Rotated Ellipsoid

An ellipse has 2D geometry and an ellipsoid has 3D geometry. An ellipse represents the intersection of a plane surface and an ellipsoid. They both have shape (eccentricity) and size (major axis). A “standard ellipsoid” has a circular midsection. A tri-axial ellipsoid has elliptic sections in all directions.

A tri-axial ellipsoid may be “tilted” in space. The tilt may be represented by turning the ellipsoid through two angles (A,B). The result is; the axes of the ellipse are not parallel to the axes of the co-ordinate system. The rotation (A,B) is not a revolution around any axis.

The ellipsoid may align with the co-ordinate system if; A = B = 0

Elliptic Co-Ordinates;

The elliptic co-ordinates are;

Eliptic Variables;	(s , t , u)	(spatial planes)
Eliptic Constants;	(a , b , c)	(properties of shape and size)
Eliptic Axis;	(a)	$a^2 = a_1^2 + a_2^2 + a_3^2$
Radial dimension:	(r)	$r^2 = s^2 + t^2 + u^2$

Spatial Co-Ordinates;

The spatial co-ordinates are;

Spatial Variables;	(x , y , z)	(Cartesian co-ordinates)
Spatial Constants;	(h , j , k)	(position)
Radial dimensions:	(r)	$r^2 = x^2 + y^2 + z^2$

Rotation;

The angles of tilt (A,B) have ratios;

$C_A = \text{Cos}(A)$ and $S_A = \text{Sin}(A)$

$C_B = \text{Cos}(B)$ and $S_B = \text{Sin}(B)$

General Equation;

The general equation of a “tilted” ellipsoid may be written as;

$$s^2/a^2 + t^2/b^2 + u^2/c^2 = 1$$

Where; a,b,c represent eliptic properties (shape and size)

s,t,u represent spatial planes;

The elliptic variables (s,t,u) are linear functions of the spatial variables (x,y,z);

$$s = (x-h)C_AC_B + (y-j)S_AC_B + (z-k)S_B$$

$$t = (y-j)C_A - (x-h)S_A$$

$$u = (z-k)C_B - (y-j)S_AS_B - (x-h)C_AS_B$$

The Standard Ellipsoid;

The standard ellipsoid is un-displaced, and has a circular midsection. The conditions for a standard ellipsoid are;

$$c = b$$

$$h = j = k = 0$$

The standard ellipsoid is;

$$s^2/a^2 + t^2/b^2 + u^2/b^2 = 1$$

The planar equations are;

$$s = xC_AC_B + yS_AC_B + zS_B$$

$$t = yC_A - xS_A$$

$$u = zC_B - yS_AS_B - xC_AS_B$$

A "cutting plane" may be defined as;

$$s = 0$$

$$0 = xC_AC_B + yS_AC_B + zS_B$$

The intersection of the cutting plane and the standard ellipsoid is a circle;

$$t^2 + u^2 = b^2$$

The Base Ellipsoid;

The base ellipsoid is an un-rotated standard ellipsoid.

The conditions for a base ellipsoid are;

$$c = b$$

$$h = j = k = 0$$

$$A = B = 0$$

$S_A = 0$ and $C_A = 1$

$S_B = 0$ and $C_B = 1$

Giving; $s = xC_AC_B + yS_AC_B + zS_B = x$

$t = yC_A - xS_A = y$

$u = zC_B - yS_AS_B - xC_AS_B = z$

The general equation of a base ellipsoid is;

$s^2/a^2 + t^2/b^2 + u^2/b^2 = 1$

$x^2/a^2 + y^2/b^2 + z^2/b^2 = 1$

A “cutting plane” is; $x = 0$

The intersection of the cutting plane and base ellipsoid is a circle; $y^2 + z^2 = b^2$

Another “cutting plane” is; $y = 0$

The intersection of this cutting plane and base ellipsoid is an ellipse; $z^2 = (b^2/a^2)(a^2 - x^2)$

Where; shape (e) is; $e = b/a$

a represents size (major axis)

Giving; $z = e(a^2 - x^2)^{½}$

Single Rotation (A) Standard Ellipsoid;

A condition for a single rotation is; $B = 0$

$S_B = 0$ and $C_B = 1$

Giving; $s = xC_A + yS_A$

$t = yC_A - xS_A$

$u = z$

The single-rotation standard ellipsoid is;

$s^2/a^2 + t^2/b^2 + u^2/b^2 = 1$

$(xC_A + yS_A)^2/a^2 + (yC_A - xS_A)^2/b^2 + z^2/b^2 = 1$

A “cutting plane” may be defined as;

$s = 0$

$xC_A + yS_A = 0$

$y = -xC_A/S_A = -x/T_A$

The intersection of the cutting plane and the single-rot ellipsoid is a circle;

$t^2 + u^2 = b^2$

$(yC_A - xS_A)^2 + z^2 = b^2$

Another cutting plane may be defined as;

$$t = 0$$

$$yC_A - xS_A = 0$$

$$y = xS_A/C_A = xT_A$$

The intersection is an ellipse;

$$s^2/a^2 + u^2/b^2 = 1$$

$$(yS_A + xC_A)^2/a^2 + z^2/b^2 = 1$$