(1) $\log(x/y) = \log(x) - \log(y)$

(2) $var(y-x) = var(x) + var(y)$

(3) $var(x+y) = var(x) + var(y) + 2\,cov(x,y)$

(4) $var(g(x)) = var(x)\left(\frac{\delta g(x)}{\delta x}\right)^2$ (delta method)

(5) $var(x) = se(x)^2$

(6) $cov(x,y) = E(x \cdot y) - E(x) \cdot E(y)$

(7)

$cov[C,e)$

$= E[ce] - E[C] \cdot E[e]$ using 6

$= E[(m_1-m_2)(p_1-p_2)] - E[m_1-m_2] \cdot E[p_1-p_2]$

$= E[m_1p_1 - m_1p_2 - m_2p_1 + m_2p_2] - (E[m_1]-E[m_2]) \cdot (E[p_1]-E[p_2])$

$= E[m_1p_1] - E[m_1p_2] - E[m_2p_1] + E[m_2p_2] - E[m_1] \cdot E[p_1] - E[m_1] \cdot E[p_2] - E[m_2] \cdot E[p_1] - E[m_2] \cdot E[p_2]$

$= (E[m_1p_1] - E[m_1]E[p_1]) - (E[m_1p_2] - E[m_1]E[p_2]) - (E[m_2p_1] - E[m_2]E[p_1]) - (E[m_2p_2] - E[m_2]E[p_2])$

$= cov[m_1, p_1] - cov[m_1, p_2] - cov[m2, p_1] + cov[m_2\ p_2]$ no relation between treatments

$= cov[m_1, p_1] + cov[m_2, p_2]$

(8) $cov[m_1, p_1]$

$= cov\left[\underbrace{\frac{1}{n}\sum x_i}_{\text{mean cost treatment 1}}, \underbrace{\frac{1}{n}\sum y_i}_{\text{mean survival rate treatment 1}}\right]$

$= \frac{1}{n^2} E[\sum x_i \sum y_i] - \frac{1}{n}E[\sum x_i] \cdot \frac{1}{n}E[\sum y_i]$ using 6

$= \frac{n}{n^2} E[x_i y_i] - \frac{n}{n}E[x_i] \cdot \frac{n}{n}E[y_i]$ the sum of a constant c is the same as $n \cdot c$

$= \frac{1}{n} E[x_i y_i] - E[x_i]E[y_i]$

$= \frac{cov(x_i, y_i)}{n}$

$se(C) = \sqrt{se(m_1)^2 + se(m_2)^2}$

$se(e) = \sqrt{se(p_1)^2 + se(p_2)^2}$

$se(p_2) = \sqrt{m_1(1-m_1)/n_1}$

$var[\log(c/e)]$

$= var[\log(c)] - \log(e)]$ using 1

$= var[\log(c)] + var[\log(e)] - 2cov(\log(C), \log(e)]$ using 2,3

$= \frac{var(c)}{c^2} + \frac{var(e)}{e^2} - 2cov(c,e) \cdot \frac{1}{c} \cdot \frac{1}{e}$ using 4

$= \frac{se(c)^2}{c^2} + \frac{se(e)^2}{e^2} - \frac{2\,cov(c,e)}{c \cdot e}$ using 5

$= \frac{se(c)^2}{c^2} + \frac{se(e)^2}{e^2} - 2\,\frac{cov(m_1, p_2) + cov(m_2, p_2)}{c \cdot e}$

this corresponds to the dataframe values

$= \frac{se(c)^2}{c^2} + \frac{se(e)^2}{e^2} - 2\,\frac{cov(x_{i1}, y_{i1})/n + cov(x_{i2}, y_{i2})/n}{c \cdot e}$ using 8