Abdullah Gul University

Math-301 (Probability & Statistics)

Fall 2022, QUIZ - VI

Name & Surname:
ID Number:

Q 1.
(30 pt.)

A random variable X is described with a probability density function of

$$f(x) = \begin{cases} \dfrac{1}{x\ln(1.5)} & 4 \le x \le 6 \\ 0 & otherwise \end{cases}$$

Find the mean and median of the random variable X.

SOLUTION:

a- To find mean:

$$mean = E[X] = \int_4^6 x\frac{1}{x\ln(1.5)}dx = \frac{x}{\ln(1.5)}\bigg|_4^6 = \frac{6}{\ln(1.5)} - \frac{4}{\ln(1.5)} = 4.94$$

b- To find median:

$$0.5 = \int_4^x f(x)\,dx = \int_4^x \frac{1}{x\ln(1.5)}dx = \frac{\ln(x)}{\ln(1.5)}\bigg|_4^x = \frac{1}{\ln(1.5)}(\ln(x) - \ln(4)) = 4.94$$

$$x = 4.90 = median$$

Q 2. Let X has the following discrete probability mass function: p(0)=0.2, p(1)=0.5, p(2)=0.3. Then, calculate the variance of the random variable X.

(30 pt.)

SOLUTION:

$$\mu = E[X] = \sum_{i=0}^{n=2} Xf(X_i) = 0(0.2) + 1(0.5) + 2(0.3) = 1.1$$

$$Var(X) = E[(X-\mu)^2] = E[X^2] - 2\mu E[X] + E[\mu^2] = E[X^2] - 2\mu\mu + \mu^2 = E[X^2] - \mu^2$$

$$E[X^2] = \sum_{i=0}^{n=2} (X_i)^2 f(X_i) = 0^2 f(0) + 1^2 f(1) + 2^2 f(2) = 0 + 0.5 + 1.2 = 1.7$$

$$Var(X) = 1.7 - (1.1)^2 = 0.49$$

Q 3.

(40 pt.)

Suppose that a fair coin is tossed in three times. So, the sample space is

$$S = \begin{Bmatrix} HHH \\ HHT \\ HTH \\ HTT \\ THH \\ THT \\ TTH \\ TTT \end{Bmatrix}$$

If the random variable X is the number of heads obtained in the first and second tosses, and if the random variable Y is the number of heads in the third toss, then the joint probability density function of X and Y is given in the table below:

	X=0	X=1	X=2
Y=0	**1/8** (due to TTT)	**2/8** (due to HTT and THT)	**1/8** (due to HHT)
Y=1	**1/8** (due to TTH)	**2/8** (due to HTH and THH)	**1/8** (due to HHH)

Thus, find the covariance of two random variables X and Y.

SOLUTION:

$$\mu_x = E(x) = \sum_{i=0}^{n=2} x_i f(x_i) = (0)\left(\frac{1}{8}+\frac{1}{8}\right) + (1)\left(\frac{2}{8}+\frac{2}{8}\right) + (2)\left(\frac{1}{8}+\frac{1}{8}\right) = 1$$

$$\mu_y = E(y) = \sum_{i=0}^{n=2} y_i f(y_i) = (0)\left(\frac{1}{8}+\frac{2}{8}+\frac{1}{8}\right) + (1)\left(\frac{1}{8}+\frac{2}{8}+\frac{1}{8}\right) = \frac{1}{2}$$

$$Cov(X,Y) = \sigma_{XY} = E[(X-\mu_X)(Y-\mu_Y)] = \sum_{i=0}^{n=2}\sum_{j=0}^{k=3}(x_j-\mu_X)(y_i-\mu_y)f(x_j,y_i)$$

$$= (0-1)\left(0-\frac{1}{2}\right)\left(\frac{1}{8}\right) + (1-1)\left(0-\frac{1}{2}\right)\left(\frac{1}{4}\right) + (2-1)\left(0-\frac{1}{2}\right)\left(\frac{1}{8}\right)$$

$$+ (0-1)\left(1-\frac{1}{2}\right)\left(\frac{1}{8}\right) + (1-1)\left(1-\frac{1}{2}\right)\left(\frac{1}{4}\right) + (2-1)\left(1-\frac{1}{2}\right)\left(\frac{1}{8}\right)$$

$$= 0$$