MSO201: PROBABILITY & STATISTICS
Problem Set #6

[1] Let X be a Poisson random variable with parameter λ. Find the probability mass function of $Y = X^2 - 5$.

[2] Let X be Binomial random variable with parameters n and p. Find the probability mass function of $Y = n - X$.

[3] Consider the discrete random variable X with the probability mass function

$$P(X=-2)=\frac{1}{5},\quad P(X=-1)=\frac{1}{6},\quad P(X=0)=\frac{1}{5},$$

$$P(X=1)=\frac{1}{15},\quad P(X=2)=\frac{10}{30},\quad P(X=3)=\frac{1}{30}.$$

Find the probability mass function of $Y = X^2$.

[4] The probability mass function of the random variable X is given by

$$P(X=x)=\begin{cases}\frac{1}{3}\left(\frac{2}{3}\right)^x & x=0,1,2,\ldots\\ 0 & \text{otherwise.}\end{cases}$$

Find the p.m.f. of $Y = X/(X+1)$.

[5] Let X be a discrete random variable with probability mass function

$$f_X(x) = P(X=x) = \begin{cases} e^{-1}, & x = 0\\ \dfrac{e^{-1}}{2\,(|x|)!}, & x = \pm 1, \pm 2, \ldots\\ 0, & \text{otherwise.}\end{cases}$$

Find the p.m.f. of $Y = |X|$ and identify the distribution.

[6] The probability density function of the random variable X is

$$f_X(x)=\begin{cases}1 & 0<x<1\\ 0 & \text{otherwise.}\end{cases}$$

i.e. $X \sim U(0,1)$. Find the distribution of the following functions of X

(a) $Y=\sqrt{X}$; (b) $Y = X^2$; (c) $Y = 2X+3$; (d) $Y = -\lambda \log X; \lambda > 0$.

[7] Let X be a random variable with $U(0,\theta)$, $\theta > 0$ distribution. Find the distribution of $Y = \min(X, \theta/2)$.

[8] The probability density function of X is given by

$$f_X(x) = \begin{cases}\frac{1}{2}, & -\frac{1}{2} < x < \frac{3}{2}\\ 0, & \text{otherwise}\end{cases}$$

Find the distribution of $Y = X^2$.

[9] The probability density function of X is given by

$$f_X(x)=\begin{cases} k\dfrac{x^{p-1}}{(1+x)^{p+q}} & x>0 \\ 0 & \text{otherwise,} \end{cases}$$

$p,q>0$. Derive the distribution of $Y=(1+X)^{-1}$.

[10] The probability density function of X is given by

$$f_X(x)=\begin{cases} k\,x^{\beta-1}\exp\left(-\alpha x^{\beta}\right) & x>0 \\ 0 & \text{otherwise,} \end{cases}$$

$\alpha,\beta>0$. Derive the distribution of $Y=X^{\beta}$

[11] According to the Maxwell-Boltzmann law of theoretical physics, the probability density function of V, the velocity of a gas molecule, is

$$f_V(v)=\begin{cases} k\,v^2\exp\left(-\beta v^2\right) & v>0 \\ 0 & \text{otherwise,} \end{cases}$$

where, $\beta>0$ is a constant which depends on the mass and absolute temperature of the molecule and $k>0$ is a normalizing constant. Derive the distribution of the kinetic energy $E=mV^2/2$.

[12] The probability density function of the random variable X is

$$f_X(x)=\begin{cases} \dfrac{3}{8}(x+1)^2 & -1<x<1 \\ 0 & \text{otherwise.} \end{cases}$$

Find the distribution of the following function of $Y=1-X^2$.

[13] Let X be a random variable with $U(0,1)$ distribution. Find the distribution function of $Y=\min(X,1-X)$ and the probability density function of $Z=(1-Y)/Y$.

[14] Suppose $X\sim N\left(\mu,\sigma^2\right)$, $\mu\in\Re,\sigma\in\Re^+$. Find the distribution of $2X-6$.

[15] Let X be a continuous random variable on (a,b) with p.d.f f and c.d.f. F. Find the p.d.f. of $Z=-\log(F(X))$.

[16] Let X be a continuous r.v. having the following p.d.f.

$$f(x)=\begin{cases} 6x(1-x) & \text{if } 0\le x\le 1 \\ 0 & \text{if otherwise} \end{cases}.$$

Derive the distribution function of X and hence find the p.d.f. of $Y=X^2(3-2X)$.

[17] Let X be distributed as double exponential with p.d.f. $f(x)=\frac{1}{2}e^{-|x|}; x\in\Re$. Find the p.d.f. of $Y=|X|$

[18] 3 balls are placed randomly in 3 boxes B_1, B_2 and B_3. Let N be the total number of boxes which are occupied and X_i be the total number of balls in the box B_i, $i=1,2,3$. Find the joint p.m.f. of (N, X_1) and (X_1, X_2). Obtain the marginal distributions of N, X_1 and X_2 from the joint p.m.f.s.

[19] The joint p.m.f. of X and Y is given by

$$p(x,y)=\begin{cases} c\,xy & \text{if } (x,y)\in\{(1,1),(2,1),(2,2),(3,1)\} \\ 0 & \text{otherwise.}\end{cases}$$

Find the constant c, the marginal p.m.f. of X and Y and the conditional p.m.f. of X given $Y=2$.

[20] The joint p.m.f. of X and Y is given by

$$p(x,y)=\begin{cases} (x+2y)/18 & \text{if } (x,y)\in\{(1,1),(1,2),(2,1),(2,2)\} \\ 0 & \text{otherwise.}\end{cases}$$

(a) Find the marginal distributions.
(b) Verify whether X and Y are independent random variables.
(c) Find $P(X<Y), P(X+Y>2)$.
(d) Find the conditional p.m.f. of Y given $X=x, x=1,2$.