Abdullah Gul University

Math-301 (Probability & Statistics)

Fall 2022, QUIZ - IV

Name & Surname:
ID Number:

Q 1. When you had flipped a coin in three times, you have observed that at least two head occurred. Then, what is the probability that you have observed at least three heads?

(30 pt.)

SOLUTION:

This is a conditional probability question with the given event A which at least two head occurs. Our target is to find the probability of the event B which at least three heads occur. So, the probability of B with given A is as follow;

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

First, let us find $P(A \cap B)$;

The set of the elements of A is that, A={THH, HTH, HHT, HHH}

The set of the elements of B is that, B={HHH}

$A \cap B$ = {HHH}

The sample space elements are {HTT, THT, TTH, THH, HTH, HHT, HHH, TTT}

$$P(A \cap B) = \frac{\neq (A \cap B)}{\neq S} = \frac{1}{8}$$

Second, let us find $P(A)$;

$$P(A) = \frac{\neq A}{\neq S} = \frac{4}{8}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/8}{4/8} = \frac{1}{4}$$

Q 2.

(30 pt.)

Consider an experiment in which 2 stones are drawn in succession from a bag, **without** replacement. Also consider there are 3 red and 4 blue stones in this bag at the initial case. So, if A is the event that the color of the selected first stone is red, and B is the event that the color of the selected second stone is blue, then these A and B events are independent or dependent? Please prove whether these events are independent or not.

SOLUTION:

If A is the event that the color of the selected first stone is red;

$A = \{R_1, R_2, R_3\}$ within the Sample Space, $S = \{R_1, R_2, R_3, B_1, B_2, B_3, B_4\}$

$$P(A) = \frac{\neq A}{\neq S} = \frac{3}{7}$$

After the selection of the first stone, if B is the event that the color of the selected second stone is blue;

$B = \{B_1, B_2, B_3, B_4\}$ within the Sample Space, $S = \{R_1, R_2, B_1, B_2, B_3, B_4\}$

$$P(B) = \frac{\neq B}{\neq S} = \frac{4}{6}$$

The elements of the second event are given below with given that the first selected stone is red color;

$B|A = \{B_1, B_2, B_3, B_4\}$ within the Sample Space, $S = \{R_1, R_2, R_3, B_1, B_2, B_3, B_4\}$

$$P(B|A) = \frac{\neq (B|A)}{\neq S} = \frac{4}{7}$$

$P(B|A) \neq P(B)$, then we can state that **A and B are dependent events**.

Q 3.

(40 pt.)

There are two local factories that produce radios. Each radio produced at factory A is defective with probability .05, whereas each one produced at factory B is defective with probability .01. Suppose you purchase two radios that were produced at the same factory, which is equally likely to have been either factory A or factory B. If the first radio that you check is defective, what is the conditional probability that the other one is also defective?

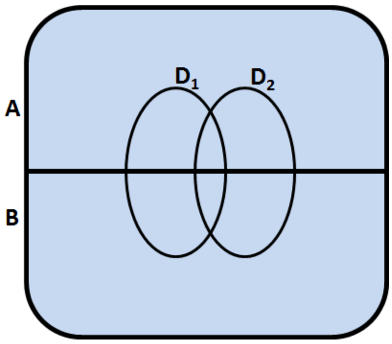
SOLUTION:

Let A be the event that both of the radios you purchase were produced at factory A.
Let B be the event that both of the radios you purchase were produced at factory B.

It is written in the question that "*Suppose you purchase two radios that were produced at the same factory, which is equally likely to have been either factory A or factory B*". Thus, P(A)=0.5 and P(B)=0.5

Let D_1 be the event that the selected first radio is defective.
Let D_2 be the event that the selected second radio is defective.

The venn diagram corresponding to the events are given below:



Our target is to find the probability of D_2 with given D_1. So, according to the Bayes' Rule;

$$P(D_2|D_1) = \frac{P(D_1 \cap D_2)}{P(D_1)} = \frac{\frac{P(D_1 \cap D_2 \cap A)P(A)}{P(A)} + \frac{P(D_1 \cap D_2 \cap B)P(B)}{P(B)}}{\frac{P(D_1 \cap A)P(A)}{P(A)} + \frac{P(D_1 \cap B)P(B)}{P(B)}}$$

$$= \frac{P(D_1 \cap D_2|A)P(A) + P(D_1 \cap D_2|B)P(B)}{P(D_1|A)P(A) + P(D_1|B)P(B)}$$

$$= \frac{(0.05)(0.05)(0.5) + (0.01)(0.01)(0.5)}{(0.05)(0.5) + (0.01)(0.5)} = 0.043$$