Abdullah Gul University

Math-301 (Probability & Statistics)

Fall 2022, QUIZ - VII

Name & Surname:
ID Number:

Q 1. (100 pt.)

If each voter is for Proposition A with probability 0.7, then a-) what is the probability that exactly 7 of 10 voters are for this proposition, b-) draw the histogram for the probability density function of that the random variable of voters are for Proposition A, c-) calculate the mean and standard deviation for the histogram.

Formulas:

$$\mathrm{b(x, n, p)} = \frac{n!}{x!\,(n-x)!}(p)^x(q)^{n-x}$$

$$\mu = E[x] = \sum_i^n x_i f(x_i)$$

$$Var(X) = E[(X-\mu)^2]$$

SOLUTION:

a-)

We have two independent results:

Success: To be for Proposition A
Failure: Not to be for Proposition A

Then,
p=0.7
q=1-0.7=0.3

According to the binomial distribution formula;

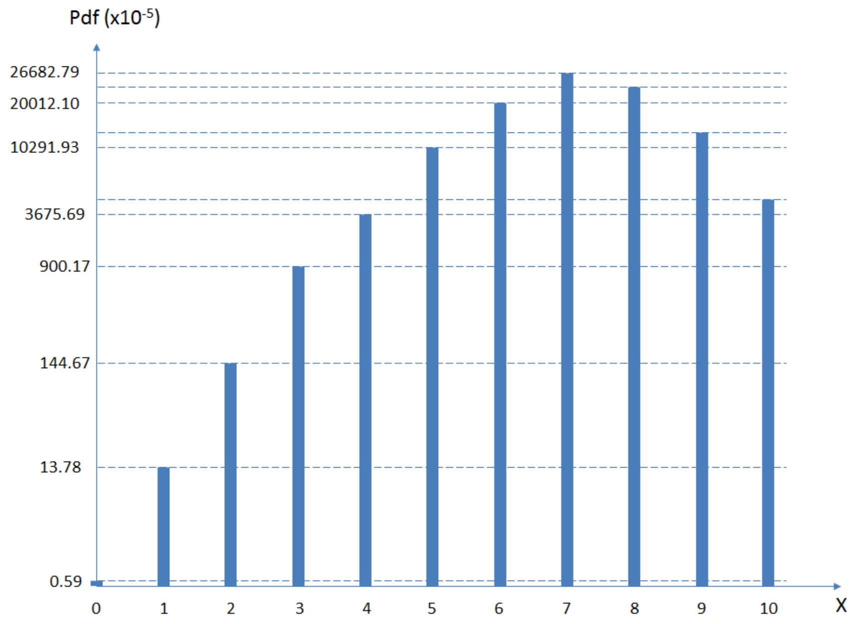
$$\mathrm{b}(7,10,0.7) = \frac{10!}{7!\,(10-7)!}(0.7)^7(0.3)^3 \approx 0.267$$

b-)

We should calculate the pdf value for each of the random variable changes from 0 to 10;
The pdf values are b(0,10,0.3), b(1,10,0.3), b(2,10,0.3), b(3,10,0.3), b(4,10,0.3), b(5,10,0.3), b(6,10,0.3), b(7,10,0.3), b(8,10,0.3), b(9,10,0.3), and b(10,10,0.3).

$$b(0,10,0.7) = \frac{10!}{0!\,(10-0)!}(0.7)^0(0.3)^{10} \approx 0.0000059$$
$$b(1,10,0.7) = \frac{10!}{1!\,(10-1)!}(0.7)^1(0.3)^{9} \approx 0.0001378$$
$$b(2,10,0.7) = \frac{10!}{2!\,(10-2)!}(0.7)^2(0.3)^{8} \approx 0.0014467$$
$$b(3,10,0.7) = \frac{10!}{3!\,(10-3)!}(0.7)^3(0.3)^{7} \approx 0.0090017$$
$$b(4,10,0.7) = \frac{10!}{4!\,(10-4)!}(0.7)^4(0.3)^{6} \approx 0.0367569$$
$$b(5,10,0.7) = \frac{10!}{5!\,(10-5)!}(0.7)^5(0.3)^{5} \approx 0.1029193$$
$$b(6,10,0.7) = \frac{10!}{6!\,(10-6)!}(0.7)^6(0.3)^{4} \approx 0.2001210$$
$$b(7,10,0.7) = \frac{10!}{7!\,(10-7)!}(0.7)^7(0.3)^{3} \approx 0.2668279$$
$$b(8,10,0.7) = \frac{10!}{8!\,(10-8)!}(0.7)^8(0.3)^{2} \approx 0.2334744$$
$$b(9,10,0.7) = \frac{10!}{9!\,(10-9)!}(0.7)^9(0.3)^{1} \approx 0.1210608$$
$$b(10,10,0.7) = \frac{10!}{10!\,(10-10)!}(0.7)^{10}(0.3)^{0} \approx 0.0282475$$

To draw the pdf function:



c-)

$$\mu = E[x] = \sum_{i=0}^{n=10} x_i f(x_i)$$
$$= (0)(0.0000059) + (1)(0.0001378) + (2)(0.0014467) + (3)(0.0090017)$$
$$+ (4)(0.0367569) + (5)(0.1029193) + (6)(0.2001210) + (7)(0.2668279)$$
$$+ (8)(0.2334744) + (9)(0.1210608) + (10)(0.0282475) = 7 = np$$
$$= (10)(0.7)$$

$$Var(X) = E[(X-\mu)^2] = E[X^2] - 2\mu E[X] + E[\mu^2] = E[X^2] - 2\mu\mu + \mu^2 = E[X^2] - \mu^2$$

$$E[X^2] = \sum_{i=0}^{n=10} (X_i)^2 f(X_i)$$
$$= 0^2 f(0) + 1^2 f(1) + 2^2 f(2) + 3^2 f(3) + 4^2 f(4) + 5^2 f(5) + 6^2 f(6) + 7^2 f(7)$$
$$+ 8^2 f(8) + 9^2 f(9) + 10^2 f(10) = 51.1$$

$$\sigma^2 = Var(X) = 51.1 - (7)^2 = 2.1 = npq = (10)(0.7)(0.3)$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{2.1} \approx 1.45$$