(1) $X \sim P(\lambda)$

p.m.f. $P(X=x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^{x}}{x!}, & x=0,1,2,\ldots \\ 0, & \text{o/w.} \end{cases}$

$Y = X^2 - 5 \Rightarrow$ Range space of $Y = \{-5, -4, -1, 4, 11, \ldots\} = \mathcal{Y}$

$P(Y=y) = P(X^2-5=y) = P(X^2 = y+5)$

p.m.f. of Y : $P(Y=y) = P(X=\sqrt{y+5}) = \begin{cases} \dfrac{e^{-\lambda}\lambda^{\sqrt{y+5}}}{(\sqrt{y+5})!}, & y \in \mathcal{Y} \\ 0, & \text{o/w.} \end{cases}$

(2) $X \sim B(n, p)$

p.m.f. $P(X=x) = \begin{cases} \binom{n}{x} p^{x} (1-p)^{n-x}, & x=0,1,\ldots n \\ 0, & \text{o/w.} \end{cases}$

$Y = n - x \Rightarrow \mathcal{Y} = \{0, 1, \ldots n\}$.

$P(Y=y) = P(n-X=y) = P(X=n-y)$

$\Rightarrow$ p.m.f. of Y

$$P(Y=y) = \begin{cases} \binom{n}{n-y} p^{n-y} (1-p)^{n-(n-y)} ; & y=0,1,\ldots n \\ 0, & \text{o/w} \end{cases}$$

$$= \begin{cases} \binom{n}{y} (1-p)^{y} p^{n-y}, & y=0,1,\ldots n \\ 0, & \text{o/w.} \end{cases}$$

$\Rightarrow Y \sim B(n, 1-p)$.

(3) $\mathcal{X} = \{-2, -1, 0, 1, 2, 3\}$

$Y = X^2 \rightarrow$ range sp $= \{0, 1, 4, 9\}$

p.m.f. $P(Y = y) = \begin{cases} P(X=0) & y=0 \\ P(X=-1)+P(X=1) & y=1 \\ P(X=-2)+P(X=2) & y=4 \\ P(X=3) & y=9 \end{cases}$

$$= \begin{cases} \frac{1}{5}, & y=0 \\ \frac{1}{6}+\frac{1}{15}, & y=1 \\ \frac{1}{5}+\frac{1}{3}, & y=4 \\ \frac{1}{30} & y=9 \end{cases}$$

(4) $P(X=x) = \begin{cases} \frac{1}{3}\left(\frac{2}{3}\right)^x, & x = 0, 1, 2, \ldots \\ 0, & \text{o/w} \end{cases}$

$Y = \frac{X}{X+1} \Rightarrow X = \frac{Y}{1-Y}$

range space of $Y = \left\{0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\right\}$

$P(Y=y) = P\left(\frac{X}{X+1} = y\right) = P\left(X = \frac{y}{1-y}\right)$

$$= \begin{cases} \frac{1}{3}\left(\frac{2}{3}\right)^{\frac{y}{1-y}}, & y = 0, \frac{1}{2}, \frac{2}{3}, \ldots \\ 0 & \text{o/w}. \end{cases}$$

(5)

p.m.f. of X

$$P(X=x) = \begin{cases} e^{-1}, & x=0 \\ \dfrac{e^{-1}}{2(|x|)!}, & x \in \{\pm 1, \pm 2, \ldots\} \\ 0, & o/w. \end{cases}$$

$Y = |X|$ $\quad \mathcal{Y} = \{0, 1, 2, \ldots\}$.

$$P(Y=0) = P(X=0) = e^{-1}$$

$$P(Y=1) = P(X=-1) + P(X=1)$$
$$= \frac{e^{-1}}{2} + \frac{e^{-1}}{2} = e^{-1}$$

$$P(Y=2) = P(X=-2) + P(X=2)$$
$$= \frac{e^{-1}}{2.2!} + \frac{e^{-1}}{2.2!} = \frac{e^{-1}}{2!}$$

Sly for K = 1, 2, ...

$$P(Y=K) = P(X=-K) + P(X=K)$$
$$= \frac{e^{-1}}{2.K!} + \frac{e^{-1}}{2.K!} = \frac{e^{-1}}{K!}$$

p.m.f. of Y

$$P(Y=y) = \begin{cases} \dfrac{e^{-1}}{y!}, & y = 0, 1, 2, \ldots \\ 0, & o/w \end{cases}$$

$Y \sim P(1)$

(6) $X \sim U(0,1)$ $\quad F_X(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$

<u>Solutions using d.f. Method</u>

(a) $Y = \sqrt{X}$

d.f. of Y : $F_Y(y) = P(\sqrt{X} \leq y)$

$$= P(X \leq y^2) = \begin{cases} 0 & y < 0 \\ y^2 & 0 \leq y \leq 1 \\ 1 & y > 1 \end{cases}$$

p.d.f. of Y is $f_Y(y) = \begin{cases} 0, & y < 0 \\ 2y, & 0 \leq y \leq 1 \\ 1, & y > 1 \end{cases} = \begin{cases} 2y, & 0 \leq y \leq 1 \\ 0, & o/w. \end{cases}$

(b) $Y = X^2$

d.f. of Y : $F_Y(y) = P(X^2 \leq y)$

$$= \begin{cases} 0, & y < 0 \\ P(-\sqrt{y} \leq X \leq \sqrt{y}), & 0 \leq y \leq 1 \\ 1, & y > 1 \end{cases}$$

for $0 \leq y \leq 1$

$$P(-\sqrt{y} \leq X \leq \sqrt{y}) = P(0 \leq X \leq \sqrt{y}) = F_X(\sqrt{y}) = \sqrt{y}$$

$$\Rightarrow \quad F_Y(y) = \begin{cases} 0, & y < 0 \\ \sqrt{y}, & 0 \leq y \leq 1 \\ 1, & y > 1 \end{cases}$$

p.d.f. of Y : $f_Y(y) = \begin{cases} \frac{1}{2\sqrt{y}} & 0 \leq y \leq 1 \\ 0 & o/w. \end{cases}$

(c) $Y = 2X + 3 \longrightarrow (3,5)$

d.f. of Y : $F_Y(y) = P(2X+3 \le y)$

$$= P\left(X \le \frac{y-3}{2}\right) = \begin{cases} 0, & y < 3 \\ \frac{y-3}{2}, & 3 \le y \le 5 \\ 1 & y > 5 \end{cases}$$

p.d.f. $f_Y(y) = \begin{cases} \frac{1}{2}, & 3 \le y \le 5 \\ 0 & o/w \end{cases} \Rightarrow Y \sim U(3,5)$

(d) $Y = -\lambda \log X \rightarrow (0, \infty)$

$F_Y(y) = P(Y \le y) = P(-\lambda \log X \le y) = P(X > e^{-y/\lambda})$

$$= 1 - P(X \le e^{-y/\lambda})$$

i.e. $F_Y(y) = \begin{cases} 0 & y < 0 \\ 1 - e^{-y/\lambda} & y \geq 0 \end{cases}$

p.d.f of Y : $f_Y(y) = \begin{cases} \frac{1}{\lambda} e^{-y/\lambda}, & y \geq 0 \\ 0 & o/w \end{cases}$

i.e $Y \sim Exp(\lambda)$ (scale λ)

(7)

$X \sim U(0, \theta)$

$Y = \min(X, \theta/2) \rightarrow (0, \theta/2) \leftarrow$ range sp of Y

$F_Y(y) = P(Y \le y) = P(\min(X, \theta/2) \le y)$

$$= 1 - P(\min(X, \theta/2) > y)$$

$$= 1 - P(X > y \cap \theta/2 > y)$$

Now $P(X > y, \theta/2 > y) = 1$ If $y < 0$

$= 0$ If $y \geqslant \theta/2$

for $0 \leq y < \theta/2$; $P(X>y, \theta/2 > y) = P(X>y)$

$$= \frac{1}{\theta}\int_{y}^{\theta} dx = \frac{\theta - y}{\theta}$$

$$\Rightarrow F_y(y) = \begin{cases} 0, & y < 0 \\ y/\theta, & 0 \leq y < \theta/2 \\ 1, & y \geqslant \theta/2 \end{cases}$$

Note : $F_y(y)$ has a jump discontinuity at $\frac{\theta}{2}$

(8)

$$f_X(x) = \begin{cases} \frac{1}{2}, & -\frac{1}{2} \leq x \leq 3/2 \\ 0, & o/w \end{cases} \qquad X \sim U\left(-\frac{1}{2}, \frac{3}{2}\right)$$

$Y = X^2 \rightarrow y \in (0, 9/4)$

$F_y(y) = P(X^2 \leq y) = P(-\sqrt{y} \leq X \leq \sqrt{y})$

For $y < 0$; $F_y(y) = 0$

& $y > \frac{9}{4}$; $F_y(y) = 1$

For, $0 \leq y \leq \frac{1}{4}$; $F_y(y) = \int_{-\sqrt{y}}^{\sqrt{y}} \frac{1}{2} dx = \sqrt{y}$

for, $\frac{1}{4} < y \leq \frac{9}{4}$; $F_y(y) = \int_{-\sqrt{y}}^{-1/2} 0 . dx + \int_{-1/2}^{\sqrt{y}} \frac{1}{2} dx = \frac{1}{2}\left(\sqrt{y} + \frac{1}{2}\right)$

$$= \left(\frac{1}{4} + \frac{\sqrt{y}}{2}\right)$$

$$\Rightarrow \quad F_Y(y) = \begin{cases} 0 & y < 0 \\ \sqrt{y} & 0 \le y \le \frac{1}{4} \\ \frac{1}{2}\left(\sqrt{y}+\frac{1}{2}\right), & \frac{1}{4} \le y \le \frac{9}{4} \\ 1, & y \geq 9/4 \end{cases}$$

$$\text{p.d.f.} \quad f_Y(y) = \begin{cases} \frac{1}{2\sqrt{y}}, & 0 \le y \le \frac{1}{4} \\ \frac{1}{4\sqrt{y}}, & \frac{1}{4} < y \le \frac{9}{4}. \end{cases}$$

(9)

$$f_X(x) = \begin{cases} k \, \frac{x^{p-1}}{(1+x)^{p+q}}, & x > 0 \\ 0 & o/w \end{cases}$$

$$Y = \frac{1}{1+X} \Rightarrow X = \frac{1-Y}{Y} = g^{-1}(Y) \; ; \qquad Y \in (0,1)$$

$$J = \frac{dx}{dy} = -\frac{1}{y^2}$$

$$f_Y(y) = \begin{cases} f_X(g^{-1}(y)) \, |J|, & 0 \le y \le 1 \\ 0 & o/w \end{cases}$$

$$= \begin{cases} k \cdot \left(\frac{1-y}{y}\right)^{p-1} \left(\frac{1}{y^{-1}}\right)^{p+q} \cdot \frac{1}{y^2}, & 0 \le y \le 1 \\ 0, & o/w \end{cases}$$

$$\text{i.e} \quad f_Y(y) = \begin{cases} k \; y^{q-1}(1-y)^{p-1}, & 0 \le x \le 1 \\ 0 & o/w. \end{cases}$$

$$K = \left(\text{Beta}(q, p)\right)^{-1}$$

$$\Rightarrow \quad Y \sim \text{Beta}(q, p)$$

(10)

$$f_X(x) = \begin{cases} K \; x^{\beta-1} \; e^{-\alpha x^{\beta}} , & x > 0 \\ 0, & \text{o/w} \end{cases}$$

$Y = X^{\beta}$

$\underline{y > 0}$

$$J = \frac{dx}{dy} = \frac{1}{\beta} \, y^{1/\beta - 1} \quad \left(x = y^{1/\beta} = g^{-1}(y) \right)$$

$$f_Y(y) = \begin{cases} f_X(g^{-1}(y)) \, |J| , & y > 0 \\ 0, & \text{o/w} \end{cases}$$

$$= \begin{cases} K . \left(y^{1/\beta}\right)^{\beta-1} e^{-\alpha y} \left(\frac{1}{\beta} \cdot y^{1/\beta - 1} \right) , & y > 0 \\ 0 & \text{o/w} \end{cases}$$

$$= \begin{cases} K \; y^{1 - 1/\beta} \; e^{-\alpha y} \; \frac{1}{\beta} \cdot y^{1/\beta - 1} , & y > 0 \\ 0 & \text{o/w} \end{cases}$$

$$= \begin{cases} K \, \frac{e^{-\alpha y}}{\beta} , & y > 0 \\ 0, & \text{o/w} . \end{cases}$$

$$K = \alpha \beta \quad \Rightarrow \quad Y \sim \text{Exp}\left(\frac{1}{\alpha}\right).$$

(11)

$$f_V(v) = \begin{cases} K \, v^2 \, e^{-\beta v^2}, & v > 0 \\ 0, & o/w \end{cases}$$

$$E = \frac{1}{2} m v^2 \; ; \quad v^2 = \frac{2E}{m} .$$

$$\frac{\partial e}{\partial v} = m v$$

$$J = \frac{\partial v}{\partial e} = \frac{1}{\sqrt{2me}} .$$

$$f_E(e) = \begin{cases} K \cdot \left(\frac{2e}{m}\right) e^{-\beta\left(\frac{2e}{m}\right)} \cdot \frac{1}{\sqrt{2me}} \; ; & e > 0 \\ 0 & o/w \end{cases}$$

$$= \begin{cases} c . \; e^{1/2} \; \exp\left(- \frac{2\beta}{m} e\right), & e > 0 \\ 0, & o/w \end{cases}$$

$$c \text{ is } \ni \int_0^\infty c \; e^{1/2} \; \exp\left(- \frac{2\beta}{m} e\right) de = 1$$

$$\text{i.e. } c . \frac{\Gamma 3/2}{\left(\frac{2\beta}{m}\right)^{3/2}} = 1 \Rightarrow c = \frac{\left(2\beta/m\right)^{3/2}}{\Gamma 3/2}$$

$\Rightarrow$ E ~ Gamma (. . .)

(12)

$$f_X(x) = \begin{cases} \frac{3}{8}(x+1)^2, & -1 < x < 1 \\ 0, & o/w. \end{cases}$$

$$Y = 1 - X^2 \quad , \quad y \in (0,1)$$

$$x^2 = 1 - y \Rightarrow x = \pm\sqrt{1-y}$$

$$x \in (-1,0) \rightarrow x = -\sqrt{1-y} = g_1^{-1}(y) \rightarrow \left|\frac{dx}{dy}\right| = \frac{1}{2\sqrt{1-y}}$$

$$x \in (0,1) \rightarrow x = \sqrt{1-y} = g_2^{-1}(y) \rightarrow \left|\frac{dx}{dy}\right| = \frac{1}{2\sqrt{1-y}}.$$

$$f_Y(y) = f_X\big(\overset{\overset{(-1,\,0)}{\downarrow}}{g_1^{-1}(y)}\big) \overset{\overset{(-1,0)}{\downarrow}}{\left|\frac{dx}{dy}\right|} + f_X\big(\overset{\overset{(0,\,1)}{\downarrow}}{g_2^{-1}(y)}\big)\overset{\overset{(-1,0)}{\downarrow}}{\left|\frac{dx}{dy}\right|} . \quad 0 < y < 1$$

$$= \frac{3}{8}\left(1-\sqrt{1-y}\right)^2 \cdot \frac{1}{2\sqrt{1-y}} + \frac{3}{8}\left(1+\sqrt{1-y}\right)^2 \cdot \frac{1}{2\sqrt{1-y}}$$

$$= \frac{3}{16\sqrt{1-y}}\left(\left(1-\sqrt{1-y}\right)^2 + \left(1+\sqrt{1-y}\right)^2\right)$$

$$= \frac{3}{16\sqrt{1-y}}\left(2\left(1+(1-y)\right)\right)$$

i.e. $$f_Y(y) = \begin{cases} \frac{3}{8}\left((1-y)^{-1/2} + (1-y)^{1/2}\right), & 0 < y < 1 \\ 0, & o/w. \end{cases}$$

(13)

$Y = \min(X, 1-X)$ → range of y $(0, \frac{1}{2})$

$$F_Y(y) = P(Y \le y) = P(\min(X, 1-X) \le y) = 1 - P(\min(X,1-X) > y)$$

$$= 1 - P(X > y, 1 - X > y)$$

$$= 1 - P(X > y, 1 - y > X)$$

$$= 1 - P(y < X < 1 - y)$$

$$P(y < X < 1-y) = \begin{cases} 1 & y \leq 0 \\ \int_y^{1-y} dx & \text{if } 0 < y < \frac{1}{2} \\ 0 & y \geq \frac{1}{2} \end{cases}$$

$$\Rightarrow F_Y(y) = \begin{cases} 0 & y \leq 0 \\ 2y & 0 < y < \frac{1}{2} \\ 1 & y \geq \frac{1}{2} \end{cases} \Rightarrow \text{p.d.f } f_Y(y) = \begin{cases} 2, & 0 < y < \frac{1}{2} \\ 0, & \text{o/w} \end{cases}$$

$$Z = \frac{1-Y}{Y} = \frac{1}{Y} - 1 \rightarrow \text{range of } Z \text{ is } (1, \infty)$$

$F_Z(z) = P(Z \leq z)$ -

If $z \leq 1$, then $F_Z(z) = 0$

If $z > 1$, then $P(Z \leq z) = P\left(\frac{1}{Y} - 1 \leq z\right) = P\left(\frac{1}{Y} \leq z+1\right)$

$$= P\left(Y \geq \frac{1}{z+1}\right) = 1 - P\left(Y < \frac{1}{z+1}\right)$$

$$= 1 - \frac{2}{z+1} \quad (\text{using d.f of } Y)$$

$$\Rightarrow F_Z(z) = \begin{cases} 0, & \text{if } z \leq 1 \\ 1 - \frac{2}{z+1}, & \text{if } z > 1 \end{cases}$$

$$\Rightarrow \text{p.d.f. of } Z \text{ is } f_Z(z) = \begin{cases} \frac{2}{(z+1)^2}, & z > 1 \\ 0 & \text{o/w} \end{cases}$$

(14) $X \sim N(\mu, \sigma^2)$

$Y = 2X - 6$; $\quad y \in (-\infty, \infty)$

$F_Y(y) = P(Y \le y) = P(2X - 6 \le y)$

$= P\left(X \le \frac{y+6}{2}\right)$

$= P\left(\frac{X-\mu}{\sigma} \le \left(\frac{y+6}{2} - \mu\right)/\sigma\right)$

$= \Phi\left(\frac{y+6-2\mu}{2\sigma}\right)$

$f_Y(y) = \phi\left(\frac{y+6-2\mu}{2\sigma}\right) \cdot \frac{1}{2\sigma}$. $\quad y \in (-\infty, \infty)$

$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-(2\mu-6)}{2\sigma}\right)^2} \cdot \frac{1}{2\sigma}$

$= \frac{1}{\sqrt{2\pi}\,(2\sigma)} \exp\left(-\frac{1}{2(4\sigma^2)}\left(y - (2\mu-6)\right)^2\right)$

$\Rightarrow Y \sim N(2\mu - 6, 4\sigma^2)$.

(15) $X \sim f_X(x)$

$Z = -\log F(X)$; $\quad z \in (0, \infty)$

$z = -\log F(x) \Rightarrow x = F^{-1}(e^{-z})$

$\left|\frac{dz}{dx}\right| = \left(\frac{f(x)}{F(x)}\right) \Rightarrow |J| = \frac{F(x)}{f(x)}$

p.d.f. of $Z = f\left(F^{-1}(e^{-z})\right) \cdot \frac{F\left(F^{-1}(e^{-z})\right)}{f\left(F^{-1}(e^{-z})\right)}$

$= F\left(F^{-1}(e^{-z})\right) = e^{-z}$;

$\Rightarrow f_Z(z) = \begin{cases} e^{-z}, & z > 0 \\ 0 & o/w. \end{cases}$

(16) $$f_X(x) = \begin{cases} 6x(1-x), & 0 \le x \le 1 \\ 0 & o/w \end{cases}$$

$$F_X(x) = P(X \le x) = \begin{cases} 0, & x < 0 \\ \int_0^x (6y - 6y^2)\,dx, & 0 \le x \le 1 \\ 1, & x > 1 \end{cases}$$

$$= \begin{cases} 0, & x < 0 \\ x^2(3-2x), & 0 \le x \le 1 \\ 1, & x > 1 \end{cases}$$

$$F_X(X) = X^2(3 - 2X)$$

If $X \sim F_X(x)$ dist^n f^n, then $Y = F(X) \sim U(0,1)$ $\downarrow$

[$\underset{\uparrow}{X} \sim f_X(x)$ p.d.f. & d.f. F

Cont r.v. $Y = F(X) \rightarrow y \in (0,1)$

$x = F^{-1}(y)$]

$$\frac{dy}{dx} = f(x) \qquad \left|\frac{dx}{dy}\right| = \left|\frac{1}{f(x)}\right|$$

$\Rightarrow$ p.d.f. of Y : $$f_Y(y) = \begin{cases} \dfrac{f_X(F^{-1}(y))}{f_X(F^{-1}(y))} = 1 & \text{If } 0<y<1 \\ 0 & o/w \end{cases}$$

$\Rightarrow Y \sim U(0,1)$

$\Rightarrow$ For the given dist^n $X^2(3-2X) = F(X) \sim U(0,1)$

(17)

$X \sim$ Double exponential

$$f_X(x) = \frac{1}{2} e^{-|x|} \quad ; \qquad -\infty < x < \infty$$

$Y = |X|$ range of Y : $(0, \infty)$

$x \in (-\infty, 0) \rightarrow x = -y \rightarrow \left|\frac{dx}{dy}\right| = 1$

$x \in (0, \infty) \rightarrow x = y \rightarrow \left|\frac{dx}{dy}\right| = 1$

$$\Rightarrow f_Y(y) = f_X(\overset{(-\infty, 0)}{g_1^{-1}(y)})\,|J| + f_X(g_2^{-1}(y))\,|J|$$

$\uparrow$
in $(-\infty, 0)$

i.e. $f_Y(y) = \begin{cases} \frac{1}{2} e^{-y} + \frac{1}{2} e^{-y}, & 0 < y < \infty \\ 0, & o/w \end{cases}$

$\therefore$ p.d.f. of Y : $f_Y(y) = \begin{cases} e^{-y}, & 0 < y < \infty \\ 0 & o/w. \end{cases}$

(18)

$X_i = 0, 1, 2, 3 \quad$ for $\quad i = 1, 2, 3$

$N = 1, 2, 3$

Possible configurations with 3 boxes and 3 balls

B_1	B_2	B_3
3	0	0
0	3	0
0	0	3
2	1	0
2	0	1
1	2	0
0	2	1
1	0	2
0	1	2
1	1	1

→ each with prob $= \frac{1}{\binom{3+3-1}{3}} = \frac{1}{10}$

→

↓N	X_1	X_2	X_3
1	3	0	0
1	0	3	0
1	0	0	3
2	2	1	0
2	2	0	1
2	1	2	0
2	0	2	1
2	1	0	2
2	0	1	2
3	1	1	1

jt p.m.f. of (N, X_1)

$N \backslash X_1$	0	1	2	3	
1	2/10	0	0	1/10	3/10
2	2/10	2/10	2/10	0	6/10
3	0	1/10	0	0	1/10
	4/10	3/10	2/10	1/10	

Marginal of N

Marginal of X_1

jt. p.m.f. of (X_1, X_2)

$X_1 \backslash X_2$	0	1	2	3	
0	1/10	1/10	1/10	1/10	4/10
1	1/10	1/10	1/10	0	3/10
2	1/10	1/10	0	0	2/10
3	1/10	0	0	0	1/10
	4/10	3/10	2/10	1/10	

Marg of X_1

Marg of X_2

(19)

$$\sum_{(x,y)} p(x,y) = c \sum_{(x,y)} (xy) = 1$$

$$\Rightarrow c\left[(1.1) + (2.1) + (2.2) + (3.1)\right] = 1$$

$$\Rightarrow c = \frac{1}{10}$$

jt p.m.f.

X \ Y	1	2	
1	1/10	0	1/10
2	2/10	4/10	6/10
3	3/10	0	3/10
	6/10	4/10	1

marginal of X

marginal of Y

Conditional p.m.f. of X given Y = 2, $\frac{p(x,2)}{p_Y(2)} = 1$ if $x = 2$

$= 0$ o/w

(20) jt p.m.f.

X \ Y	1	2	
1	3/18	5/18	8/18
2	4/18	6/18	10/18
	7/18	11/18	

marg of X

marg of Y

(b) $P(X=1, Y=1) = \frac{3}{18} \neq P(X=1) \cdot P(Y=1) = \frac{8}{18} \cdot \frac{7}{18}$

$\Rightarrow$ X & Y not ind.

(c) $P(X < Y) = P(X=1, Y=2) = \frac{5}{18}$

$P(X+Y>2) = P(X=1, Y=2) + P(X=2, Y=1) + P(X=2, Y=2)$

$= \frac{15}{18}$

(d) marg of X : $p_X(x) = P(X=x) = \frac{x+3}{9}$; $x = 1, 2$

$$p_{Y|X=x} = \frac{\frac{1}{18}(x+2y)}{\frac{1}{18}(2x+6)} = \frac{x+2y}{2x+6} \quad ; \quad y = 1, 2$$