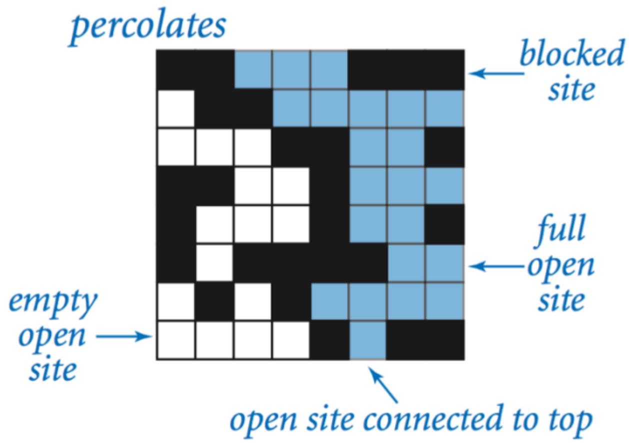
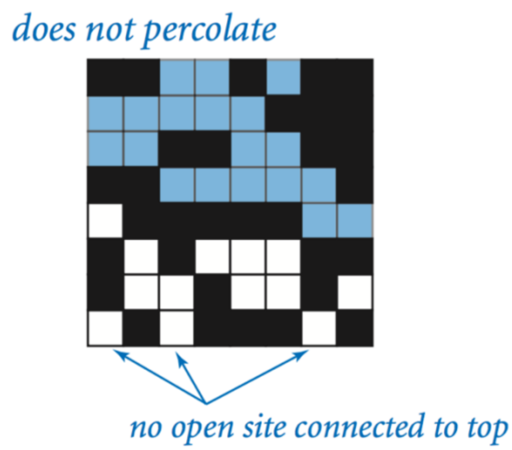
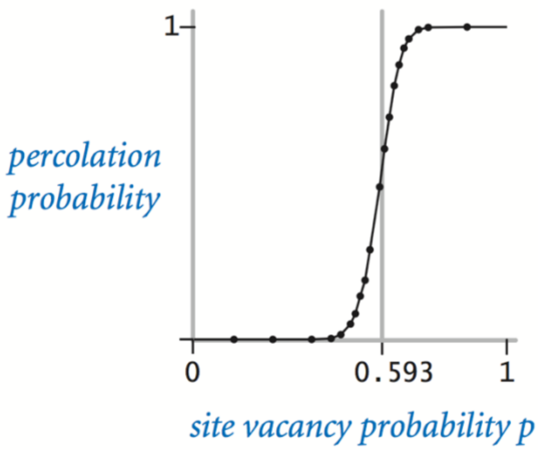
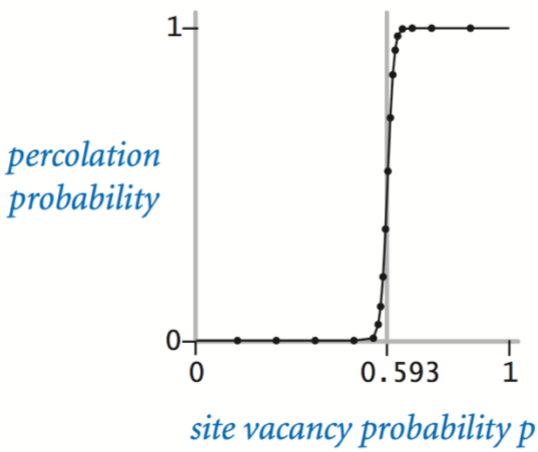
The model. This model is a percolation system using an n-by-n grid of *sites*. Each site is either *open* or *blocked*. A *full* site is an open site that can be connected to an open site in the top row via a chain of neighboring (left, right, up, down) open sites. We say the system *percolates* if there is a full site in the bottom row. In other words, a system percolates if we fill all open sites connected to the top row and that process fills some open site on the bottom row. (For the insulating/metallic materials example, the open sites correspond to metallic materials, so that a system that percolates has a metallic path from top to bottom, with full sites conducting. For the porous substance example, the open sites correspond to empty space through which water might flow, so that a system that percolates lets water fill open sites, flowing from top to bottom.)





The problem. In a famous scientific problem, researchers are interested in the following question: if sites are independently set to be open with probability p (and therefore blocked with probability $1 - p$), what is the probability that the system percolates? When p equals 0, the system does not percolate; when p equals 1, the system percolates. The plots below show the site vacancy probability p versus the percolation probability for 20-by-20 random grids (left) and 100-by-100 random grids (right).



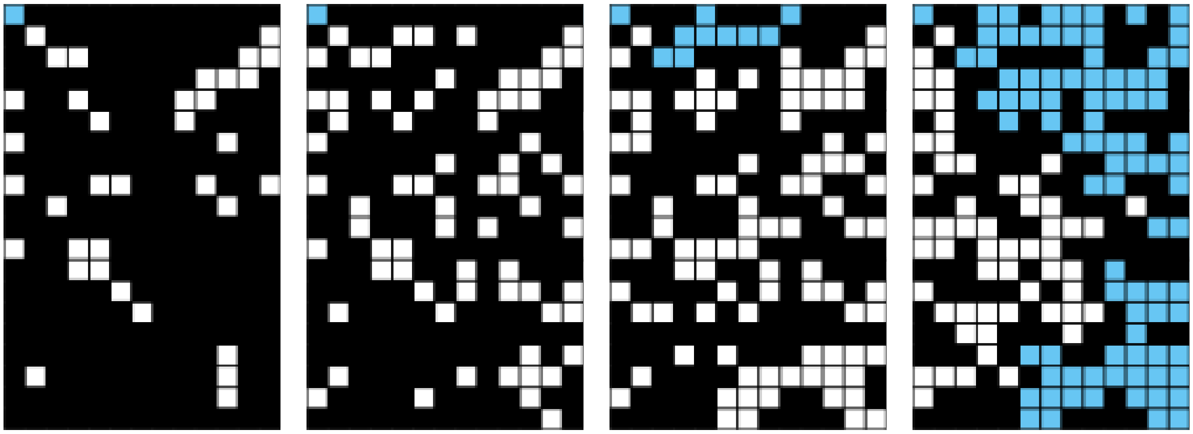


When n is sufficiently large, there is a *threshold* value p^* such that when $p < p^*$ a random n-by-n grid almost never percolates, and when $p > p^*$, a random n-by-n grid almost always percolates. No mathematical solution for determining the percolation threshold p^* has yet been derived. This project is a computer program to estimate p^*.

Monte Carlo simulation. To estimate the percolation threshold, this project considers the following computational experiment:

- Initialize all sites to be blocked.
- Repeat the following until the system percolates:
    - Choose a site uniformly at random among all blocked sites.
    - Open the site.
- The fraction of sites that are opened when the system percolates provides an estimate of the percolation threshold.

For example, if sites are opened in a 20-by-20 grid according to the snapshots below, then our estimate of the percolation threshold is 204/400 = 0.51 because the system percolates when the 204th site is opened.



50 open sites *100 open sites* *150 open sites* *204 open sites*

By repeating this computation experiment T times and averaging the results, we obtain a more accurate estimate of the percolation threshold. Let x_t be the fraction of open sites in computational experiment t. The sample mean $\bar{x}$ provides an estimate of the percolation threshold; the sample standard deviation s measures the sharpness of the threshold.

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_T}{T}, \quad s^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_T - \bar{x})^2}{T-1}$$

Assuming T is sufficiently large (say, at least 30), the following provides a 95% confidence interval for the percolation threshold:

$$\left[\bar{x} - \frac{1.96 s}{\sqrt{T}}, \bar{x} + \frac{1.96 s}{\sqrt{T}}\right]$$