are rejected then another adjustment will be required.

The residual (v_k) = ΔH_{ij} – (M_{jk} - M_{ik}) (9)

Where k = observation set number between Pins i and j.

Example 2: Calculating the residuals.

The residuals are the differences between the observations and the adjusted height differences between the pins. Observations containing large residuals should be flagged for possible rejection. However a more reliable method for detecting systematic errors is to analyse the standardised residuals as shown in example (4).

The residuals for the observations between Pins i and j are:
v_1=0.246893 - 0.24695 = 0.00006 metres
v_2=0.246893 - 0.24687 = 0.00002 metres
v_3=0.246893 - 0.24693 = 0.00016 metres
v_4=0.246893 - 0.24693 = 0.00016 metres
v_5=0.246893 - 0.24686 = 0.00009 metres
v_6=0.246893 - 0.24683 = 0.00036 metres
v_7=0.246893 - 0.24697 = 0.00064 metres
v_8=0.246893 - 0.24690 = 0.00001 metres

The standardised residual is the residual divided by its standard deviation

Δv_k = Standardised residual = $v_k / \sigma v_k$ standard deviations (10)

Where σv_k is the a posteriori standard deviation of the residual. σv_k can be calculated from the variance-covariance of the adjusted quantities and the a priori weight matrix P.

$$\sigma_{Vk} = \sqrt{(P^{-1} - \sigma_{LL})} \qquad (11)$$

Where σ_{LL} = variance-covariance matrix of the adjusted observations $A(A^TPA)^{-1}A^T$.
Equation (11) can be simplified as follows:

$$\sigma_{Vk} = \sqrt{(\sigma_k^2 - \sigma_H^2)} \qquad (12)$$

where σ_H = a posteriori standard deviation of the adjusted height difference
and σ_k = a priori standard deviation of the observation set.

Example 3: Calculating the a posteriori standard deviation of the residuals.

The a posteriori standard deviations indicate the uncertainty of the adjusted height differences between the pins. These values are generally converted to a confidence level of 95% by multiplying the following values by a coverage factor of 1.96. They are also required for the computation of the standardised residuals.

$\sigma v_1 = \sqrt{(0.00011^2 - 0.00003925^2)}$ = 0.000106 metres
$\sigma v_2 = \sqrt{(0.00007^2 - 0.00003925^2)}$ = 0.000058 metres
$\sigma v_3 = \sqrt{(0.00011^2 - 0.00003925^2)}$ = 0.000106 metres
$\sigma v_4 = \sqrt{(0.00014^2 - 0.00003925^2)}$ = 0.000134 metres
$\sigma v_5 = \sqrt{(0.00007^2 - 0.00003925^2)}$ = 0.000058 metres
$\sigma v_6 = \sqrt{(0.00011^2 - 0.00003925^2)}$ = 0.000106 metres
$\sigma v_7 = \sqrt{(0.00011^2 - 0.00003925^2)}$ = 0.000106 metres
$\sigma v_8 = \sqrt{(0.00011^2 - 0.00003925^2)}$ = 0.000106 metres

Example 4: Calculating the standardised residuals.

The standardised residuals are generally used to detect gross and systematic errors. All observations whose standardised residuals exceed a predetermined value could contain systematic or gross errors. This predetermined value is known as the rejection criterion and this value can either be estimated by the user or computed using statistical functions.

Compute the standardised residuals using equation (10)

Δv_1= 0.00006 / 0.000106 = 0.6 standard deviations
Δv_2= 0.00002 / 0.000058 = 0.3 standard deviations
Δv_3= 0.00016 / 0.000106 = 1.5 standard deviations
Δv_4= 0.00016 / 0.000134 = 1.2 standard deviations
Δv_5= 0.00009 / 0.000058 = 1.6 standard deviations
Δv_6= 0.00036 / 0.000106 = 3.4 standard deviations
Δv_7= 0.00064 / 0.000106 = 6.0 standard deviations
Δv_8= 0.00001 / 0.000106 = 0.1 standard deviations.

In this example if the rejection criterion is set at 3 standard deviations then observations 6 and 7 could contain systematic errors.

7. Rejection Criterion

Observations with very large standardised residuals should be rejected. For smaller standardised residuals one has to decide whether they are caused by:

- large random errors, in which case you should retain the observations, or
- systematic errors, in which case they should be rejected.

A value known as the rejection criterion can be computed and used to flag observations for possible rejection when their standardised residuals exceed this value. This criterion should reduce the possibility of rejecting good observations (Type I errors) or leaving systematic errors in the data (Type II errors)

Standardised residual < residual rejection criterion (13)

By increasing the residual rejection criterion you reduce the possibility of rejecting good observations but increase the possibility of leaving gross errors in the data. The residual rejection criterions are generally based on statistical functions and are sensitive to the degrees of freedom and the number of observations. A confidence level of 95% is generally adopted to determine the probability of detecting gross errors. A considerable number of methods exist for the computations of this criterion.

For the calibration of the Boya Range, only a limited number of observation sets are used for each height difference and these statistical functions do not work very well for only these few observations.
For the Boya Range, the analysis is based on flagging the observation sets with the largest standardised residuals and the largest residuals.

8. Conclusion

The calibration range and software developed by DLI enables surveyors using modern digital levels and barcode staves to calibrate their staves with a simple and cost effective method for use in geodetic and other high order levelling. The calibration of the range with invar staves is performed using the least squares method and is mathematically rigorous. Surveyors in Western Australia can be confident of the calibration constants of production barcode staves obtained on this range using DLI's methods and software.

Bibliography

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