```python
In [1]: import warnings
        warnings.filterwarnings("ignore")
        from sklearn.datasets import load_boston
        from random import seed
        from random import randrange
        from csv import reader
        from math import sqrt
        from sklearn import preprocessing
        import pandas as pd
        import numpy as np
        import matplotlib.pyplot as plt
        from prettytable import PrettyTable
        from sklearn.linear_model import SGDRegressor
        from sklearn import preprocessing
        from sklearn.metrics import mean_squared_error
        from sklearn.preprocessing import StandardScaler
        from sklearn import linear_model
        import seaborn as sns
        from sklearn.model_selection import train_test_split
        from numpy import c_
```

```python
In [2]: dataset = load_boston()
        X = pd.DataFrame(dataset.data)
        y = dataset.target
```

```python
In [3]: X_tr, X_te, Y_train, Y_test = train_test_split(X, y, test_size = 0.33, random_
        state = 1)
```

```python
In [4]: # Standardize the data
        sc = StandardScaler()
        X_train = sc.fit_transform(X_tr)
        X_test = sc.transform(X_te)
```

```python
In [5]: print(X_train.shape)
        print(X_test.shape)
```

```
(339, 13)
(167, 13)
```

SGD Implementation of our own:

In [6]:
```python
iterations = 1000
learning_rate = 0.01
length = 339
# print(length)

X_tr = np.c_[np.ones((len(X_train),1)),X_train]
X_train = X_tr
X_te = np.c_[np.ones((len(X_test),1)),X_test]
X_test = X_te

weights = np.random.normal(0,1,X_train.shape[1])
weights = np.c_[weights]
# weights = weights.reshape(1,13)

for k in range(iterations):
    for i in range(length):
        rand_size = np.random.randint(0,length)

        X_bat_size = X_train[rand_size,:].reshape(1,X_train.shape[1])
        y_bat_size = Y_train[rand_size].reshape(1,1)

        prediction = np.dot(X_bat_size,weights)
        weights = weights -(2/length)*learning_rate*( X_bat_size.T.dot((predic
tion - y_bat_size)))
    preds = X_test.dot(weights)
    Mse_cust = mean_squared_error(Y_test, preds)
    print("Mean Squared Error (MSE) = ",Mse_cust)

print(X_bat_size.shape)
print(y_bat_size.shape)
```

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Mean Squared Error (MSE) =  20.485053252905633
Mean Squared Error (MSE) =  20.487883106822682
Mean Squared Error (MSE) =  20.492132590711588
Mean Squared Error (MSE) =  20.518727919142087
Mean Squared Error (MSE) =  20.549352183438366
Mean Squared Error (MSE) =  20.546593175820856
Mean Squared Error (MSE) =  20.624948567330833
Mean Squared Error (MSE) =  20.547002028324282
Mean Squared Error (MSE) =  20.546568216895025
Mean Squared Error (MSE) =  20.560535083320737
Mean Squared Error (MSE) =  20.585954743467035
Mean Squared Error (MSE) =  20.608339328405574
Mean Squared Error (MSE) =  20.59578221150627
Mean Squared Error (MSE) =  20.65529355283774
Mean Squared Error (MSE) =  20.7837783876146
Mean Squared Error (MSE) =  20.791337846148096
Mean Squared Error (MSE) =  20.76940188781533
Mean Squared Error (MSE) =  20.773994389976888
Mean Squared Error (MSE) =  20.705215116206684
Mean Squared Error (MSE) =  20.65236389440742
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Mean Squared Error (MSE) =  20.658321139692877
Mean Squared Error (MSE) =  20.6628401102745
Mean Squared Error (MSE) =  20.646618133430042
Mean Squared Error (MSE) =  20.646072393112426
Mean Squared Error (MSE) =  20.586961398508524
Mean Squared Error (MSE) =  20.610605279320826
Mean Squared Error (MSE) =  20.61953196095278
Mean Squared Error (MSE) =  20.648628562331158
Mean Squared Error (MSE) =  20.707564443833835
Mean Squared Error (MSE) =  20.623855075834946
Mean Squared Error (MSE) =  20.679890843166255
Mean Squared Error (MSE) =  20.745818132701498
Mean Squared Error (MSE) =  20.747712642420247
Mean Squared Error (MSE) =  20.760767599811157
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Mean Squared Error (MSE) =   20.74893938344189
Mean Squared Error (MSE) =   20.666025217368194
Mean Squared Error (MSE) =   20.68670455144605
Mean Squared Error (MSE) =   20.630998777456796
Mean Squared Error (MSE) =   20.66052515282273
Mean Squared Error (MSE) =   20.76490475776353
Mean Squared Error (MSE) =   20.675022135728415
Mean Squared Error (MSE) =   20.7110767893866
Mean Squared Error (MSE) =   20.771390613148764
Mean Squared Error (MSE) =   20.750909419536633
Mean Squared Error (MSE) =   20.70389763781305
Mean Squared Error (MSE) =   20.81284830468502
Mean Squared Error (MSE) =   20.753029881430713
Mean Squared Error (MSE) =   20.79071623880769
Mean Squared Error (MSE) =   20.753616857980266
Mean Squared Error (MSE) =   20.758870854598907
Mean Squared Error (MSE) =   20.697383183284163
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Mean Squared Error (MSE) =   20.67450883239283
Mean Squared Error (MSE) =   20.721148529464973
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Mean Squared Error (MSE) =   20.750271375466728
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Mean Squared Error (MSE) =   20.91440399837191
Mean Squared Error (MSE) =   20.830107694057304
Mean Squared Error (MSE) =   20.767021872234803
Mean Squared Error (MSE) =   20.779214853563236
Mean Squared Error (MSE) =   20.771924343573357
Mean Squared Error (MSE) =   20.792987977507675
Mean Squared Error (MSE) =   20.75490752998343
Mean Squared Error (MSE) =   20.73875120785374
Mean Squared Error (MSE) =   20.710886417432114
Mean Squared Error (MSE) =   20.781570429685633
Mean Squared Error (MSE) =   20.752032743540894
Mean Squared Error (MSE) =   20.720561520499853
Mean Squared Error (MSE) =   20.70202123105787
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Mean Squared Error (MSE) =   20.736811713887544
Mean Squared Error (MSE) =   20.866864807237217
Mean Squared Error (MSE) =   20.977976431977325
Mean Squared Error (MSE) =   20.98343975241473
Mean Squared Error (MSE) =   20.91218871121724
Mean Squared Error (MSE) =   20.798776056349155
Mean Squared Error (MSE) =   20.817952582582404
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Mean Squared Error (MSE) =   20.776652777884554
Mean Squared Error (MSE) =   20.759497369544714
Mean Squared Error (MSE) =   20.81190405416719
Mean Squared Error (MSE) =   20.804213387226675
Mean Squared Error (MSE) =   20.75568415051758
Mean Squared Error (MSE) =   20.79195540198538
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Mean Squared Error (MSE) =  20.7546010363114
Mean Squared Error (MSE) =  20.759254695138114
Mean Squared Error (MSE) =  20.705752146936184
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Mean Squared Error (MSE) =  20.830709268463153
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Mean Squared Error (MSE) =  20.768513994855155
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Mean Squared Error (MSE) =  20.70857438946268
Mean Squared Error (MSE) =  20.756627081983115
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Mean Squared Error (MSE) =  20.70444687672887
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Mean Squared Error (MSE) =  20.79683663027793
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Mean Squared Error (MSE) =  20.891485545539926
Mean Squared Error (MSE) =  20.816598352673054
Mean Squared Error (MSE) =  20.859813837228334
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Mean Squared Error (MSE) =   20.787281627357025
Mean Squared Error (MSE) =   20.86522058514859
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Mean Squared Error (MSE) =   20.81319005780998
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Mean Squared Error (MSE) =   20.875715104159575
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Mean Squared Error (MSE) =   20.741067982313496
Mean Squared Error (MSE) =   20.784417043982458
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Mean Squared Error (MSE) =   20.71675987697902
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Mean Squared Error (MSE) =   20.603956333284252
Mean Squared Error (MSE) =   20.61698912509679
Mean Squared Error (MSE) =   20.556012524651525
Mean Squared Error (MSE) =   20.52092601871389
Mean Squared Error (MSE) =   20.515270390812304
Mean Squared Error (MSE) =   20.521311274276467
```

```
Mean Squared Error (MSE) =  20.529706159579472
Mean Squared Error (MSE) =  20.550087405307586
Mean Squared Error (MSE) =  20.726340545801467
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Mean Squared Error (MSE) =  20.86071108690615
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Mean Squared Error (MSE) =  20.46848899190441
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Mean Squared Error (MSE) =  20.469386053520566
Mean Squared Error (MSE) =  20.455190219965445
Mean Squared Error (MSE) =  20.49384307544728
Mean Squared Error (MSE) =  20.410846343116475
Mean Squared Error (MSE) =  20.37978596953981
(1, 14)
(1, 1)
```
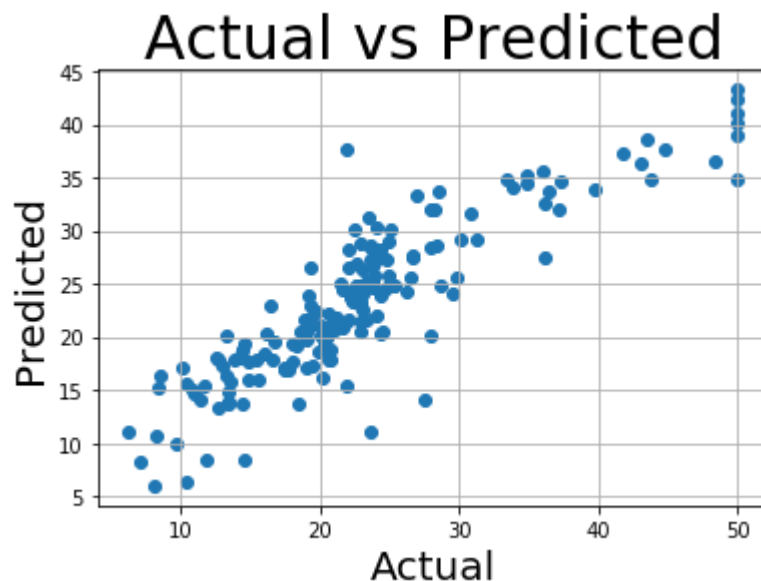
In [7]: `learning_rate`

Out[7]: `0.01`

Now we'll test the obtained weight vector on test data

In [8]: `preds = X_test.dot(weights)`

```
In [9]: plt.scatter(Y_test, preds)
        plt.xlabel("Actual",size=20)
        plt.ylabel("Predicted",size=20)
        plt.title("Actual vs Predicted",size=30)
        plt.grid()
        plt.show()
```



```
In [10]: Mse_cust = mean_squared_error(Y_test, preds)
         print("Mean Squared Error (MSE) = ",Mse_cust)
```

```
Mean Squared Error (MSE) =  20.37978596953981
```
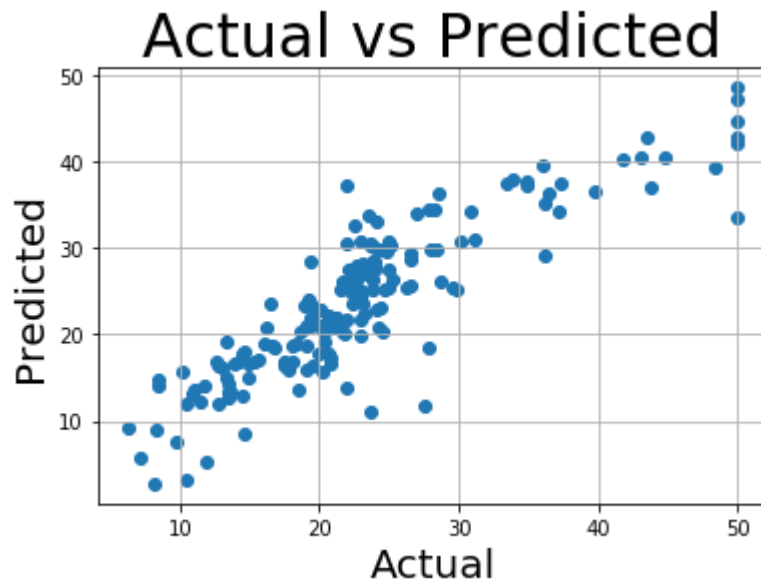
SGD Implementation of our SKlearn:

```
In [11]: model = SGDRegressor(penalty='none', max_iter=1000, learning_rate='constant')
         model.fit(X_train,Y_train)
         sklearn_preds = model.predict(X_test)
```

```
In [12]: sklearn_weights = model.coef_
         sklearn_weights
```

```
Out[12]: array([ 1.13359583e+01, -1.10742345e+00,  1.59415681e+00,  3.59770477e-01,
                 5.16550232e-01, -2.32462123e+00,  2.94777051e+00,  6.28280769e-03,
                -3.10766346e+00,  2.43379864e+00, -2.25728874e+00, -2.78892010e+00,
                 6.54925267e-01, -4.01530486e+00])
```

In [13]:
```python
plt.scatter(Y_test, sklearn_preds)
plt.xlabel("Actual",size=20)
plt.ylabel("Predicted",size=20)
plt.title("Actual vs Predicted",size=30)
plt.grid()
plt.show()
```

Actual vs Predicted

In [14]:
```python
Mse_sk = mean_squared_error(Y_test, sklearn_preds)
print("Mean Squared Error (MSE) = ",Mse_sk)
```

Mean Squared Error (MSE) = 21.59324101490106

In [ ]:

Comparing both the models

In [15]:
```python
from prettytable import PrettyTable
table = PrettyTable()
no = [1,2,3,4,5,6,7,8,9,10,11,12,13,14]
table.add_column("NO.",no)
table.add_column("Weights of our SGD",weights)
table.add_column("Weights of Sklear SGD",sklearn_weights)
print(table)
```

```
+-----+--------------------+-----------------------+
| NO. | Weights of our SGD | Weights of Sklear SGD |
+-----+--------------------+-----------------------+
|  1  |    [22.29042014]   |   11.335958278348123  |
|  2  |    [-0.7972116]    |   -1.1074234507749452 |
|  3  |    [1.23065787]    |    1.5941568063071851 |
|  4  |    [0.18183622]    |   0.35977047699695636 |
|  5  |    [0.86004567]    |    0.5165502319342093 |
|  6  |    [-2.29272973]   |   -2.3246212335848235 |
|  7  |    [2.51781706]    |    2.9477705051101926 |
|  8  |    [0.02236938]    |   0.006282807694412557|
|  9  |    [-2.96594944]   |   -3.107663463965207  |
|  10 |    [1.89243197]    |    2.4337986358257564 |
|  11 |    [-1.13183467]   |   -2.2572887351652997 |
|  12 |    [-2.05394956]   |   -2.7889201033734343 |
|  13 |    [0.69544039]    |    0.6549252673247172 |
|  14 |    [-3.76369195]   |   -4.015304862939994  |
+-----+--------------------+-----------------------+
```

In [16]:
```python
from prettytable import PrettyTable

table = PrettyTable()
table.field_names = ["SGD","MSE"]
table.add_row(["Custom SGD",Mse_cust])
table.add_row(["Sklearn SGD",Mse_sk])
print(table)
```

```
+-------------+-------------------+
|     SGD     |        MSE        |
+-------------+-------------------+
|  Custom SGD |  20.37978596953981 |
| Sklearn SGD | 21.59324101490106 |
+-------------+-------------------+
```

In [ ]:

In [ ]: