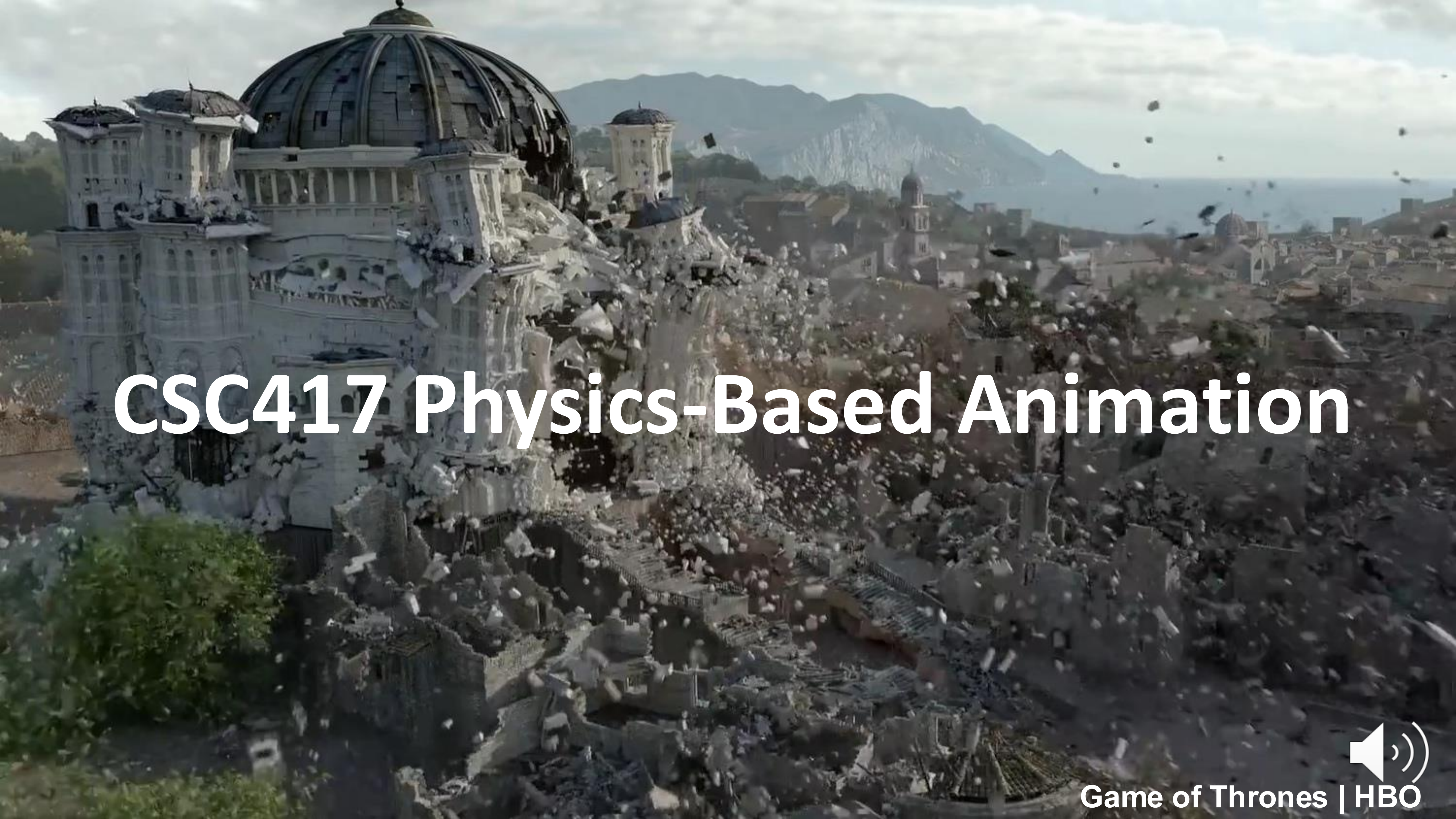
CSC417 Physics-Based Animation

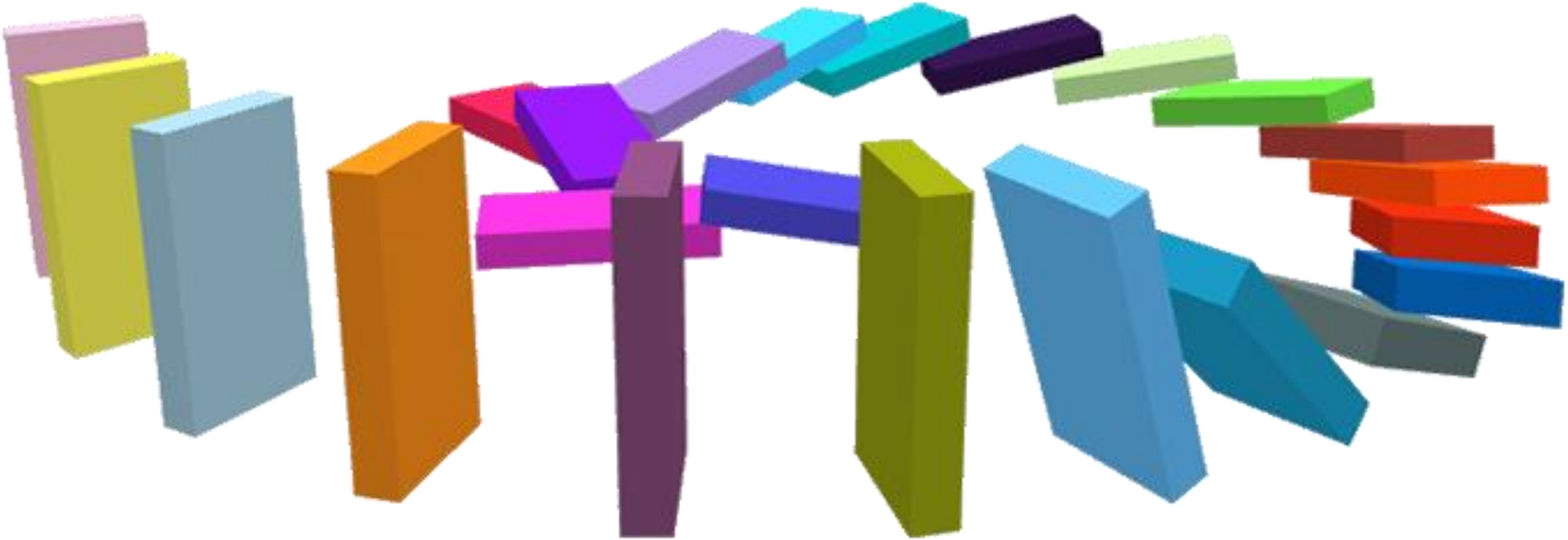


Game of Thrones | HBO

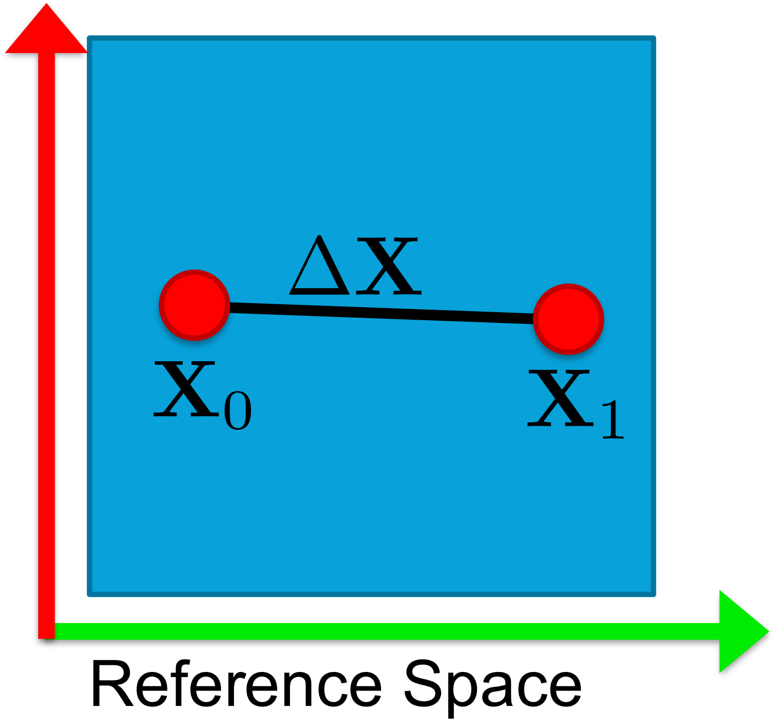
Rigid Body Simulation

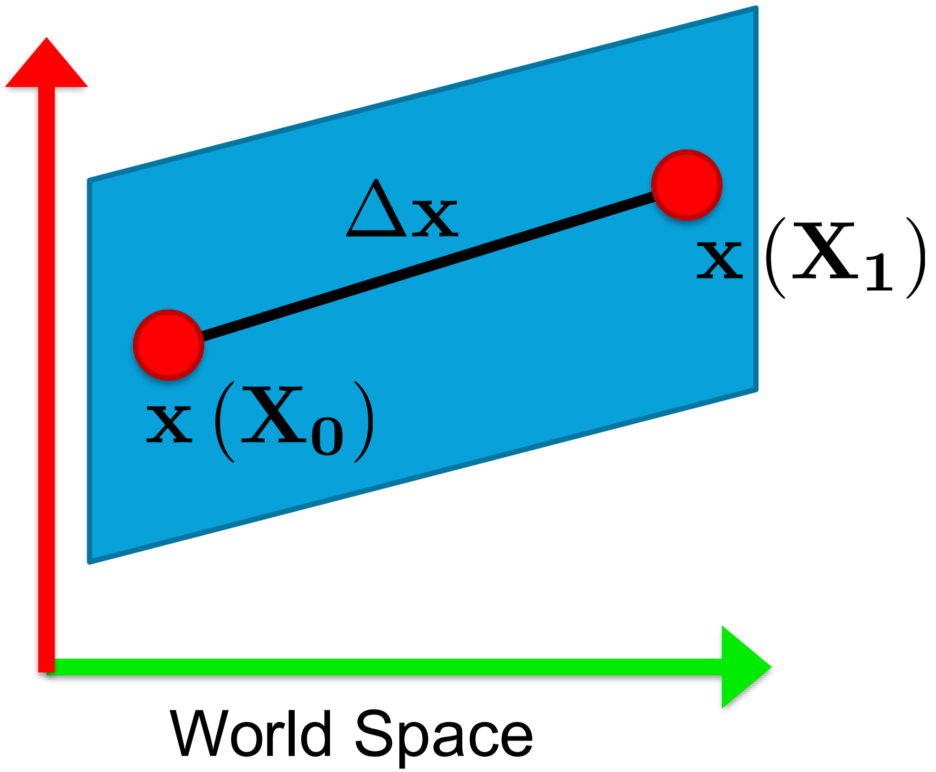
Questions from Previous Lecture ?

What Makes an Object Rigid ?



What Makes an Object Rigid ?





$$\Delta\mathbf{x} \approx \cancel{\mathbf{x}(\mathbf{X}_0)} + \underbrace{\frac{\partial\mathbf{x}}{\partial\mathbf{X}}}_{\text{deformation gradient } \mathrm{F}}\Delta\mathbf{X} - \cancel{\mathbf{x}(\mathbf{X}_0)}$$

What Makes an Object Rigid ?

Strain $\Delta \mathbf{x}^T \Delta \mathbf{x} - \Delta \mathbf{X}^T \Delta \mathbf{X}$

$$\Delta \mathbf{X}^T \mathrm{F}^T \mathrm{F} \Delta \mathbf{X} - \Delta \mathbf{X}^T \Delta \mathbf{X}$$

$$\Delta \mathbf{X}^T \underbrace{\left(\overbrace{\mathrm{F}^T \mathrm{F}}^{\text{Right Cauchy Green Deformation}} - \mathrm{I}\right)}_{\text{Green Lagrange Strain}} \Delta \mathbf{X} = \mathbf{0}$$

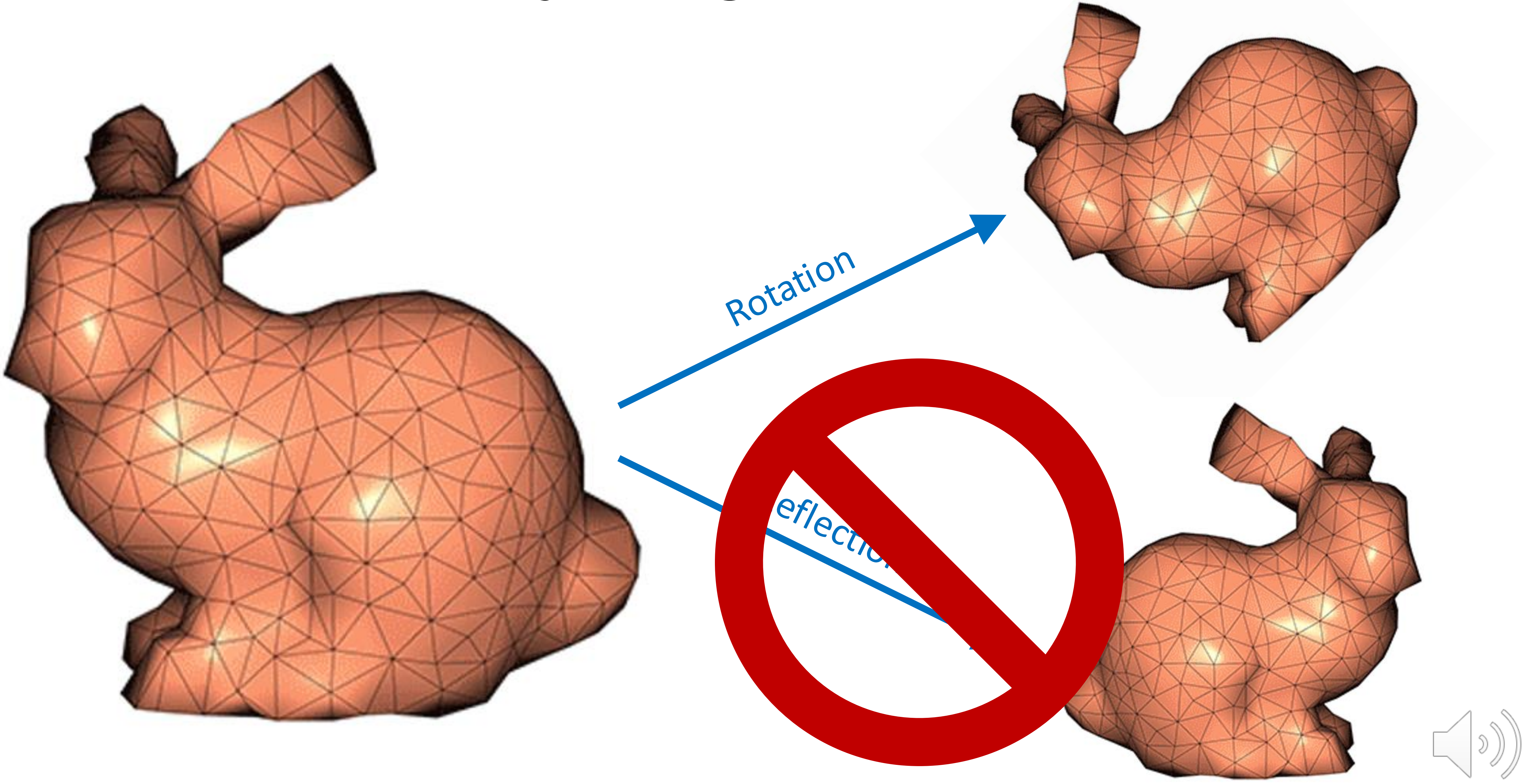
What Makes an Object Rigid ?

$$\Delta\mathbf{X}^T\left(\mathrm{F}^T\mathrm{F}-\mathrm{I}\right)\Delta\mathbf{X}=\mathbf{0}$$

Implies

$$\underbrace{\mathrm{F}^T\mathrm{F}}_{\text{Orthogonal}}=\mathrm{I}$$

What Makes an Object Rigid ?



What Makes an Object Rigid ?

$$\Delta\mathbf{X}^T\left(\mathrm{F}^T\mathrm{F}-\mathrm{I}\right)\Delta\mathbf{X}=\mathbf{0}$$

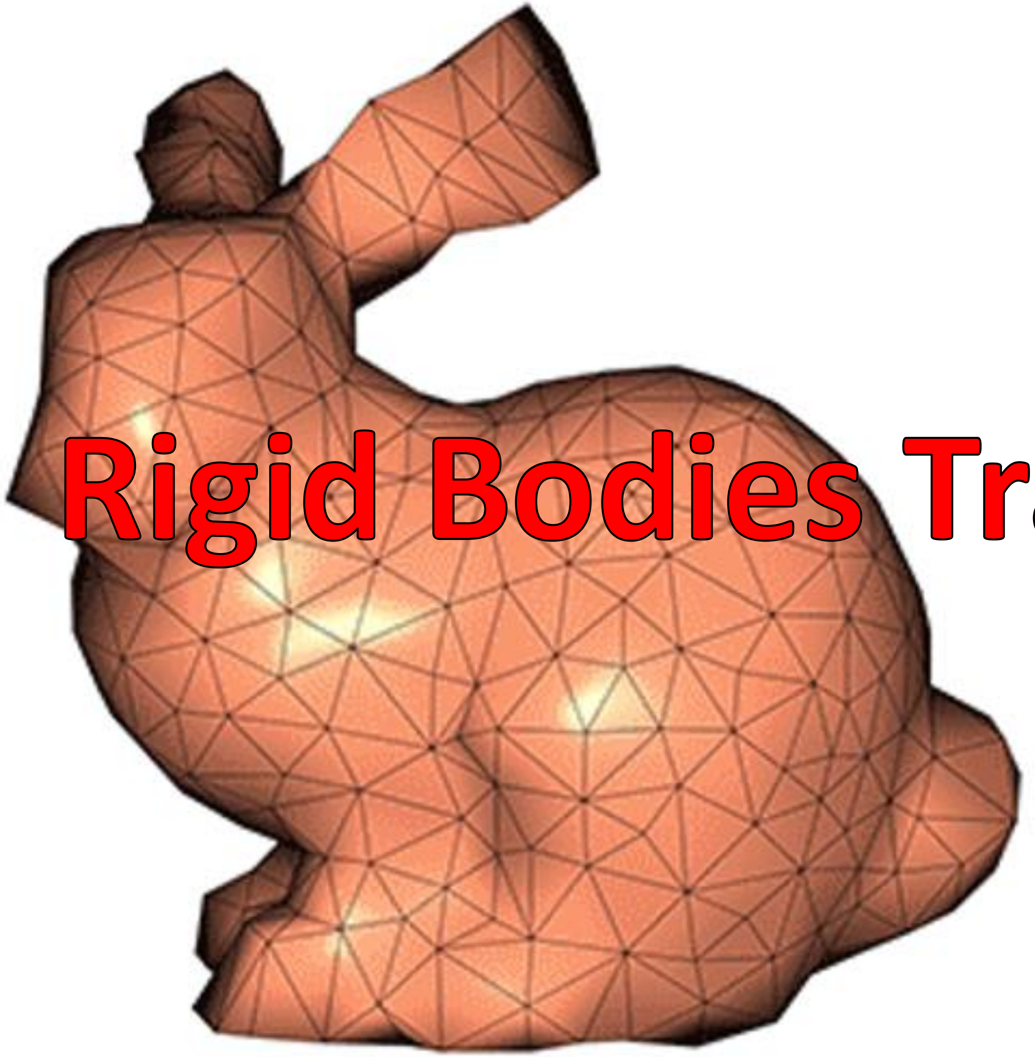
Implies

$$\underbrace{\mathrm{F}^T\mathrm{F}}_{\text{Orthogonal}}=\mathrm{I}$$

Rigid Bodies Rotate

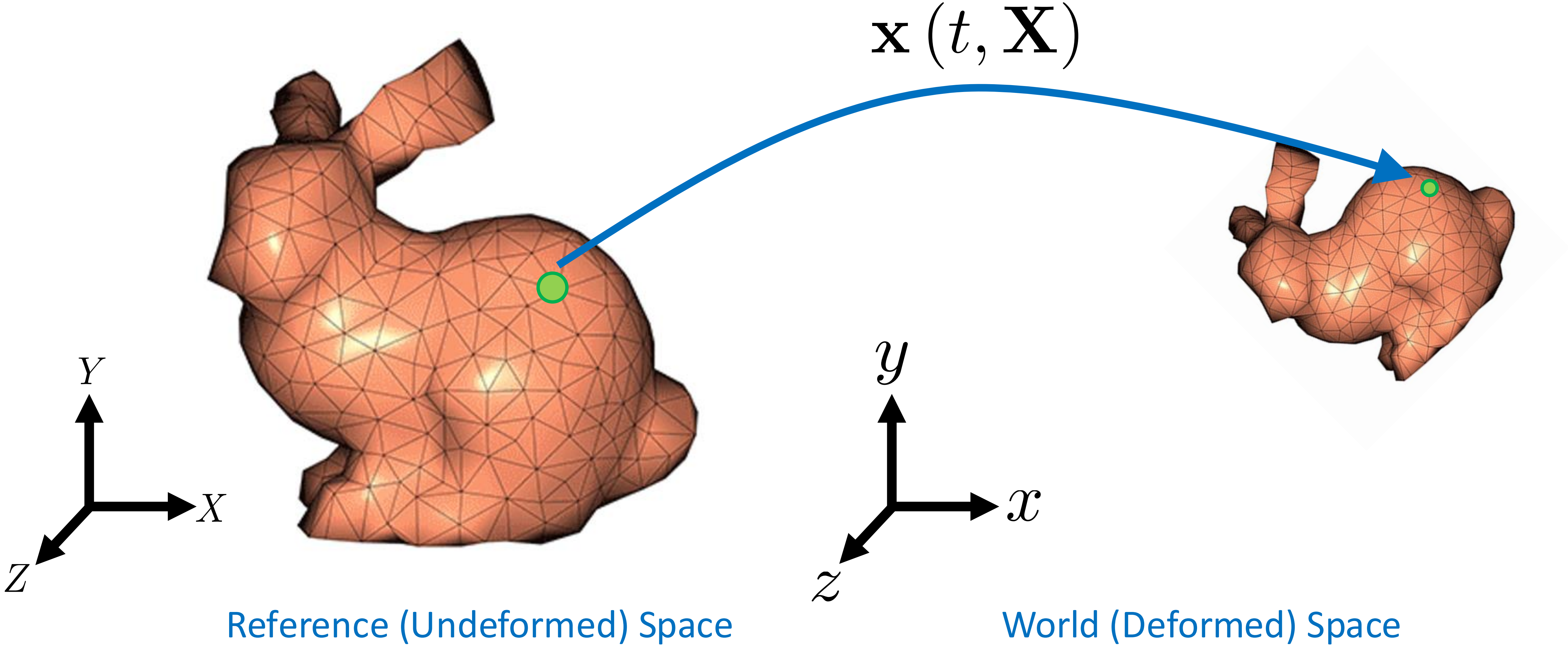
$$\mathrm{F}\in\underbrace{SO\left(3\right)}_{\text{Special Orthogonal Group (Rotations) !}}$$

What Makes an Object Rigid ?



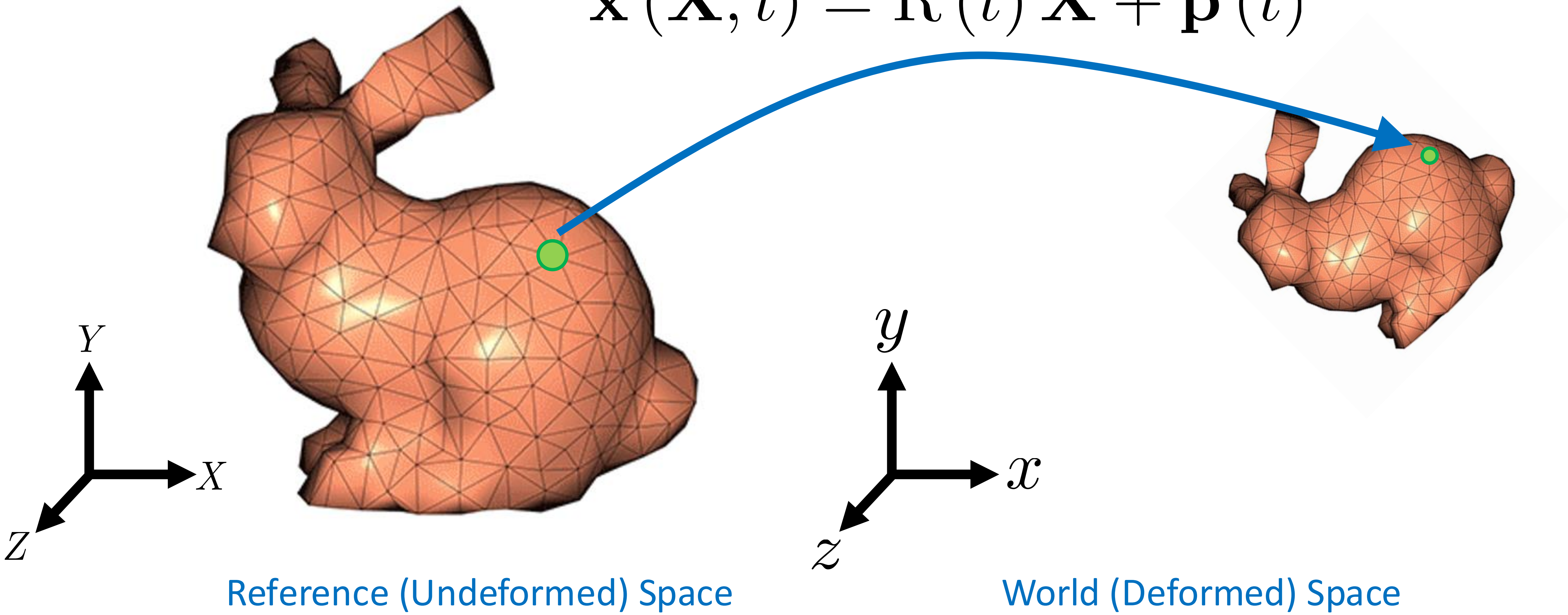
Rigid Bodies Translate

The Rigid Body Mapping

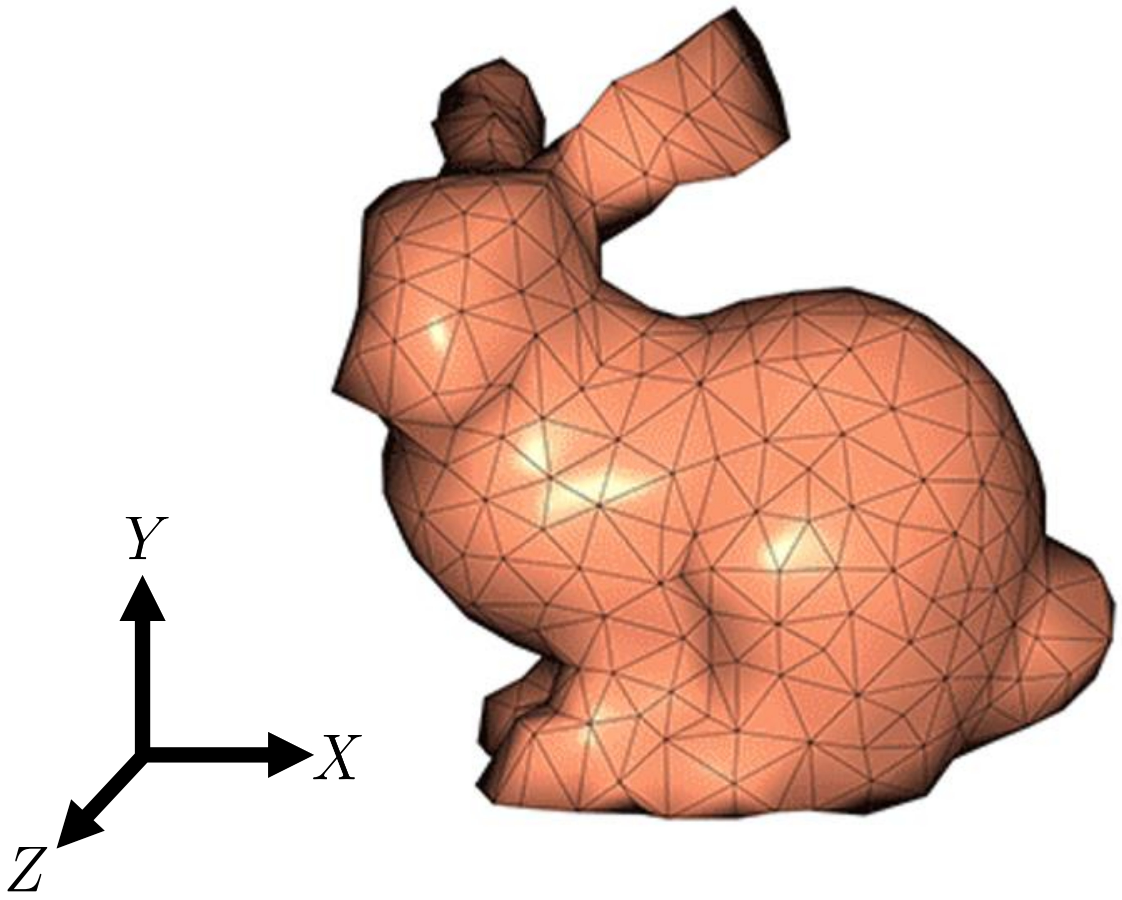


The Rigid Body Mapping

$$\mathbf{x}(\mathbf{X}, t) = \mathrm{R}(t)\,\mathbf{X} + \mathbf{p}(t)$$



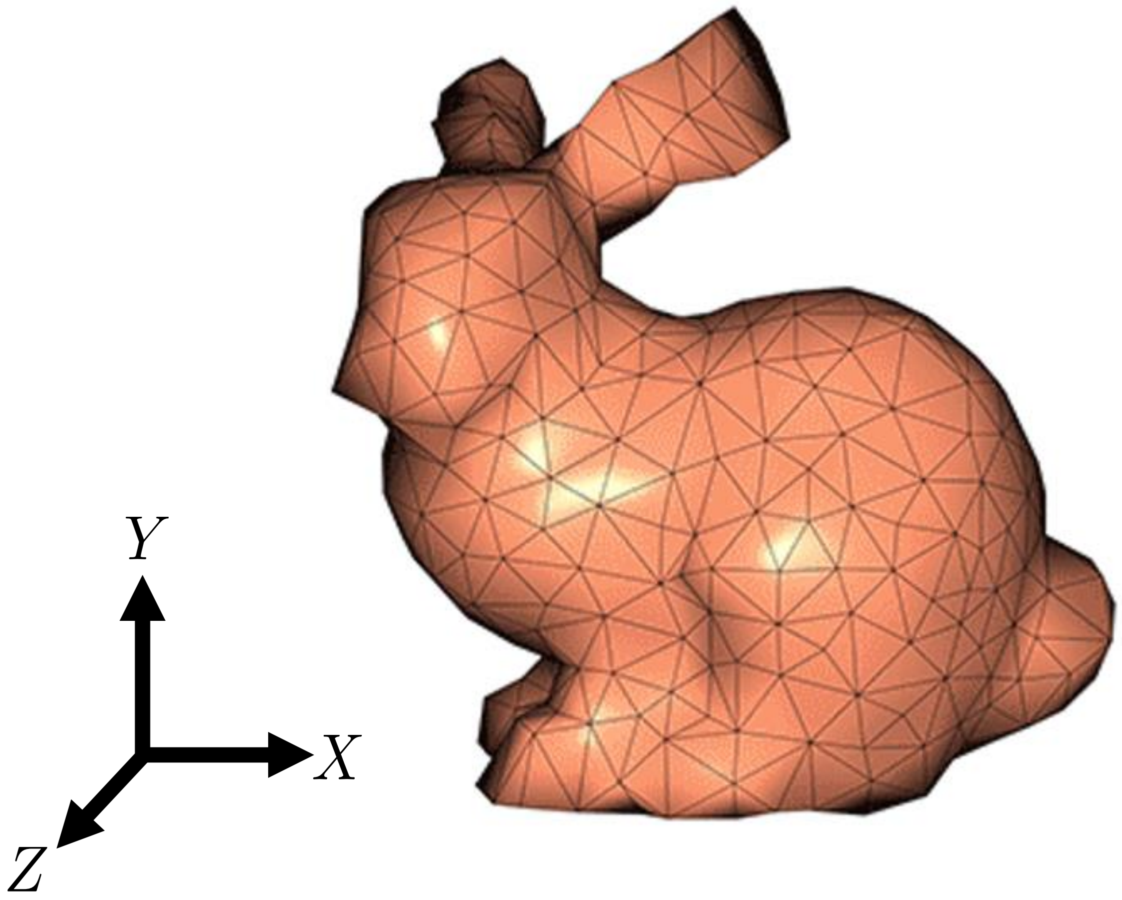
Generalized Coordinates of a Rigid Body



Reference (Undeformed) Space

$$\mathbf{x}\left(\mathbf{X}, t\right) = \underbrace{\mathrm{R}\left(t\right)}_{\text{Rotation Matrix } \in \mathbb{R}^{3\times 3}} \mathbf{X} + \overbrace{\mathbf{p}\left(t\right)}^{\text{Translation } \in \mathbb{R}^{3\times 1}}$$

Generalized Coordinates of a Rigid Body

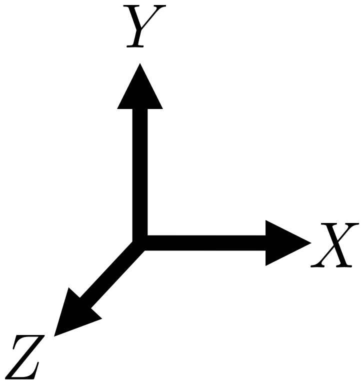


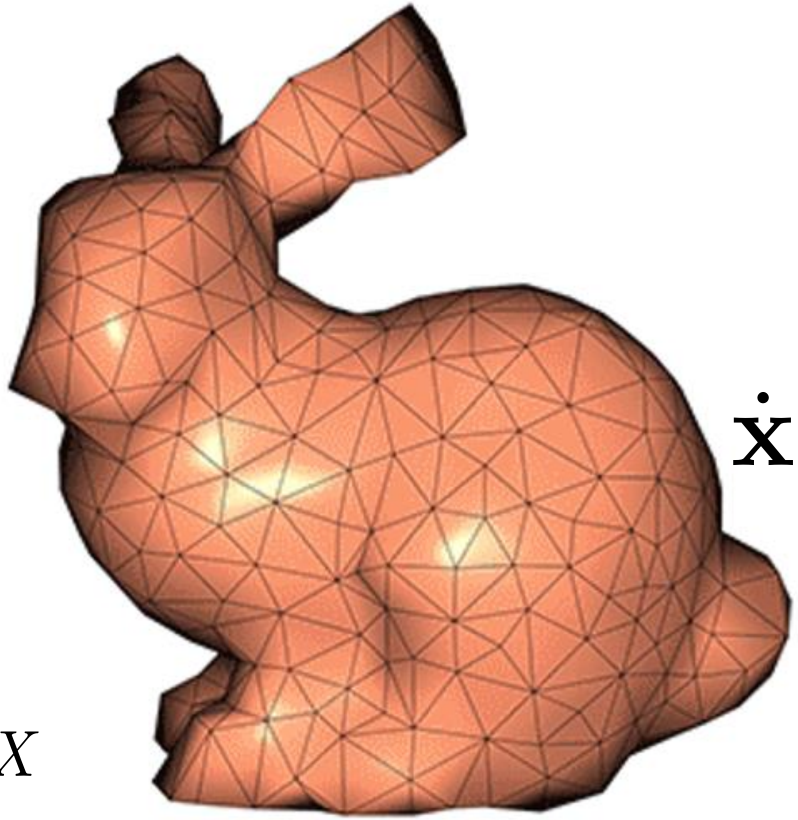
Reference (Undeformed) Space

$$\mathbf{q} = \{\mathrm{R}, \mathbf{p}\}$$

Set with rotation and translation

Generalized Velocity of a Rigid Body

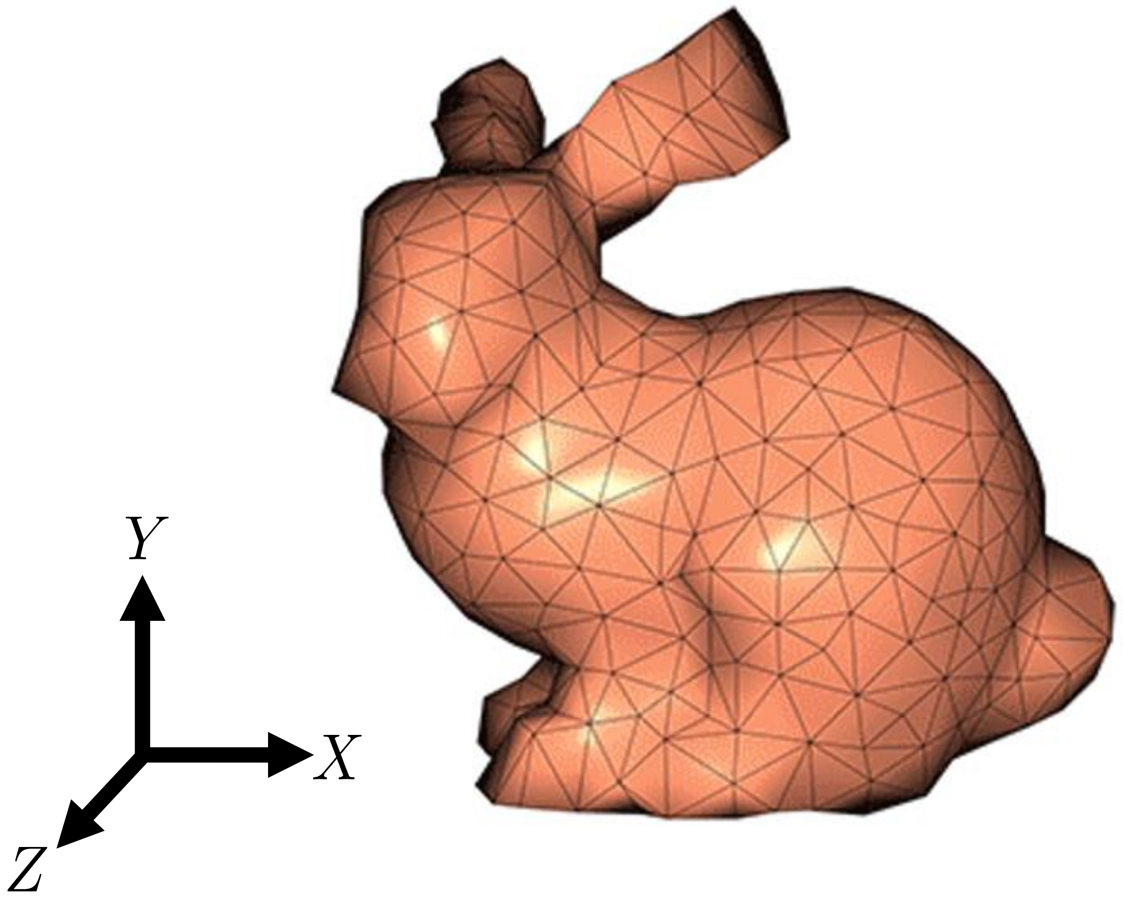




$$\dot{\mathbf{x}}\left(\mathbf{X},t\right) = \frac{d}{dt}\left(\underbrace{\mathrm{R}\left(t\right)}_{\text{Rotation Matrix } \in \mathbb{R}^{3\times3}}\mathbf{X} + \overbrace{\mathbf{p}\left(t\right)}^{\text{Translation } \in \mathbb{R}^{3\times1}}\right)$$

Reference (Undeformed) Space

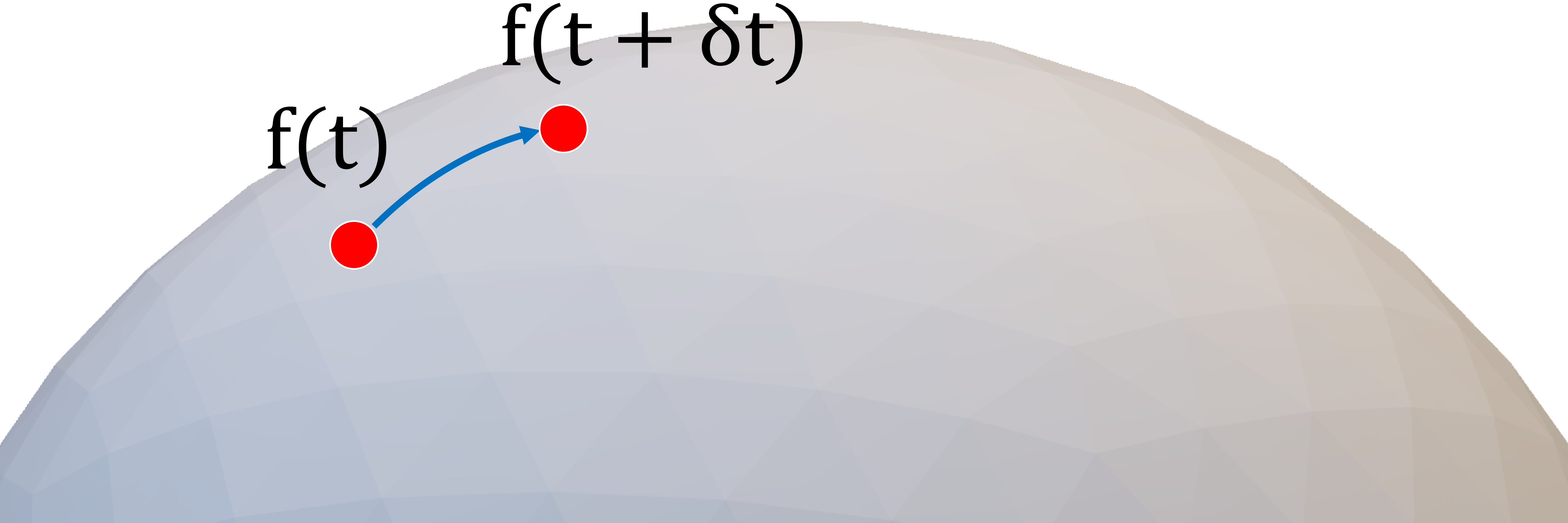
Generalized Velocity of a Rigid Body



$$\dot{\mathbf{x}}\left(\mathbf{X},t\right) = \underbrace{\dot{\mathrm{R}}\left(t\right)}_{\text{Time Derivative of Rotation Matrix}}\mathbf{X} + \overbrace{\dot{\mathbf{p}}}^{\text{Linear Velocity}}\left(t\right)$$

Reference (Undeformed) Space

Aside: Using Geometry to Compute Derivatives



Aside: Using Geometry to Compute Derivatives

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Time Derivatives of Rotation Matrices

$$R(t) \in SO(3)$$

$$R(t + \delta t)$$ **Next Rotation in SO(3) (Yikes !)**

$$\exp([\omega]\delta t)R(t)$$

Angular velocity 3-dimensional vector

Time Derivatives of Rotation Matrices

$$R(t) \in SO(3)$$

$$R(t + \delta t)$$ **Next Rotation in SO(3) (Yikes !)**

$$\exp([\omega]\delta t)R(t)$$

Square brackets = skew-symmetric matrix

Time Derivatives of Rotation Matrices

$$R(t) \in SO(3)$$

$$R(t + \delta t)$$ **Next Rotation in SO(3) (Yikes !)**

$$\exp([\omega]\delta t)R(t)$$

Matrix Exponential

Step 1: d/dt

$$\frac{d}{d\delta t} exp([\omega]\delta t)R(t)$$

Infinite Series for Exponential

$$\frac{d}{d\delta t}(\mathrm{I} + [\omega]\delta \mathrm{t} \; + \; \frac{1}{2}[\omega][\omega]\delta \mathrm{t}^2 \; \ldots) \, R(t)$$

Step 1: d/dt

$$\frac{d}{d\delta t}\left(\mathrm{I} + [\omega]\delta t + \frac{1}{2}[\omega][\omega]\delta t^2 \;\ldots\right) R(t)$$

$$\underbrace{\left([\omega] + [\omega][\omega]\delta t \ldots\right) R(t)}_{\text{Yuck, still an infinite series}}$$

Step 2: Limit !!!

$$([\omega] + [\omega][\omega]\delta t \ldots) R(t)$$

Yuck, still an infinite series

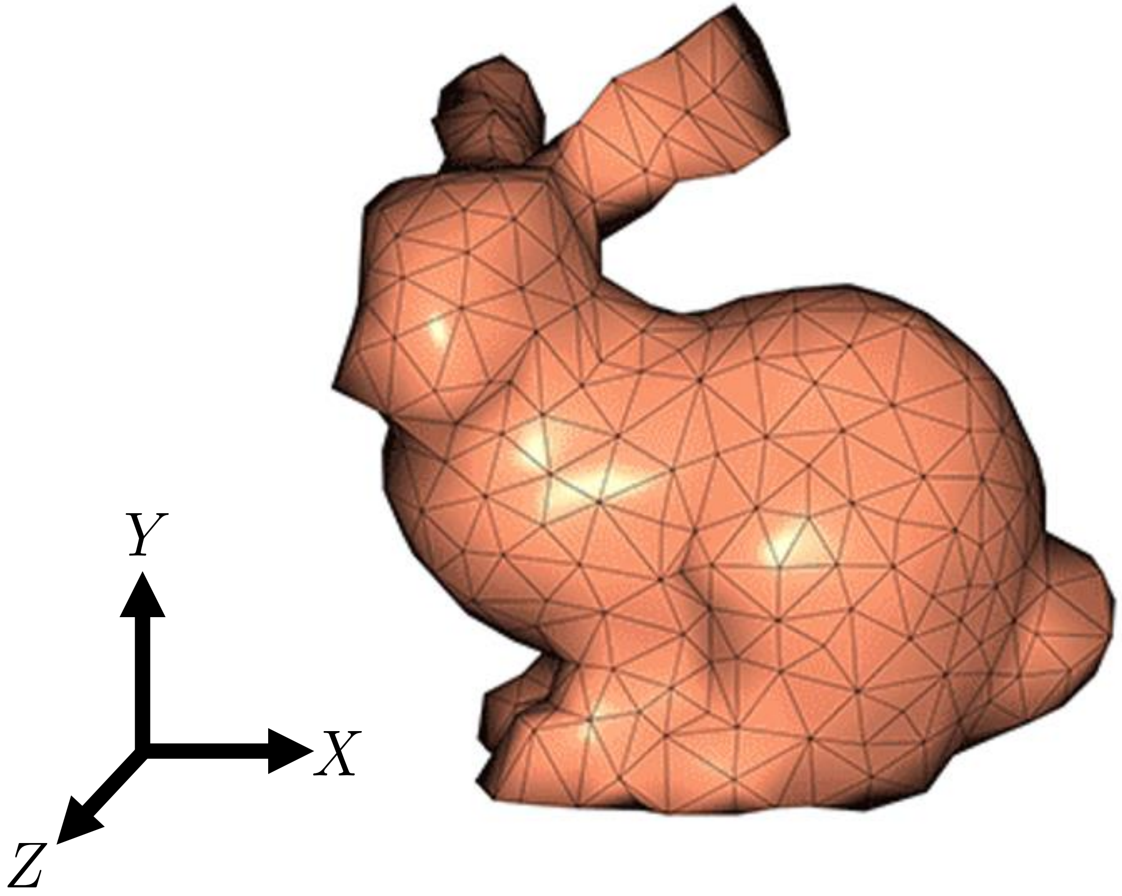
$$\lim_{\delta t \to 0} ([\omega] + [\omega][\omega]\delta t \ldots) R(t)$$

Only constant terms remain

Step 3: Finale

$$\dot{\mathrm{R}(\mathrm{t})} = [\omega]R(t)$$

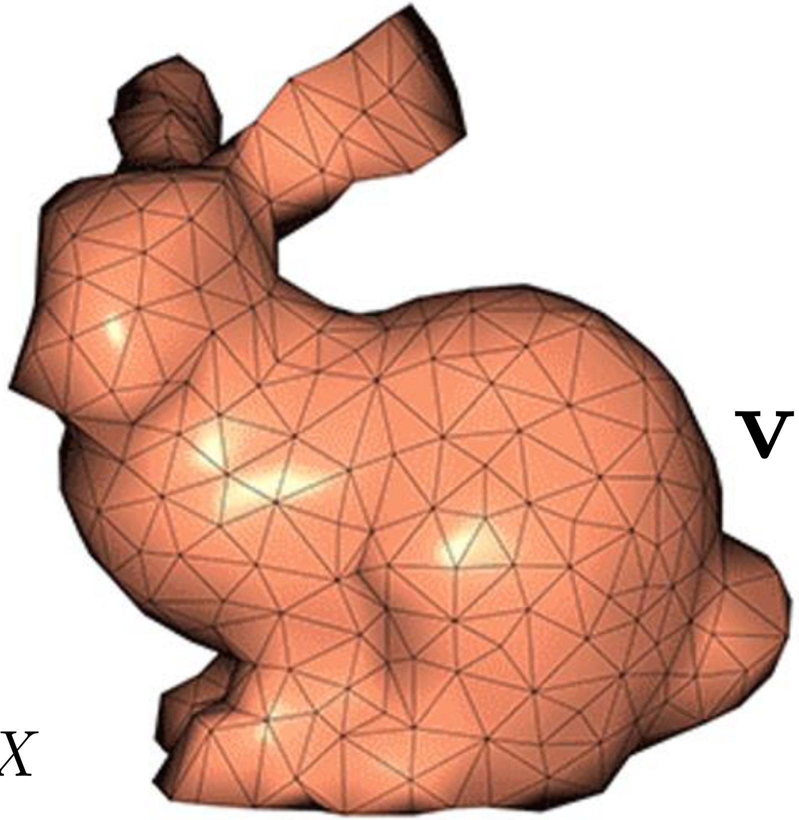
Generalized Velocity of a Rigid Body

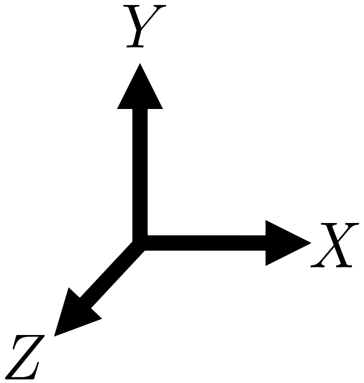


Reference (Undeformed) Space

$$\mathbf{x}(\mathbf{X}, t) = \underbrace{[\omega]R(t)\mathbf{X}}_{\text{Can be understood as } \omega \times (R(t)\mathbf{X})} + \overbrace{\dot{\mathbf{p}}(t)}^{\text{Linear Velocity}}$$

Generalized Velocity of a Rigid Body





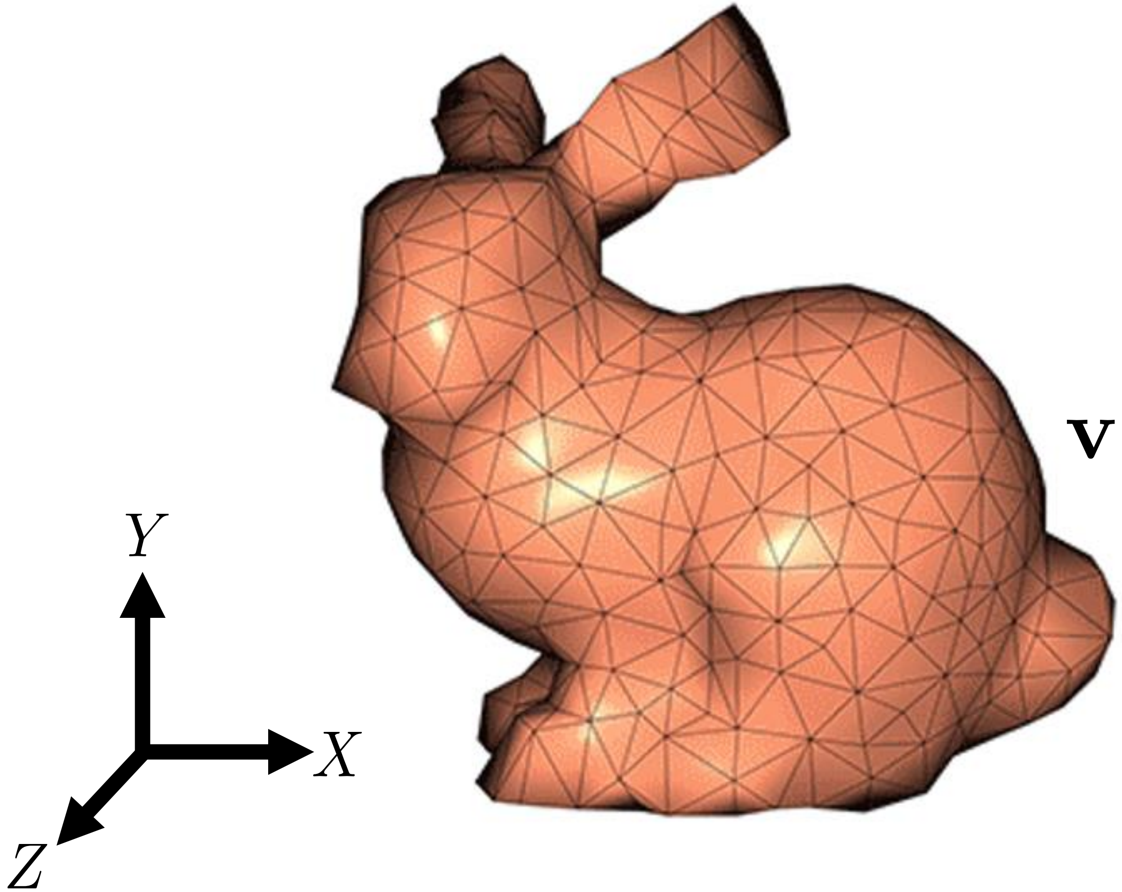
Reference (Undeformed) Space

$$\mathbf{v}(\mathbf{X}, t) = \mathrm{R}\,[\mathbf{X}]^T\,\mathrm{R}^T\underbrace{\omega}_{\text{Angular Velocity } \in \mathbb{R}^3} + \overbrace{\dot{\mathbf{p}}(t)}^{\text{Linear Velocity}}$$

$$[\mathbf{X}] = \begin{pmatrix} 0 & -X_z & X_y \\ X_z & 0 & -X_x \\ -X_y & X_x & 0 \end{pmatrix}$$

Cross Product Matrix

Generalized Velocity of a Rigid Body



$$\mathbf{v}\left(\mathbf{X},t\right) = \mathrm{R}\begin{pmatrix}[\mathbf{X}]^T & \mathrm{I}\end{pmatrix}\begin{pmatrix}\mathrm{R}^T & 0\\ 0 & \mathrm{I}\end{pmatrix}\underbrace{\begin{pmatrix}\omega\\ \dot{\mathbf{p}}\end{pmatrix}}_{\dot{\mathbf{q}}\in\mathbb{R}^6}$$

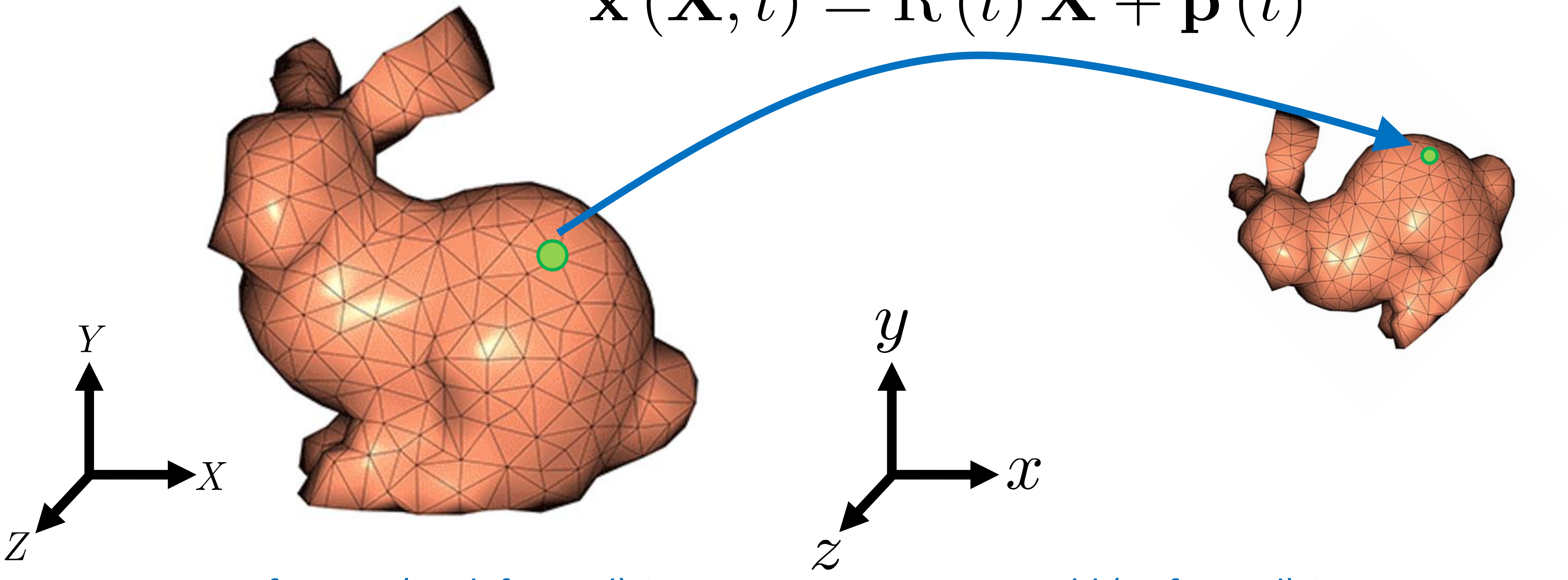
Reference (Undeformed) Space

Ok Who Wants to do the Second Derivates for Acceleration ?

Me Neither, Let’s Do Something Else

The Rigid Body Mapping

$$\mathbf{x}\left(\mathbf{X}, t\right) = \mathrm{R}\left(t\right)\mathbf{X} + \mathbf{p}\left(t\right)$$



Reference (Undeformed) Space

World (Deformed) Space

Why did we use this mapping ?

What Makes an Object Rigid ?

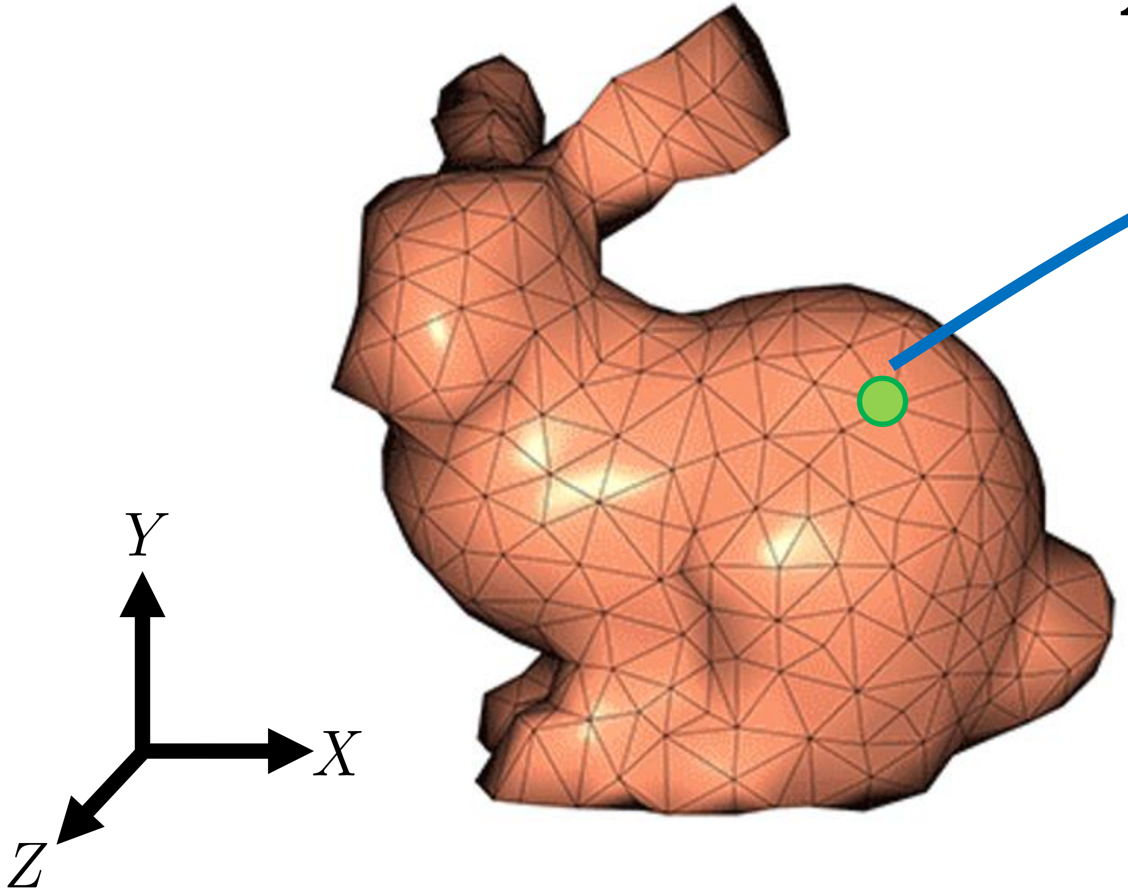
$$\Delta\mathbf{X}^T\left(\mathrm{F}^T\mathrm{F}-\mathrm{I}\right)\Delta\mathbf{X}=\mathbf{0}$$

Implies

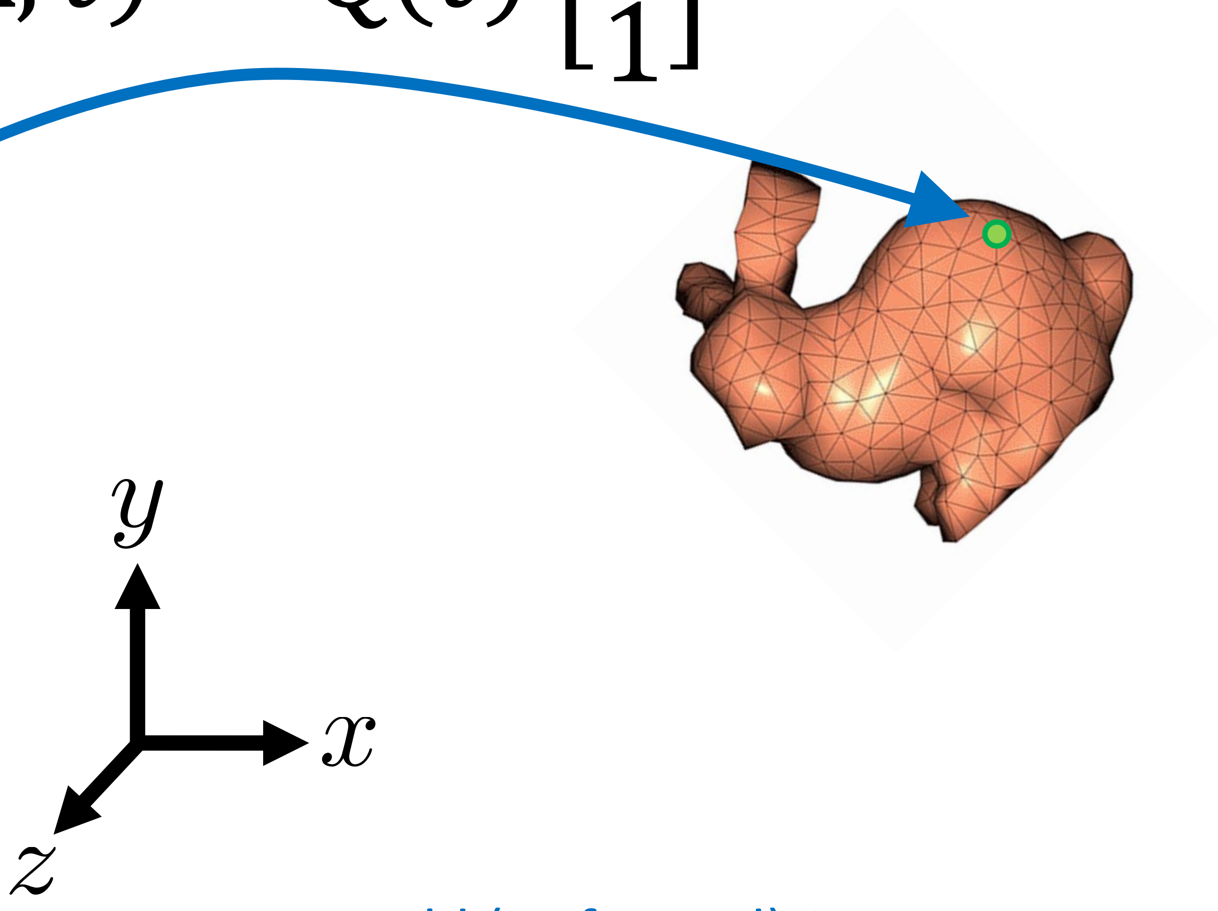
$$\underbrace{\mathrm{F}^T\mathrm{F}}_{\text{Orthogonal}}=\mathrm{I}$$

The Affine Body Mapping

$$\mathbf{x}(\mathbf{X}, t) = \mathrm{Q}(t)\begin{bmatrix}\mathbf{X}\\ 1\end{bmatrix}$$

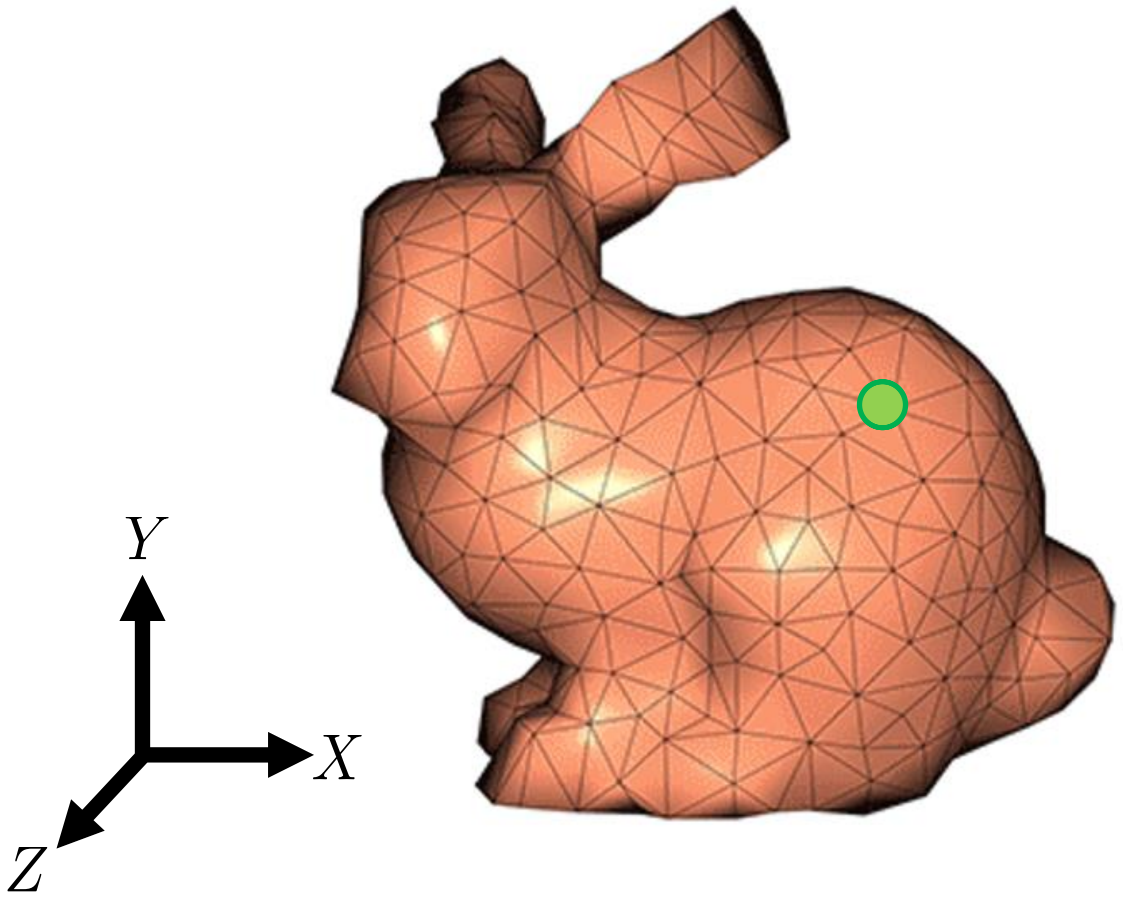


Reference (Undeformed) Space



World (Deformed) Space

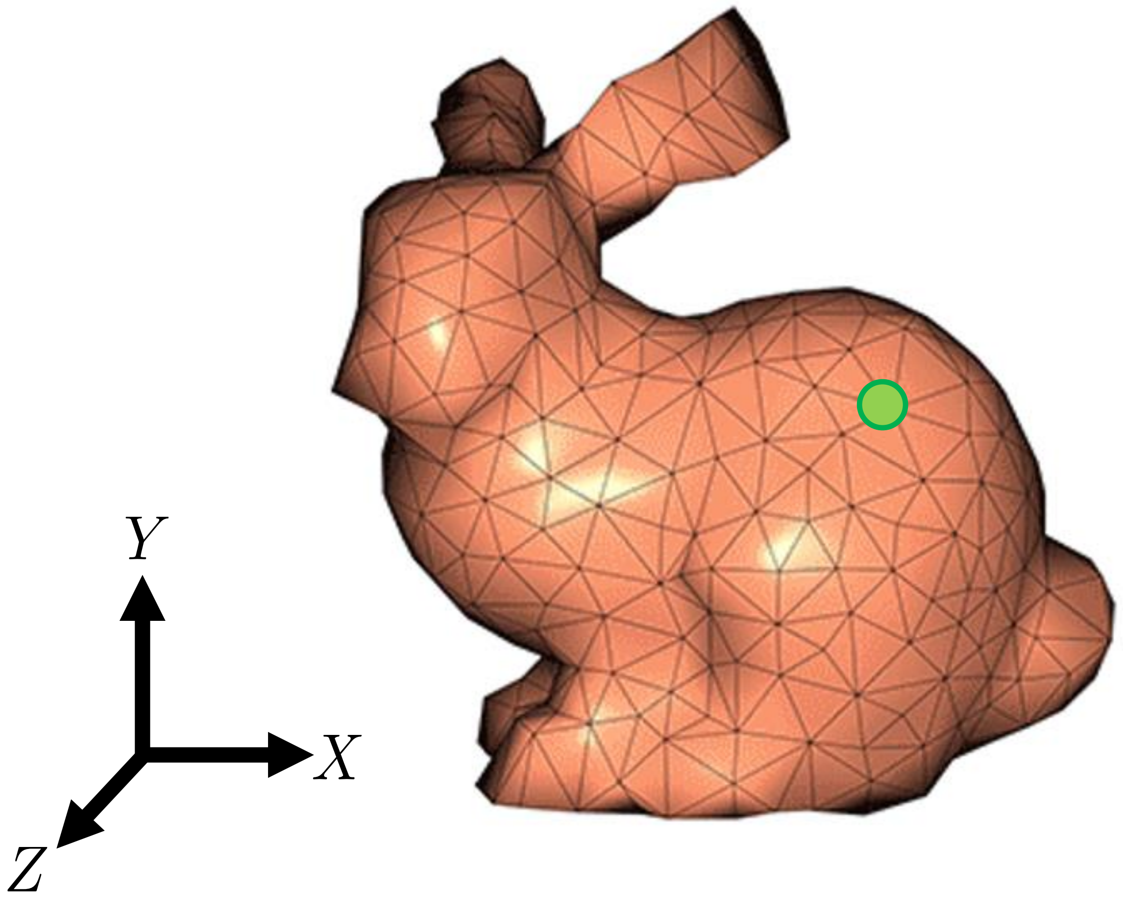
The Affine Body Mapping



Reference (Undeformed) Space

$$\mathbf{x}(\mathbf{X}, t) = \underbrace{\mathrm{Q}(t)}_{\text{Affine Transform}} \begin{bmatrix}\mathbf{X}\\ 1\end{bmatrix}$$

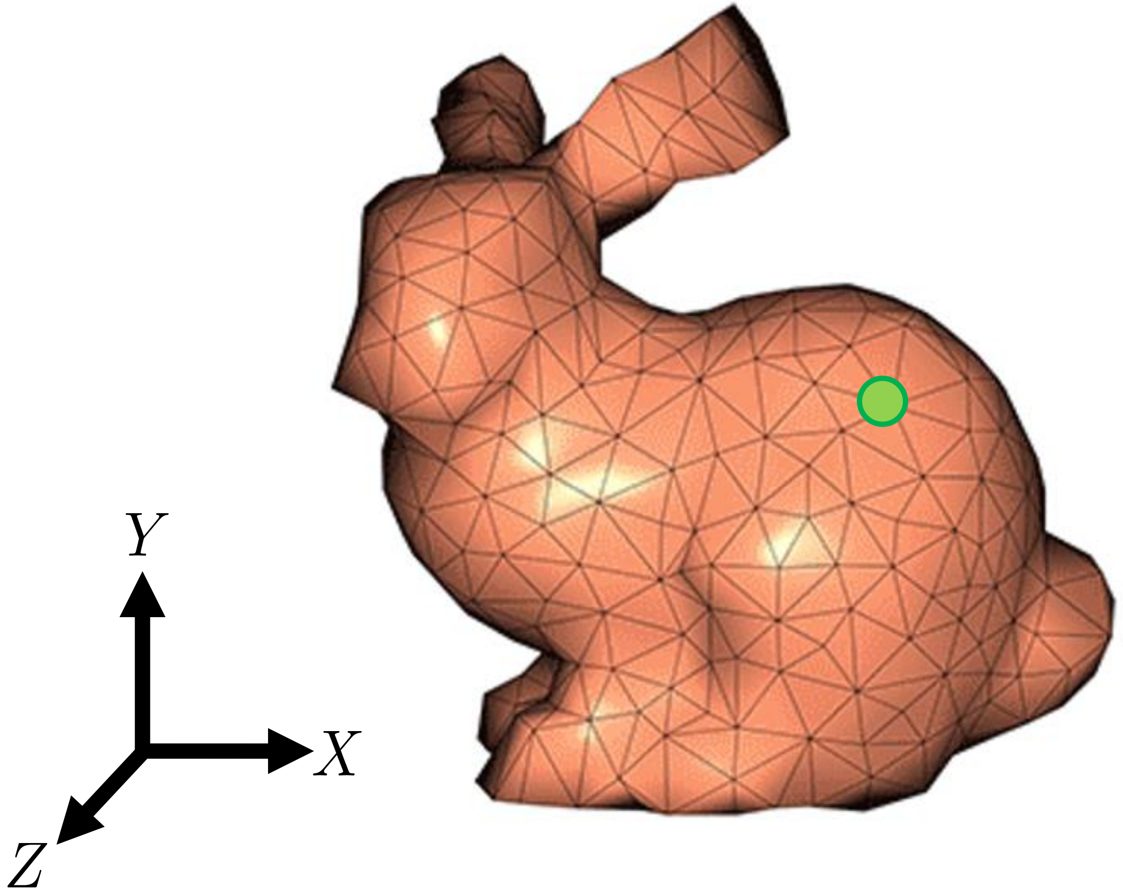
The Affine Body Mapping



Reference (Undeformed) Space

$$\mathbf{x}(\mathbf{X}, t) = \underbrace{\mathrm{Q}(t)}_{\begin{pmatrix} q_0 & q_1 & q_2 & q_3 \\ q_4 & q_5 & q_6 & q_7 \\ q_8 & q_9 & q_{10} & q_{11} \end{pmatrix}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

The Affine Body Mapping



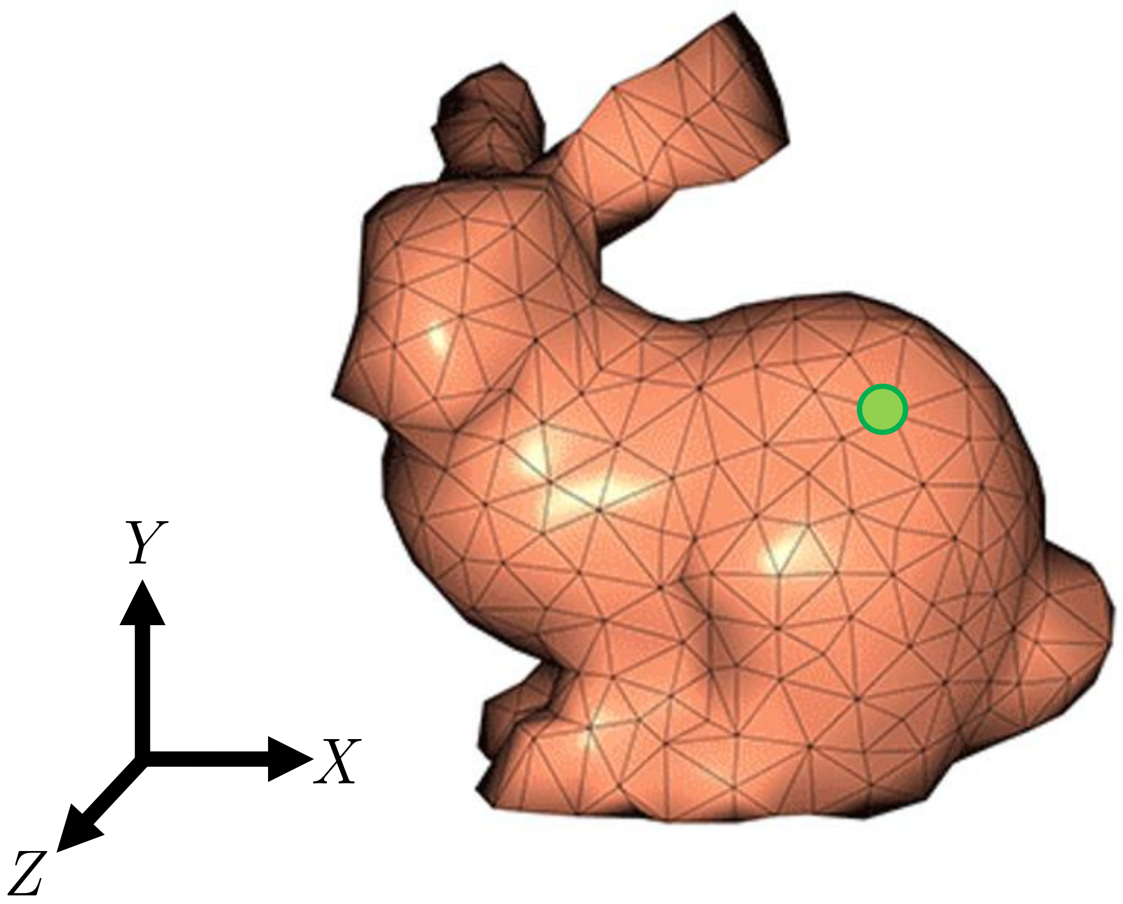
Reference (Undeformed) Space

$$\mathbf{x}(\mathbf{X}, t) = \mathrm{Q}(t) \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

$$\begin{pmatrix} q_0 & q_1 & q_2 & q_3 \\ q_4 & q_5 & q_6 & q_7 \\ q_8 & q_9 & q_{10} & q_{11} \end{pmatrix}$$

Linear Transform

The Affine Body Mapping



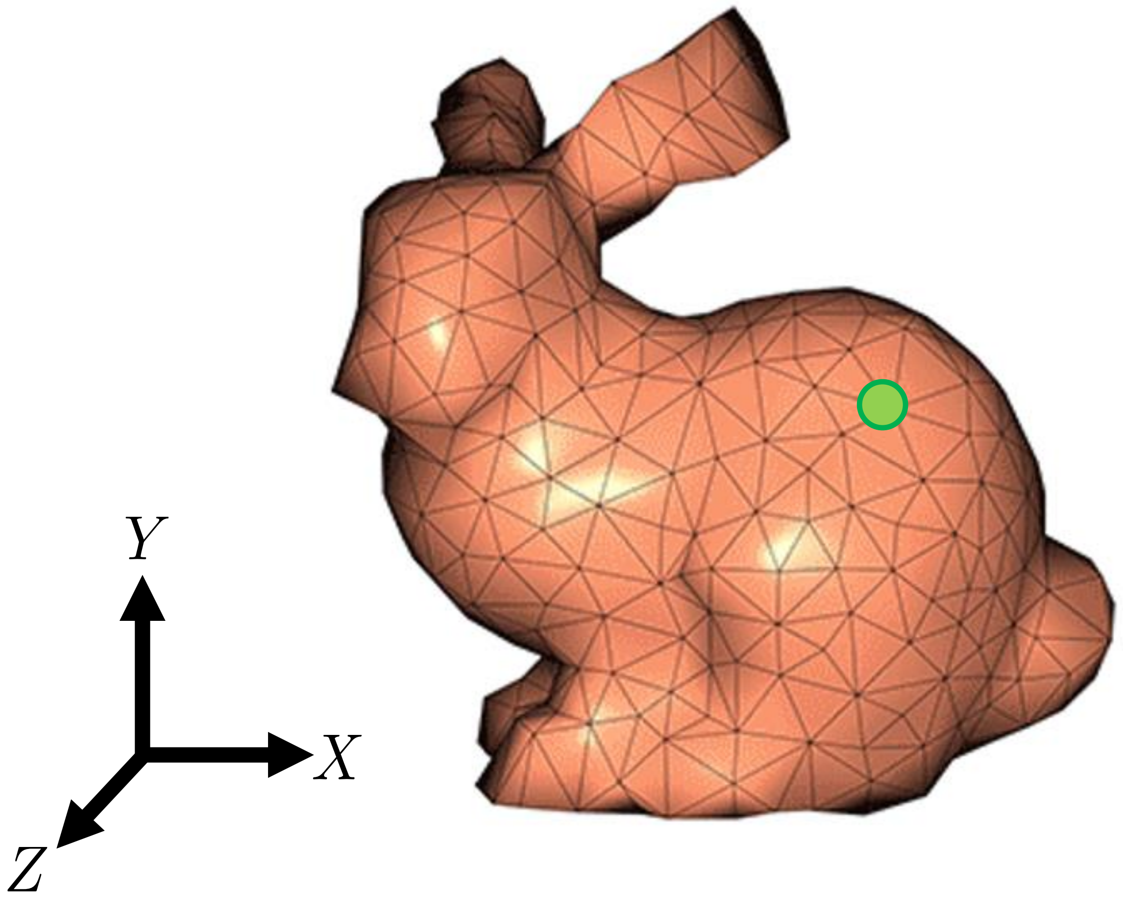
Reference (Undeformed) Space

$$\mathbf{x}(\mathbf{X}, t) = \underbrace{\mathrm{Q}(t)}_{} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

$$\begin{pmatrix} q_0 & q_1 & q_2 & q_3 \\ q_4 & q_5 & q_6 & q_7 \\ q_8 & q_9 & q_{10} & q_{11} \end{pmatrix}$$

Translation

Vectorized Generalized Coordinates



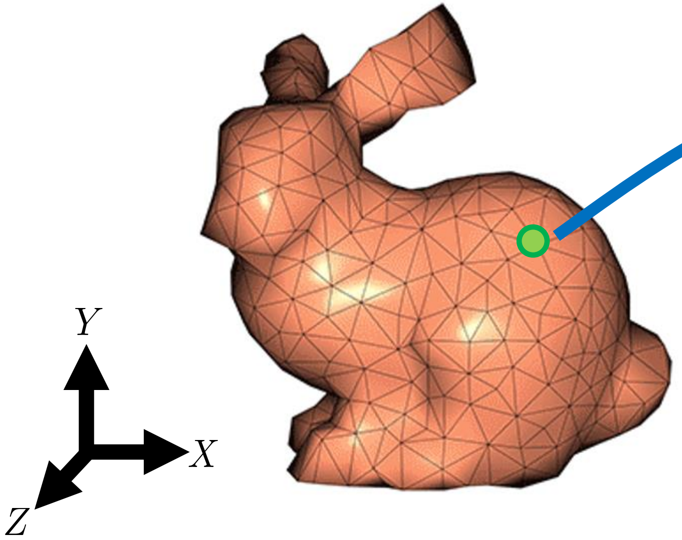
Reference (Undeformed) Space

J is ?? x ??

$$\mathbf{x}(\mathbf{X}, t) = \text{mat}\left(\begin{bmatrix}\mathbf{X} \\ 1\end{bmatrix}\right) \text{q(t)}$$

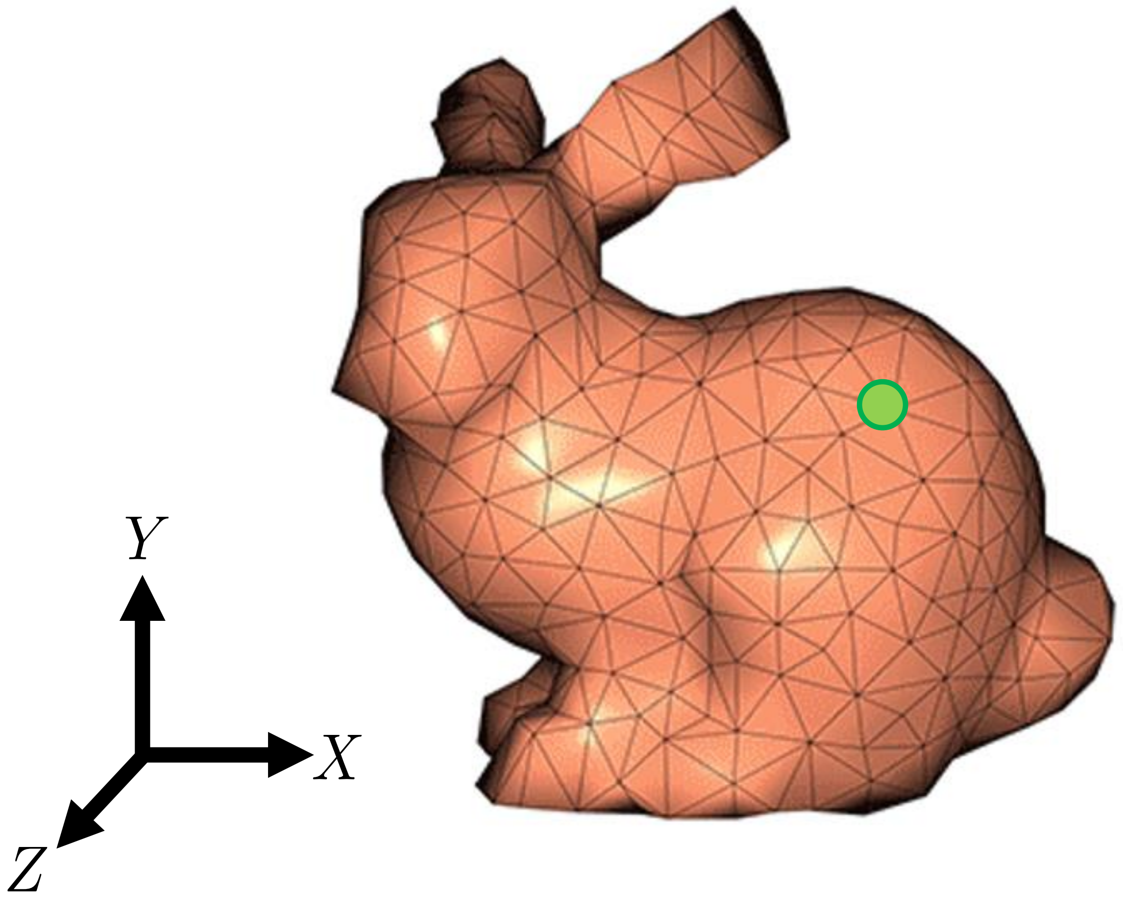
Row-wise Flatten

The Kinematic Jacobian



$$\underbrace{\begin{bmatrix} X & Y & Z & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & X & Y & Z & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & Y & Z & 1 \end{bmatrix}}_{\mathbf{J}}$$

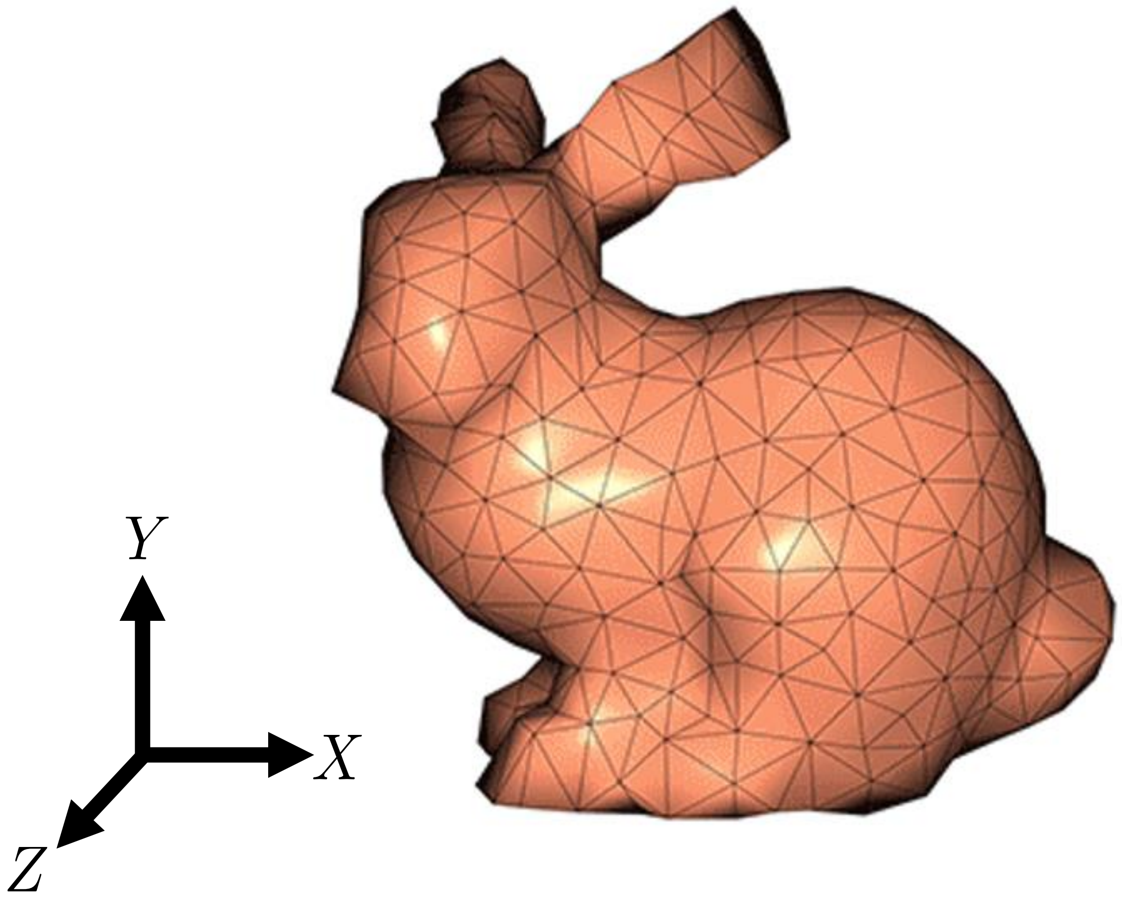
Vectorized Generalized Coordinates



Reference (Undeformed) Space

$$\mathbf{x}(\mathbf{X}, t) = \mathrm{J}(\boldsymbol{X})\mathrm{q}(\mathrm{t})$$

Generalized Velocity of an Affine Body



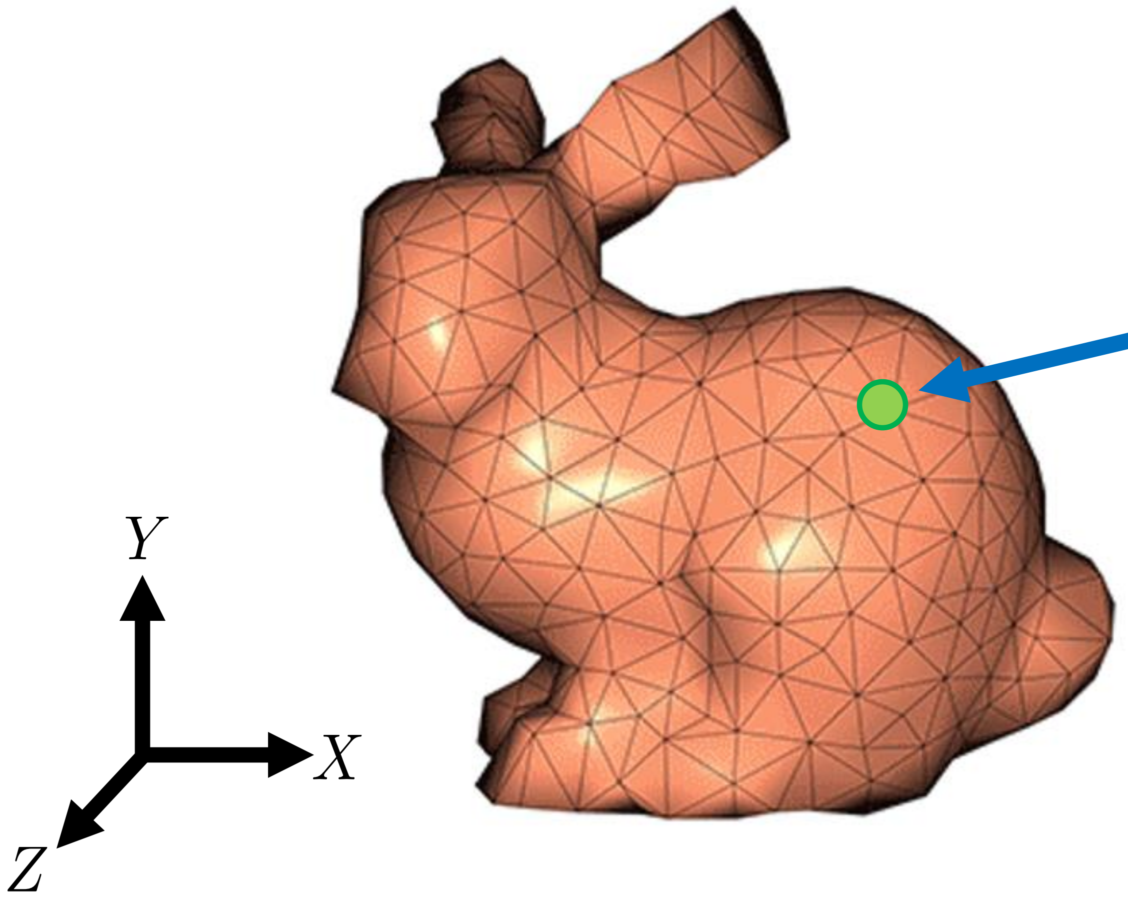
Reference (Undeformed) Space

$$\boldsymbol{v}(\mathbf{X}, t) = \mathrm{J}(\boldsymbol{X})\dot{\mathrm{q}}(\mathrm{t})$$

Equations of Motion

$$M\ddot{\mathbf{q}} = -\frac{\partial V}{\partial \mathbf{q}}$$

Kinetic Energy of an Affine Body



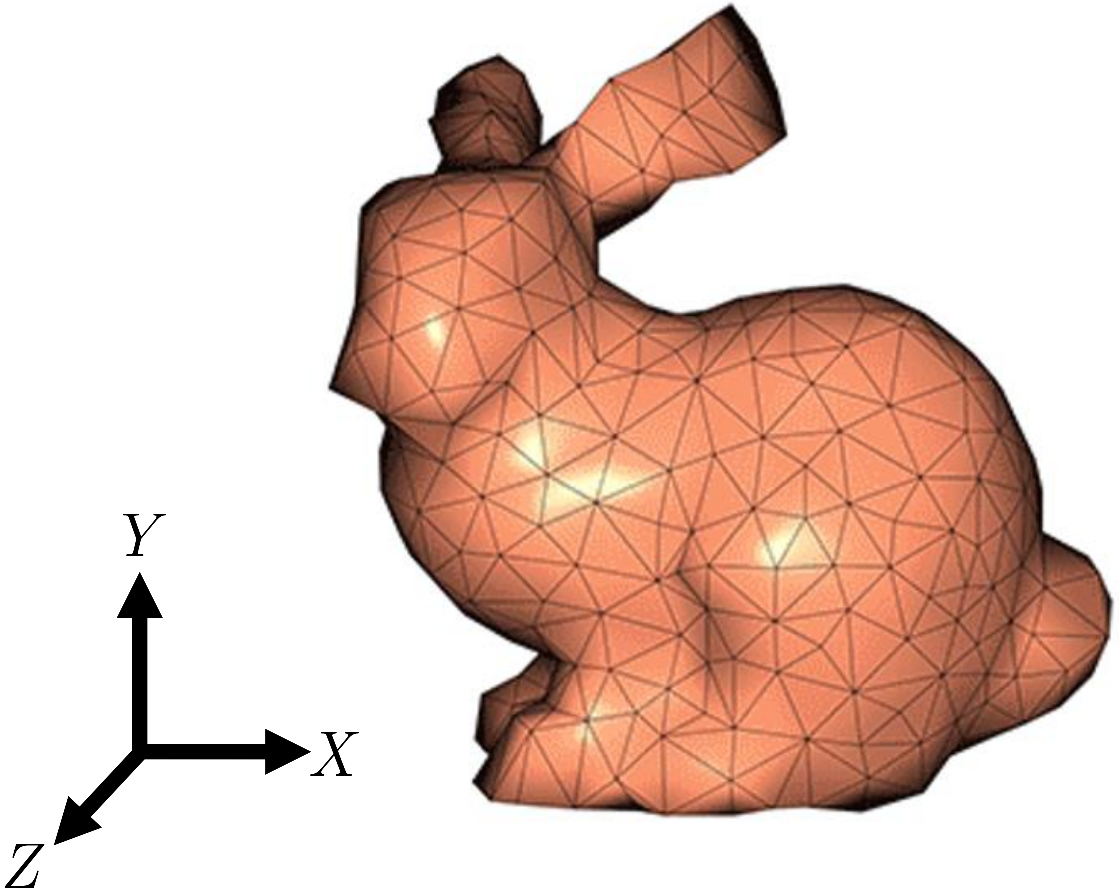
infinitesimal volume

$$\frac{1}{2}\rho\|\mathbf{v}\left(\mathbf{X}\right)\|_2^2 d\Omega$$

density ($\frac{kg}{m^3}$)

Reference (Undeformed) Space

Kinetic Energy of an Affine Body



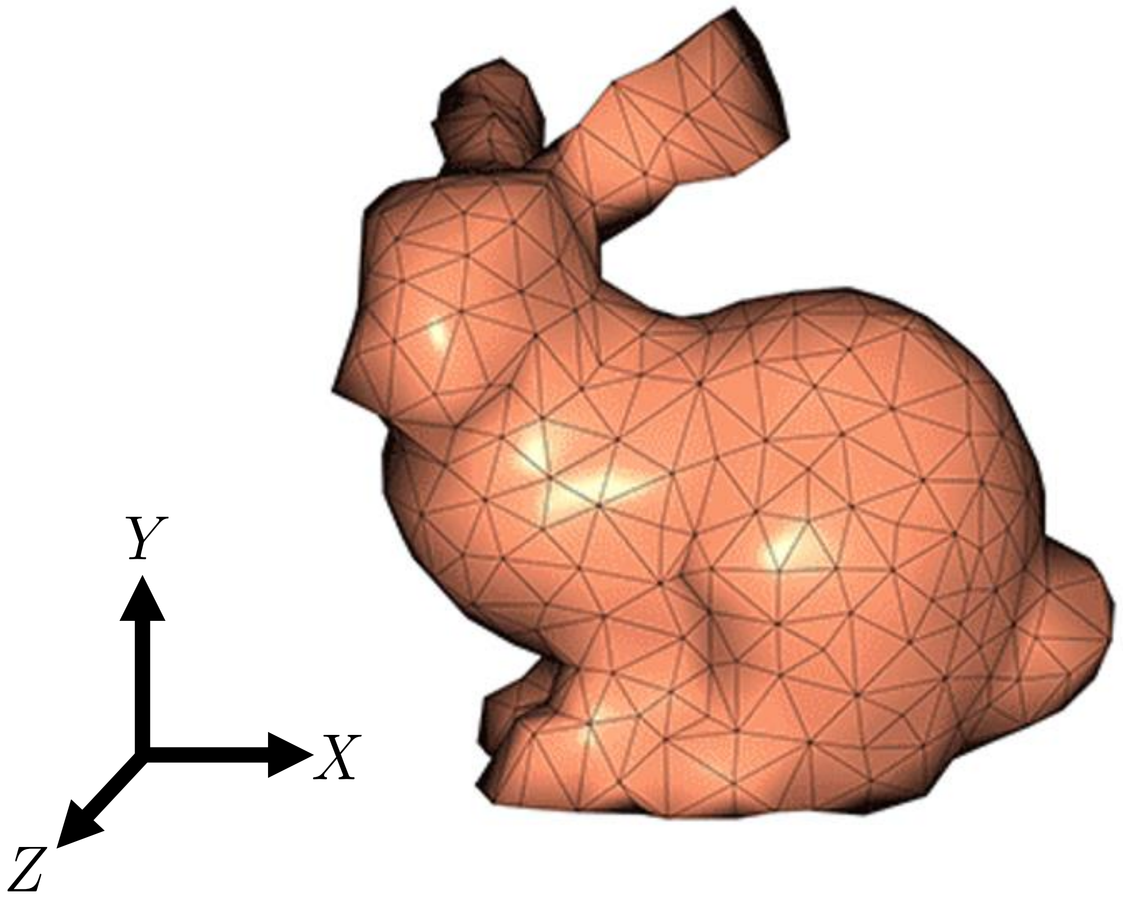
Reference (Undeformed) Space

infinitesimal volume

$$\frac{1}{2}\int_{\Omega} \rho \| \mathbf{v}\left(\mathbf{X}\right) \|_2^2 \, d\Omega$$

entire rigid body

Kinetic Energy of an Affine Body

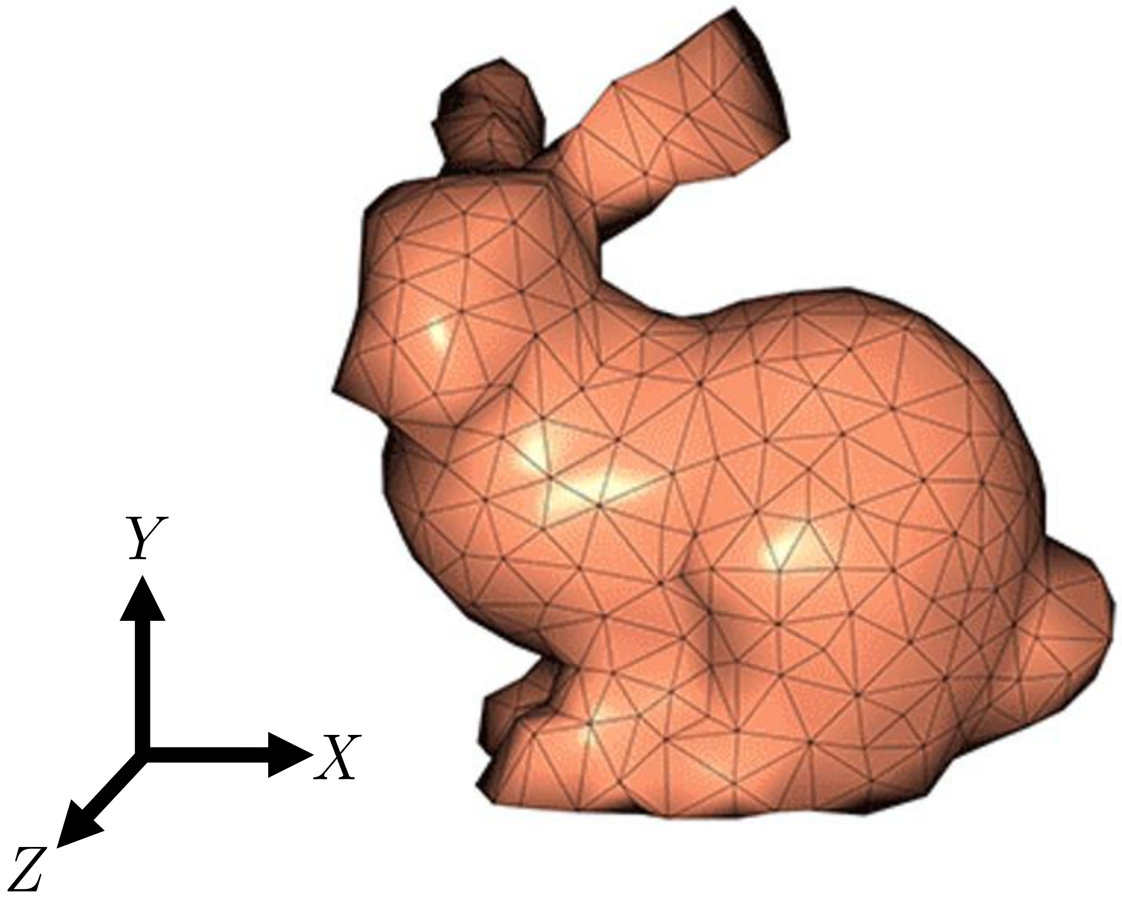


Reference (Undeformed) Space

$$\frac{1}{2}\int_{\Omega} \rho \mathbf{v}^T \mathbf{v} \, d\Omega$$

entire rigid body

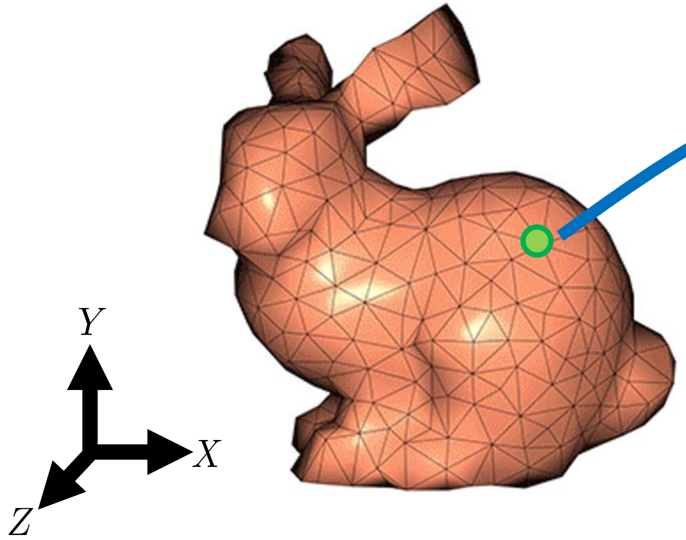
Kinetic Energy of an Affine Body



Reference (Undeformed) Space

$$\frac{1}{2}\,\dot{\boldsymbol{q}}^T \overbrace{\left(\rho \underbrace{\int_\Omega \mathrm{J}(\mathrm{X})^\mathrm{T}\mathrm{J}(\mathrm{X})\,d\Omega}_{\mathrm{M}_0}\right)}^{\text{Mass Matrix } \mathrm{M}} \dot{\boldsymbol{q}}$$

Recall J … What does this tell us about M_0 ?

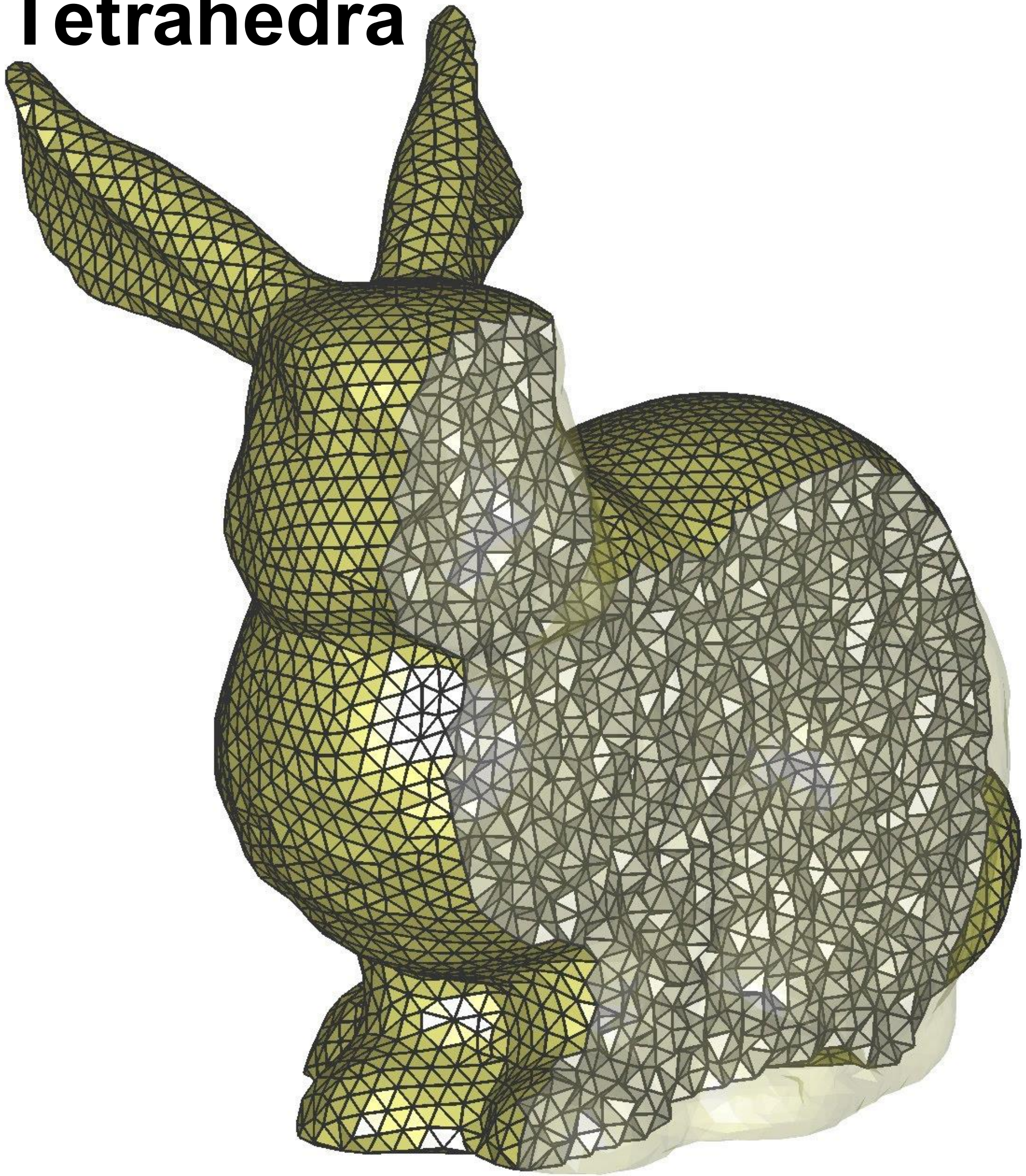


$$\underbrace{\begin{bmatrix} X & Y & Z & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & X & Y & Z & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & Y & Z & 1 \end{bmatrix}}_{\mathrm{J}}$$

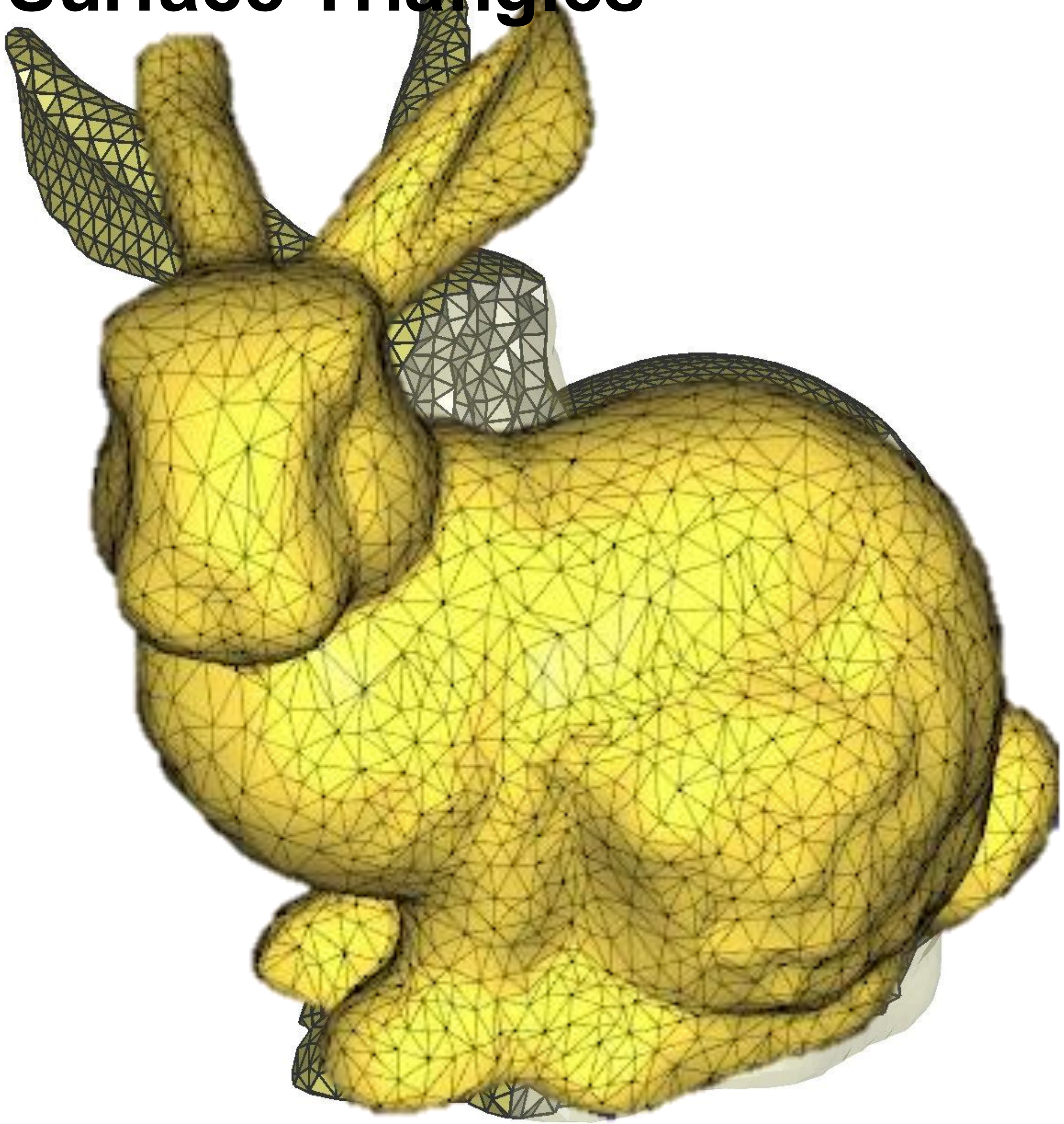
$$
\mathrm{M}_0 = \begin{pmatrix}
X^2 & XY & XZ & X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
XY & Y^2 & YZ & Y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
XZ & YZ & Z^2 & Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
X & Y & Z & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & X^2 & XY & XZ & X & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & XY & Y^2 & YZ & Y & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & XZ & YZ & Z^2 & Z & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & X & Y & Z & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X^2 & XY & XZ & X \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & XY & Y^2 & YZ & Y \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & XZ & YZ & Z^2 & Z \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & Y & Z & 1
\end{pmatrix}
$$

$$
M_0 = \begin{pmatrix}
X^2 & XY & XZ & X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
XY & Y^2 & YZ & Y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
XZ & YZ & Z^2 & Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
X & Y & Z & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \boxed{X^2} & XY & XZ & X & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & XY & Y^2 & YZ & Y & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & XZ & YZ & Z^2 & Z & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & X & Y & Z & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X^2 & XY & XZ & X \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & XY & Y^2 & YZ & Y \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & XZ & YZ & Z^2 & Z \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & Y & Z & 1
\end{pmatrix}
\longrightarrow \int_{\underbrace{\Omega}_{\text{Volume}}} X^2 \, d\Omega
$$

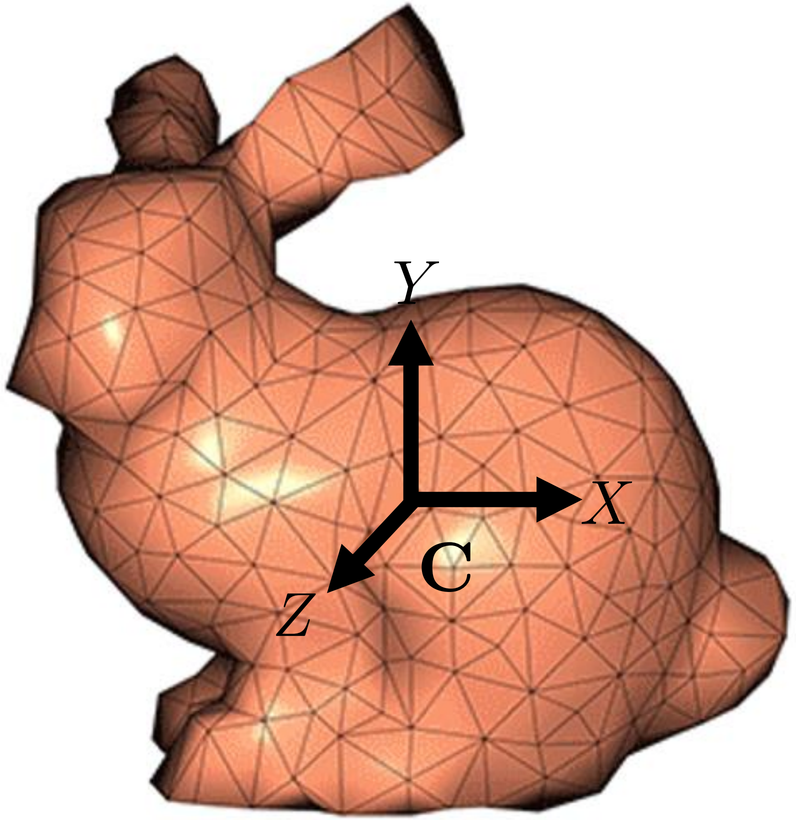
Integrate over Tetrahedra



Integrate over Surface Triangles



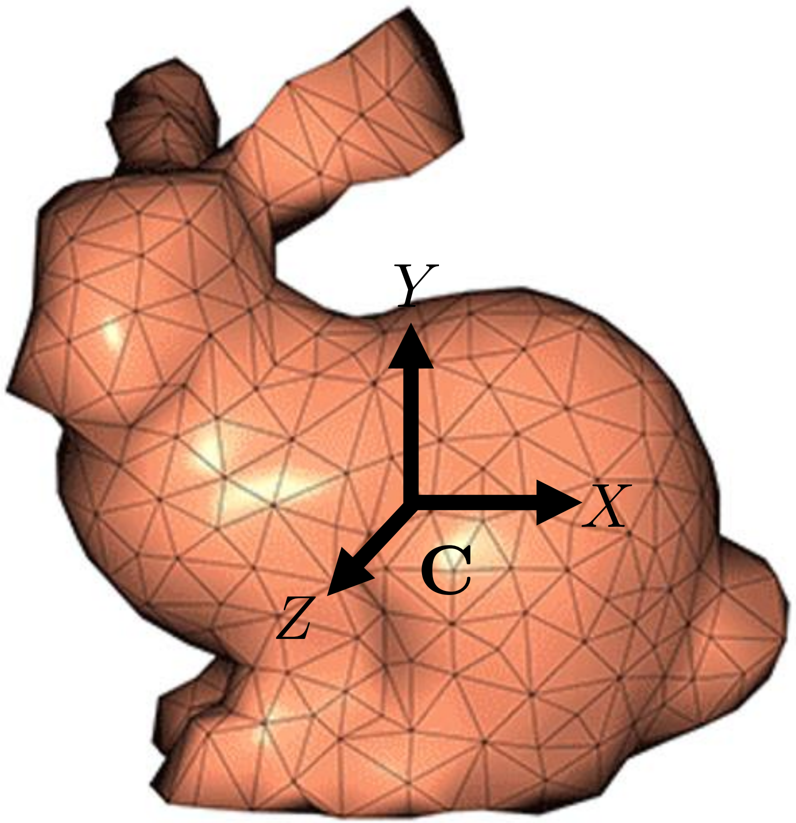
Aside: Divergence Theorem



Reference (Undeformed) Space

$$\underbrace{\int_\Omega \nabla \cdot \overbrace{\mathbf{f}}^{\in \mathbb{R}^3}\left(\bar{\mathbf{X}}\right) d\Omega}_{\text{volume integral}} = \underbrace{\int_\Gamma f\left(\bar{\mathbf{X}}\right) \cdot \overbrace{\mathbf{n}}^{\text{surface normal}} d\Gamma}_{\text{surface integral}}$$

Integrate over Volume



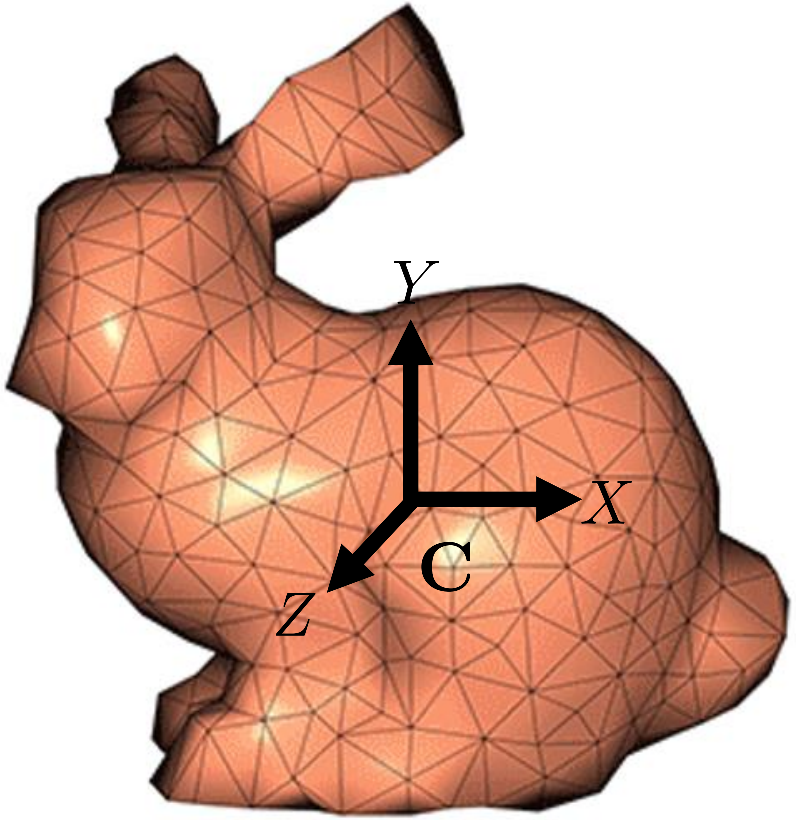
Express using divergence

$$\int_{\Omega} \mathrm{X}^2 d\Omega = \int_{\Omega} \nabla \cdot \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix} d\Omega$$

Reference (Undeformed) Space

Reminder $\nabla \cdot \begin{bmatrix} \mathrm{a} \\ \mathrm{b} \\ \mathrm{c} \end{bmatrix} = \frac{\partial a}{\partial x} + \frac{\partial \mathrm{b}}{\partial \mathrm{y}} + \frac{\partial \mathrm{c}}{\partial \mathrm{z}}$

Integrate over Volume



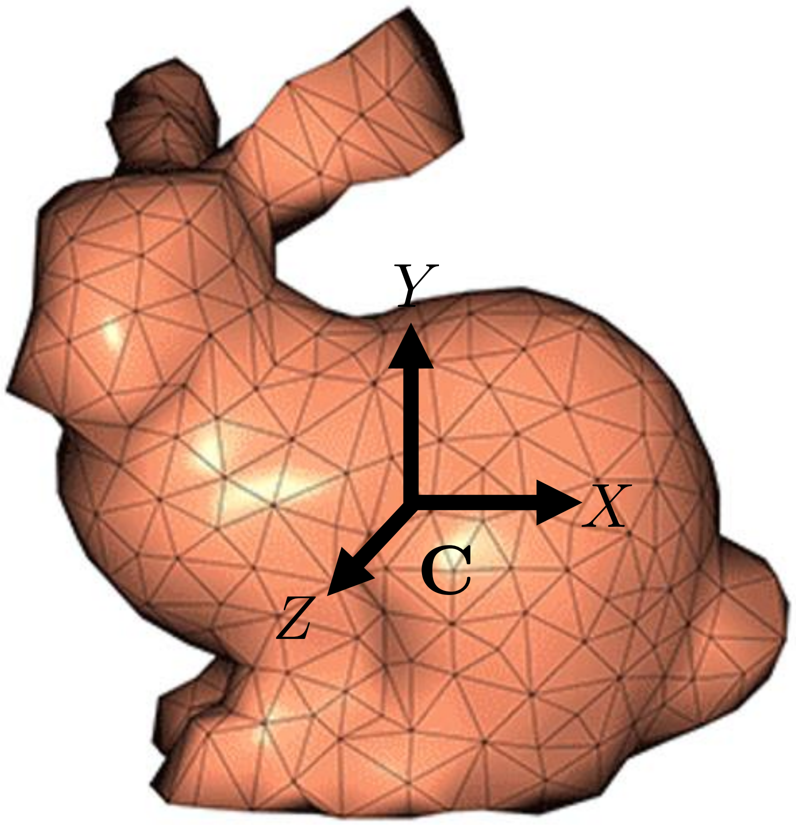
Express using divergence

$$\int_{\Omega} X^2 d\Omega = \int_{\Omega} \nabla \cdot \begin{bmatrix} \frac{1}{3} X^3 \\ 0 \\ 0 \end{bmatrix} d\Omega$$

Reference (Undeformed) Space

Reminder $\nabla \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{\partial a}{\partial x} + \frac{\partial b}{\partial y} + \frac{\partial c}{\partial z}$

Integrate over Surface



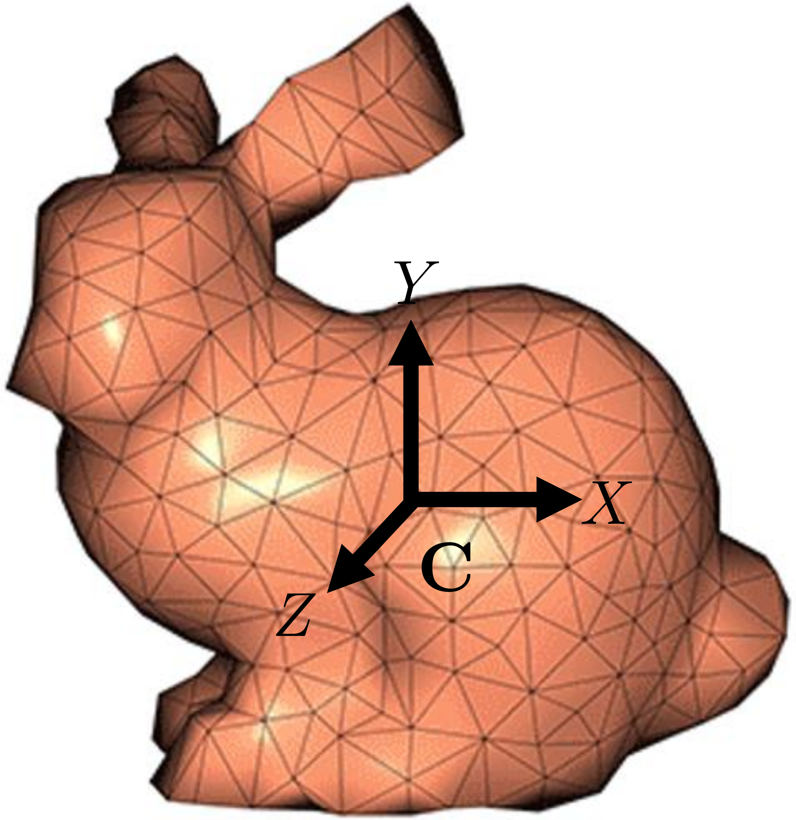
Reference (Undeformed) Space

Convert to Surface Integral

$$\int_{\Gamma} \frac{1}{3} X^{3} \mathrm{n}_x \, d\Gamma$$

Little surface area

Integrate over Triangles !!!!



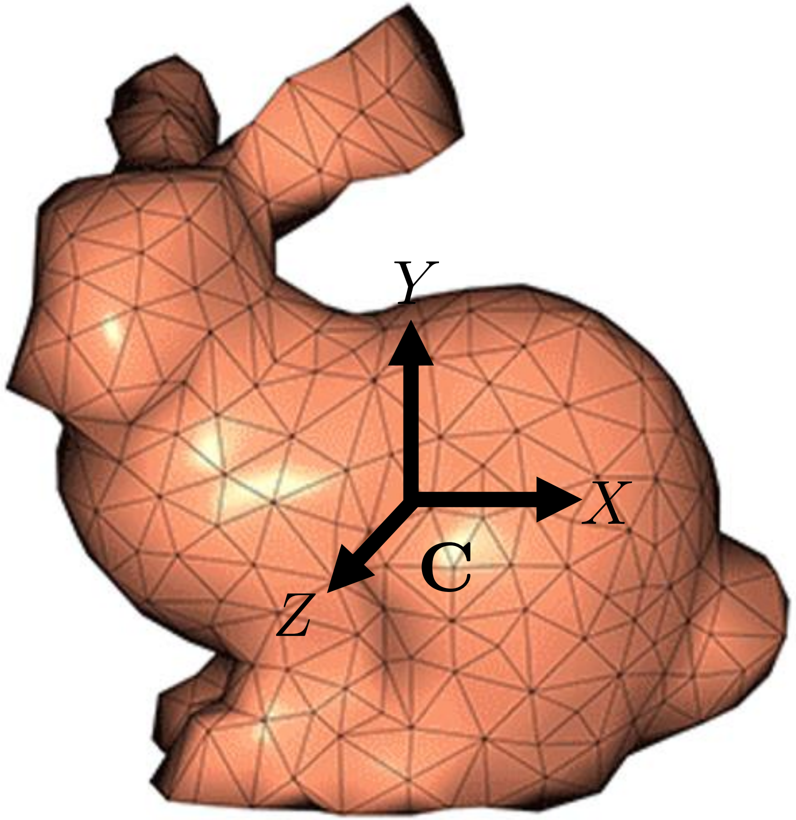
Reference (Undeformed) Space

Convert to Surface Integral

$$\sum_{i=0}^{|T|} \int_{T\_i} \frac{1}{3} X^3 n_x dT$$

Triangle Area

Barycentric Integration for Each Triangle

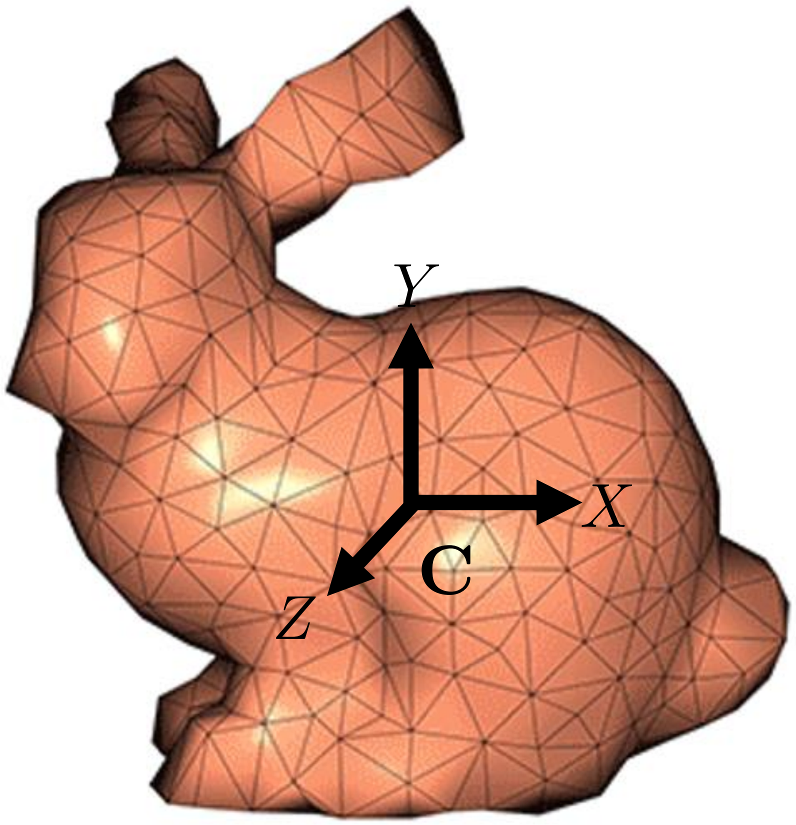


Reference (Undeformed) Space

Convert to Surface Integral

$$\int_{\mathrm{T\_i}} \frac{1}{3} X^3 \mathrm{n}_x d\mathrm{T}$$

Barycentric Integration for Each Triangle

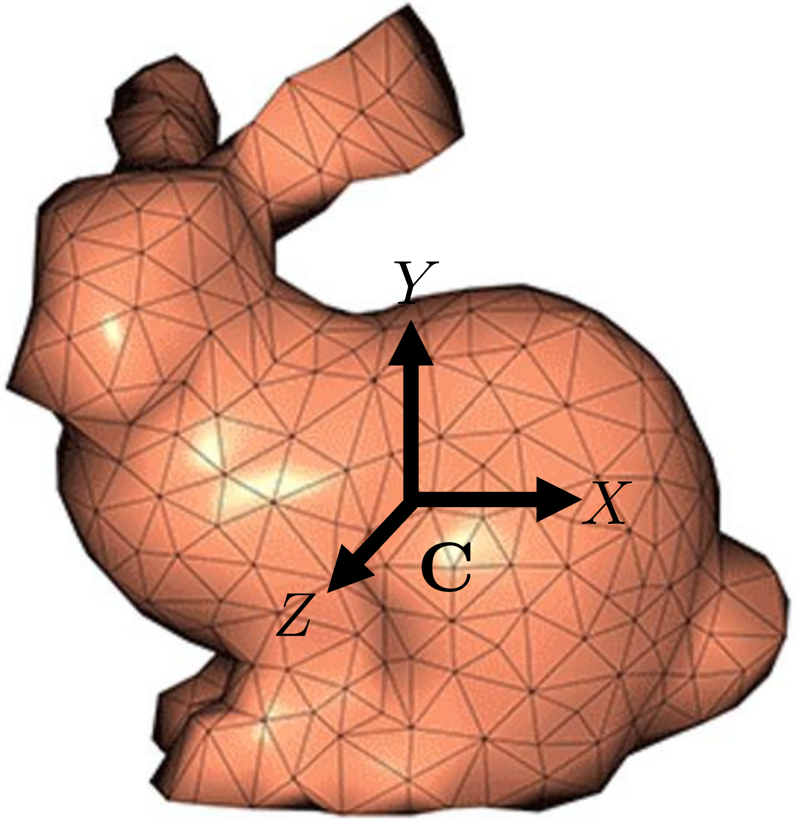


Reference (Undeformed) Space

Replace X (resp Y, Z) with:

$$X = \sum_{\mathrm{i}=0}^{2} X_i \, \underbrace{\phi_i}_{\text{Barycentric Coordinates}}(\boldsymbol{X})$$

Barycentric Integration for Each Triangle

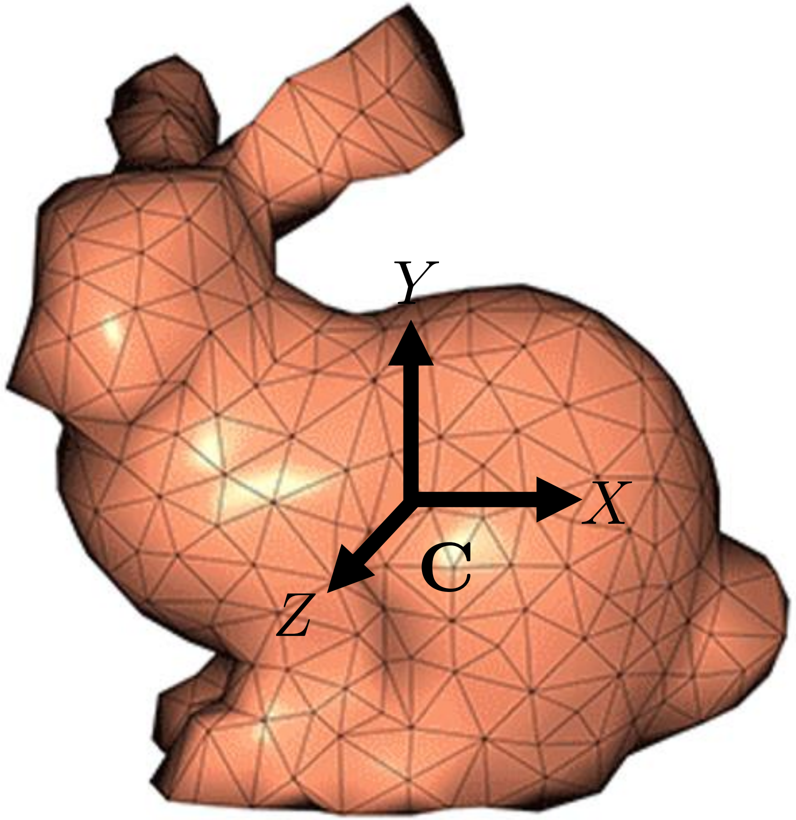


Reference (Undeformed) Space

Integrate using Barycentric Coordinates

$$\int_0^1 \int_0^{1-\phi_1} \frac{1}{3} X(\phi_1, \phi_2))^3 \mathrm{n}_x \mathrm{d}\phi_1 \, \mathrm{d}\phi_2$$

Add up to form full Mass Matrix

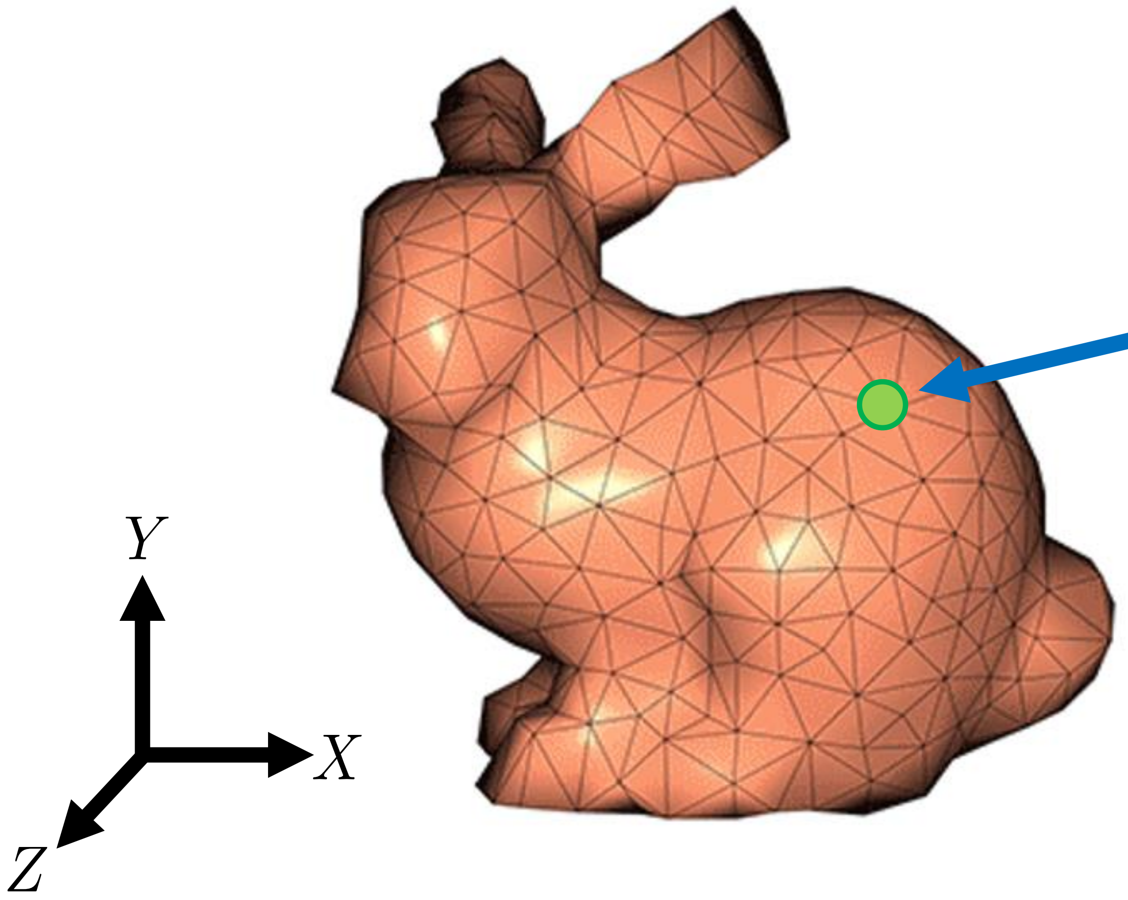


Reference (Undeformed) Space

Convert to Surface Integral

$$\sum_{i=0}^{|T|} \int_{T\_i} \frac{1}{3} X^3 \underbrace{n_x}_{\text{Triangle Area}} dT$$

Potential Energy of Affine Body



$$\psi(\mathrm{q}) = ????$$

How do we keep the object rigid ?

Reference (Undeformed) Space

What Makes an Object Rigid ?

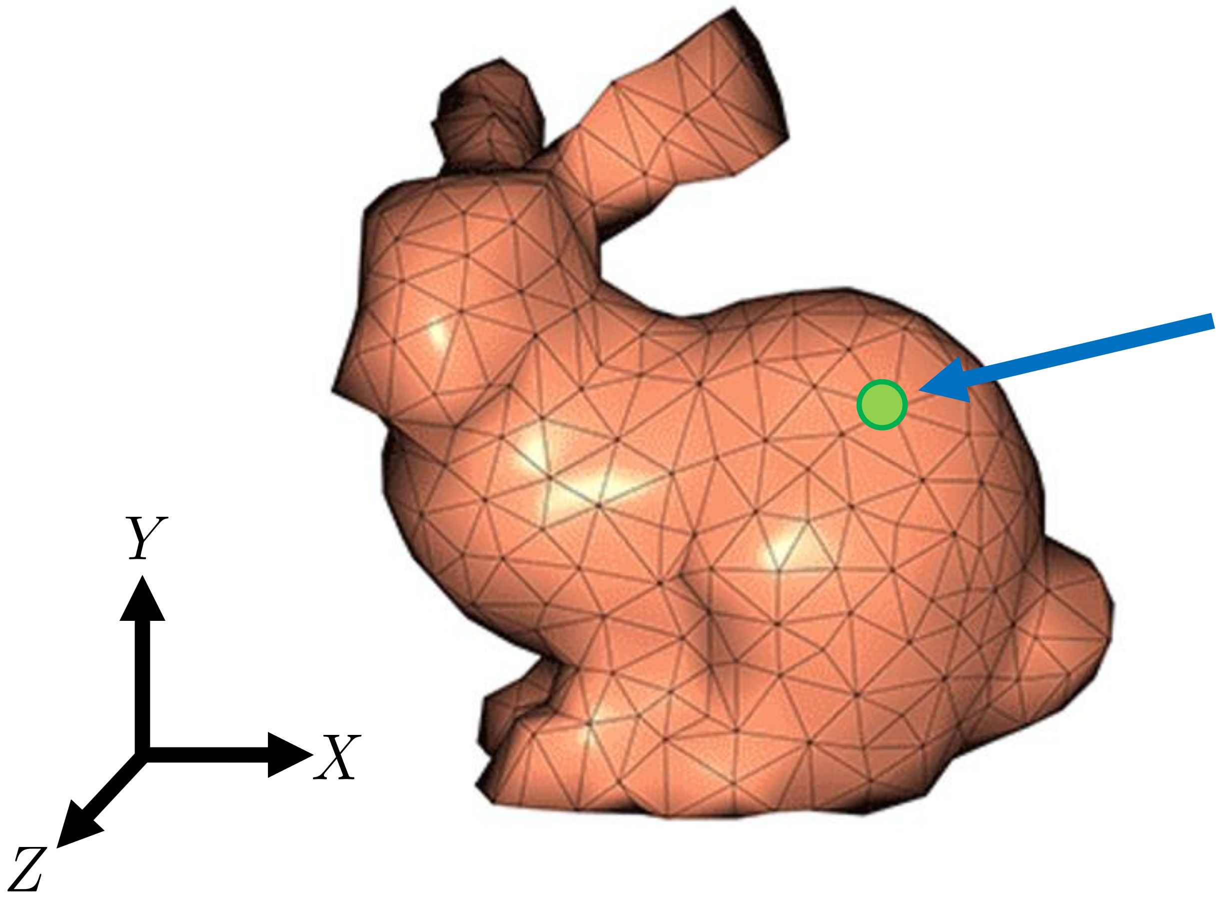
$$\Delta\mathbf{X}^T\left(\mathrm{F}^T\mathrm{F}-\mathrm{I}\right)\Delta\mathbf{X}=\mathbf{0}$$

Implies

$$\mathrm{F}^T\mathrm{F}=\mathrm{I}$$

Orthogonal

Potential Energy of Affine Body



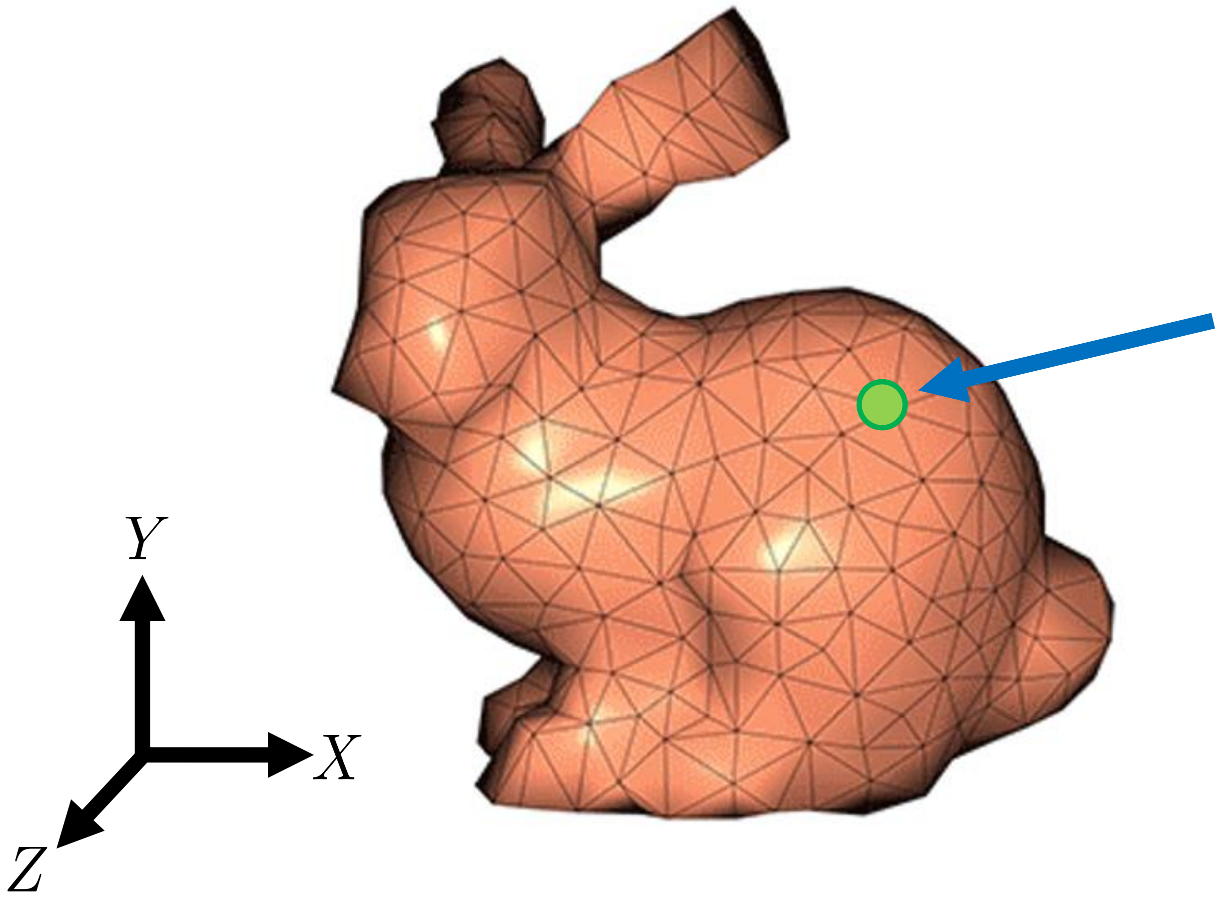
Reference (Undeformed) Space

Set this to a big value

$$\psi = \kappa \|F^T F - I\|_F^2 d\Omega$$

How do we keep the object rigid ?

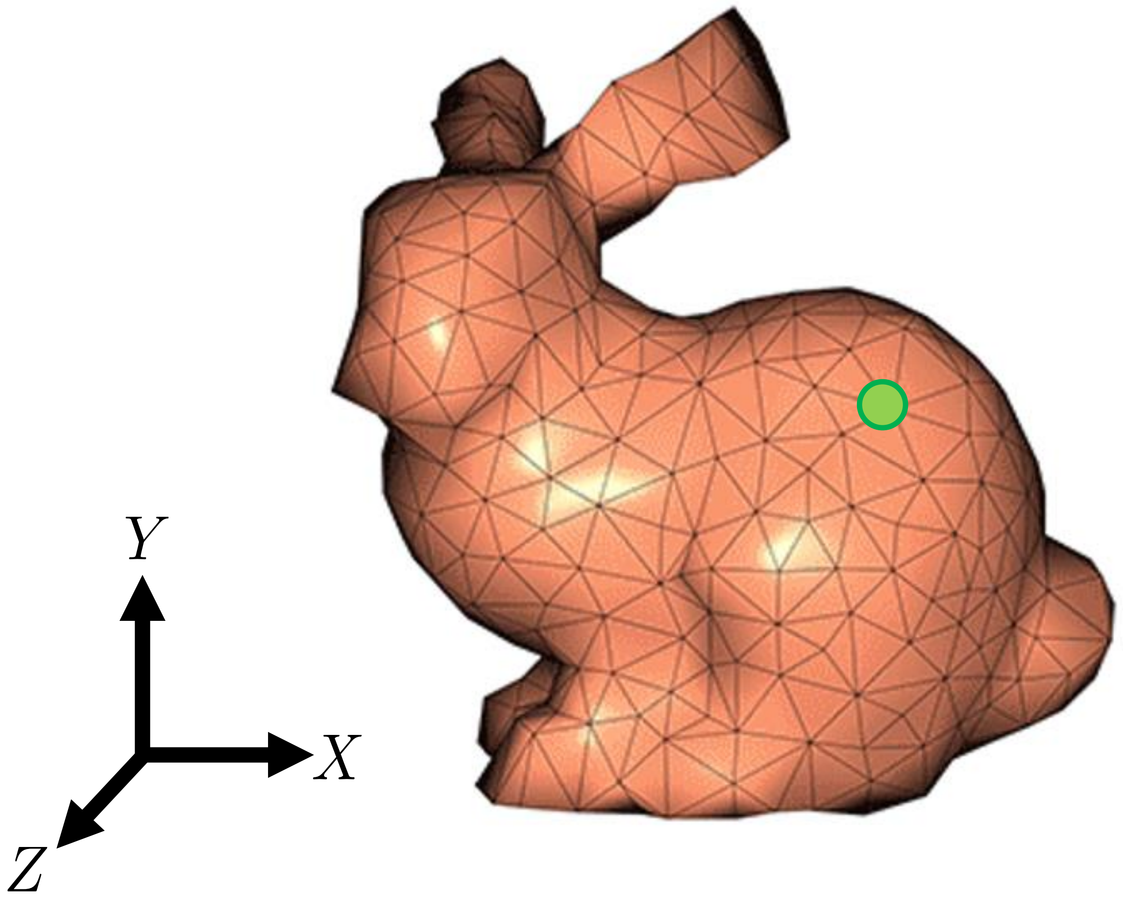
Potential Energy of Affine Body



Reference (Undeformed) Space

$$\mathrm{V} = \int \kappa \left|\left|F^T F - I\right|\right|_F^2 d\Omega$$

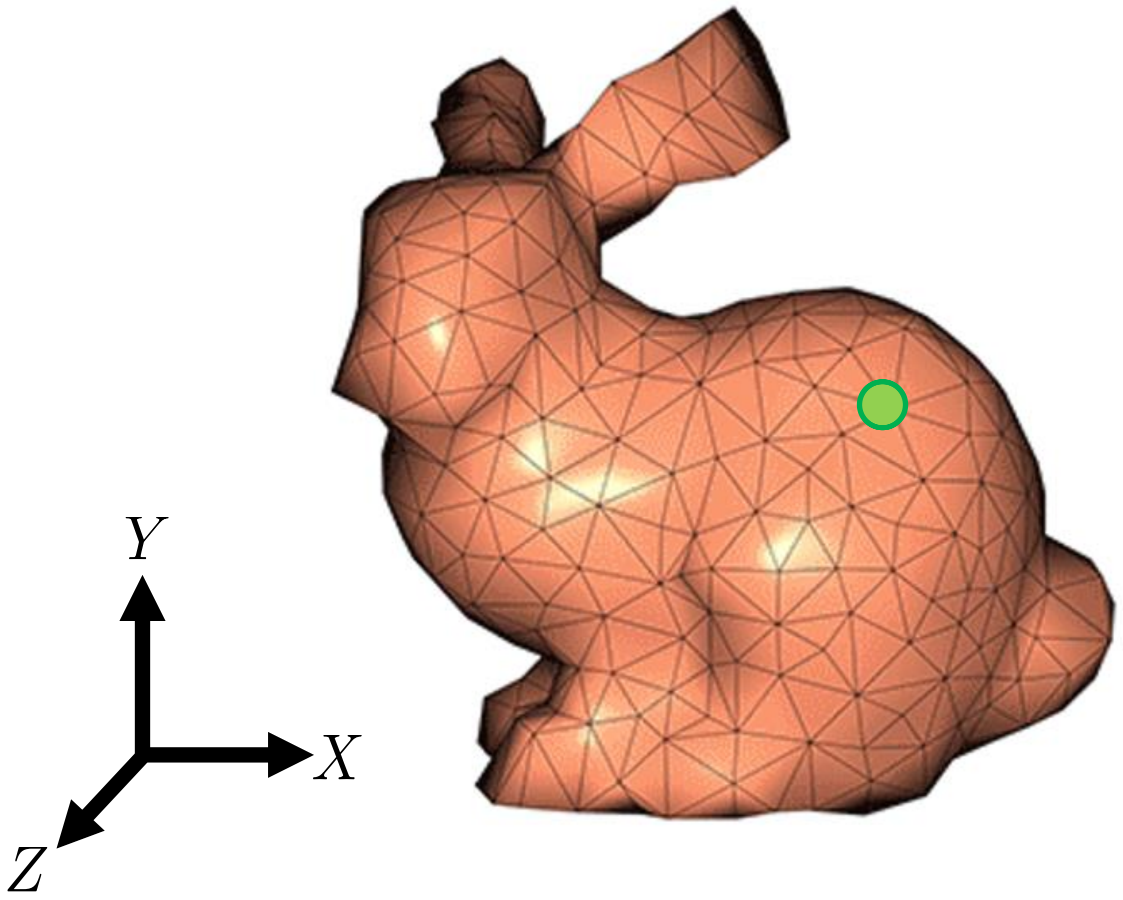
But what is the Deformation Gradient ?



Reference (Undeformed) Space

$$\mathbf{x}(\mathbf{X}, t) = \mathrm{J}(\boldsymbol{X})\mathrm{q}(\mathrm{t})$$

But what is the Deformation Gradient ?



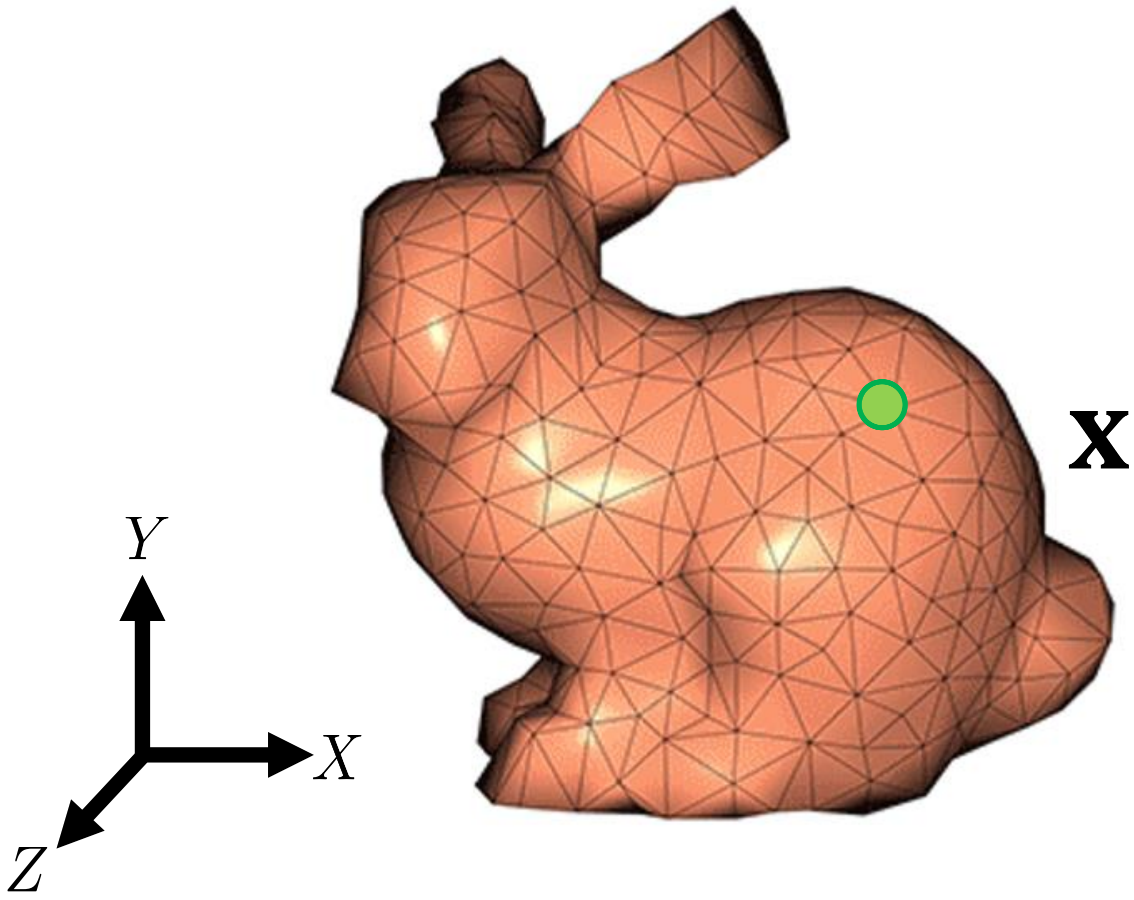
Reference (Undeformed) Space

$$\mathbf{x}(\mathbf{X}, t) = \mathrm{Q}(t) \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

$$\begin{pmatrix} q_0 & q_1 & q_2 & q_3 \\ q_4 & q_5 & q_6 & q_7 \\ q_8 & q_9 & q_{10} & q_{11} \end{pmatrix}$$

Linear Transform Translation

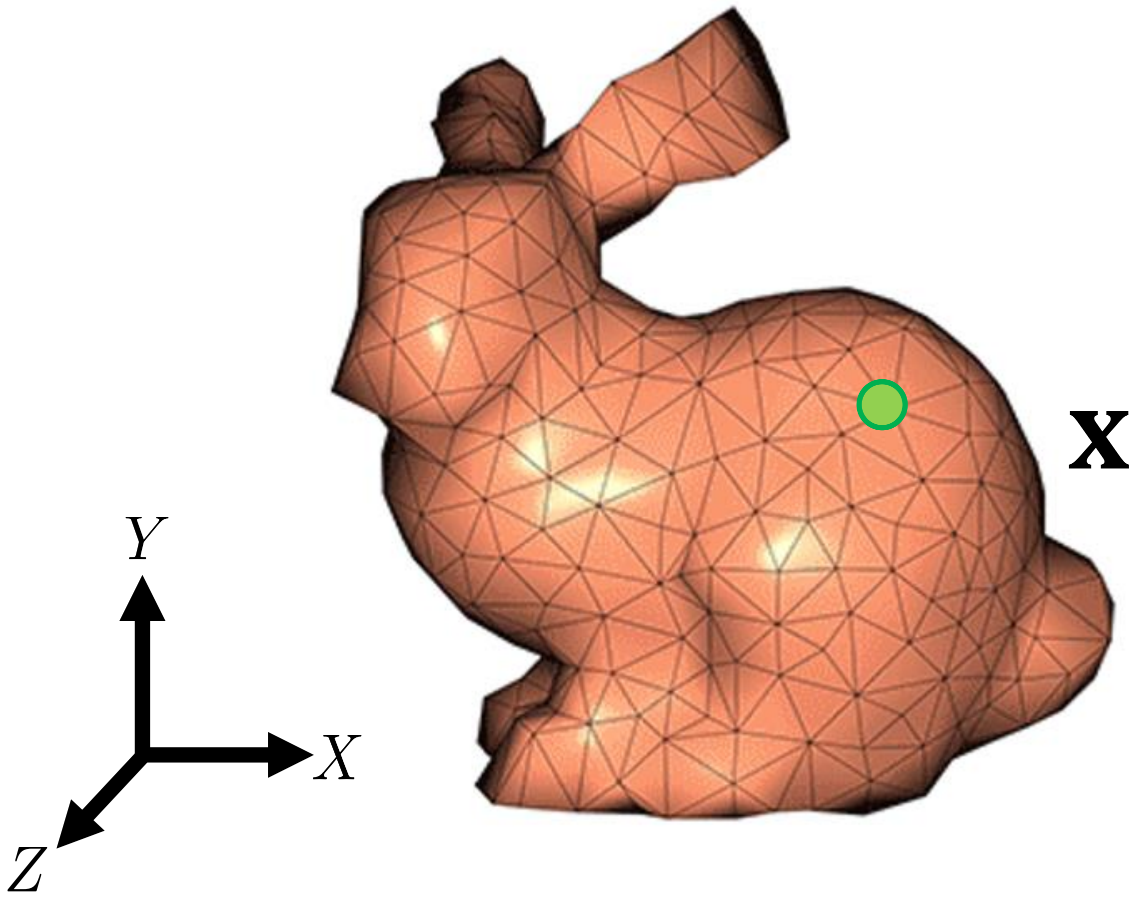
But what is the Deformation Gradient ?



$$\mathbf{x}(\mathbf{X}, t) = \begin{bmatrix} q_0 & q_1 & q_2 \\ q_4 & q_5 & q_6 \\ q_8 & q_9 & q_{10} \end{bmatrix} \mathrm{X} + \begin{bmatrix} q_3 \\ q_7 \\ q_{11} \end{bmatrix}$$

Reference (Undeformed) Space

But what is the Deformation Gradient ?

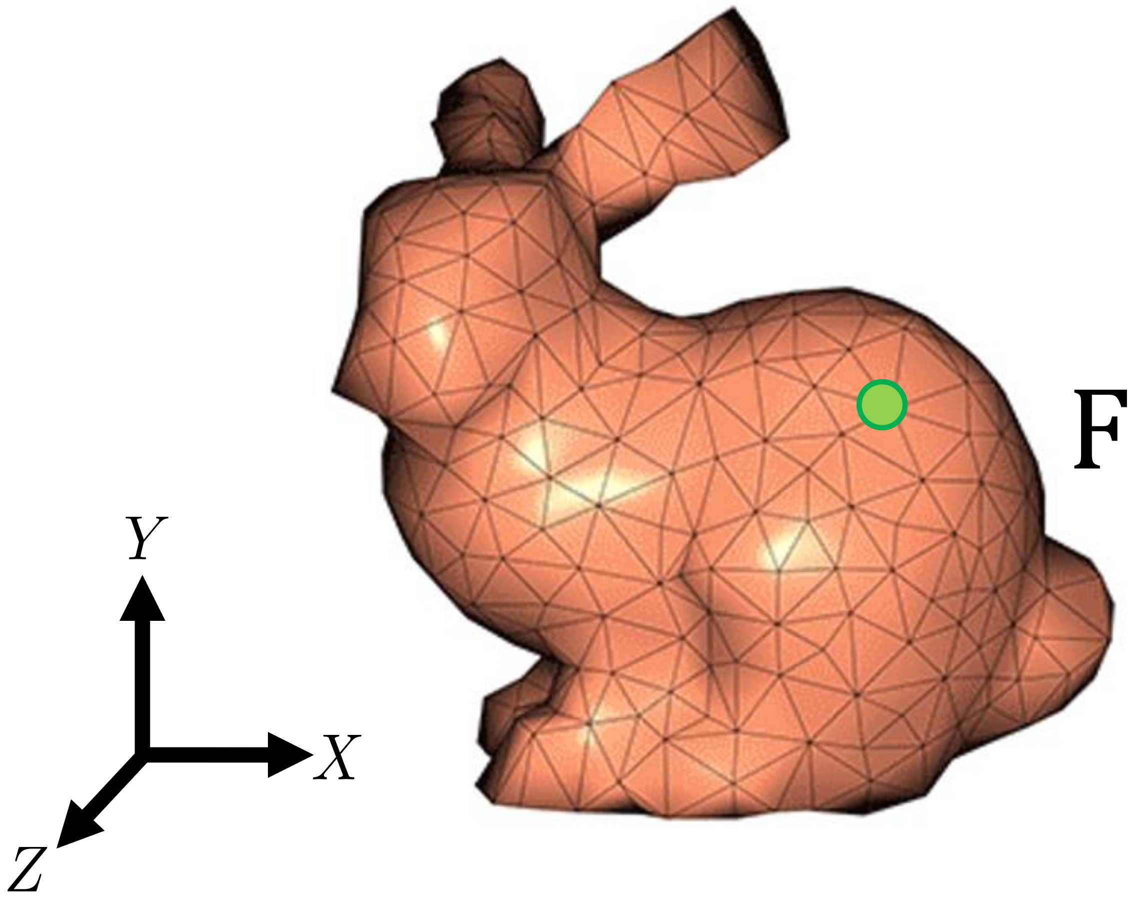


$$\mathbf{x}(\mathbf{X}, t) = \begin{bmatrix} q_0 & q_1 & q_2 \\ q_4 & q_5 & q_6 \\ q_8 & q_9 & q_{10} \end{bmatrix} \mathrm{X} + \begin{bmatrix} q_3 \\ q_7 \\ q_{11} \end{bmatrix}$$

Reference (Undeformed) Space

$$F = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}$$

But what is the Deformation Gradient ?

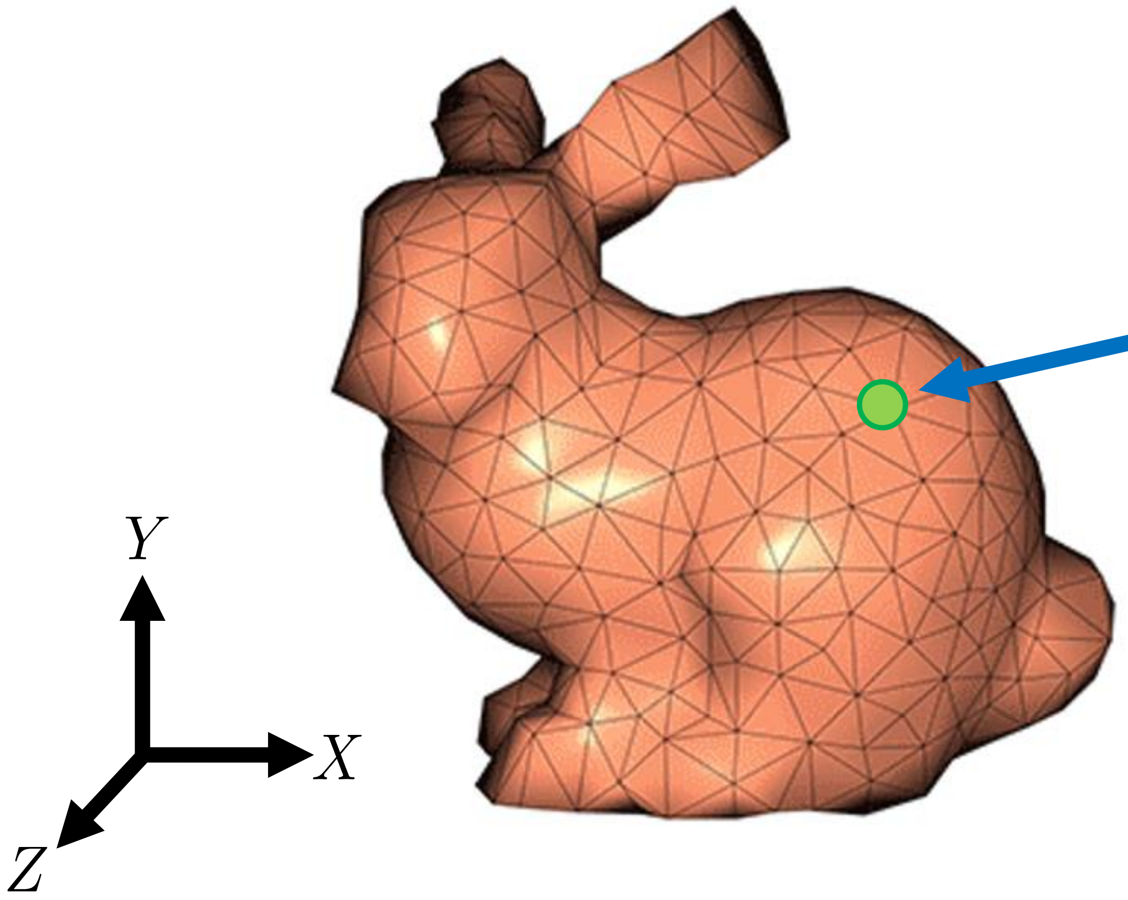


Reference (Undeformed) Space

$$\mathrm{F}(\mathbf{X}, t) = \begin{bmatrix} q_0 & q_1 & q_2 \\ q_4 & q_5 & q_6 \\ q_8 & q_9 & q_{10} \end{bmatrix}$$

$$F = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}$$

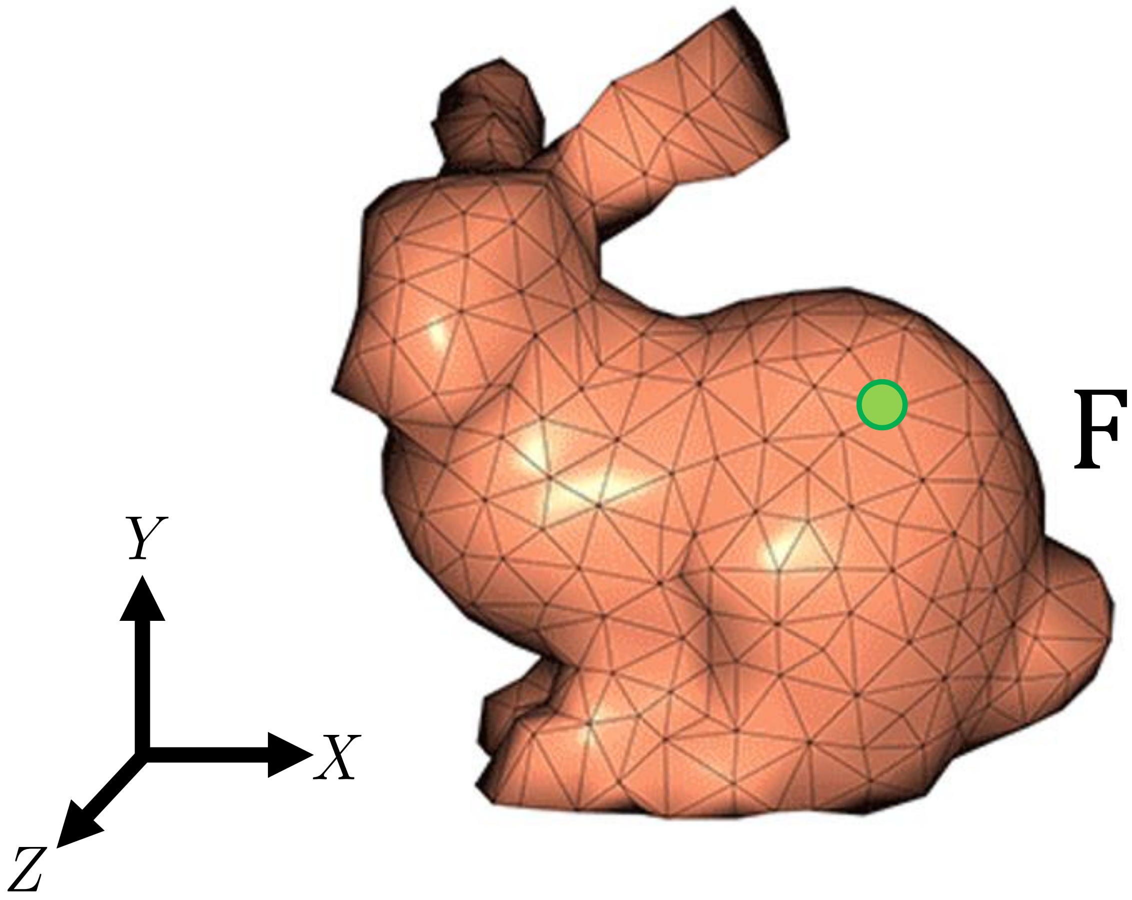
Potential Energy of Affine Body



$$\mathrm{V}(\mathbf{q}) = \int \kappa \left|\left|F^T F - I\right|\right|_F^2 d\Omega$$

Reference (Undeformed) Space

But what is the Deformation Gradient ?

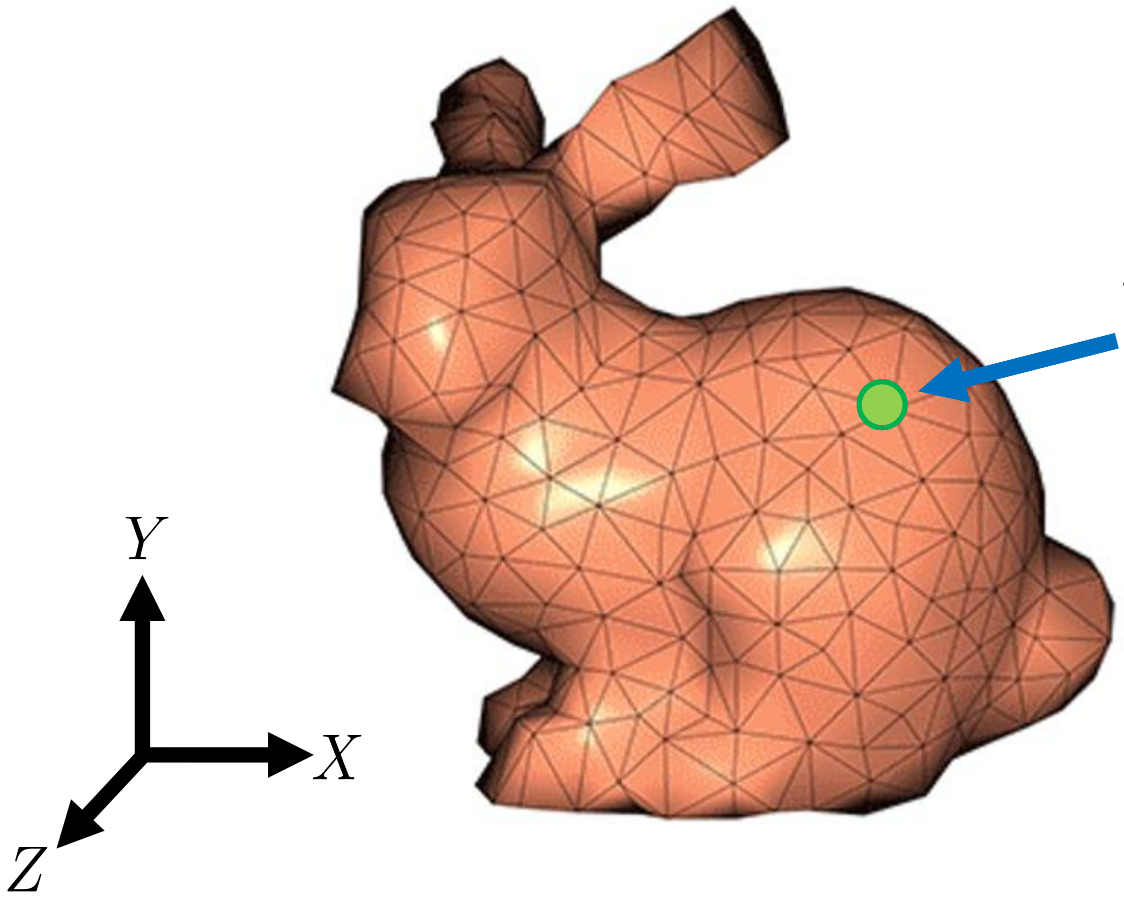


$$\mathrm{F}(\mathbf{X}, t) = \begin{bmatrix} q_0 & q_1 & q_2 \\ q_4 & q_5 & q_6 \\ q_8 & q_9 & q_{10} \end{bmatrix}$$

Reference (Undeformed) Space

How does this change as a function of X?

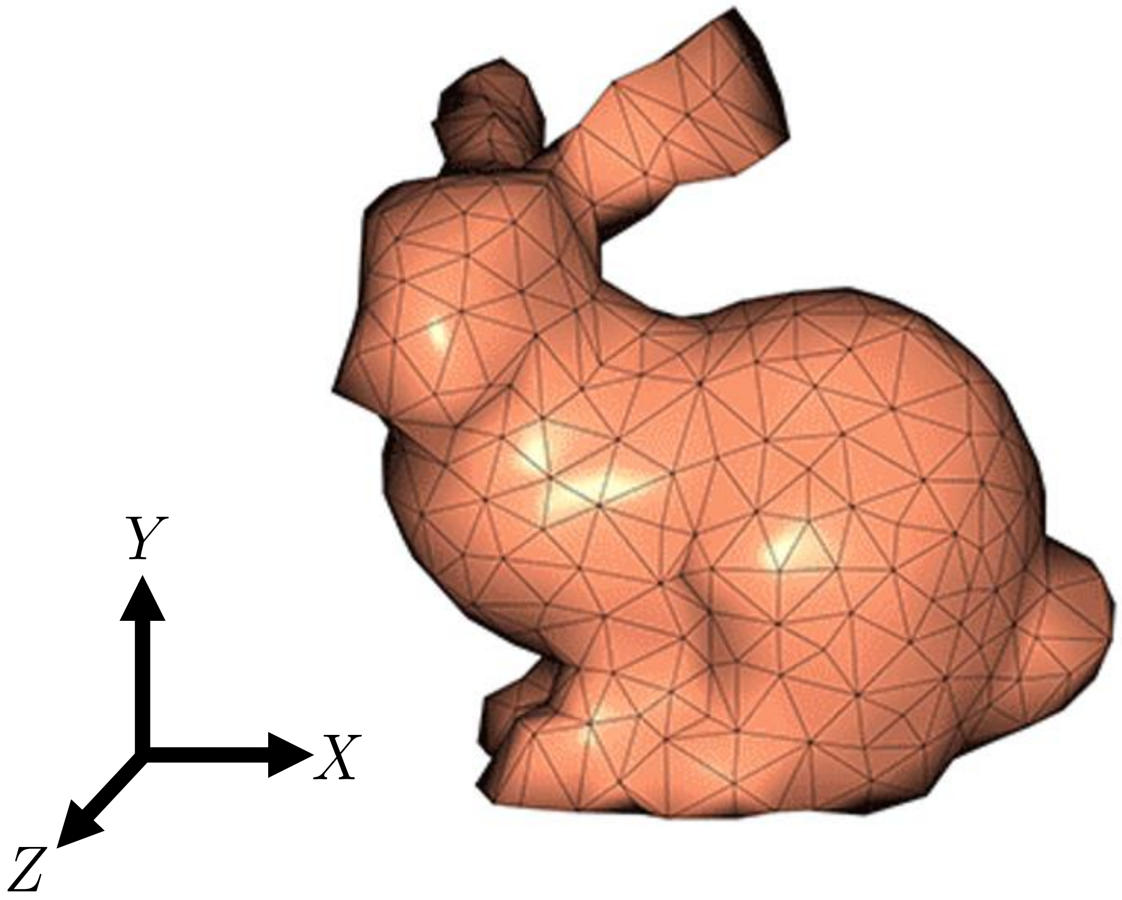
Potential Energy of Affine Body



Reference (Undeformed) Space

$$\mathrm{V}(\mathbf{q}) = \kappa \cdot \underbrace{\mathrm{vol}}_{\text{Volume of Rigid Body}} \cdot \left|\left|F^T F - I\right|\right|_F^2$$

Kinetic Energy of an Affine Body



Reference (Undeformed) Space

$$\text{vol} = \int_{\Omega} 1 d\Omega$$

We computed this when we computed the mass matrix

Equations of Motion

$$M\ddot{\mathbf{q}} = -\frac{\partial V}{\partial \mathbf{q}}$$

Solve using Optimization via Newton's Method

$$E\left(\mathbf{q}^{\mathbf{i+1}}\right) = \frac{1}{2}\left(\mathbf{q}^{\mathbf{i+1}} - \widetilde{\mathbf{q}}^{\mathbf{i}}\right)^{\mathrm{T}} M\left(\mathbf{q}^{\mathbf{i+1}} - \widetilde{\mathbf{q}}^{\mathbf{i}}\right) + h^{2} V\left(\mathbf{q}^{\mathrm{i+1}}\right)$$