Abdullah Gul University

Math-301 (Probability & Statistics)

Fall 2022, QUIZ - V

Name & Surname:
ID Number:

Q 1.

(30 pt.)

Let X be a discrete random variable with the following PMF (probability mass function):

$$P_X(x) = \begin{cases} 0.2 & for\ x = 1 \\ 0.2 & for\ x = 2 \\ 0.1 & for\ x = 3 \\ 0.1 & for\ x = 4 \\ 0.4 & for\ x = 5 \\ 0.0 & otherwise \end{cases}$$

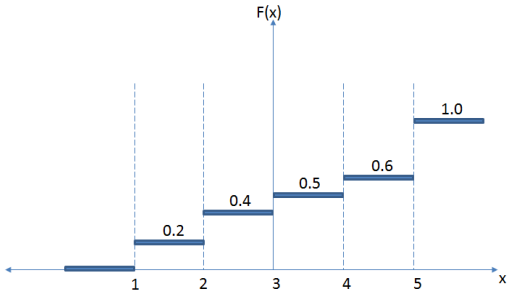
Find and plot the CDF (cumulative density function) of X.

SOLUTION:

The formula of the CDF is below;
$F(x) = P(X \leq x) = \sum_{t \leq x} f(t)$,
Then;

$$F(x) = P(X \leq x) = \begin{cases} \sum_{t \leq x} f(t) = 0 & x < 1 \\ \sum_{t \leq x} f(t) = 0.2 & 1 \geq x < 2 \\ \sum_{t \leq x} f(t) = 0.2 + 0.2 = 0.4 & 2 \geq x < 3 \\ \sum_{t \leq x} f(t) = 0.2 + 0.2 + 0.1 = 0.5 & 3 \geq x < 4 \\ \sum_{t \leq x} f(t) = 0.2 + 0.2 + 0.1 + 0.1 = 0.6 & 4 \geq x < 5 \\ \sum_{t \leq x} f(t) = 0.2 + 0.2 + 0.1 + 0.1 + 0.4 = 1.0 & 5 \leq x \end{cases}$$

To plot the CDF;



Let X be a random variable with PDF

Q 2.

(30 pt.)

$$P_X(x) = \begin{cases} 2/3 & 0 \le x < 1.5 \\ 0 & otherwise \end{cases}$$

Find the CDF (cumulative density function) of X. Then, draw the cumulative density function in a cartesian coordinate system.

SOLUTION:

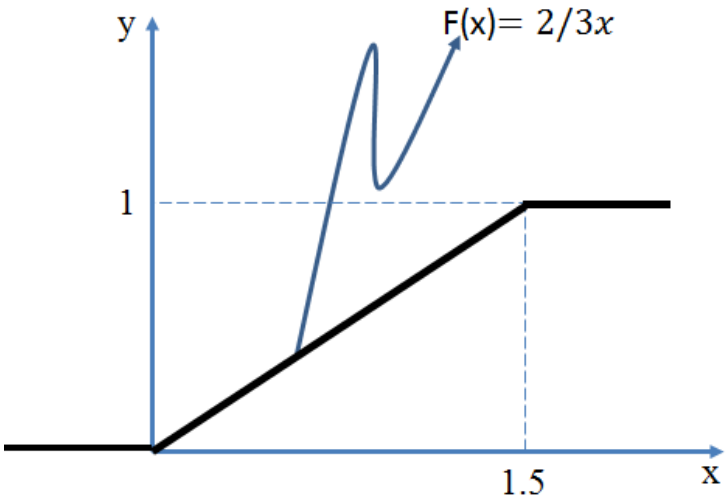
Find the continues CDF (cumulative density function) of X;

$$F(x) = \int \frac{2}{3} dx = \frac{2x}{3}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{2x}{3} & 0 \le x < 1.5 \\ 1 & x \ge 1.5 \end{cases}$$

To draw the cumulative density function in a cartesian coordinate system;

F(x) is a linear function in the cartesian coordinate system while x is a value between zero and 1.5. So;



Q 3.

(40 pt.)

Let X and Y be jointly continuous random variables with joint PDF;

$$f(x,y) = \begin{cases} 3x+1 & x,y \geq 0, x+y<1 \\ 0 & otherwise \end{cases}$$

So, evaluate the cumulative function of the joint probability mass function while $0 \leq x < 1$, in terms of the variable of X.

SOLUTION:

We know that;
$x \geq 0$,
$y \geq 0$,
$x + y < 1$

Then, we can write that;

$0 \leq y < 1 - x$
For $0 \leq x < 1$

$$F(x,y) = \int_0^1 \int_0^{1-x} f(x,y)\, dydx$$
$$= \int_0^1 \int_0^{1-x} (3x+1)\, dydx$$
$$= \int_0^1 (3xy+y)|_0^{1-x}\, dx$$
$$= \int_0^1 \big(3x(1-x)+(1-x)\big)dx$$
$$= \int_0^1 (3x-3x^2-x+1)dx$$
$$= \frac{3x^2}{2} - \frac{3x^3}{3} - \frac{x^2}{2} + x$$