COMP 301 Analysis of Algorithms, Fall 2022
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HW 4

Submit your answers to Canvas for the problems given below.

1.

Indicate, for each pair of expressions (A, B) in the table below, whether A is O, o, Ω, ω, or Θ of B. Assume that $k \geq 1$, $\epsilon > 0$, and $c > 1$ are constants. Your answer should be in the form of the table with "yes" or "no" written in each box.

	A	B	O	o	Ω	ω	Θ
a.	$\lg^k n$	n^ϵ					
b.	n^k	c^n					
c.	$\sqrt{n}$	$n^{\sin n}$					
d.	2^n	$2^{n/2}$					
e.	$n^{\lg c}$	$c^{\lg n}$					
f.	$\lg(n!)$	$\lg(n^n)$					

2. Consider a modification to merge sort in which n/k sublists of length k are sorted using insertion sort and then merged using the standard merging mechanism (i.e. you only do insertion sort at one level of recursion tree), where k is a value to be determined.

(a) Show that the insertion sort can sort the n/k sublists each of length k in $\Theta(nk)$ worst-case time.

(b) Show that the n/k sublists can be merged in $\Theta(n\lg(n/k))$ worst-case time

(c) Given that the modified algorithm runs in $\Theta(nk + n\lg(n/k))$ worst-case time, what is the largest value of k as a function of n and in Θ-notation for which the modified algorithm has the same running time as merge sort in Θ-notation?

(d) How should we choose k in practice? Hint: consider the list lengths for which insertion sort is better than merge sort, which is a range of integers to choose from. Then consider which of these values is the best option to start with when combined with merge sort.

3. Show that if $f(n)$ and $g(n)$ are monotonically increasing functions, then so is the function $f(g(n))$, and if $f(n)$ and $g(n)$ are in addition nonnegative, then $f(n)g(n)$ is monotonically increasing.