MSO201: PROBABILITY & STATISTICS
Problem Set #1

[1] A coin is tossed until for the first time the same result appear twice in succession. To an outcome requiring n tosses assign a probability 2^{-n}. Describe the sample space. Evaluate the probability of the following events:
(a) A = The experiment ends before the 6^{th} toss.
(b) B = An even number of tosses are required.
(c) $A \cap B$, $A^C \cap B$

[2] Three tickets are drawn randomly without replacement from a set of tickets numbered 1 to 100. Show that the probability that the number of selected tickets are in (i) arithmetic progression is $\frac{1}{66}$ and (ii) geometric progression is $\frac{105}{\binom{100}{3}}$.

[3] Three players A, B and C play a series of games, none of which can be drawn and their probability of winning any game are equal. The winner of each game scores 1 point and the series is won by the player who first scores 4 points. Out of the first three games *A* won 2 games and *B* won 1 game. Find the probability that *C* will win the series.

[4] A point *P* is randomly placed in a square with side of 1 cm. Find the probability that the distance from *P* to the nearest side does not exceed x cm.

[5] Let there be *n* people in a room and p denote the probability that there are no common birthdays. Find an approximate value of p for $n = 10$.

[6] Suppose a lift has 3 occupants A, B and C and there are three possible floors (1,2 and 3) on which they can get out. Assuming that each person acts independently of the others and that each person has an equally likely chance of getting off at each floor, calculate the probability that exactly one person will get out on each floor.

[7] If n men, among whom are A and B, stand in a row, what is the probability that there will be exactly r men between A and B?

[8] In a town of $n+1$ inhabitants, a person tells a rumor to a second person, who in turn tells it to a third person, and so on. At each step the recipient of the rumor is chosen at random from the n people available. Find the probability that the rumor will be told r times without
(a) returning to the originator,
(b) being repeated to any person.
Do the same problem when at each step the rumor is told to a gathering of N randomly chosen people.

[9] 2 points are taken at random and independently of each other on a line segment of length *m*. Find the probability that the distance between the 2 points is less than $m/3$.

[10] n points are taken at random and independently of one another inside a sphere of radius R. What is the probability that the distance from the centre of the sphere to the nearest point is not less than r?

[11] A car is parked among N cars in a row, not at either end. On his return, the owner finds that exactly r of the N places are still occupied. What is the probability that both neighboring places are empty?

[12] 3 points X, Y, Z are taken at random and independently of each other on a line segment AB. What is the probability that Y will lie between X and Z?

[13] The coefficients of the equation $ax^2+bx+c=0$ are determined by throwing an ordinary die. Find the probability that the framed equation will have real roots.

[14] Let $\Omega=\{1,2,3,4\}$. Check whether any of the following is a σ-field of subsets of Ω

$\mathcal{F}_1=\{\phi,\{1,2\},\{3,4\}\}$

$\mathcal{F}_2=\{\phi,\Omega,\{1\},\{2,3,4\},\{1,2\},\{3,4\}\}$

$\mathcal{F}_3=\{\phi,\Omega,\{1\},\{2\},\{1,2\},\{3,4\},\{2,3,4\},\{1,3,4\}\}$

[15] Prove that if $\mathcal{F}_1$ and $\mathcal{F}_2$ are σ-fields of subsets of Ω, then $F_1 \cap F_2$ is also a σ-field. Give a counter example to show that similar result for union of σ-fields does not hold.

[16] Let $\mathcal{F}$ be a σ-field of subsets of the sample space Ω and let $A\in\mathcal{F}$ be fixed. Show that $\mathcal{F}_A=\{C : C = A\cap B,\, B\in\mathcal{F}\}$ is a σ-field of subsets of A.