Abdullah Gul University

Math-301 (Probability & Statistics)

Fall 2022, QUIZ - IV

Name & Surname:
ID Number:

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| Q 1. (30 pt.) | When you had flipped a coin in three times, you have observed that at least one head occurred. Then, what is the probability that you have observed at least two heads? |

**SOLUTION:**

This is a conditional probability question with the given event A which at least one head occurs. Our target is to find the probability of the event B which at least two heads occur. So, the probability of B with given A is as follow;

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

First, let us find $P(A \cap B)$;

The set of the elements of A is that, A={HTT, THT, TTH, THH, HTH, HHT, HHH}

The set of the elements of B is that, B={THH, HTH, HHT, HHH}

$A \cap B$ = {THH, HTH, HHT, HHH}

The sample space elements are {HTT, THT, TTH, THH, HTH, HHT, HHH, TTT}

$$P(A \cap B) = \frac{\neq (A \cap B)}{\neq S} = \frac{4}{8}$$

Second, let us find $P(A)$;

$$P(A) = \frac{\neq A}{\neq S} = \frac{7}{8}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{4/8}{7/8} = \frac{4}{7}$$

Q 2.

(30 pt.)

Consider an experiment in which 2 stones are drawn in succession from a bag, with replacement. Also consider there are 3 red and 4 blue stones in this bag at the initial case. So, if A is the event that the color of the selected first stone is red, and B is the event that the color of the selected second stone is blue, then these A and B events are independent or dependent? Please prove whether these events are independent or not.

**SOLUTION:**

If A is the event that the color of the selected first stone is red;

$A = \{R_1, R_2, R_3\}$ within the Sample Space, $S = \{R_1, R_2, R_3, B_1, B_2, B_3, B_4\}$

$$P(A) = \frac{\neq A}{\neq S} = \frac{3}{7}$$

After the replacement of the first selected stone, if B is the event that the color of the selected second stone is blue;

$B = \{B_1, B_2, B_3, B_4\}$ within the Sample Space, $S = \{R_1, R_2, R_3, B_1, B_2, B_3, B_4\}$

$$P(B) = \frac{\neq B}{\neq S} = \frac{4}{7}$$

Even if we know that the first selected stone is red color, the elements of the second event are given below;

$B|A = \{B_1, B_2, B_3, B_4\}$ within the Sample Space, $S = \{R_1, R_2, R_3, B_1, B_2, B_3, B_4\}$

$$P(B|A) = \frac{\neq (B|A)}{\neq S} = \frac{4}{7}$$

$P(B|A) = P(B)$, then we can state that **A and B are independent events**.

Q 3.

(40 pt.)

A laboratory blood test is 99 percent effective in detecting a certain disease when it is, in fact, present (That is, if a real sick person is tested, the test result will imply with probability .99 that he or she has the disease). However, the test also yields a "positive" result for 3 percent of the healthy people tested (That is, if a healthy person is tested, then, with probability .03, the test result will imply he or she has the disease). If .5 percent of the population has the disease in fact, what is the probability a person has the disease given that his test result is positive?

**SOLUTION:**

Let D be the event that the tested person has the disease.
Let Y be the event that the test result is positive.

The given probability values follow;

$P(Y|D) = 0.99$
$P(D) = 0.005$
$P(D') = 0.995$
$P(Y|D') = 0.03$

Our target is to calculate $P(D|Y)$. So, according to the Bayes' Rule;

$$P(D|Y) = \frac{P(D \cap Y)}{P(Y)} = \frac{P(Y|D)P(D)}{P(Y \cap D) + P(Y \cap D')} = \frac{P(Y|D)P(D)}{P(Y|D)P(D) + P(Y|D')P(D')}$$

$$P(D|Y) = \frac{(0.99)(0.005)}{(0.99)(0.005) + (0.03)(0.995)} = \frac{0.00495}{0.00495 + 0.02985} = 0.142 \approx 14\%$$