Abdullah Gul University

Math-301 (Probability & Statistics)

Fall 2022, QUIZ - V

Name & Surname:
ID Number:

Q 1.

(30 pt.)

Let XX be a discrete random variable with the following PMF (probability mass function):

$$P_X(x) = \begin{cases} 0.2 & for\ x = -2 \\ 0.3 & for\ x = -1 \\ 0.2 & for\ x = 0 \\ 0.2 & for\ x = 1 \\ 0.1 & for\ x = 2 \\ 0 & otherwise \end{cases}$$

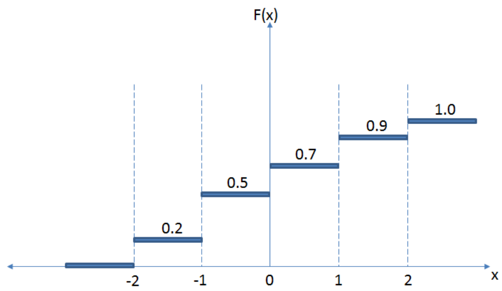
Find and plot the CDF (cumulative density function) of X.

SOLUTION:

The formula of the CDF is below;
$F(x) = P(X \leq x) = \sum_{t \leq x} f(t),$
Then;

$$F(x) = P(X \leq x) = \begin{cases} \sum\limits_{t \leq x} f(t) = 0 & x < -2 \\ \sum\limits_{t \leq x} f(t) = 0.2 & -2 \geq x < -1 \\ \sum\limits_{t \leq x} f(t) = 0.2 + 0.3 = 0.5 & -1 \geq x < 0 \\ \sum\limits_{t \leq x} f(t) = 0.2 + 0.3 + 0.2 = 0.7 & 0 \geq x < 1 \\ \sum\limits_{t \leq x} f(t) = 0.2 + 0.3 + 0.2 + 0.2 = 0.9 & 1 \geq x < 2 \\ \sum\limits_{t \leq x} f(t) = 0.2 + 0.3 + 0.2 + 0.2 + 0.1 = 1.0 & 2 \leq x \end{cases}$$

To plot the CDF;



Let XX be a continuous random variable with PDF

Q 2.

(30 pt.)

$$P_X(x) = \begin{cases} x^2 + \frac{2}{3} & 0 \leq x < 1 \\ 0 & otherwise \end{cases}$$

Find the CDF (cumulative density function) of X. Then, use it to evaluate P(0≤X1).

SOLUTION:

Find the CDF (cumulative density function) of X;

$$Ff(x) = \int \left(x^2 + \frac{2}{3}\right) dx = \frac{x^3 + 2x}{3}$$

To evaluate P(0≤x1);

$$P(0 \leq X < 1) = F(1) - F(0) = \frac{1^3 + 2(1)}{3} - \frac{0^3 + 2(0)}{3} = 1$$

Q 3. Let X and Y be jointly continuous random variables with joint PDF;

(40 pt.)

$$f(x,y) = \begin{cases} cx+1 & x,y \geq 0, x+y<1 \\ 0 & otherwise \end{cases}$$

So, find the value of the constant c in the equation above. *Hint:* $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = 1$

SOLUTION:

We know that;

$x \geq 0,$

$y \geq 0,$

$x + y < 1$

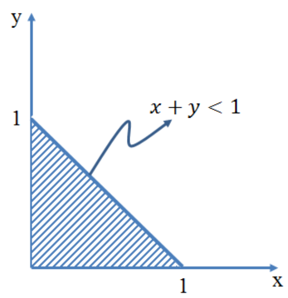
Then, we can write that;

$\mathbf{0 \leq y < 1 - x}$

$0 \leq x < 1 - y$

$x_{max} < 1 - y_{min} = 1 - 0 = 1$

$\mathbf{0 \leq x < 1}$



The cumulative density function should be equal to 1.0, while $\mathbf{0 \leq y < 1 - x}$, and $\mathbf{0 \leq x < 1}$.

$$\begin{aligned}
1 = \quad & \int_0^1 \int_0^{1-x} f(x,y)\, dydx \\
&= \int_0^1 \int_0^{1-x} (cx+1)\, dydx \\
&= \int_0^1 (cxy+y)\Big|_0^{1-x} dx \\
&= \int_0^1 \big(cx(1-x)+(1-x)\big)dx \\
&= \int_0^1 (cx - cx^2 - x + 1)dx \\
&= \left(\frac{cx^2}{2} - \frac{cx^3}{3} - \frac{x^2}{2} + x\right)\Big|_0^1 \\
&= \frac{c(1)^2}{2} - \frac{c(1)^3}{3} - \frac{(1)^2}{2} + (1) = \frac{c}{6} + \frac{1}{2}
\end{aligned}$$

$$\frac{c}{6} + \frac{1}{2} = 1$$

$$c = 3$$