Abdullah Gul University

Math-301 (Probability & Statistics)

Fall 2022, QUIZ - VII

Name & Surname:
ID Number:

Q 1.
(100 pt.)

A non-fair die is rolled in 8 times. After tossing the die, the probabilities of getting 1, 2, 3, 4, and 5 are 0.15, 0.18, 0.20, 0.12, and 0.20, respectively. Then, a-) what is the probability of getting 6 in a total of 4 times, b-) draw the histogram for the probability density function of getting 6 for randomly varying times (from zero to 8), c-) calculate the mean and standard deviation for the histogram.

***Formulas:***

$$\mathrm{b(x, n, p)} = \frac{n!}{x!\,(n-x)!}(p)^x(q)^{n-x}$$

$$\mu = E[x] = \sum_i^n x_i f(x_i)$$

$$Var(X) = E[(X-\mu)^2]$$

**SOLUTION:**

**a-)**

We have two independent results:

Success: To get 6 after tossing the die
Failure: Not to get 6 after tossing the die

Then,
p=1-(0.15+0.18+0.20+0.12+0.20)=0.15
q=1-0.15=0.85

According to the binomial distribution formula;

$$\mathrm{b}(4{,}8{,}0.15) = \frac{8!}{4!\,(8-4)!}(0.15)^4(0.85)^4 \approx 0.018$$

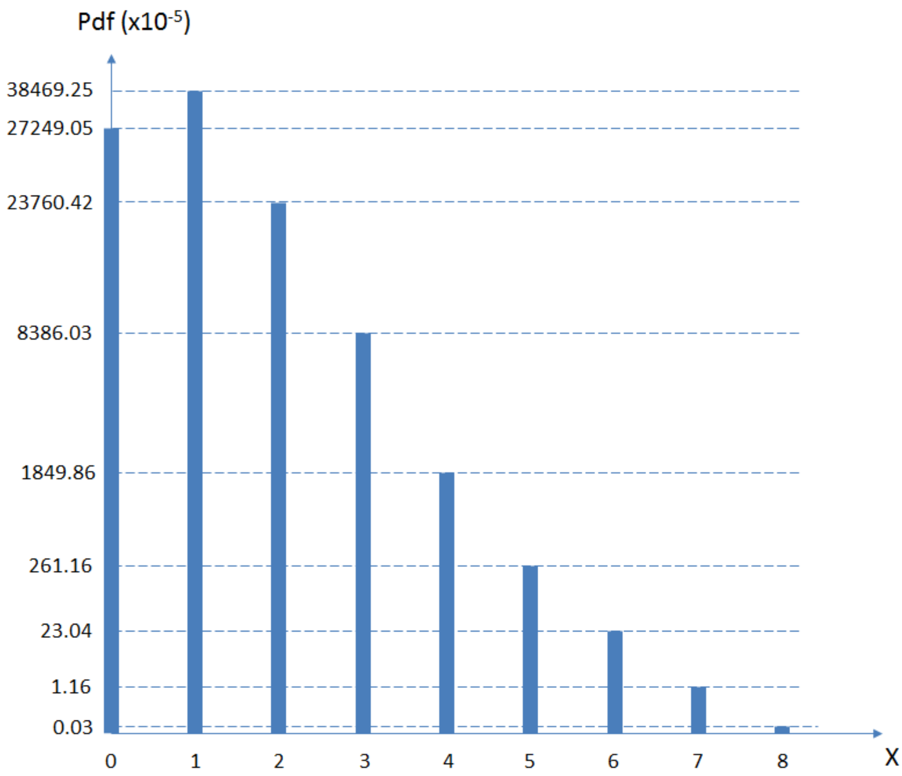
**b-)**

We should calculate the pdf value for each of the random variable changes from 0 to 8;

The pdf values are b(0,8,0.15), b(1,8,0.15), b(2,8,0.15), b(3,8,0.15), b(4,8,0.15), b(5,8,0.15), b(6,8,0.15), b(7,8,0.15), and b(8,8,0.15).

$$b(0,8,0.15) = \frac{8!}{0!\,(8-0)!}(0.15)^0(0.85)^8 \approx 0.2724905$$
$$b(1,8,0.15) = \frac{8!}{1!\,(8-1)!}(0.15)^1(0.85)^7 \approx 0.3846925$$
$$b(2,8,0.15) = \frac{8!}{2!\,(8-2)!}(0.15)^2(0.85)^6 \approx 0.2376042$$
$$b(3,8,0.15) = \frac{8!}{3!\,(8-3)!}(0.15)^3(0.85)^5 \approx 0.0838603$$
$$b(4,8,0.15) = \frac{8!}{4!\,(8-4)!}(0.15)^4(0.85)^4 \approx 0.0184986$$
$$b(5,8,0.15) = \frac{8!}{5!\,(8-5)!}(0.15)^5(0.85)^3 \approx 0.0026116$$
$$b(6,8,0.15) = \frac{8!}{6!\,(8-6)!}(0.15)^6(0.85)^2 \approx 0.0002304$$
$$b(7,8,0.15) = \frac{8!}{7!\,(8-7)!}(0.15)^7(0.85)^1 \approx 0.0000116$$
$$b(8,8,0.15) = \frac{8!}{8!\,(8-8)!}(0.15)^8(0.85)^0 \approx 0.0000003$$

To draw the pdf function:



**c-)**

$$\mu = E[x] = \sum_{i=0}^{n=8} x_i f(x_i)$$
$$= (0)(0.2724905) + (1)(0.3846925) + (2)(0.2376042) + (3)(0.0838603)$$
$$+ (4)(0.0184986) + (5)(0.0026116) + (6)(0.0002304) + (7)(0.0000116)$$
$$+ (8)(0.0000003) = 1.2 = np = (8)(0.15)$$

$$Var(X) = E[(X-\mu)^2] = E[X^2] - 2\mu E[X] + E[\mu^2] = E[X^2] - 2\mu\mu + \mu^2 = E[X^2] - \mu^2$$

$$E[X^2] = \sum_{i=0}^{n=8} (X_i)^2 f(X_i)$$
$$= 0^2 f(0) + 1^2 f(1) + 2^2 f(2) + 3^2 f(3) + 4^2 f(4) + 5^2 f(5) + 6^2 f(6) + 7^2 f(7)$$
$$+ 8^2 f(8) = 2.46$$

$$\sigma^2 = Var(X) = 2.46 - (1.2)^2 = 1.02 = npq = (8)(0.15)(0.85)$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{1.02} \approx 1.01$$