COORDINATE SYSTEM PLOTTING FOR ANTENNA MEASUREMENTS

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ABSTRACT

Antenna measurement data is collected over a surface as a function of position relative to the antenna. The data collection coordinate system directly affects how data is mapped to the surface: planar, cylindrical, spherical or other types. Far-field measurements are usually mapped or converted to spherical surfaces from which directivity, polarization and patterns are calculated and projected. Often the collected coordinate system is not the same as the final-mapped system, requiring special formulas for proper conversion. In addition, projecting this data in two and three-dimensional polar or rectangular plots presents other problems in interpreting data. This paper presents many of the most commonly encountered coordinate system formulas and shows how their mapping directly affects the interpretation of pattern and polarization data in an easily recognizable way.

Keywords: CAD, Coordinate systems, pattern, polarization, mapping, directivity, conversion.

1.0 Introduction

Antenna measurements are made to show the performance of antenna: gain, pattern, directivity, cross-pol *etc*.. Data collection of performance characteristics come in the form of printed patterns, exported files and interactive computer displays. Various formats have been designed to allow the user to quickly compare antenna performance to expected results. These comparisons are often in the form of overlaid patterns, pass-fail spec lines or require additional computation by other computer programs.

It is important to understand the details of the measurement coordinate system prior to comparing between measurement data to expected results or data taken on another range. The rotation of the antenna and/or probe when making measurements will directly affect the patterns produced. In addition, natural polarization vectors are produced by a positioner system and these can by quite different if compared to those of a different type of positioner.

Figure 1 shows a classical Roll-over-Elevation-over-Azimuth positioner. This is a very common type of positioner because it supports the three standard types of spherical coordinate systems. For the purposes of this paper the authors will confine formulas and geometries to this type of positioner. It is left to the reader to apply the formulas supplied here to their particular positioner system.

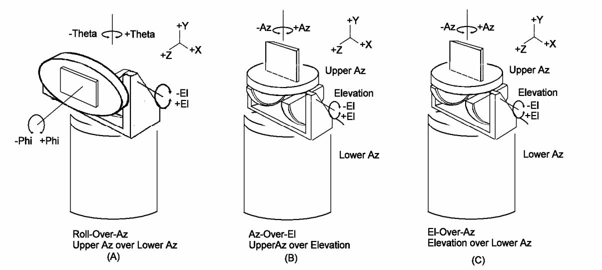


Figure 1 Classical Roll-over-Elevation-over-Azimuth positioner

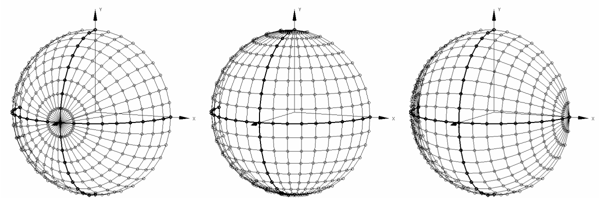
Figure 1A shows the antenna-under-test (AUT) mounted on the Roll axis with the Elevation axis at 90°. This is a standard Theta-Phi coordinate system. Figure 1B and Figure 1C show how the antenna is mounted for the two other standard coordinate systems. Each system consists of two movable axes defined for that system and one fixed axis that is not part of the system. Each system has a natural pole in a different direction. The pole is where the AUT does not change its pointing angle in space when one of the two defined axes is rotated, *i.e.* a singularity. Table 1 shows how the positioner axes are set up to make each coordinate system.

Table 1 Coordinate system definition for 3-axis positioner

System	Pole	Roll (Upper-Az)	Elevation	Azimuth (Lower-AZ)
θ- ϕ	Z-axis	Phi	Fixed at 90.0	Theta
Az/El	Y-axis	Azimuth	Elevation	Fixed at 0.0
El/Az	X-axis	Fixed at 0.0	Elevation	Azimuth

Figure 2 shows the angles a far-field probe makes with respect to the AUT as it is rotated. Note: it is important to remember that there is a difference between the angle the antenna points in space and the angle made between the probe and positioner. In Figure 2 A, B and C, the cardinal cuts, which as was described above are equivalent, can be seen plotted in bold.

Each coordinate system has two angles and two poles. Angle-1 is measured relative to the pole axis. A complete circle of Angle-1 will go through each pole. The other angle (Angle-2) moves around the pole. The size of the circle for Angle-2 is a function of Angle-1. Figure 2 shows the angles for each coordinate system. Table 2 shows the relationship between Angle-1 and Angle-2 for each coordinate system.



(A) θ-ϕ, (B) Az-over-El (C) El-over-Az

Figure 2 Probe-to-AUT rotation angles

Table 2 Relationship of two angles to coordinate system

System	Pole	Angle-1 (Moves through pole)	Angle-2 (Moves around pole)
θ- ϕ	Z-axis	θ	ϕ
Az/El	Y-axis	Elevation	Azimuth
El/Az	X-axis	Azimuth	Elevation

Proper antenna positioner rotation is important to making sure that the range is properly defined. Far-field positioners often include encoders whose polarity can be changed based on various needs. The standard definition for positioners is that when looking from the top of the rotation platen, a clockwise rotation should produce a positive-going angle (*e.g.* -170 to -150 or +10 to +20). In the case of the Elevation positioner, positive angles will expose the upper side of the AUT to the source antenna. Figure 1 shows the rotation of each axis to produce a positive angle. Often the polarity can be reversed by reversing the leads on two of the encoder wires.

2.0 Two-Dimensional Antenna Measurements

The simplest antenna patterns or cuts are made by rotating only one axis and recording the amplitude and/or phase at defined locations while the other axis is fixed. Even though each coordinate system is different, it turns out that there are two identical cuts that can be made with each system. These are called the Cardinal cuts and they correspond to a Horizontal and Vertical cut as seen from the probe. Table 3 shows how the axes are moving in the three coordinate systems to take the Cardinal cuts. The patterns are often display as 2-dimensional polar or rectangular plots (see Figure 3); hence the name two-dimensional (2D) antenna measurements.

It is important to note that there are no other cuts that are the same between the two coordinate systems.

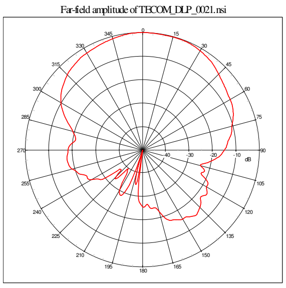
Table 3 Cardinal cut definition vs. coordinate system

Cardinal cut	System	Roll (Upper-Az)	Elevation	Azimuth (Lower-AZ)
Horizontal	θ- ϕ	Phi = 0.0	Fixed at 90.0	Theta-cut
Vertical	θ- ϕ	Phi = 90.0	Fixed at 90.0	Theta-cut
Horizontal	Az/El	Az-cut	Fixed at 0.0	Fixed at 0.0
Vertical	Az/El	Fixed at 0.0	El-cut	Fixed at 0.0
Horizontal	El/Az	Fixed at 0.0	Fixed at 0.0	Az-cut
Vertical	El/Az	Fixed at 0.0	El-cut	Fixed at 0.0

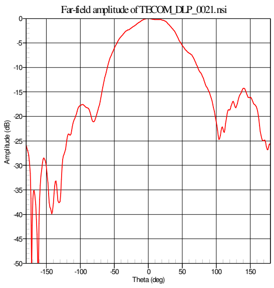
3.0 Three-Dimensional Antenna Measurements

Three-dimensional (3-D) antenna measurements are made by rotating two axes to sweep out a full sphere or section thereof and recording the amplitude and/or phase at defined locations. In practice, it is usually not possible to measure the complete sphere without some blockage due to the positioner. Nonetheless, a complete sphere can be measured by rotating one axis through 180° and the other through 360°. Note: Some configurations, such as Elevation-over-azimuth may have additional restrictions due to the mechanical make up of the positioner. In the positioner configuration shown in Figure 1C, the Elevation axis is restricted to -45° < El < +90°

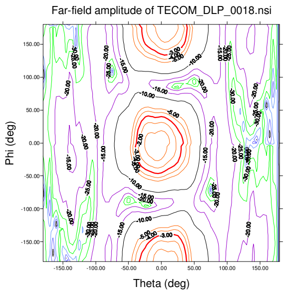
When 3-D data is collected it is often printed in various formats to show the complete data set. Figure 3 shows various formats (*e.g.* Contour, Waterfall *etc.*) In Contour and Waterfall plots there will be a Horizontal and Vertical axis for the plot. If the center of the plot corresponds to the point where the coordinate system's defined axes are both at exactly zero degrees, then the plot will include the Cardinal cuts. Sometimes a section of the sphere will be magnified (zoomed-in) to analyze the pattern. In this case the Cardinal cuts may not be included. When this is the case the 3D patterns can look quite different between the three coordinate systems. The further away from the Cardinal cuts that the points are, the more different the plots look.



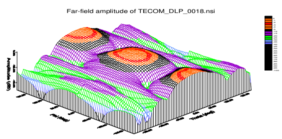
Polar cut (A)



Rectangular cut (B)



Contour plot (C)



Waterfall plot (D)

Figure 3 2D and 3D plots

In order to show how different the patterns can be, consider a map of the Earth plotted in each of the three coordinate systems. In each case the H = 0, V = 0, point is on the equator in the Atlantic ocean off the West coast of Africa. As shown in Figure 4, the image between Elevation = ±90° is easily recognizable as a Mercator projection [1], *i.e.* azimuth equates to longitude whilst elevation equates to latitude. The image beyond Elevation = ±90°shows the alternate sphere (remember, the data is from a double sphere).

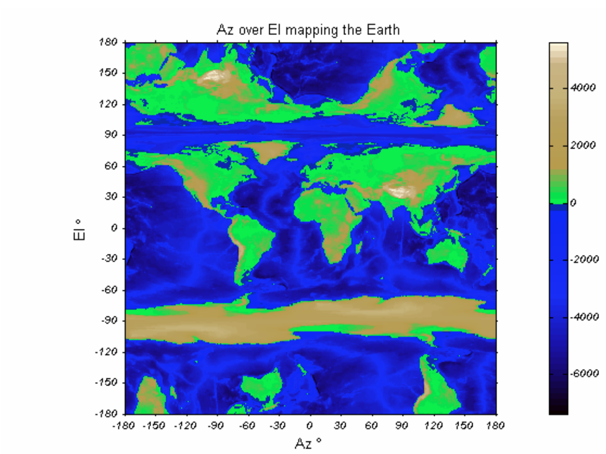


Figure 4 Earth mapped using an Az/El positioner system

This map corresponds to an Az/El positioner system whose pole is at the Y-axis. Note the distortion at the North and South poles. This is because the point at Elevation = ±90° is the same point in space irrespective of the Azimuth angle. By plotting the data in this way , each of the poles (which is a single point in space) has been stretched out until it becomes a line as long as the equator. This causes the map to be stretched near the pole. As can be seen in Figure 5, a similar thing happens in the El/Az projection although here the distortion appears near the X-axis pole (Azimuth = ±90°).

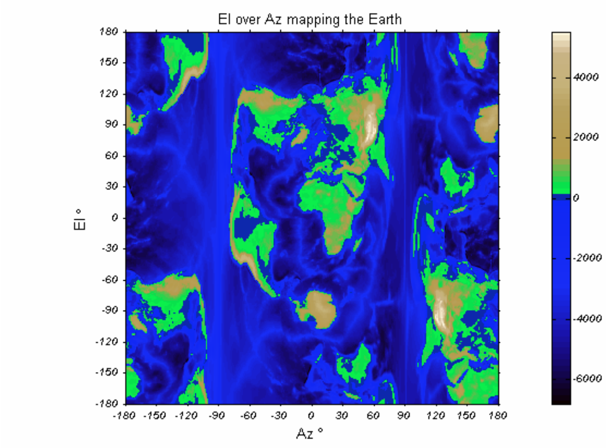


Figure 5 Earth mapped using an El/Az positioner system

In the third case, there are two ways to show the map. One is to display the θ axis along the Horizontal axis of the plot, with the ϕ axis as the Vertical axis of the plot forming a rectangular plot as shown in Figure 6. In this case distortion appears greatest along the Z-axis pole (Theta = 0°, 180°).

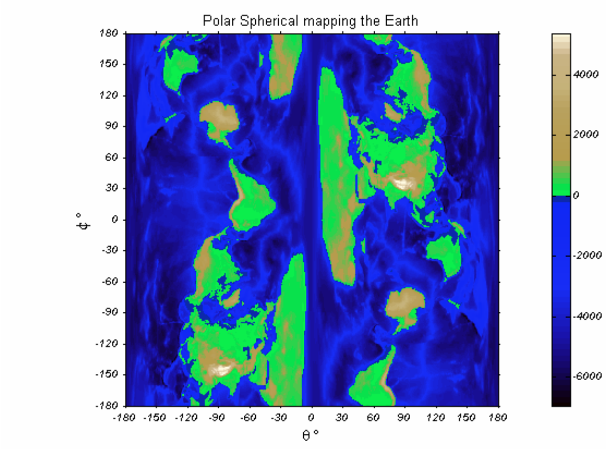


Figure 6 Earth mapped using a polar spherical roll over theta positioning system

The alternative possibility is to show the θ cuts in the form of a polar diagram. Figure 7 shows the θ cuts plotted radially with each cut being rotated by an amount determined by the ϕ angle. Here, parts of the pattern with θ angles < 0 correspond to the alternate sphere. When the angles are in radians, the mapping from the alternate sphere to the conventional sphere can be expressed mathematically as,

$$\theta \to -\theta \tag{1}$$

$$\phi \to \phi - \pi \tag{2}$$

which is obvious from inspection this figure.

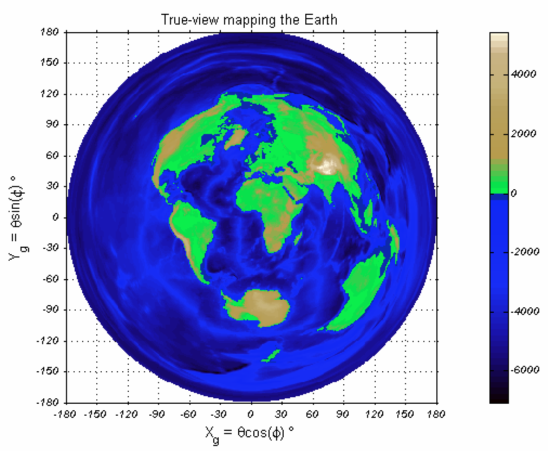


Figure 7 Earth mapped using a polar spherical roll over theta positioning system

It is important to note that the Earth's continents have not changed, this is a given. Their relative spacing to one another is also the same. It is the spherical projection onto a flat piece of paper that distorts the image. This distortion is an unavoidable consequence of trying to represent a three-dimensional object on a two-dimensional piece of paper. Flat maps could not exist without map projections because a sphere cannot be laid flat over a plane without distortion. This can be seen mathematically as a consequence of Gauss's Theorema Egregium [2] which essentially states that it is not possible to bend a finite sized, *i.e. not* infinitesimal, piece of paper onto the surface of a sphere.

Overlaying one of the three projections onto the others, as in the case of overlaying antenna patterns from ranges with different positioning systems, is worthless in identifying differences between the patterns except on, or *very* near, the Cardinal cuts. To compare data at other cuts between the three systems, a conversion, *i.e.* transformation, must be implemented between one coordinate system and another.

Figure 7 above differs from the previous plots as this is not achieved by plotting the measured data using a rectangular, *i.e.* raster, format. Relaxing the rigid connection between plotting system and positioner enables the introduction of other alternative plotting systems that can potentially offer advantages when interpreting the patterns. By way of illustration, Figure 8 contains an Earth map when plotted using a direction cosine coordinate system. This system is essentially the same as the k-space coordinate system since the two are related by a linear scaling of the free-space propagation constant k_0. However, the direction cosine system has the inherent advantage that the system is not dependant upon (scaled by) the frequency of the radiated field.

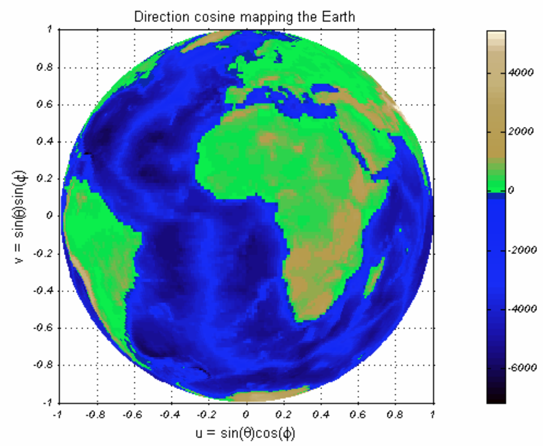


Figure 8 Earth mapped using a direction cosine plotting system

Clearly, the direction cosine system corresponds to an orthographic projection in which the sphere is projected onto a tangent, or secant, plane. Here, only a half space is visible at any one time and points on the plot for which,

$$u^2 + v^2 > 1 \tag{3}$$

correspond to real ϕ angles and complex θ angles. If this system is used to plot the angular spectrum of plane waves, then propagating field will be contained within the parts of the pattern when $u^2 + v^2 \leq 1$, *i.e.* visible space, and the reactive field will be contained with parts of the pattern when $u^2 + v^2 > 1$. Thus, when plotting true asymptotic far-field patterns the field will be identically zero outside the unit circle. Another useful plotting system, which is sometimes favoured by designers of active electronically scanned array antennas, can be obtained by taking the arcsine of the x and y plotting axes. This is illustrated in Figure 9 below. Here, anything outside of the unit square corresponds to invisible space.

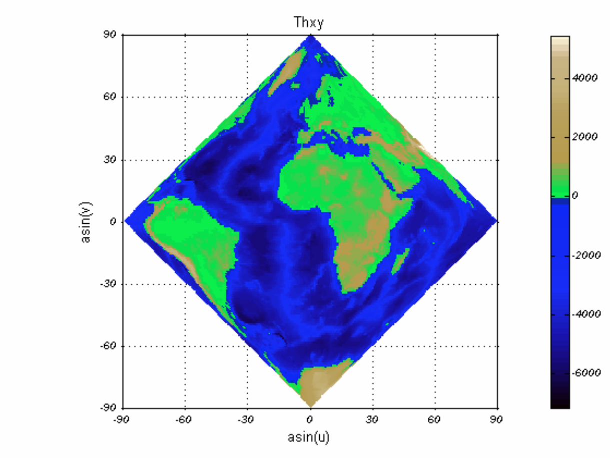


Figure 9 Earth mapped using a arcsin-space plotting system

More recently, antenna patterns have been presented in a virtual three-dimensional space with the radius equating to the amplitude or phase of the patterns at a given direction in space. Figure 10 contains a three-dimensional representation of the Earth with the topographical features having been plotted in the radial direction.

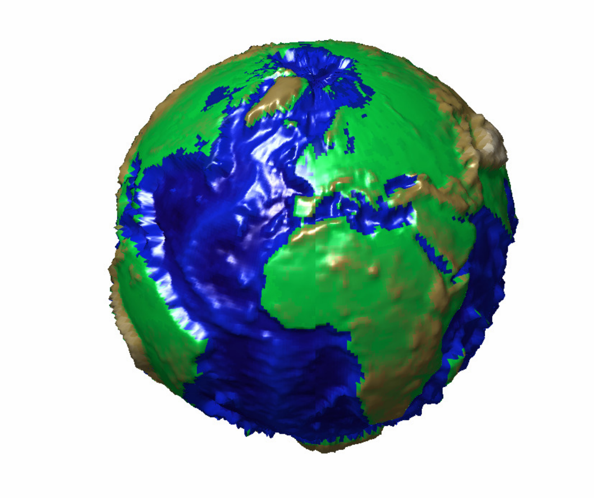


Figure 10 Three dimensional “virtual-reality” view of the Earth showing topographical data

Although the Earth is actually an oblate spheroid, for the purposes of visualizing antenna patter function, the writers have assumed that the Earth is a perfect sphere.

4.0 Coordinate System Transformation Formulas

In general, a straightforward reliable method for transforming from one coordinate system to another is by means of equating Cartesian direction cosines. The goal is to create a map such that the horizontal and vertical dimensions of the plot (H and V respectively) can be related between coordinate systems. The H and V axes will be described such that they are plaid, monotonic and equally spaced. This is described with the following expressions,

$$H = H_0 + \Delta H(n-1) \tag{4}$$

$$V = V_0 + \Delta V(p-1) \tag{5}$$

Here n and p are positive integers, n = 1, 2, 3, …N and p = 1, 2, 3, … P. Here, V_0 and H_0 are the starting values of the grid in the h- and v-plotting axes respectively, ΔH, ΔV are the incrementing values in the h- and v- plotting axes respectively.

4.1 Direction Cosines and Angles

Direction cosines use three direction angles α, β and γ to identify a point in space. The point is described by a vector r measured from the origin to the point. Direction cosines relate the vector r to Cartesian co-ordinates where,

$$r = u\underline{\hat{a}}_x + v\underline{\hat{a}}_y + w\underline{\hat{a}}_z \tag{6}$$

u, v and w, which are called the direction cosines are the weightings for each unit vector. They are described by the following expressions,

$$u = \cos\alpha = a\Big/\sqrt{a^2+b^2+c^2} \tag{7}$$

$$v = \cos\beta = b\Big/\sqrt{a^2+b^2+c^2} \tag{8}$$

$$w = \cos\gamma = c\Big/\sqrt{a^2+b^2+c^2} \tag{9}$$

Each angle α, β and γ is measured from the X, Y and Z axes respectively. Using these expressions, a straight-forward conversion can be done between any of the three typical antenna measurement co-ordinates systems: Theta-Phi, Azimuth-over-Elevation, Elevation-over-Azimuth. This is done by setting the length or $|r|$ = 1 and relating the two angles in each antenna coordinate system to the three direction cosines.

4.2 Polar-Spherical (Theta-Phi)

As shown in Figure-1A, the rotation of the ϕ angle is made around the θ angle. This means that a pole is produced at the Theta points 0°, 180°, 360°, … . The Pole is along Z-axis, thus $\theta = \gamma$ and ϕ will be a combination of α, β angles in the following way,

$$u = \cos\alpha = \sin\theta\cos\phi \tag{10}$$

$$v = \cos\beta = \sin\theta\sin\phi \tag{11}$$

$$w = \cos\gamma = \cos\theta \tag{12}$$

4.3 Azimuth-Over-Elevation

As shown in Figure-1B, the rotation of the Azimuth angle is made around the Elevation angle. This means that a pole is produced at two Elevation points -90° and 90°. The Pole is along Y-axis, thus El= -β +90° and Az will be a combination of α and γ angles in the following way,

$$u = \cos\alpha = \sin Az \cos El \tag{13}$$

$$v = \cos\beta = \sin El \tag{14}$$

$$w = \cos\gamma = \cos Az \cos El \tag{15}$$

4.4 Elevation-Over-Azimuth

As shown in Figure 1C, the rotation of the Elevation angle is made around the Azimuth angle. This means that a pole is produced at two Azimuth points -90° and 90°. The Pole is along X-axis, thus Az= -γ +90° and Az will be a combination of α and γ angles in the following way,

$$u = \cos\alpha = \sin Az \tag{16}$$

$$v = \cos\beta = \cos Az \sin El \tag{17}$$

$$w = \cos\gamma = \cos Az \cos El \tag{18}$$

Thus the three can be combined into one table as shown in Table 4.

Table 4 Direction cosines for three coordinate systems

System	u	v	w
θ- ϕ	Sin(θ)Cos(ϕ)	Sin(θ)Sin(ϕ)	Cos(θ)
Az/El	Cos(El)Sin(Az)	Sin(El)	Cos(El)Cos(Az)
El/Az	Sin(Az)	Cos(Az)Sin(El)	Cos(El)Cos(Az)

4.5 True-View

Unlike the coordinate systems described above, the true-view coordinate system is not a rectangular raster plot of data recorded as a function of two co-ordinates. Instead, it is a polar representation of the polar spherical (θ, ϕ) coordinate system. Thus, the *x*- and *y*-axes of the plot, denoted by X_g and Y_g respectively, are related to the spherical angles through,

$$\theta = \sqrt{X_g^2 + Y_g^2} \tag{19}$$

$$\phi = \arctan\left(\frac{Y_g}{X_g}\right) \tag{20}$$

Here, arctan is used to denote the four-quadrant arc tangent function which has a range of $-\pi$ to π. Thus,

$$u = \cos\alpha = \sin\left(\sqrt{X_g^2 + Y_g^2}\right)\cos\left(\arctan\left(\frac{Y_g}{X_g}\right)\right) \tag{21}$$

$$v = \cos\beta = \sin\left(\sqrt{X_g^2 + Y_g^2}\right)\sin\left(\arctan\left(\frac{Y_g}{X_g}\right)\right) \tag{22}$$

$$w = \cos\gamma = \cos\left(\sqrt{X_g^2 + Y_g^2}\right) \tag{23}$$

4.6 Direction Cosine

Direction cosine coordinate system has no direct analogy with an arrangement of rotation stages. However, we are still not completely free to chose the values of *u*, *v* and *w* as the length of the unit vector which these components represent has, by definition, a length of unity. Thus,

$$1 = u^2 + v^2 + w^2 \tag{24}$$

4.7 θ_{xy}

Again, the θ_{xy} coordinate system has no direct analogy with an arrangement of rotation stages and is instead most closely related to the direction-cosine system as,

$$u = \cos\alpha = \sin X_g \tag{25}$$

$$v = \cos\beta = \sin Y_g \tag{26}$$

$$w = \cos\gamma = \sqrt{\sin^2 X_g + \sin^2 Y_g} \tag{27}$$

4.8 Viewing Angle

When making a projection, another important idea is that of a viewing angle. This is related to the H and V-axes of the plot, discussed in Section 4.0 The viewing angle is related to where the H and V-axes are centered and how they are aligned to the original coordinate system. The plot center (H_0, V_0) is the angle relative to the original coordinate system and is not just a shift in the rotation of the sphere. Because of this, the pattern at the center of the plot could be distorted (*e.g.* H= 90°, V = -90° in an Az/El coordinate system). The angles are allowed to wrap but the distortion at the poles of the plotting system will still apply. To illustrate this, Figure 11 contains a plot of the Earth tabulated on a regular azimuth over elevation coordinate system, *c.f.* Figure 4 above. However, in this case the Earth has been rotated through -90° about the positive x-axis so that Antarctica, which is Earth's most southerly continent is now plotted at the equator of the plotting coordinate system. Essentially, this is similar to viewing the Earth from sub-satellite latitude =-90°, sub satellite longitude = 0°. Note also, that although Earth map has been rotated, the poles of the plotting system are still located at ±90° in elevation and the equator is still at 0° and ±180° in elevation.

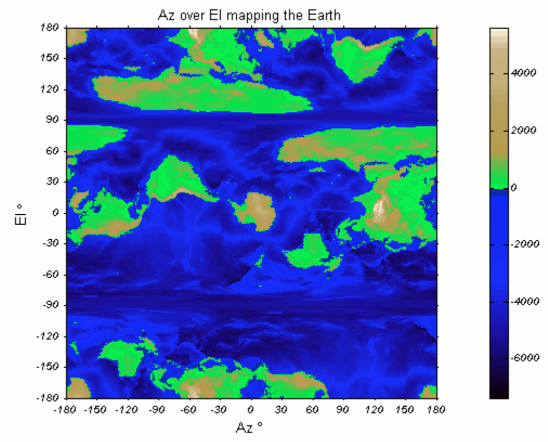


Figure 11 Earth mapped using an Az/El positioner system with the Earth rotated about the *x*-axis by -90°

Such isometric rotations are easily implemented by using a transformation matrix to rotate the triad of direction cosines. Transformation matrices are matrices that post-multiply a column point vector to produce a new column point vector. A series of transformation matrices may be concatenated into a single matrix by matrix multiplication. A transformation matrix may represent each of the operations of translation, scaling, and rotation. However, if A is a three by three orthogonal, normalised, square matrix, it may be used to specify an isometric rotation that can be used to relate two frames of reference, *i.e.* two coordinate systems. Here, an isometric rotation is taken to mean a transformation in which the distance between any two points on an object remains invariant under the transformation. Any number of angular definitions for describing the relationship between the two coordinate systems are available. However, if the angles azimuth, elevation and roll are used, where the rotations are applied in this order, we may write that a point in one frame of reference can be specified in terms of a point in the other frame of reference as,

$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = [A] \cdot \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{28}$$

Here primed coordinates are used to denote the rotated coordinate system. The necessary direction cosine transformation matrix can be obtained from a concatenation of a series of any number of rotations. Such transformation matrices can be easily derived either from geometry, or from trigonometric identities. Here, in accordance with the rules of linear algebra, the first rotation matrix must be written to the right with the next rotation being written to it's left and so on. Such rotations are termed passive as each successive rotation is applied to the newly rotated system. In this instance, a rotation of ψ about the positive x-axis can be expressed as,

$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{bmatrix} \cdot \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{29}$$

4.9 Antenna Pattern Viewing

An antenna pattern is not a physical boundary as are the continents of the world. It is a transparent snapshot at a particular radius. For this reason the observer can view the pattern that the antenna makes on the sphere from inside the sphere or from outside it. This orientation will change the H and V-axes slightly based on how they are related to the original angles. It is necessary then to add two additional constants to the formulas presented above. These constants are designated as $\boldsymbol{l}$ and $\boldsymbol{m}$. They are only permitted to take on the values ± 1 depending upon where the observer is situated with respect to the AUT. The three possible cases are as follows:

- l = 1, m = 1, observer facing the AUT. Thus, looking in the $-Z$ direction (into the page), the $+X$ axis is horizontal and increases towards the right and the $+Y$ axis is vertical and increases upwards.
- l =-1, m = 1, observer standing behind the AUT. Thus, looking in the $+Z$ direction (into the page), the $+X$ axis is horizontal and increases towards the left and the $+Y$ axis is vertical and increases upwards.
- l = 1, m =-1, observer standing behind the AUT. Thus, looking in the $+Z$ direction (into the page), the $+X$ axis is horizontal and increases towards the right and the $+Y$ axis is vertical and increases downwards.

Case 2 is commonly used within the space industry when plotting antenna patterns over Earth maps to demonstrate antenna performance compliance with a given coverage region, which are often specified in terms of geopolitical boundaries. Case 3 is commonly used within the RCS measurement community as targets are routinely mounted upside down on low RCS pylons. In our maps of the world concept, Case-1 would be looking from space at the earth. Case-2 would be looking from the center of the earth out with your feet pointing at the South Pole. Case-3 would be would be looking from the center of the earth out with your feet pointing at the North Pole. This can be expressed compactly in terms of a transformation matrix as follows,

$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} l & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{30}$$

5.0 Polarization

Thus far we have been able to describe a point in space with a particular antenna coordinate system and convert it to any other antenna coordinate system. Now it is time to introduce the concept of polarization. In the far-field, an antenna will produce an electric field perpendicular to the direction of propagation in some orientation on a plane. That orientation can be described in two spherical angles, similar to those shown here:

$$\underline{E}(r,\theta,\phi)=\left[A(\theta,\phi)\underline{\hat{e}}_{\theta}+B(\theta,\phi)\underline{\hat{e}}_{\phi}\right]e^{-jk_0 r} \tag{31}$$

$$\underline{E}(r,Az,El)=\left[A(Az,El)\underline{\hat{e}}_{Az}+B(Az,El)\underline{\hat{e}}_{El}\right]e^{-jk_0 r} \tag{32}$$

$$\underline{E}(r,\alpha,\varepsilon)=\left[A(\alpha,\varepsilon)\underline{\hat{e}}_{\alpha}+B(\alpha,\varepsilon)\underline{\hat{e}}_{\varepsilon}\right]e^{-jk_0 r} \tag{33}$$

Here, *A* and *B* are complex quantities. Any two unit vectors can be used to describe the polarization. There are two special cases for *A* and *B* that are important to consider and those are: Case-1, Linearly polarized (*A* or *B* = 0). In this case the resulting electric field is indicative of a field that is polarized in one direction with respect to the direction of propagation. Case-2, Elliptical polarization (*A* and *B* are 90° out-of-phase with each other). In this case the resulting field appears to rotate around the direction of propagation. In the special case where the magnitudes of *A* and *B* are equal, circular polarization results.

When an antenna pattern is measured either in receive or transmit mode, its pattern is a function of the polarization of the far-field probe that is used to measure it. For example, if the far-field probe, or source antenna as it is sometimes called, is predominantly polarized as that matching the AUT (co-polarized) then the pattern will have higher values than when it is mis-matched to the AUT polarization (cross-polarized). With the far-field probe antenna down range, pointing at the AUT positioner, at least two orientations of the probe are required to completely measure the AUT polarization. For convenience, the two orientations should be orthogonal to each other and perpendicular to the direction of propagation. In most ranges this is done by making a measurement and rotating the probe by 90° about its boresight direction, and repeating the measurement. If only a linear copolar and cross-polar pattern are required, the operator may decide to locate the peak of the pattern and then rotate the probe's angle so that the lowest value is received. This angle is the cross-polar angle. The copolar pattern is then measured 90° from this angle.

If a complete characterization of the polarization is required then the operator must measure both the amplitude and phase for two orthogonal polarizations. With these two measurements and a perfect far-field probe, any polarization can be synthesized. If the probe's polarization is not perfect additional correction must be done to take out the probe's polarization effects. This correction is straight forward but not part of the scope for this paper.

As was stated previously, each positioner configuration has a natural basis of polarization vectors. These vectors can be converted to other bases using a series of transformations.

5.1 LI, Cartesian Polarization Basis

The Cartesian polarization basis, Ludwig's definition I, [3] corresponds to resolving the electric field onto three unit vectors aligned with each of the three Cartesian axes. This can be expressed as,

$$\underline{E}(\underline{\hat{r}})=E_x(\underline{\hat{r}})\underline{\hat{e}}_x+E_y(\underline{\hat{r}})\underline{\hat{e}}_y+E_z(\underline{\hat{r}})\underline{\hat{e}}_z \tag{34}$$

This arrangement is illustrated in Figure 12 below which shows the "co-polar" (magenta) and "cross-polar" (red) unit vectors depicted as arrows placed over the surface of a unit sphere.

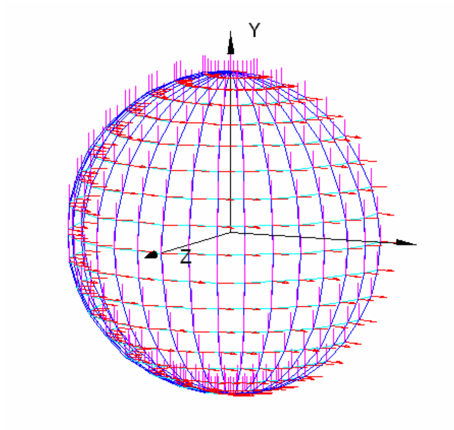


Figure 12 Cartesian Polarization Basis (LI), E_x, E_y

5.2 LII, E_{az} over E_{el} Polarization Basis

If instead the electric field is resolved onto a spherical polarization basis then it is possible to define three further polarization bases, each corresponding to placing the pole along the *x*-, *y*- or *z*-axes respectively with each corresponding to one of the positioner arrangements described above. Ludwig's definition II therefore has three variants although only two are useful in terms of a "co-polar" and "cross-polar" value on boresight. The azimuth-over-elevation polarization basis being the first of these which can be expressed as,

$$\underline{E}(\underline{\hat{r}})=E_{az}(\underline{\hat{r}})\underline{\hat{e}}_{az}+E_{el}(\underline{\hat{r}})\underline{\hat{e}}_{el} \tag{35}$$

As we are primarily concerned in this paper with plotting far-field antenna patterns the radial orientated field component is not considered, as this is identically zero which is a direct consequence of the plane wave condition. This arrangement is illustrated schematically in Figure 13 below which shows the "co-polar" (magenta) and "cross-polar" (red) unit vectors depicted as arrows placed over the surface of a unit sphere and correspond to the case where the pole is placed along the y-axis.

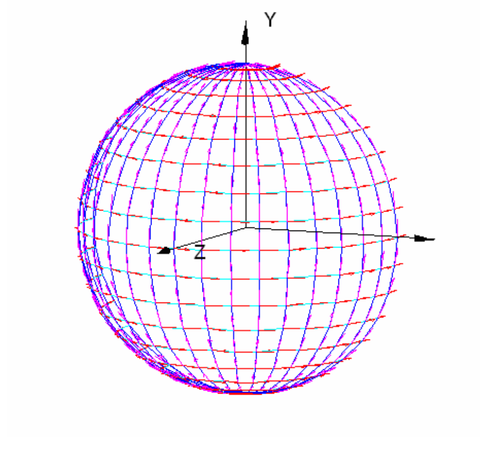


Figure 13 Az/El Polarization basis (LII)

5.3 LII E_{el} over E_{az} Polarization Basis

If one resolves the electric field onto a spherical polarization basis with the pole aligned with the *x*-axis then the elevation-over-azimuth polarization basis is obtained, this is the second of Ludwig's definition II which can be expressed as,

$$\underline{E}(\underline{\hat{r}}) = E_\alpha(\underline{\hat{r}})\underline{\hat{e}}_\alpha + E_\varepsilon(\underline{\hat{r}})\underline{\hat{e}}_\varepsilon \tag{36}$$

Again, this arrangement can be seen illustrated schematically in Figure 14,

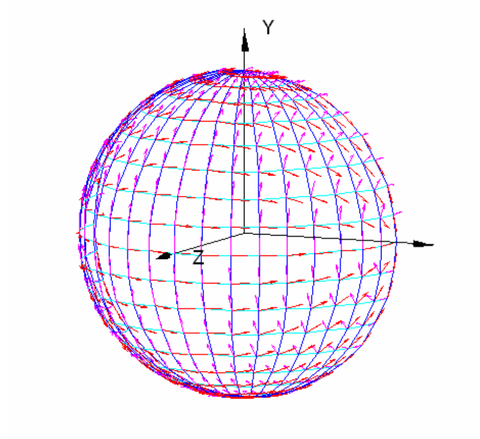


Figure 14 El/Az Polarization basis (LII)

5.4 Polar Spherical Polarization Basis

The third possibility involves placing the pole of the Polarization basis along the z-axis. This results in there not being a convenient principal copolar and cross-polar field value. However, since this is the Polarization basis that is most closely associated with the useful roll-over-azimuth positioning system, it is commonly encountered when making either near- or far-field spherical antenna pattern measurements. When the electric field is resolved onto this Polarization basis this can be expressed mathematically as,

$$\underline{E}(\underline{\hat{r}}) = E_\theta(\underline{\hat{r}})\underline{\hat{e}}_\theta + E_\phi(\underline{\hat{r}})\underline{\hat{e}}_\phi \tag{37}$$

This Polarization basis can be seen depicted in Figure 15.

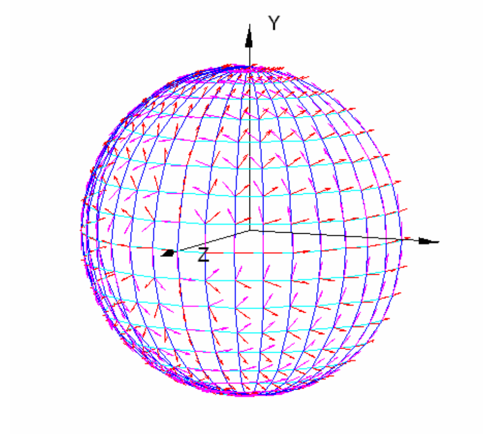


Figure 15 Polar-Spherical Polarization Basis

5.5 LIII E_{co}, E_{cross} Polarization Basis

The deficiencies of the polar-spherical Polarization basis described above can be resolved if we adopt Ludwig's definition III. This corresponds to taking measurements using a roll-over-azimuth positioner as described above and by utilizing an additional rotation stage mounted behind the probe. In this way, the probe can be counter-rotated so that as the AUT rotated in ϕ, the probe and AUT remain Polarization matched. This has the desired effect of removing the singularity on the positive Z-axis which is the deficiency of the polar spherical system. This can be expressed mathematically as,

$$\underline{E}(\underline{\hat{r}}) = E_{co}(\underline{\hat{r}})\underline{\hat{e}}_{co} + E_{cr}(\underline{\hat{r}})\underline{\hat{e}}_{cr} \tag{38}$$

This can be seen illustrated below in Figure 16. In practice when making pattern measurements, rather than pattern cut measurements, a third counter-rotating Polarization stage is seldom used instead, the necessary change of polarisation basis is implemented mathematically using Equation (44).

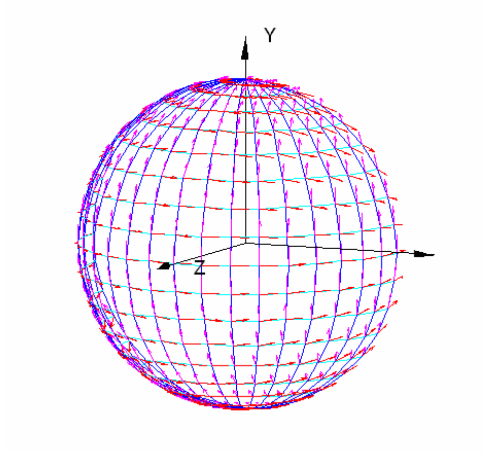


Figure 16 Ludwig III co-polar and cross-polar Polarization basis

5.6 Polarization synthesis

Two far-field orthogonal complex (amp/phase) measurements via probe rotation, will allow the polarization to be synthesized to any two orthogonal components. In general, the two desired components match those of the positioning system. This is because the amplitude of the measurement represents the copolar and cross-polar components even without polarization synthesis. Sometimes however, it is desired to convert from one set of polarization vectors to another; for example converting from E_θ-E_ϕ to E_{az}-E_{el} or vice-versa. In addition, as described above, sometimes it is convenient to convert from E_θ-E_ϕ to Ludwig-III definition. The transformation from Cartesian to polar-spherical field components can be accomplished by,

$$\begin{bmatrix} E_\theta \\ E_\phi \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta & \cos\phi & 0 \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \quad (39)$$

Once the polar-spherical field components are known it is a simple matter to transform to any of the other Polarization basis as,

$$\begin{bmatrix} E_{Az} \\ E_{El} \end{bmatrix} = \frac{1}{\cos El} \begin{bmatrix} \cos\phi & -\cos\theta\sin\phi \\ \cos\theta\sin\phi & \cos\phi \end{bmatrix} \cdot \begin{bmatrix} E_\theta \\ E_\phi \end{bmatrix} \quad (40)$$

$$\begin{bmatrix} E_\alpha \\ E_\varepsilon \end{bmatrix} = \frac{1}{\cos\alpha} \begin{bmatrix} \cos\theta\cos\phi & -\sin\phi \\ \sin\phi & \cos\theta\cos\phi \end{bmatrix} \cdot \begin{bmatrix} E_\theta \\ E_\phi \end{bmatrix} \quad (41)$$

Where α and ε stand for Az and El respectively in an El/Az system and from the identity $\cos^2 a + \sin^2 a = 1$, then clearly,

$$\cos El = \sqrt{1 - \sin^2\theta \sin^2\phi} \quad (42)$$

$$\cos\alpha = \sqrt{1 - \sin^2\theta \cos^2\phi} \quad (43)$$

Assuming a horizontal copolar definition the transform from spherical to LIII can be expressed as,

$$\begin{bmatrix} E_{co} \\ E_{cross} \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \cdot \begin{bmatrix} E_\theta \\ E_\phi \end{bmatrix} \quad (44)$$

The inverse transformations can be obtained by inverting these expressions thereby enabling us to transform from any one to any other basis.

5.7 Polarization pattern data

Polarization patterns include direction information about the antenna's performance in a particular orientation. For example, perhaps it is desired to know how sensitive the antenna is to signals oriented along the horizon as opposed to those vertical. In general, the AUT's pattern will not have the same polarization at all angles. In fact, at some angles the polarization could be directly opposite of the desired pattern. Some of these become design issues if cross-pol pattern rejection is of major concern. Understanding, polarization conversion and comparison between ranges using different positioner configurations is very important in determining cross-pol performance

6.0 Conclusions

The goal of this paper was to present concepts of antenna coordinate systems and polarization in a way that is easy to recognize. The differences in projected world maps on flat paper have been at issue for many years. The nuances in coordinate systems can sometimes cause the engineer to misinterpret important information or comparison data in the antenna pattern because it was acquired in a different coordinate system. In addition, it was desired that there be one place where all of the far-field coordinate conversion and polarization formulas could be found.

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[3] A.C. Ludwig, "The Definition of Cross-Polarization", IEEE Trans. Antennas Propagation, vol. AP-21 no. 1, pp. 116-119, Jan. 1973.

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