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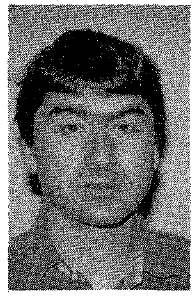

Christophe Granet

Editor's introduction

In this issue, we consider the design of axially symmetric dual reflectors, that is, Cassegrain and Gregorian reflectors. It seems that every time I perform a design-trade study on these reflectors, the initial parameters come in a different form, and I feel I have to start over and derive new equations. Thanks to Christophe Granet, of CSIRO in Epping, NSW 2121, Australia, I now have a complete set of equations for the geometry of these reflectors. This column contains more equations than normal, but every case is given. If you want to contact Christophe, his e-mail address is Christophe.Granet@tip.csiro.au, and I am sure he would enjoy feedback.

Recently, he has sent me extensions to the cases given below. You can look forward to the next issue of the *Magazine*, where he will present part two. Included in the next issue will be another software offer, available by e-mail, when I will finish reducing these equations and the next set to subroutines.

Designing Axially Symmetric Cassegrain or Gregorian Dual-Reflector Antennas from Combinations of Prescribed Geometric Parameters

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1. Abstract

A procedure to design axially symmetric Cassegrain or Gregorian dual-reflector antennas from various combinations of prescribed geometric parameters is presented. From these input parameters, the overall geometry of the antenna is derived in closed form.

2. Introduction

Axially symmetrical dual-reflector antennas (Cassegrain or Gregorian[1], classical or shaped) are of interest in radio astronomy and in Earth-station antenna technology. The design of such systems is often restricted by some mechanical constraints, the type of feed horn used, and the budget of the project (closely related to the size of the reflectors). Taking into account all of the above, various sets of input parameters, representing various solutions, are considered. From these input parameters, the overall geometry of the antenna is derived in closed form. From this starting geometry, the designer can shape the reflectors to obtain the desired radiation pattern.

3. Antenna geometry

Consider a classical Cassegrain or Gregorian dual-reflector antenna. The geometry of such a system is given in Figure 1, where Dm and Ds are the diameters of the main reflector and subreflector, respectively. The focal distance of the main reflector is F, and the distances between the apex of the main reflector (subreflector) and the phase center of the feed (the position of the secondary focus) are Lm (Ls), respectively. The angle between the z axis and the edge ray on the subreflector is θ_e.

The main-reflector profile, $z_{mr}(x_{mr}, y_{mr})$, depends on the real parameter F, and is of the form

[1]Cassegrain systems are derived from the seventeenth-century optical telescope, devised by the Abbé Cassegrain, while Gregorian systems are variants due to Gregory and Newton.

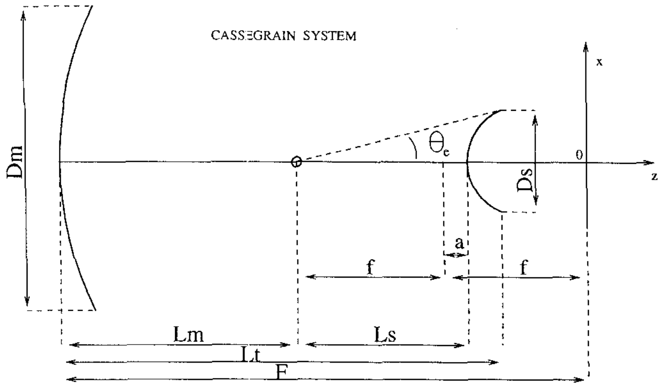

Figure 1a. The antenna geometry for the Cassegrain dual-reflector system.

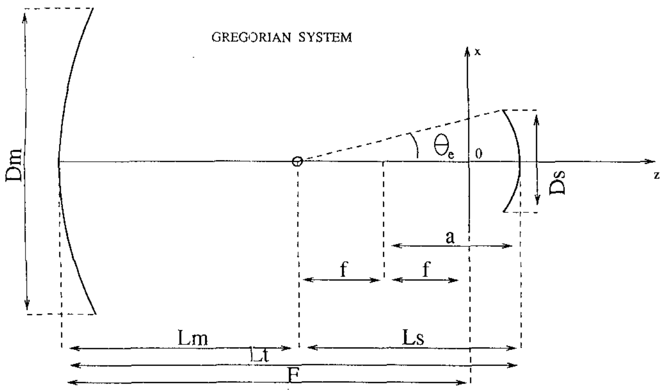

Figure 1b. The antenna geometry for the Gregorian dual-reflector system.

$$z_{mr}\left(x_{mr}, y_{mr}\right) = \frac{x_{mr}^2 + y_{mr}^2}{4F} - F$$

$$\text{with } \left(x_{mr}^2 + y_{mr}^2\right) \le \frac{Dm^2}{4}. \tag{1}$$

The subreflector profile, $z_{sr}(x_{sr}, y_{sr})$, depends on the two real parameters a and f (see Figure 1), and is of the form

$$z_{sr}\left(x_{sr}, y_{sr}\right) = a\sqrt{1 + \frac{x_{sr}^2 + y_{sr}^2}{f^2 - a^2}} - f$$

$$\text{with } \left(x_{sr}^2 + y_{sr}^2\right) \le \frac{Ds^2}{4}. \tag{2}$$

Depending on the range of the parameters a and f, different types of surfaces result:

$0 < a < f$: In this case, z_{sr} represents a hyperboloid, with axis of symmetry about the z axis, and with a focus at the origin of coordinates. The parameter f is half the distance between the focus and its image, and a is half the distance between the hyperboloid and its image, measured along the z axis. The eccentricity is $e = \frac{f}{a}$.

$a > f > 0$: In this case, z_{sr} represents an ellipsoid, with axis of symmetry about the z axis, and with one focus at the origin of coordinates. The parameter f is half the distance between the foci, and a is half the major axis of the ellipse. The eccentricity is $e = \frac{f}{a}$.

$a = 0$: In this case, z_{sr} represents a plane parallel to the (x,y) plane, with all z coordinates equal to $-f$.

4. A few design tips

- The subtended angle of the main reflector as a function of its focal-length-versus-diameter ratio, F/D, is given in Figure 2; refer to this figure if this angle is one of the antenna specifications (usually, antennas have a F/D ratio between 0.25 and 0.8).
- Try to make $Ds >> 5\lambda$, to avoid excessive diffraction losses.
- Try to make $Ds \le 0.1Dm$ for $>> 99\%$ blockage efficiency, as shown in Figure 3, and to ensure that the sidelobe levels are not excessive.
- Choose Ls keeping in mind the feed pattern to be used (near field or far field) for the analysis or synthesis.

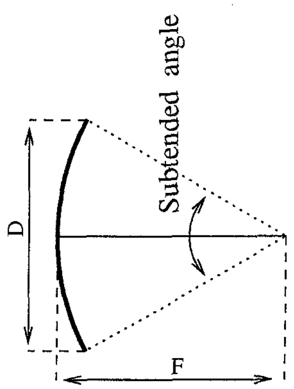

Figure 2a. The definition of the subtended angle of the main reflector.

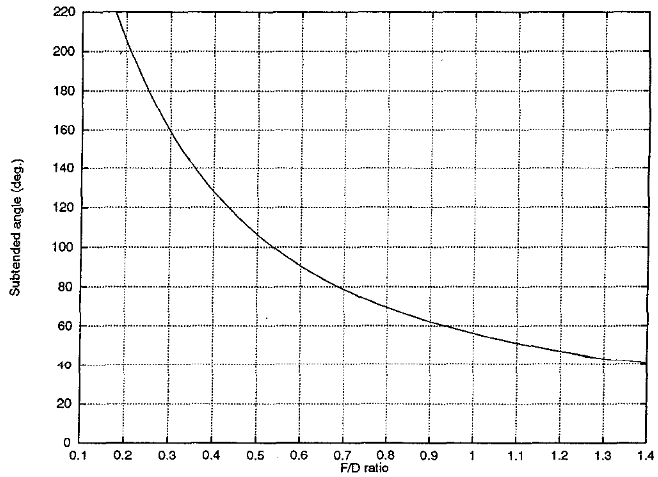

Figure 2b. The subtended angle of the main reflector as a function of its focal-length-versus-diameter ratio (F/D).

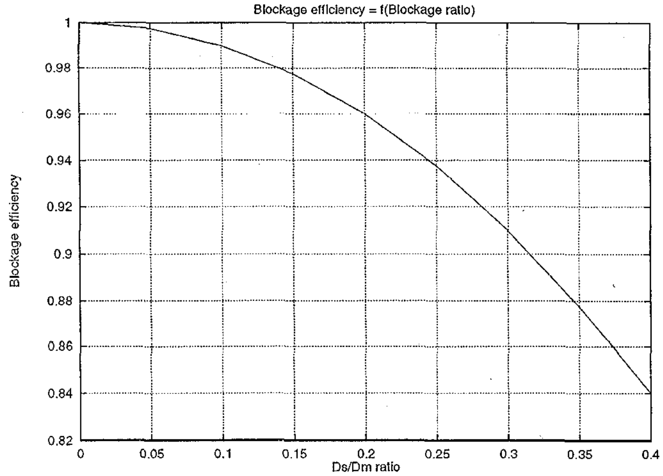

Figure 3. The blockage efficiency as a function of the blockage ratio (Ds/Dm).

- Choose Lm (which can be negative) so that the feed support will not lead to mechanical problems. Restrictions on feed location may include length or weight of the feed-support cone, space available behind the main reflector vertex, the lateral space available for the feed aperture, and the feed-system components.
- Choose θ_e to minimize the spillover, i.e., to have an edge illumination on the subreflector of the order of at least -10 to -15 dB.
- The gain of a Cassegrain or Gregorian antenna is given by

$$G = \eta \frac{\pi^2\left(Dm^2 - Ds^2\right)}{\lambda^2}, \tag{3}$$

where η is the overall antenna efficiency (the maximum gain of the antenna at a given frequency is for $\eta = 1$). This efficiency figure includes estimation of effects such as aperture-illumination efficiency, feed spillover losses, surface losses, etc. Antenna gain is usually a main requirement as well as the antenna efficiency (η) at a given frequency, so Equation (3) will constitute a condition on Dm and Ds. Of course, this gain formula does not include the blockage by the struts supporting the subreflector: this effect must be included in the overall antenna efficiency, η.

5. Design procedure

The following design procedure is based on Brown and Prata [1], but a number of different input parameters is considered. We are dealing with a system of eight parameters, defining the overall geometry of the antenna: Dm, F, Lm, Ds, Ls, a, f, and θ_e (see Figure 1). However, these parameters cannot be specified arbitrarily.

In the following procedure, we will consider various combinations of some of these parameters, and show that the remaining parameters can be calculated.

Another important parameter is Lt, the total length of the system, defined as

$$Lt = F + a\sqrt{1 + \frac{Ds^2}{4\left(f^2 - a^2\right)}} - f$$

for a Cassegrain antenna, and as

$$Lt = F + a - f$$

for a Gregorian antenna.

Different authors use different sets of parameters, depending on the antenna specifications they are given. For example, (Dm, Lm, Ls, θ_e) are used in Brown and Prata [1]; (Dm, F, Ds, θ_e) are used in Jensen [2]; (Dm, F, f, θ_e) are used in Hannan [3]; and (Dm, Ds, Lt, θ_e) are used in Lee et al. [4].

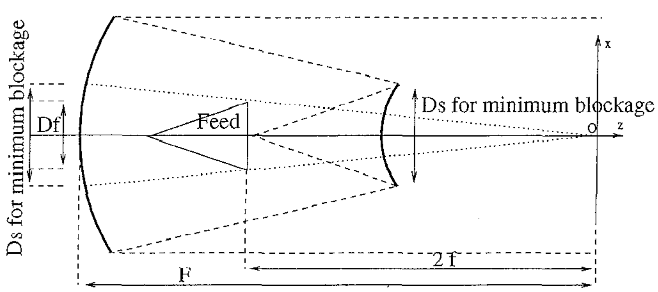

Figure 4. The condition for minimum blockage by the subreflector and feed.

Table 1. Seven cases where four input parameters are used, where there is no condition for the blockage.

Parameters	Dm	F	Lm	Ds	Ls	a	f	θ_e
Set No 1	Dm	(ii) Eq. 5	Lm	(iv) Eq. 7	Ls	(iii) Eq. 6	(i) Eq. 4	θ_e
Set No 2	Dm	F	Lm	(iv) Eq. 11	(ii) Eq. 9	(i) Eq. 8	(iii) Eq. 10	θ_e
Set No 3	Dm	F	(i) Eq. 12	(ii) Eq. 13	Ls	(iii) Eq. 14	(iv) Eq. 15	θ_e
Set No 4	(i) Eq. 16	F	(ii) Eq. 12	Ds	Ls	(iv) Eq. 14	(iii) Eq. 15	θ_e
Set No 5	(ii) Eq. 18	(iii) Eq. 5	Lm	Ds	Ls	(iv) Eq. 6	(i) Eq. 17	θ_e
Set No 6	Dm	F	(iv) Eq. 22	Ds	(i) Eq. 19	(ii) Eq. 20	(iii) Eq. 21	θ_e
Set No 7	Dm	(i) Eq. 24	(iv) Eq. 12	Ds	Ls	(iii) Eq. 14	(ii) Eq. 15	θ_e

Table 2. Seven cases where three input parameters are used, where the minimum-blockage conditon is specified.

Parameters	Dm	F	Lm	Ds	Ls	a	f	θ_e
Set No 1	Dm	F	Lm	(ii) Eq. 25	(iv) Eq. 27	(v) Eq. 6	(i) Eq. 10	(iii) Eq. 26
Set No 2	Dm	F	(ii) Eq. 29	(iii) Eq. 25	(iv) Eq. 27	(v) Eq. 6	(i) Eq. 28	θ_e
Set No 3	Dm	F	(ii) Eq. 29	Ds	(iv) Eq. 27	(v) Eq. 6	(i) Eq. 30	(iii) Eq. 26
Set No 4	Dm	(ii) Eq. 5	Lm	Ds	(iv) Eq. 27	(v) Eq. 6	(i) Eq. 31	(iii) Eq. 26
Set No 5	Dm	(i) Eq. 32	(iii) Eq. 29	Ds	(iv) Eq. 27	(v) Eq. 6	(ii) Eq. 30	θ_e
Set No 6	Dm	(i) Eq. 33	Lm	(iii) Eq. 25	(iv) Eq. 27	(v) Eq. 6	(ii) Eq. 10	θ_e
Set No 7	Dm	(i) Eq. 34	(iv) Eq. 29	(iii) Eq. 25	Ls	(v) Eq. 6	(ii) Eq. 35	θ_e

In designing a dual-reflector antenna, the starting point is often the known radiation pattern of a feed. Therefore, most of the sets of proposed input parameters use θ_e, allowing the antenna designer to define the edge illumination on the subreflector (typically $\approx$-15 dB). Sometimes, the antenna requirements include the use of a special type of feed. It is then possible to design the antenna to have the minimum feed and subreflector blockage.

We have two choices:

(i) no condition is specified for the blockage, i.e., no constraint is placed on the feed;

(ii) the minimum-blockage condition is required, where the feed used is known, including its physical dimensions.

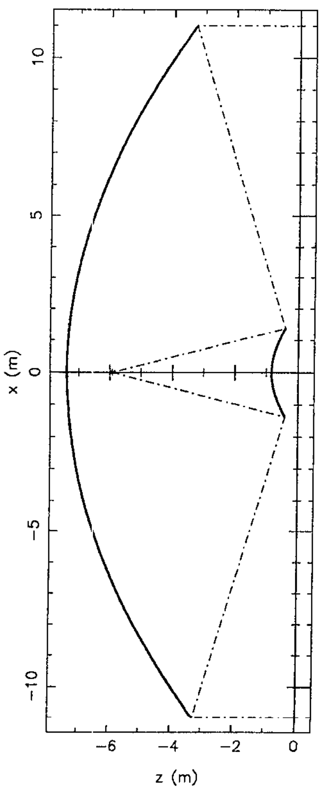

Figure 5a. The input parameters for the Cassegrain system (Dm, F, Ds, and θ_e), when there is no condition for the blockage (side view of the antenna).

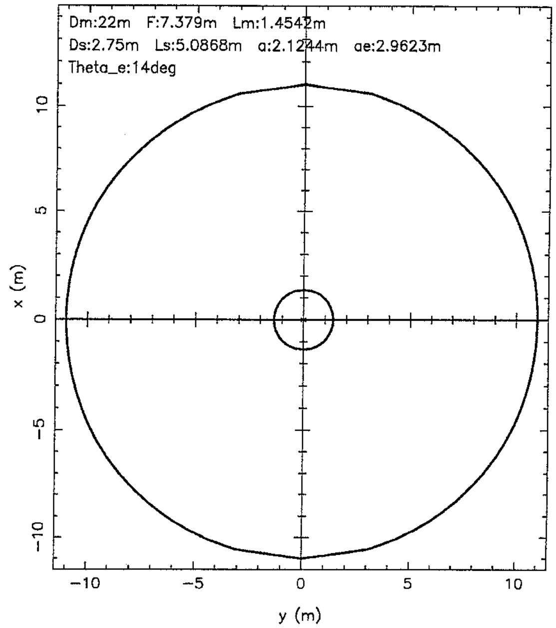

Figure 5b. The input parameters for the Cassegrain system (Dm, F, Ds, and θ_e), when there is no condition for the blockage (front view of the antenna).

5.1 No condition for the blockage

Table 1 presents seven cases where four input parameters are used, and the unknown parameters are determined using the appropriate equations, solved in the specified order ((i) to (iv)).

5.2 Minimum-blockage condition

If the type of feed to be used is one of the constraints for the antenna design, it is possible to design the antenna to have minimum blockage. In this case, the overall diameter of the feed aperture (including the flange) needs to be known. The overall diameter will be referred to as D_f (see Figure 4). The condition for minimum blockage (the shadow of the subreflector equals the shadow of the feed, see Figure 4) is given in Hannan [3] as $\frac{F}{2f} = \frac{Ds}{D_f}$. Introducing this new condition, it is then possible to design the antenna system with only three input parameters.

Table 2 presents seven cases where three input parameters are used, and the unknown parameters are determined using the appropriate equations solved in the specified order ((i) to (v)).

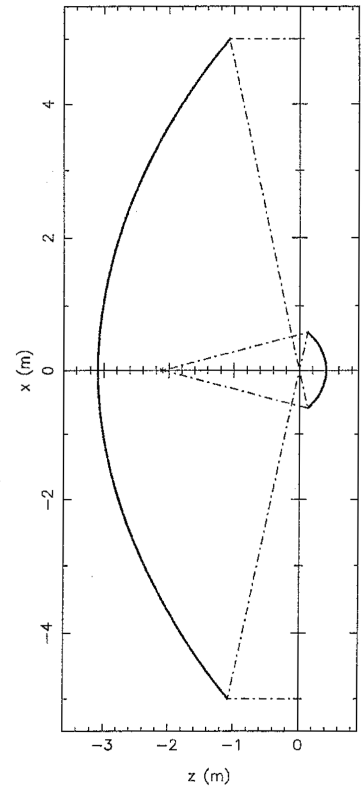

Figure 6a. The input parameters for the Gregorian system (Dm, Lm, Ls, and θ_e), when there is no condition for the blockage (side view of the antenna).

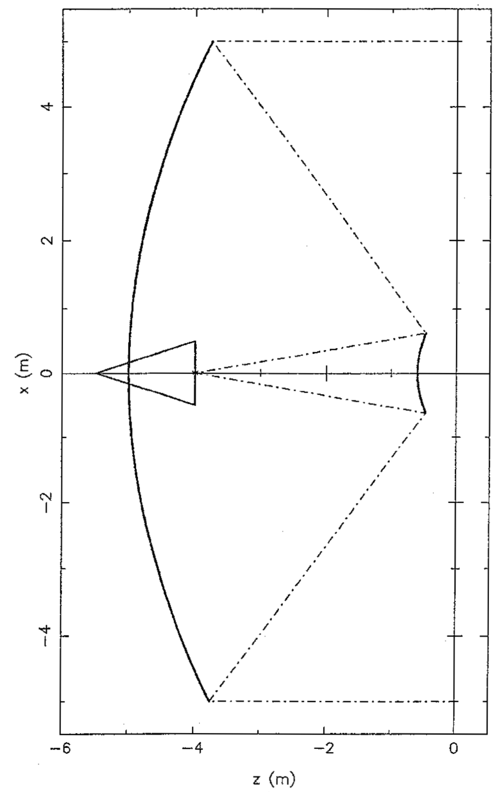

Figure 7a. The input parameters for the Cassegrain system (Dm, F, and Lm) Cassegrain system, with the condition for minimum blockage and $D_f = 1$ m (side view of the antenna).

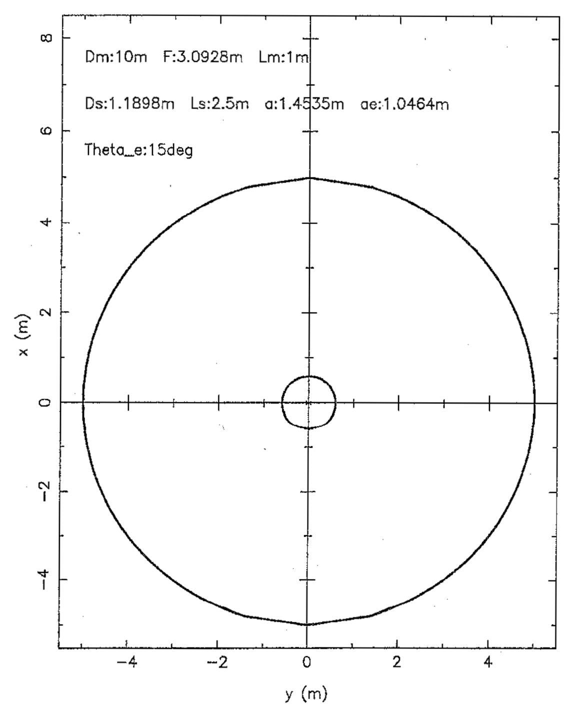

Figure 6b. The input parameters for the Gregorian system (Dm, Lm, Ls, and θ_e), when there is no condition for the blockage (front view of the antenna).

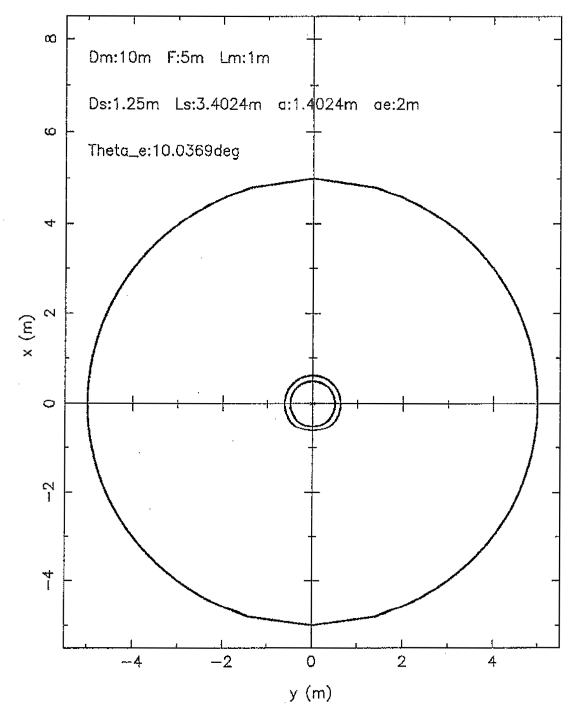

Figure 7b. The input parameters for the Cassegrain system (Dm, F, and Lm) Cassegrain system, with the condition for minimum blockage and $D_f = 1$ m (front view of the antenna).

6. Examples

A theoretical example of a Cassegrain antenna, designed with no condition for the blockage, is presented in Figure 5, while a Gregorian system is presented in Figure 6.

A theoretical example of a Cassegrain antenna, designed with the condition for the minimum blockage, is presented in Figure 7. The overall diameter of the feed is taken as $D_f = 1\,\text{m}$.

7. Conclusions

An easy procedure to design classical Cassegrain or Gregorian dual-reflector antennas has been presented. This procedure allows the antenna designer to fully define the antenna geometry with different sets of input parameters, depending on the requirements of the antenna size and its performance.

This procedure can be used as the starting point of a synthesis procedure, where both main reflector and subreflector are shaped to create the desired aperture field distribution.

8. References

1. K. W. Brown, A. Prata Jr., "Elliptical Beam Closed-Form Dual-Reflector Antenna Efficiently Illuminated by a Feed with an Axially-Symmetric Radiation Pattern," IEEE International Symposium on Antennas and Propagation *Digest Volume 2* (Newport Beach, California, June 18-23, 1995), pp. 877-880.

2. P. A. Jensen, "Cassegrain Systems," in A. W. Rudge, K. Milne, A. D. Olver, P. Knight (eds.), *The Handbook of Antenna Design, Volume 1*, London, Peter Peregrinus, 1982, Section 3.2.

3. P. W. Hannan, "Microwave Antennas Derived from the Cassegrain Telescope," *IRE Transactions on Antennas and Propagation*, March 1961, pp. 140-153.

4. Y. H. Lee, K. W. Brown, A. Prata Jr., RASCAL *Version 2.1, Interactive Reflector Antenna Synthesis & Analysis Software User's Manual*, Applied Electromagnetics Group, University of Southern California.

9. Appendix: list of equations

The parameter σ, which appears in the following equations, is set to $\sigma = -1$ for a Cassegrain system, and $\sigma = +1$ for a Gregorian system.

$$f = Ls \frac{\sigma Dm - 4\,Lm \tan\left(\frac{\theta_e}{2}\right)}{2\sigma Dm + 8\,Ls \tan\left(\frac{\theta_e}{2}\right)} \tag{4}$$

$$F = Lm + 2f \tag{5}$$

$$a = Ls - f \tag{6}$$

$$Ds = \frac{4(Ls - f)}{\frac{1}{\sin(\theta_e)} + \frac{\sigma\left(16F^2 + Dm^2\right)}{8\,F\,Dm}} \tag{7}$$

$$a = -\frac{1}{2}\frac{(Lm - F)\left[\sigma Dm + 4F \tan\left(\frac{\theta_e}{2}\right)\right]}{\sigma Dm - 4F\tan\left(\frac{\theta_e}{2}\right)} \tag{8}$$

$$Ls = -\frac{\sigma Dm(Lm - F)}{\sigma Dm - 4F\tan\left(\frac{\theta_e}{2}\right)} \tag{9}$$

$$f = \frac{1}{2}(F - Lm) \tag{10}$$

$$Ds = \frac{X_1}{X_2 + X_3} \tag{11}$$

where

$$X_1 = -16\sin(\theta_e)\,Dm\,F\,(Lm - F)\left[\sigma Dm + 4F\tan\left(\frac{\theta_e}{2}\right)\right]$$

$$X_2 = 8\,F\,Dm\left[\sigma Dm - 4F\tan\left(\frac{\theta_e}{2}\right)\right]$$

and

$$X_3 = \left(Dm^2 + 16F^2\right)\sin(\theta_e)\left[Dm - 4\sigma F\tan\left(\frac{\theta_e}{2}\right)\right]$$

$$Lm = -\frac{\sigma Dm(Ls - F) - 4\,Ls\,F\tan\left(\frac{\theta_e}{2}\right)}{\sigma Dm} \tag{12}$$

$$Ds = \frac{16\sin(\theta_e)\,F\,Ls\left[\sigma Dm + 4F\tan\left(\frac{\theta_e}{2}\right)\right]}{8\sigma F\,Dm + \sin(\theta_e)\left(Dm^2 + 16F^2\right)} \tag{13}$$

$$a = \frac{Ls\left[\sigma Dm + 4F\tan\left(\frac{\theta_e}{2}\right)\right]}{2\sigma Dm} \tag{14}$$

$$f = \frac{Ls\left[\sigma Dm - 4F\tan\left(\frac{\theta_e}{2}\right)\right]}{2\sigma Dm} \tag{15}$$

$$Dm = \text{Root of}\left\{\begin{array}{l}\left[Ds\sin(\theta_e)\right]Z^2 \\ +\left\{8F\sigma\left[Ds - 2\sin(\theta_e)Ls\right]\right\}Z \\ +16F^2\sin(\theta_e)\left[Ds - 4\,Ls\tan\left(\frac{\theta_e}{2}\right)\right]\end{array}\right\} \tag{16}$$

$$
f = \text{Root of}\left\{\begin{array}{l}\left\{4\sin(\theta_e)\left[4\,Ls\tan\left(\frac{\theta_e}{2}\right)-Ds\right]\right\}Z^2 \\ +\left\{4\,Ds\,Ls\left[\sin(\theta_e)+\tan\left(\frac{\theta_e}{2}\right)\right]-24\,Ls^2\tan\left(\frac{\theta_e}{2}\right)\sin(\theta_e)\right\}Z \\ +\left[\begin{array}{l}-Ls^2\,Ds\left[2\tan\left(\frac{\theta_e}{2}\right)+\sin(\theta_e)\right]+ \\ Ls^2\tan\left(\frac{\theta_e}{2}\right)\sin(\theta_e)\left[8\,Ls-\tan\left(\frac{\theta_e}{2}\right)Ds\right]\end{array}\right]\end{array}\right\} \tag{17}
$$

$$
Dm = \frac{4\,Ls\tan\left(\frac{\theta_e}{2}\right)(Lm+2f)}{\sigma(Ls-2f)} \tag{18}
$$

$$
Ls = \frac{Ds\,\sigma\,Y_2}{16\sin(\theta_e)F(\sigma\,Dm+4Y_1)} \tag{19}
$$

$$
a = \frac{Ds\,Y_2}{32\sin(\theta_e)Dm\,F} \tag{20}
$$

$$
f = \frac{Ds\,Y_2\,(\sigma\,Dm-4Y_1)}{32\sin(\theta_e)F(\sigma\,Dm+4Y_1)Dm} \tag{21}
$$

$$
Lm = -\frac{1}{16}\,\frac{Y_3}{Dm\sin(\theta_e)F(\sigma\,Dm+4Y_1)} \tag{22}
$$

where

$$
Y_1 = \tan\left(\frac{\theta_e}{2}\right)F
$$

$$
Y_2 = 8\,F\,Dm+\sigma\sin(\theta_e)\left(16\,F^2+Dm^2\right)
$$

and

$$
\begin{aligned} Y_3 &= 8\,\sigma\,Dm^2\,F\left[Ds-2\sin(\theta_e)F\right]-32\tan\left(\frac{\theta_e}{2}\right)F^2\,Dm\left[Ds\right. \\ &\left.+2\sin(\theta_e)F\right]+Ds\sin(\theta_e)\left(16\,F^2+Dm^2\right)\left[Dm-4\,\sigma\tan\left(\frac{\theta_e}{2}\right)F\right] \end{aligned} \tag{23}
$$

$$
F = \text{Root of}\left\{\begin{array}{l}\left\{16\sin(\theta_e)\left[4\,Ls\tan\left(\frac{\theta_e}{2}\right)-Ds\right]\right\}Z^2 \\ +\left\{8\,Dm\,\sigma\left[2\sin(\theta_e)\,Ls-Ds\right]\right\}Z \\ -Ds\,Dm^2\sin(\theta_e)\end{array}\right\} \tag{24}
$$

$$
Ds = \frac{F\,D_f}{2f} \tag{25}
$$

$$
\theta_e = \arctan\left[\frac{8\,F\,Dm\,Ds}{32\,f\,F\,Dm+\sigma\,Ds\left(16\,F^2-Dm^2\right)}\right] \tag{26}
$$

$$
Ls = \frac{2\,\sigma\,Dm\,f}{\sigma\,Dm-4\,F\tan\left(\frac{\theta_e}{2}\right)} \tag{27}
$$

$$
f = \sqrt{\frac{8\,F\,Dm\,D_f-\sigma\,D_f\tan(\theta_e)\left(16\,F^2-Dm^2\right)}{64\tan(\theta_e)Dm}} \tag{28}
$$

$$
Lm = F-2f \tag{29}
$$

$$
f = \frac{F\,D_f}{2\,Ds} \tag{30}
$$

$$
f = \frac{Lm\,D_f}{2\left(Ds-D_f\right)} \tag{31}
$$

$$
F = \text{Root of}\left\{\left[16\left(D_f\,Dm+\sigma\,Ds^2\right)\right]Z^2-\frac{8\,Dm\,Ds^2}{\tan(\theta_e)}Z-\sigma\,Ds^2\,Dm^2\right\} \tag{32}
$$

$$
F = \mathit{Root\ of}\left\{\begin{array}{l}\left[16\tan(\theta_e)\left(Dm+\sigma\,D_f\right)\right]Z^2 \\ +\left\{-8\,Dm\left[4\,Lm\tan(\theta_e)+D_f\right]\right\}Z \\ +\left[\tan(\theta_e)\,Dm\left(16\,Lm^2-\sigma\,D_f\,Dm\right)\right]\end{array}\right\} \tag{33}
$$

$$
F = \text{Root of}\left\{\begin{array}{l}\left\{16\tan(\theta_e)\left[16\,Ls^2\tan^2\left(\frac{\theta_e}{2}\right)+\sigma\,D_f\,Dm\right]\right\}Z^2 \\ +\left\{-8\,Dm\left[16\,Ls^2\,\sigma\tan(\theta_e)\tan\left(\frac{\theta_e}{2}\right)+D_f\,Dm\right]\right\}Z \\ +\left[\tan(\theta_e)\,Dm^2\left(16\,Ls^2-\sigma\,D_f\,Dm\right)\right]\end{array}\right\} \tag{34}
$$

$$
f = -\frac{1}{2}Ls\frac{4\,F\tan\left(\frac{\theta_2}{2}\right)-\sigma\,Dm}{\sigma\,Dm} \tag{35}
$$

Introducing the Author

Christophe Granet was born in Châteauroux, France, in 1967. He received his Diplôme D'étude Approfondie (DEA) from the University of Limoges (France), and his PhD from the University of Orléans (France), in 1990 and 1995, respectively. His main research project has been the design and manufacture of the new dual-reflector feed system for the Nançay radio telescope (his PhD project with the Paris Observatory). Since 1995, he has been with the Division of Telecommunications & Industrial Physics, Commonwealth Scientific and Research Organization (CSIRO), Sydney, Australia, where he has been mainly involved with the design and manufacture of reflectors and feed systems for radio astronomy and satellite communications.