Abdullah Gul University

Math-301 (Probability & Statistics)

Fall 2022, QUIZ - VIII

Name & Surname:
ID Number:

Q 1. (100 pt.)

Suppose that the mean value for the number of defective internet router boxes produced by a firm is 1 **in a year**. Thus, **a-)** considering the number (from zero to 10) of defectives produced **during three years** to be a random variable, draw the histogram for the probability distribution values of these variables, and **b-)** find the mean and standard deviation for the random variables given in the histogram.

Formulas:

$$\mathrm{b}(\mathrm{x}, \mathrm{n}, \mathrm{p}) = \frac{n!}{x!\,(n-x)!}(p)^x(q)^{n-x}$$

$$b^-(k, x, p) = \frac{(x-1)!}{(x-k)!\,(k-1)!}(p)^k(q)^{x-k}$$

$$g(x, p) = p(q)^{x-1}$$

$$p(x, \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$\mu = E[x] = \sum_{i}^{n} x_i f(x_i)$$

$$Var(X) = E[(X-\mu)^2]$$

SOLUTION:

a-)

We have two independent results:

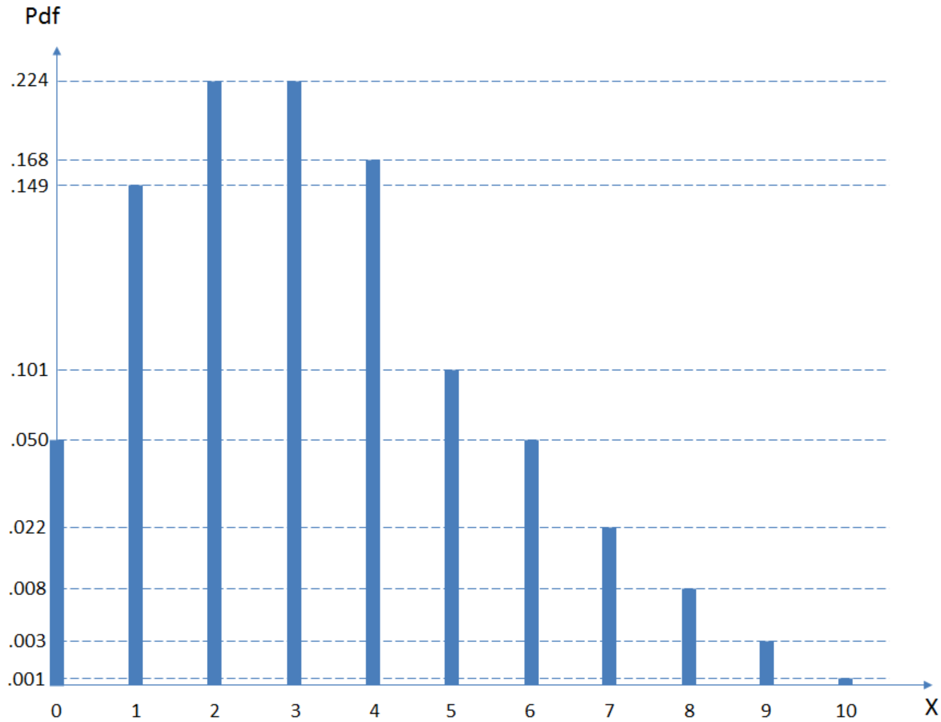
Success: To be defective
Failure: Not to be defective

In the question, we have only the information about the mean number of the defective products. Then, we can use the Poisson distribution formula. Additionally, the mean number of defectives is given as 1 in a year. But according to the question, we should focus on the mean of the number of defectives during 3 years, that is why, the lamda (λ) value will be 3 in the poisson distribution formula.

We should calculate the pdf value for each of the random variable changes from 0 to 10;
The pdf values are p(0,3), p(1,3), p(2,3), p(3,3), p(4,3), p(5,3), p(6,3), p(7,3), p(8,3), p(9,3), and p(10,3).

$$p(0,3) = \frac{2.718^{-3}3^{0}}{0!} \approx 0.050$$
$$p(1,3) = \frac{2.718^{-3}3^{1}}{1!} \approx 0.149$$
$$p(2,3) = \frac{2.718^{-3}3^{2}}{2!} \approx 0.224$$
$$p(3,3) = \frac{2.718^{-3}3^{3}}{3!} \approx 0.224$$
$$p(4,3) = \frac{2.718^{-3}3^{4}}{4!} \approx 0.168$$
$$p(5,3) = \frac{2.718^{-3}3^{5}}{5!} \approx 0.101$$
$$p(6,3) = \frac{2.718^{-3}3^{6}}{6!} \approx 0.050$$
$$p(7,3) = \frac{2.718^{-3}3^{7}}{7!} \approx 0.022$$
$$p(8,3) = \frac{2.718^{-3}3^{8}}{8!} \approx 0.008$$
$$p(9,3) = \frac{2.718^{-3}3^{9}}{9!} \approx 0.003$$
$$p(10,3) = \frac{2.718^{-3}3^{10}}{10!} \approx 0.001$$

To draw the pdf function:



b-)

$$\mu = E[x] = \sum_{i=0}^{n=10} x_i f(x_i)$$

$$= (0)(0.050) + (1)(0.149) + (2)(0.224) + (3)(0.224) + (4)(0.168) + (5)(0.101) + (6)(0.050) + (7)(0.022) + (8)(0.008) + (9)(0.003) + (10)(0.001) = 3.001 \approx 3 = \mu$$

$$Var(X) = E[(X-\mu)^2] = E[X^2] - 2\mu E[X] + E[\mu^2] = E[X^2] - 2\mu\mu + \mu^2 = E[X^2] - \mu^2$$

$$E[X^2] = \sum_{i=0}^{n=10} (X_i)^2 f(X_i)$$

$$= 0^2 f(0) + 1^2 f(1) + 2^2 f(2) + 3^2 f(3) + 4^2 f(4) + 5^2 f(5) + 6^2 f(6) + 7^2 f(7) + 8^2 f(8) + 9^2 f(9) + 10^2 f(10) = 12.007$$

$$\sigma^2 = Var(X) = 12.007 - (3)^2 = 3.001$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{3.001} \approx 1.73$$