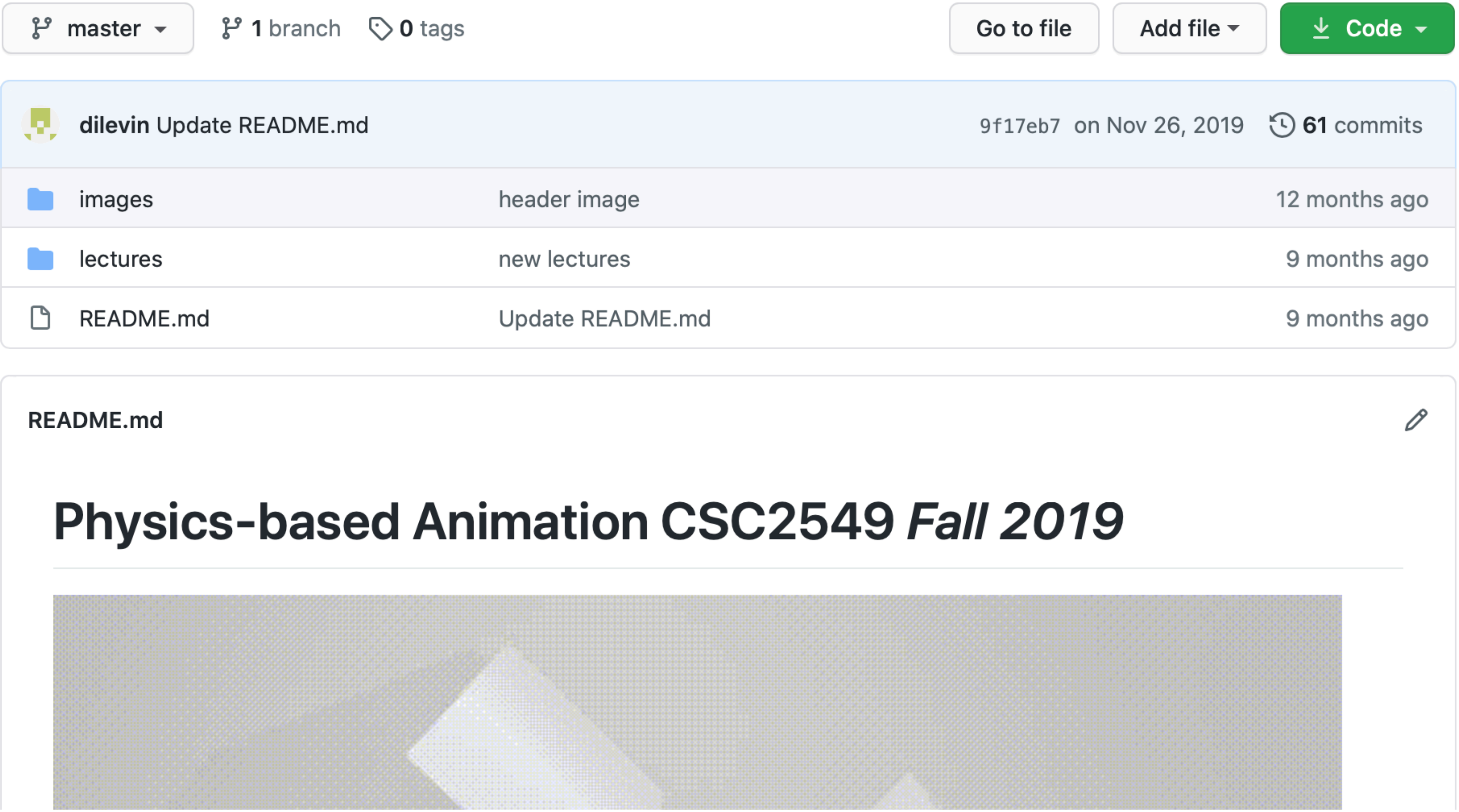
Physics-Based Animation

Lord of the Rings | Weta Digital

CSC2549 Physics-Based Animation



Course web site

https://github.com/dilevin/CSC417-physics-based-animation

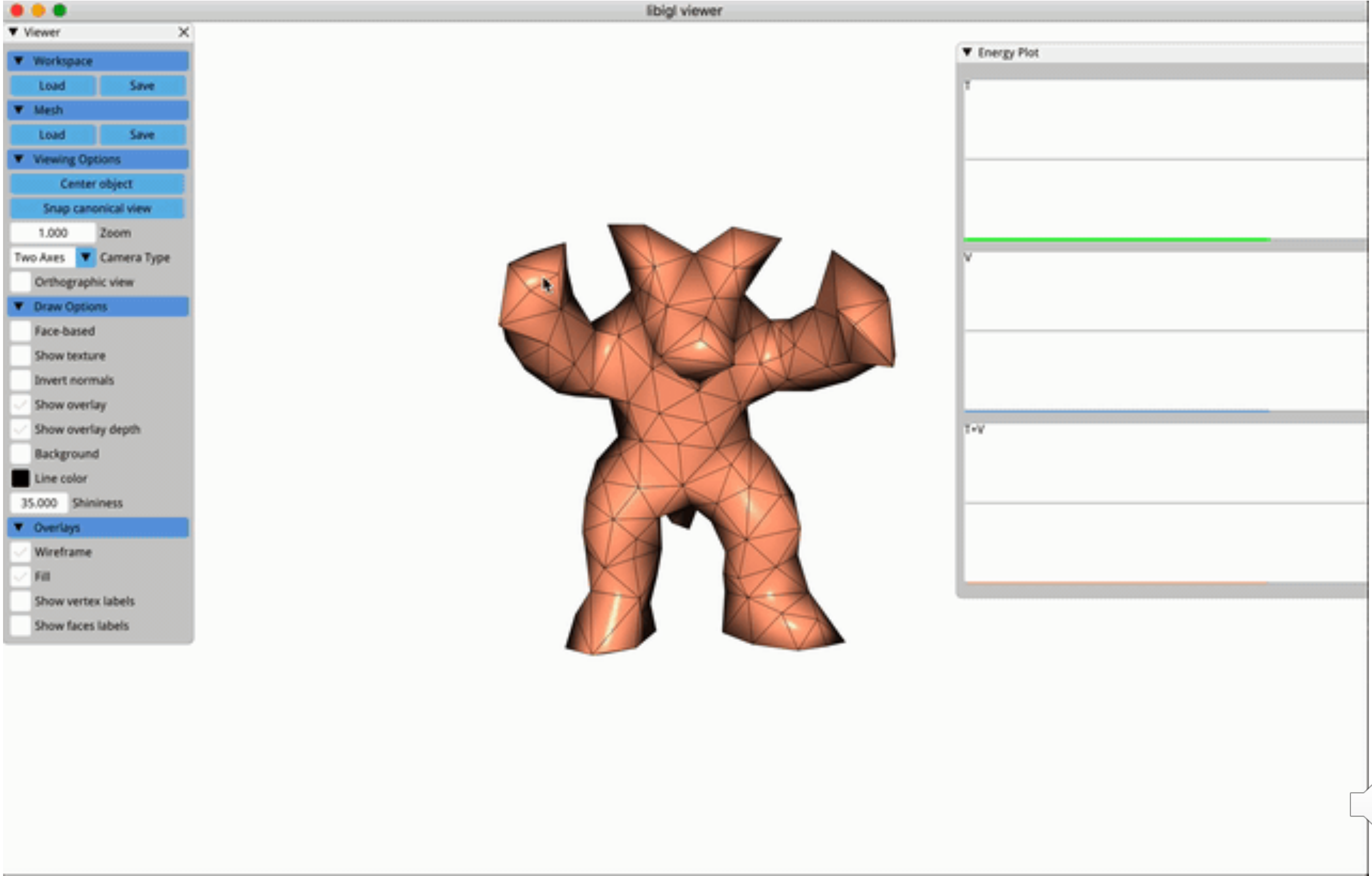
Reasons you might be taking this course

1. Just curious

2. You might want to use physics simulation in your research

3. You want to do research in physics simulation

Today: Introduction and Springs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Workspace
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Viewing Options
Center object
Snap canonical view
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Zoom
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Camera Type
Orthographic view
Draw Options
Face-based
Show texture
Invert normals
Show overlay
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Show vertex labels
Show faces labels
Energy Plot
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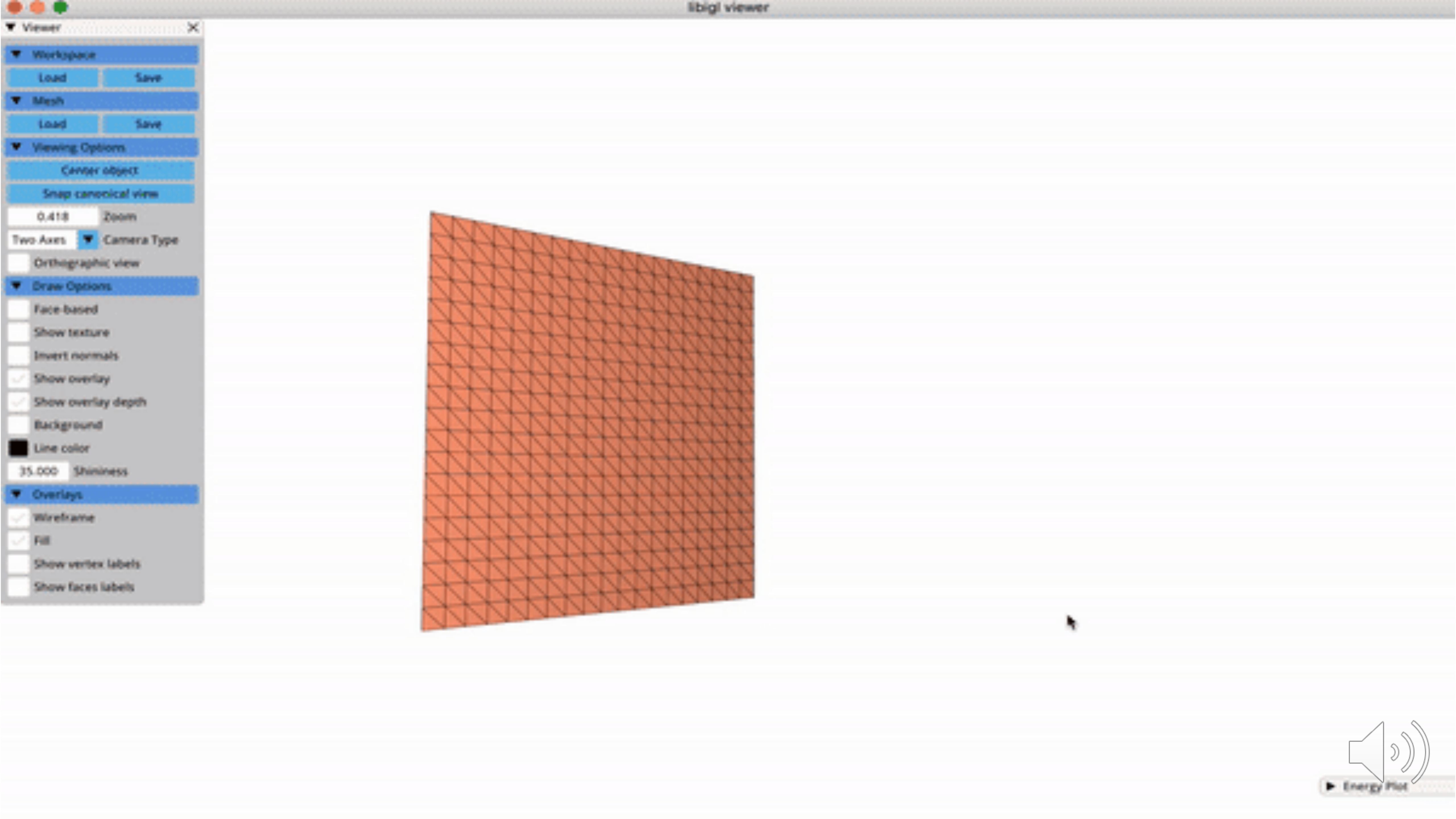
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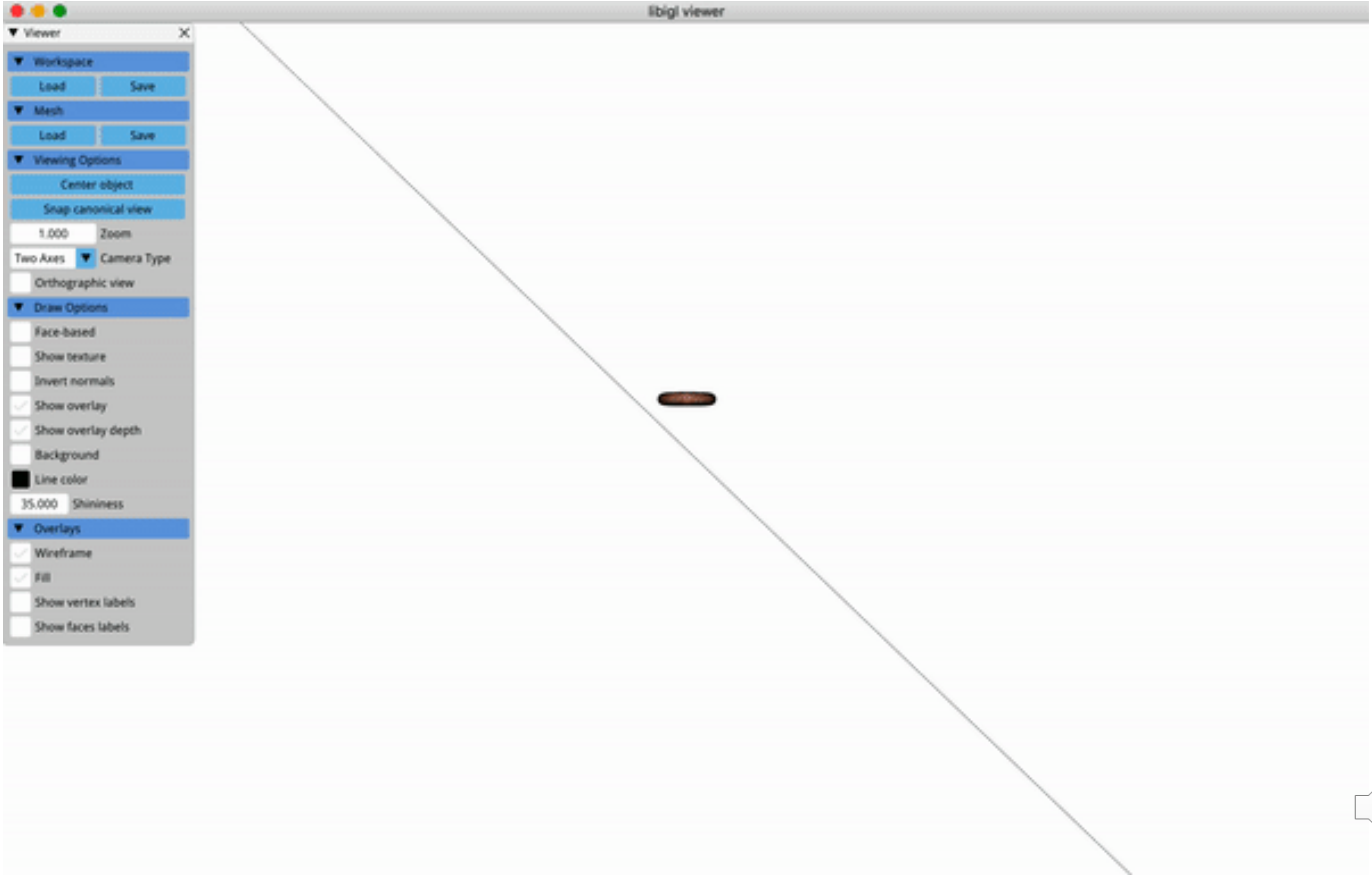
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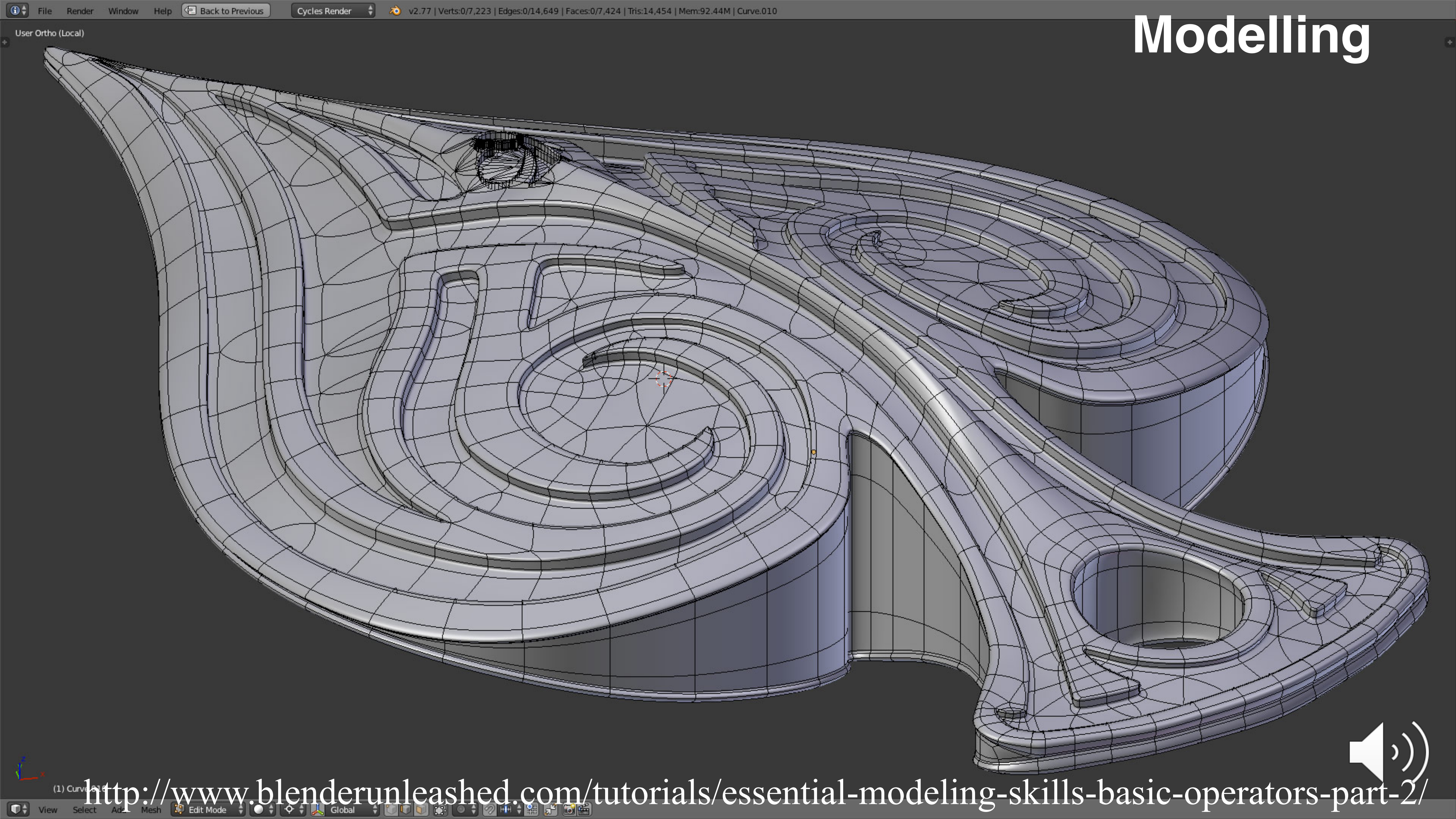
“Core” Areas of Computer Graphics

Modeling

Rendering

Animation

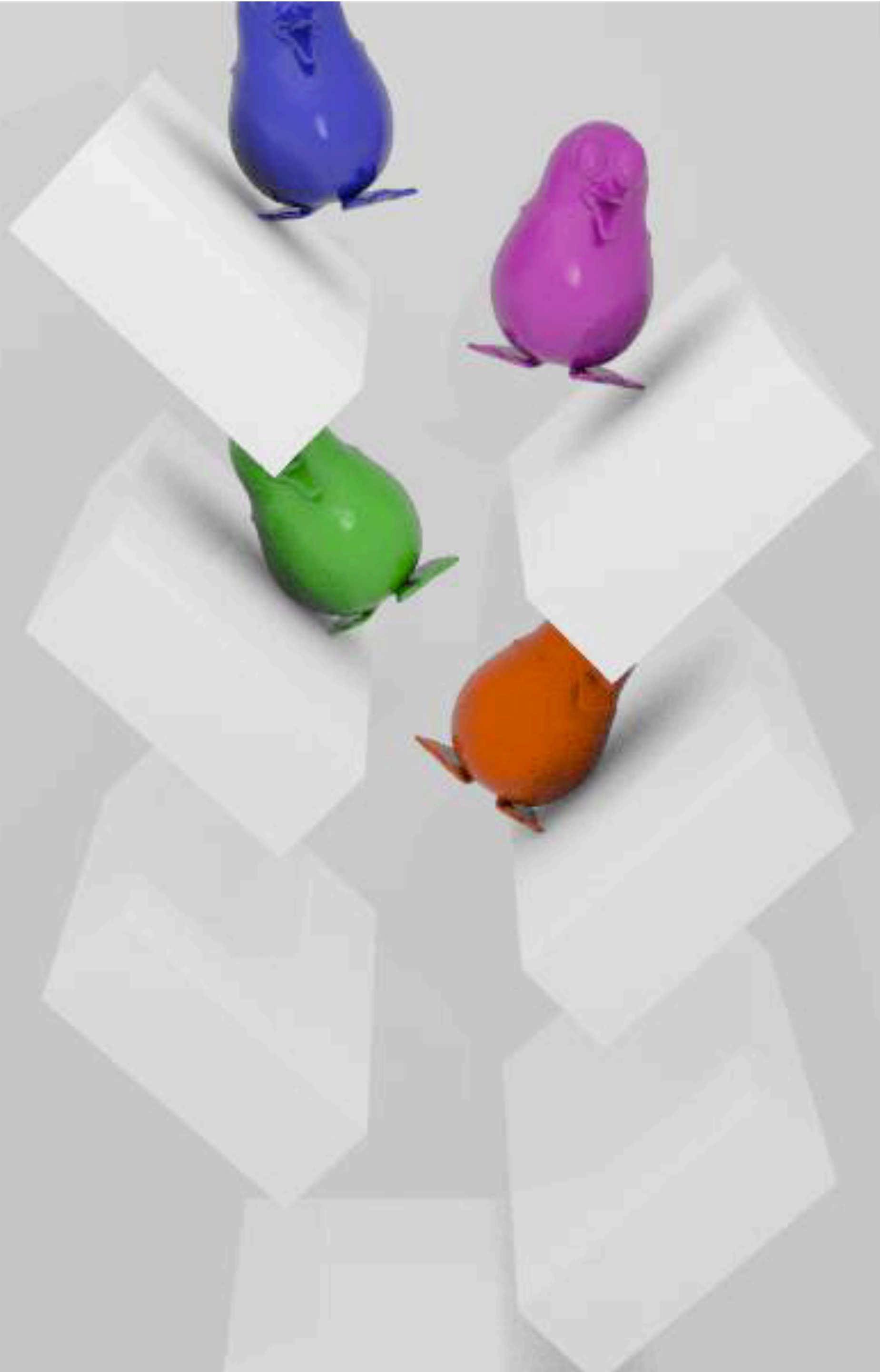
Modelling



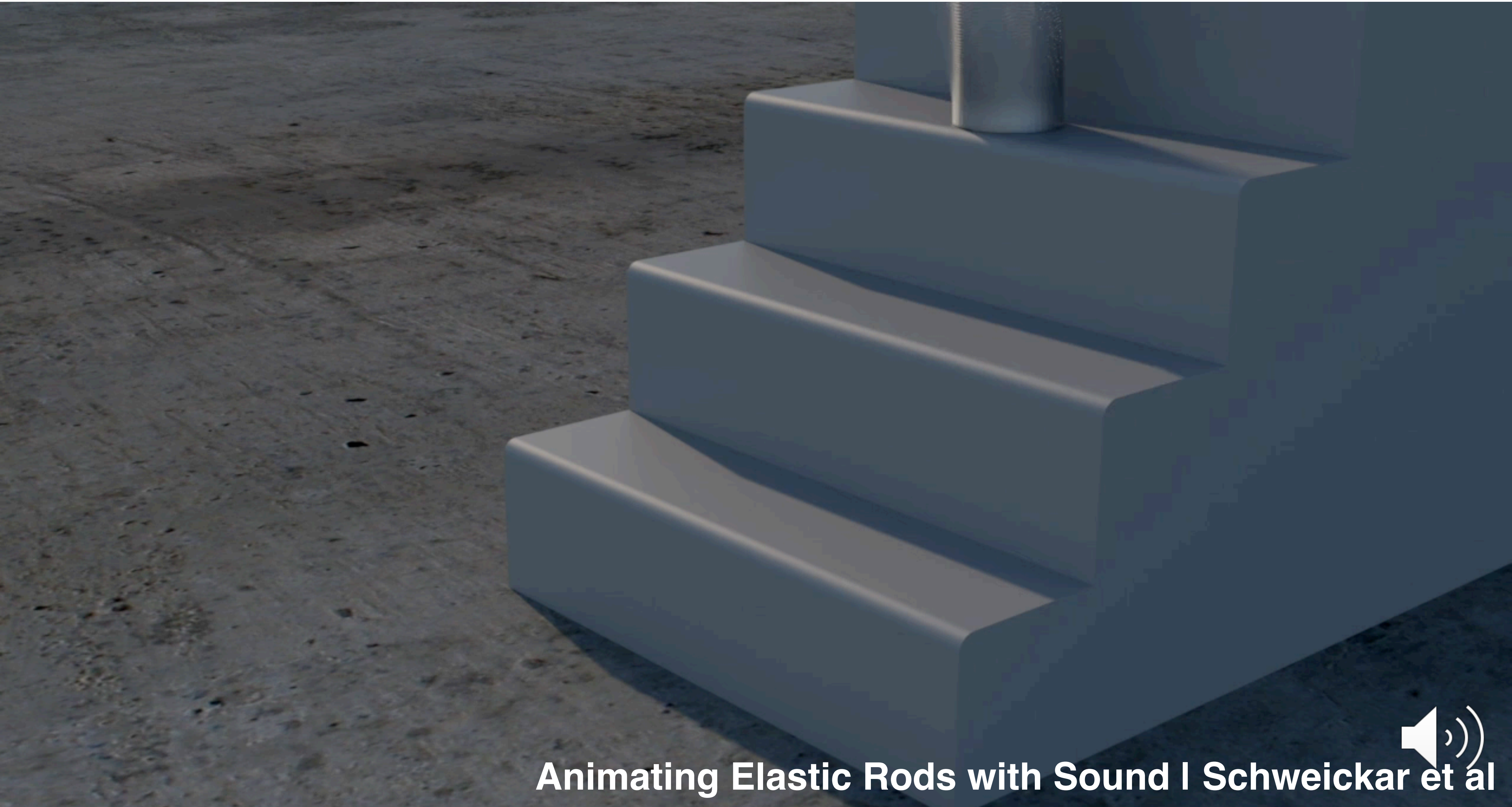
http://www.blenderunleashed.com/tutorials/essential-modeling-skills-basic-operators-part-2/

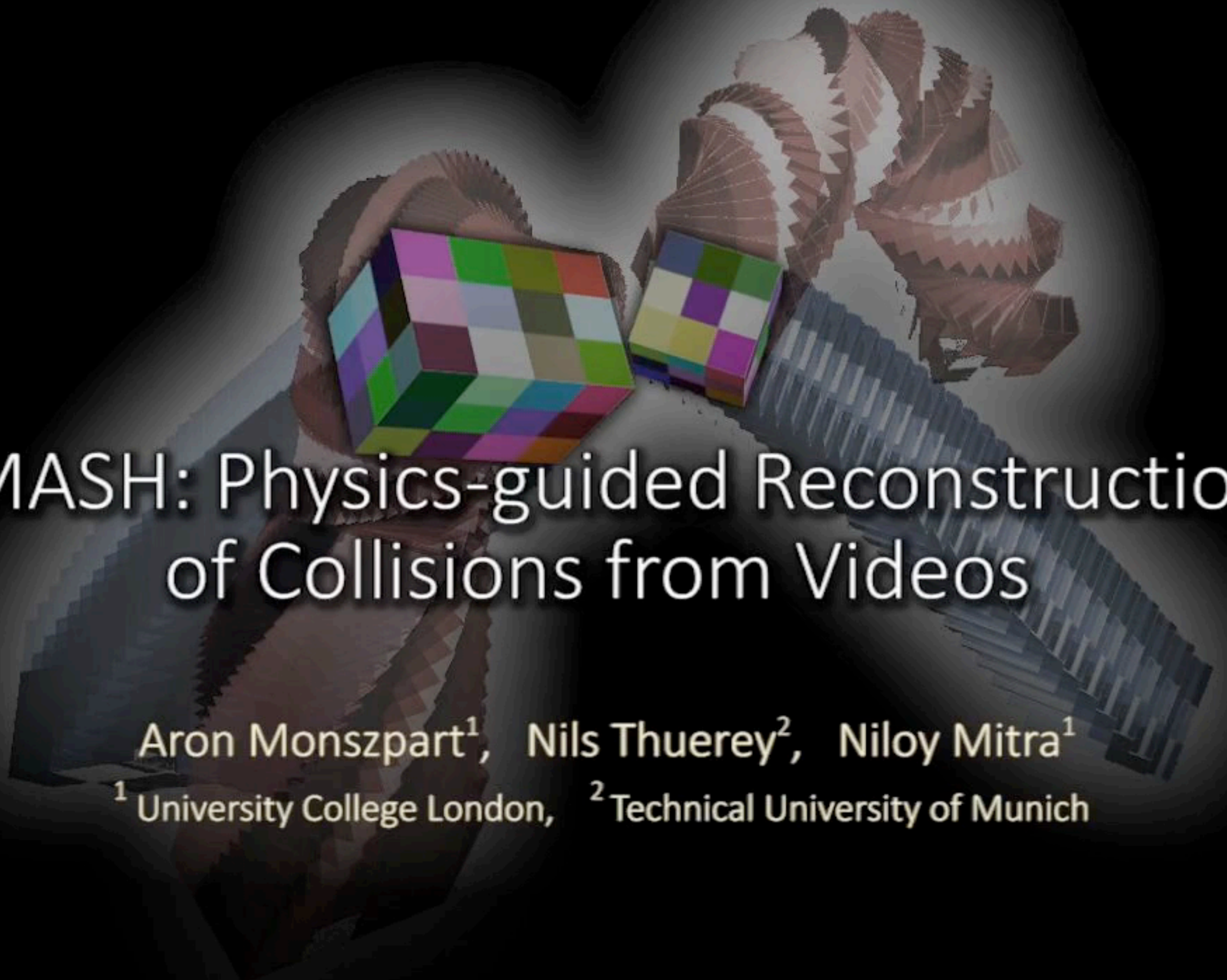
Rendering

Animation




skeleton
Image courtesy of Weta Digital


Animating Elastic Rods with Sound | Schweickar et al



SMASH: Physics-guided Reconstruction of Collisions from Videos

Aron Monszpart[1], Nils Thuerey[2], Niloy Mitra[1]
[1] University College London, [2] Technical University of Munich

Ready Player One | Warner Bros. Pictures

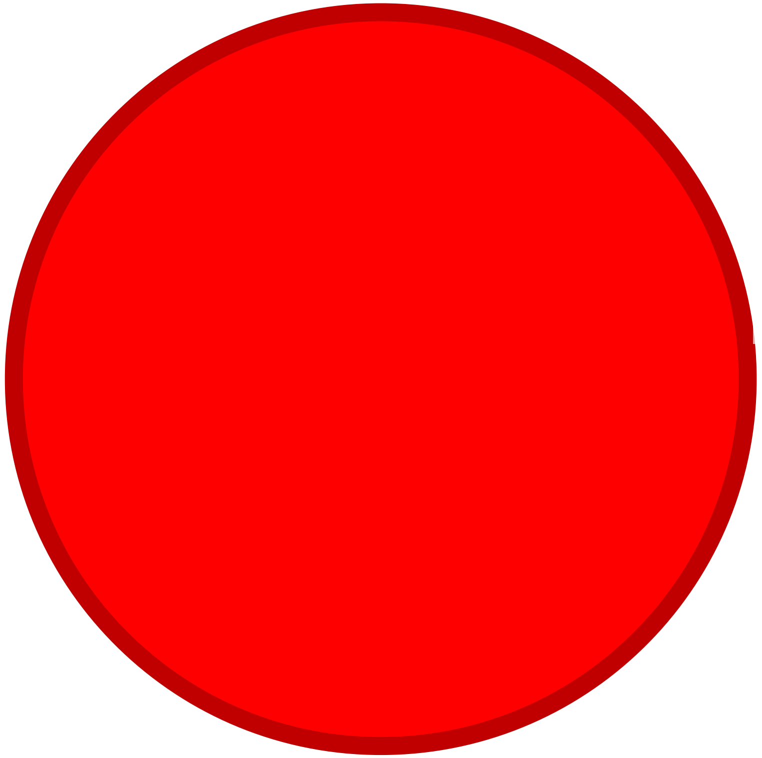
Newton's Laws

1. Every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force

2. The force acting on an object is equal to the time rate-of-change of the momentum

3. For every action there is an equal and opposite reaction

Example Physical System



Particle

Position in space (m) $\mathbf{x}(t)$

Velocity in space (m/s) $\mathbf{v}(t) = \frac{d\mathbf{x}}{dt}(t)$

Acceleration in space (m/s²) $\mathbf{a}(t) = \frac{d^2\mathbf{x}}{dt^2}(t)$

Mass (kg) m

Newton’s Laws

1. Every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force

2. The **force** acting on an object is equal to the **time rate-of-change of the momentum**

3. For every action there is an equal and opposite reaction

Newton's Laws

momentum $\underbrace{m}_{\text{mass}}\overset{\text{velocity}}{\mathbf{v}}$

time rate-of-change of the momentum **force**

$$\frac{d}{dt}(m\mathbf{v}) = \mathbf{f}$$

for constant mass

$$m\overset{\text{acceleration}}{\frac{d\mathbf{v}}{dt}} = \mathbf{f}$$

Vectorial Mechanics

Newton's Laws

momentum $m\mathbf{v}$

velocity

mass

time rate-of-change of the momentum **force**

$$\frac{d}{dt}(m\mathbf{v}) = \mathbf{f}$$

for constant mass

acceleration

$$m\frac{d\mathbf{v}}{dt} = \mathbf{f}$$

Variational Mechanics

or *Analytical Mechanics*

- Based on two fundamental energies rather than two vectorial quantities

Kinetic and Potential Energy

Kinetic Energy: Energy due to motion

Potential Energy: Energy “held within” an object due to its position, internal stresses, electrical charge etc …

Potential energy has the *potential* to become kinetic energy

Kinetic and Potential Energy

Potential Energy from Gravity

$$m \cdot g \cdot h$$

acceleration due to gravity

Kinetic and Potential Energy

Potential Energy from Gravity

$$m \cdot g \cdot h$$

height above ground

Variational Mechanics

Also called ”Analytical Mechanics”

Based on two fundamental energies rather than two vectorial quantities

Motion defined using a variational principle

$$\underbrace{e}_{\text{functional}}\left(\overbrace{\mathbf{f}(t), \dot{\mathbf{f}}(t), \ldots}^{\text{functions of time and derivatives}}\right) \to \underbrace{\mathbb{R}}_{\text{real numbers}}$$

Generalized Coordinates

$$\mathbf{x}(t) = \mathbf{f}(\mathbf{q}(t))$$

generalized coordinates

Jacobian (lots of things are going to get called Jacobians)

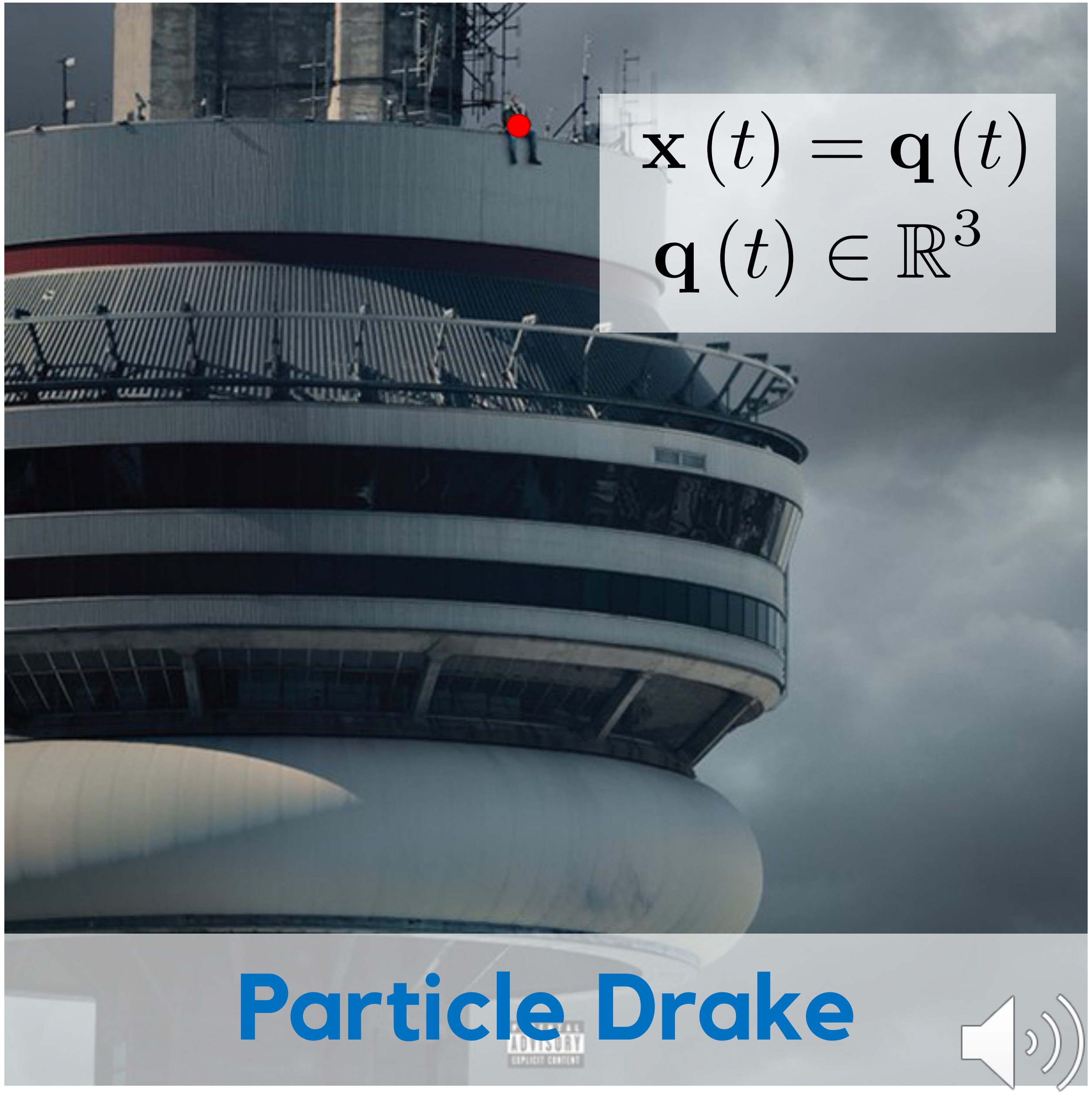
$$\frac{d\mathbf{x}}{dt}(t) = \frac{d\mathbf{f}}{d\mathbf{q}}\dot{\mathbf{q}}(t)$$

generalized velocity

Generalized Coordinates

$$\mathbf{x}(t) = \mathbf{f}(\underbrace{\mathbf{q}(t)}_{\text{generalized coordinates}})$$

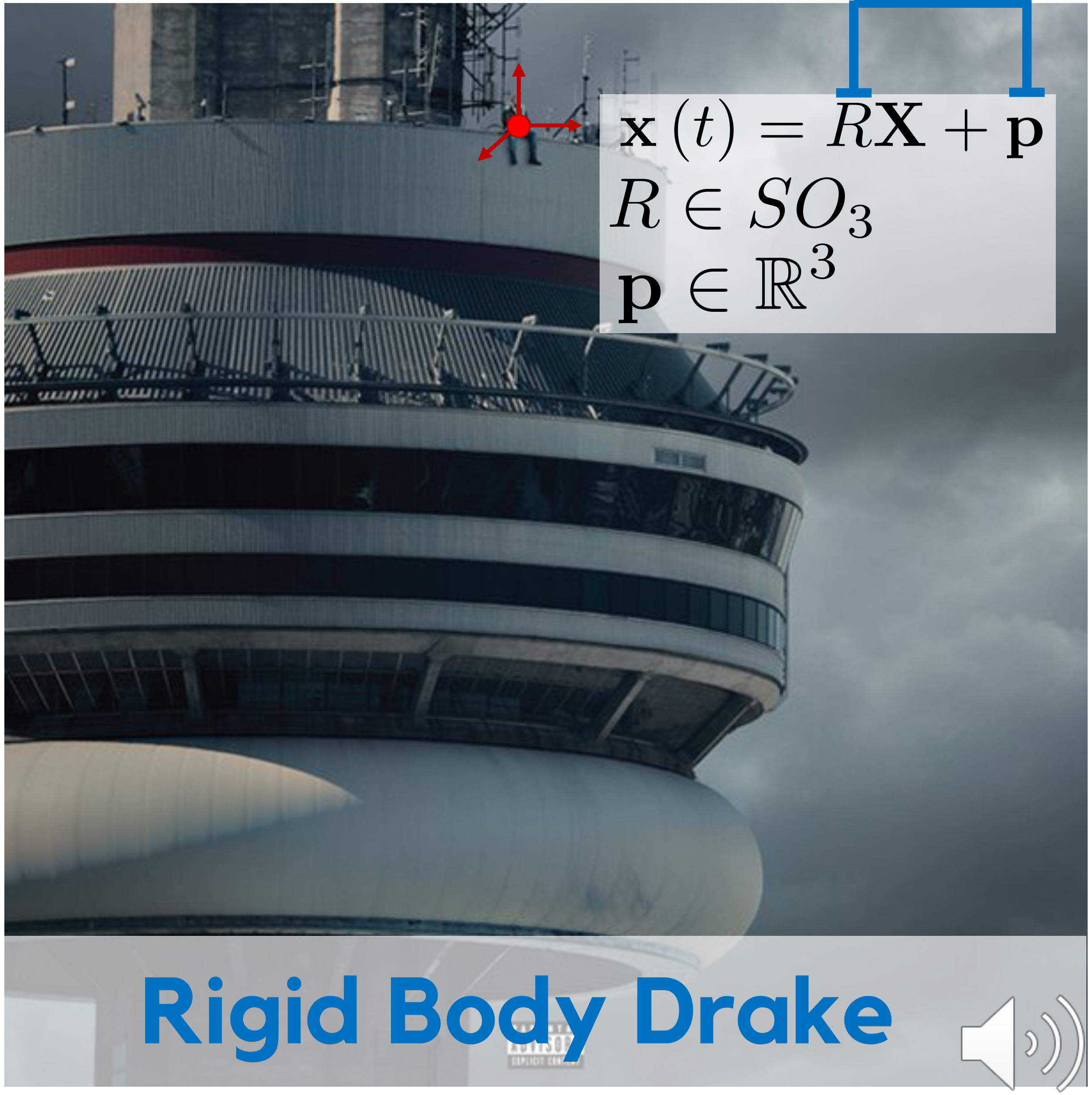
$$\frac{d\mathbf{x}}{dt}(t) = \overbrace{\frac{d\mathbf{f}}{d\mathbf{q}}}^{\text{Jacobian}} \underbrace{\dot{\mathbf{q}}(t)}_{\text{generalized velocity}}$$



Generalized Coordinates

$$\mathbf{x}(t) = \mathbf{f}(\underbrace{\mathbf{q}(t)}_{\text{generalized coordinates}})$$

$$\frac{d\mathbf{x}}{dt}(t) = \overbrace{\frac{d\mathbf{f}}{d\mathbf{q}}}^{\text{Jacobian}} \underbrace{\dot{\mathbf{q}}(t)}_{\text{generalized velocity}}$$

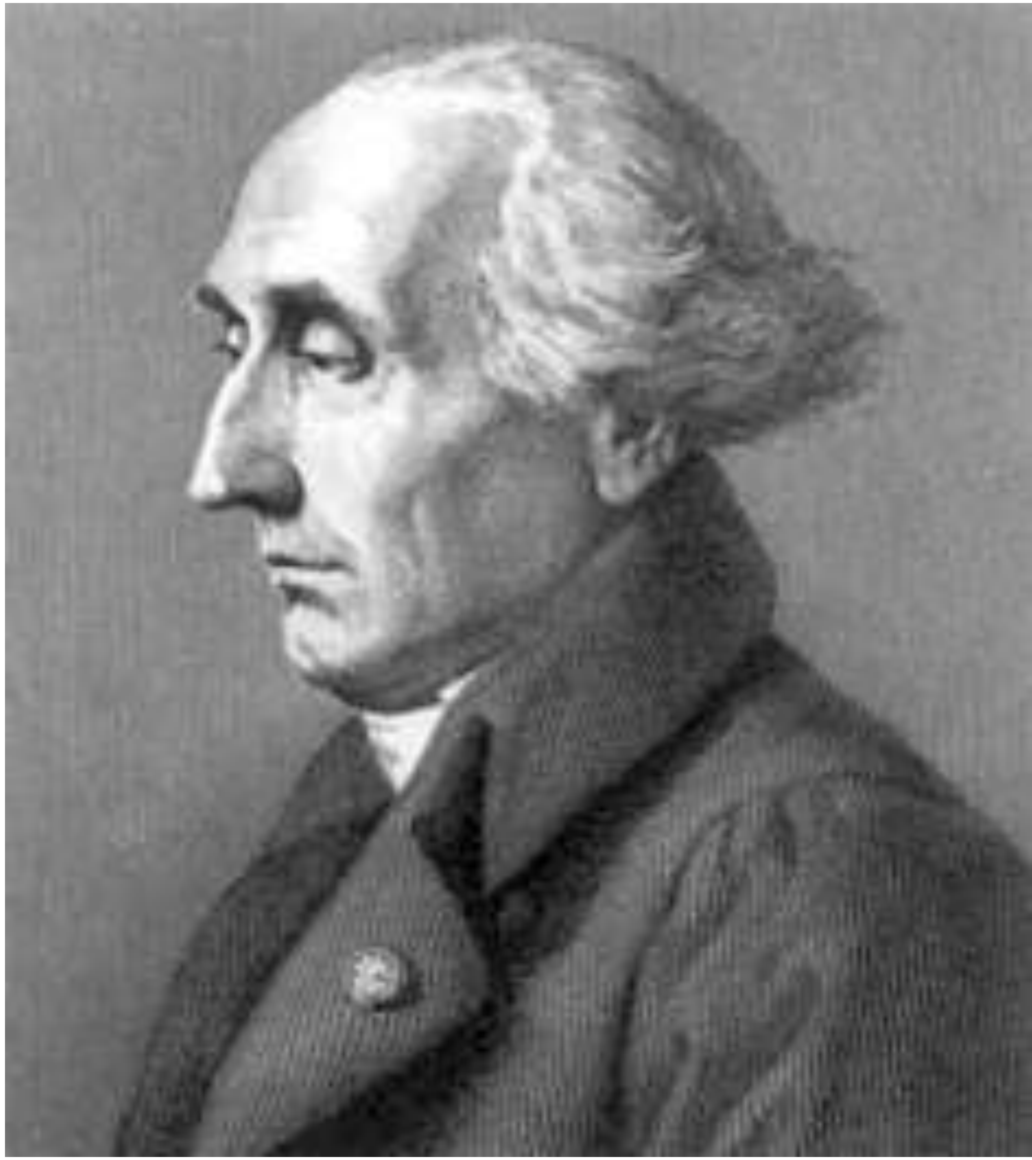


The Lagrangian

Potential Energy

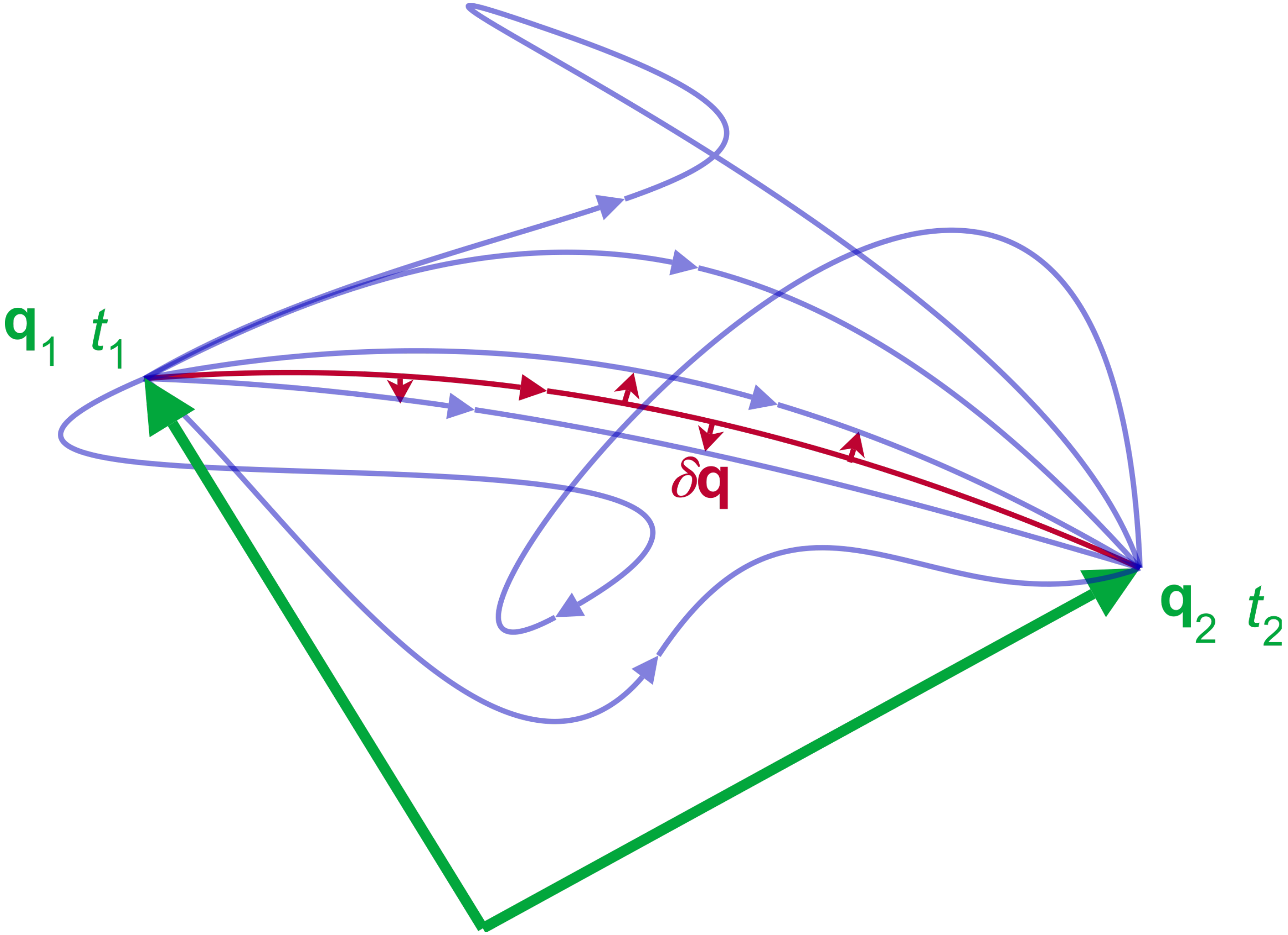
$$L = T - V$$

Kinetic Energy



Joseph-Louis Lagrange
(was pretty good at math)

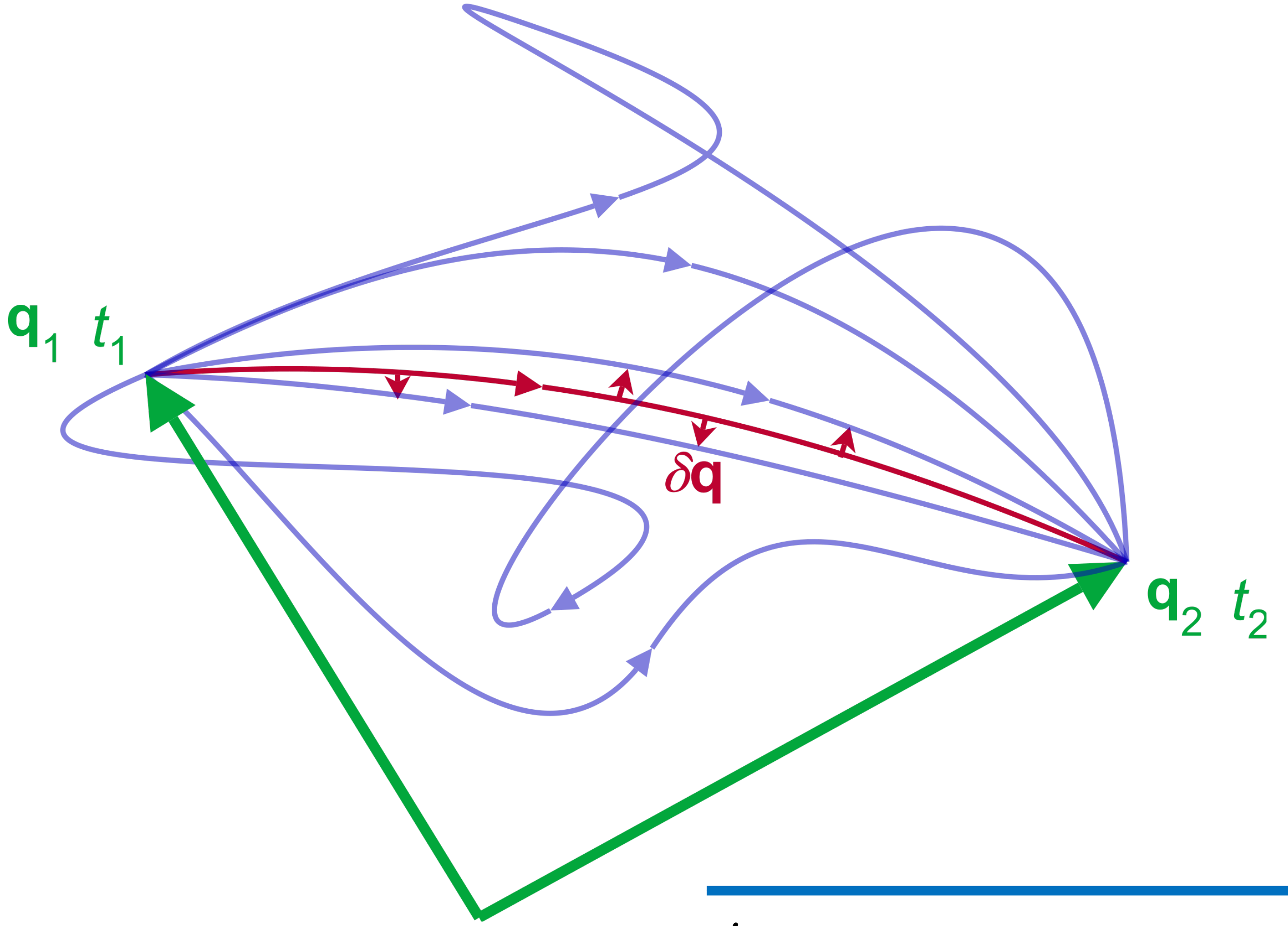
The Principle of Least Action



Gottfried Wilhelm Leibniz
(was pretty good at math)

Assume you know the end points, find the path between them by finding a stationary point of the **ACTION**

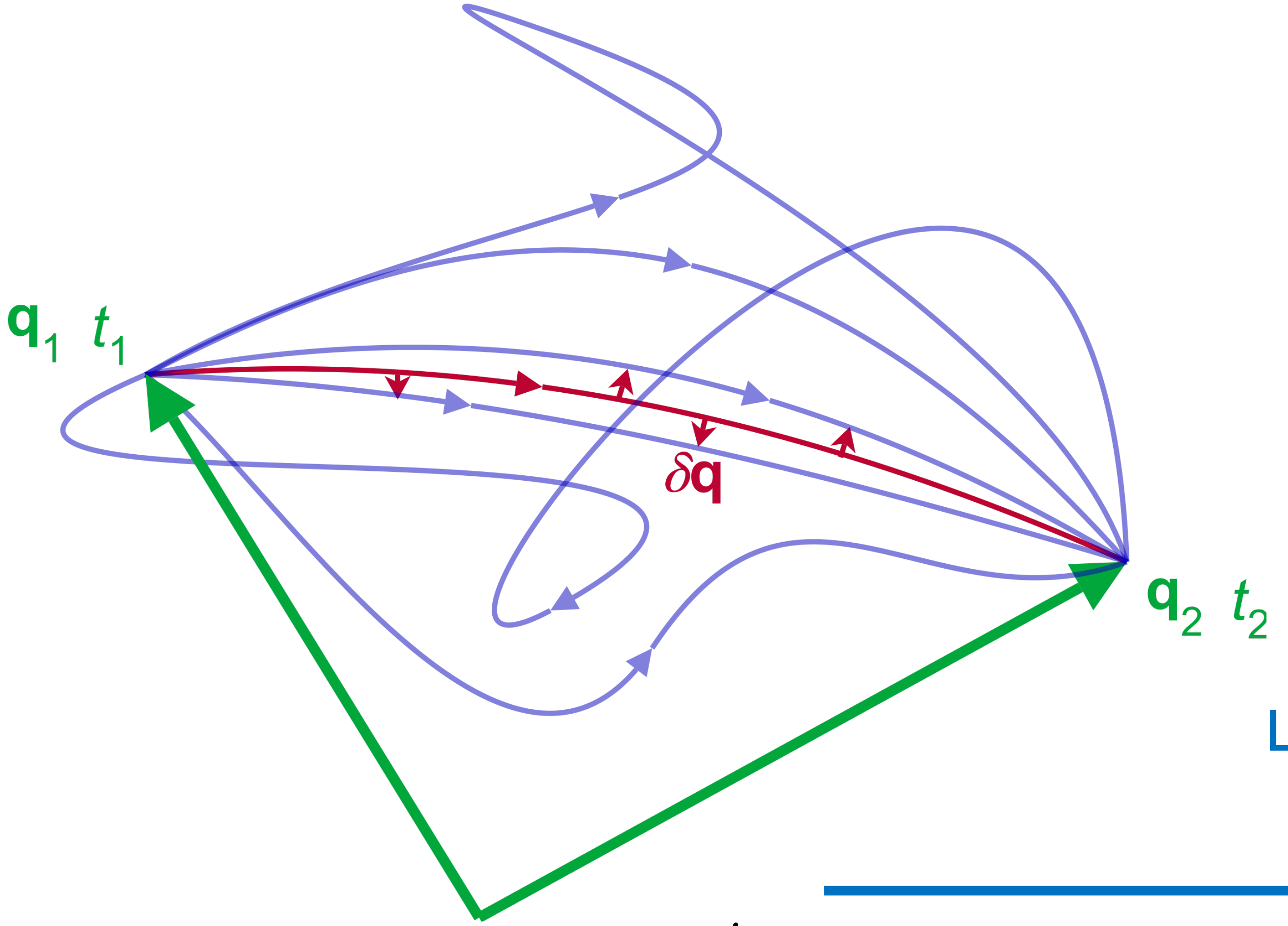
The Principle of Least Action



functional

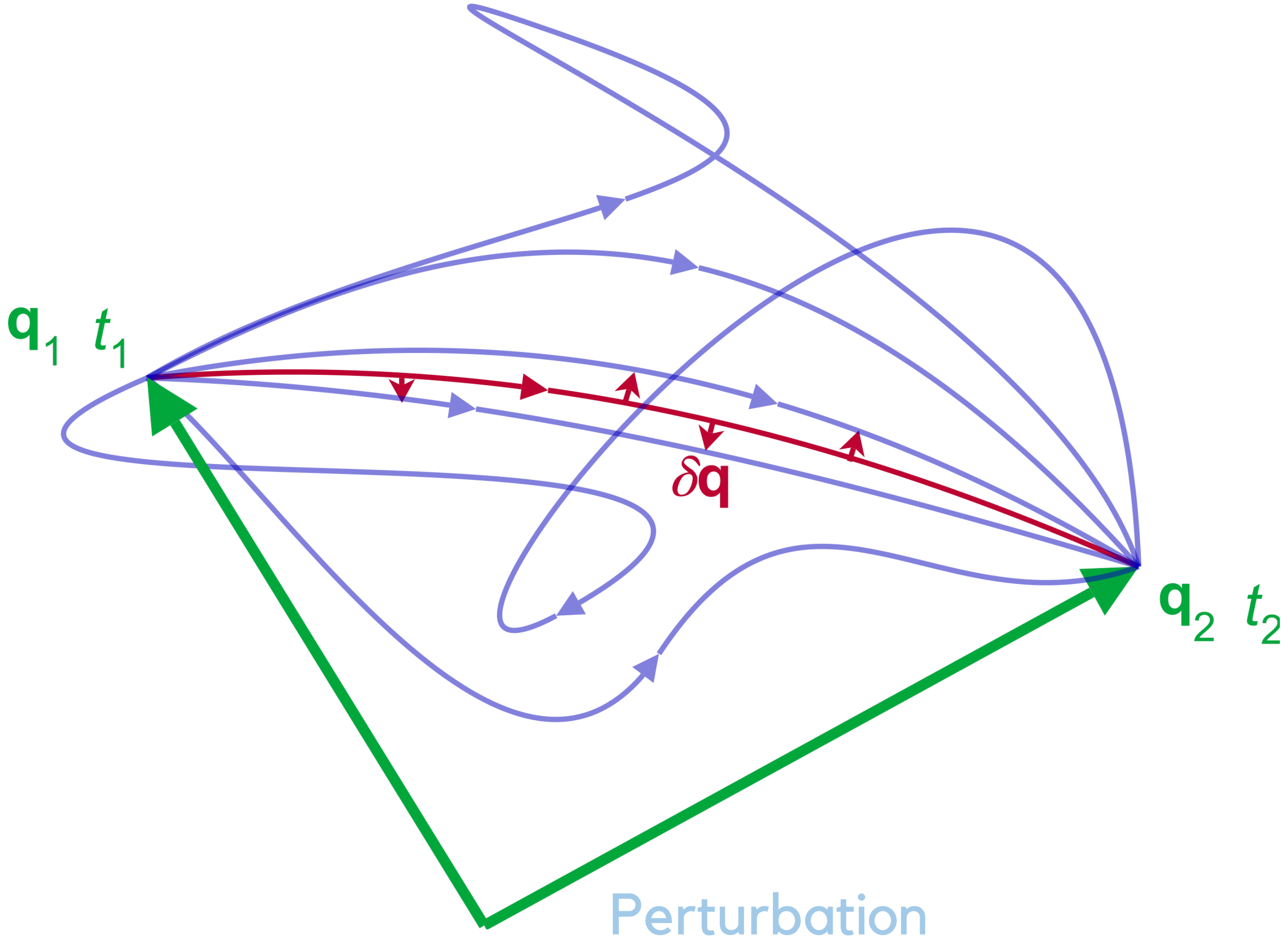
$$S\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right)=\int_{t_1}^{t_2}T\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right)-V\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right)dt$$

The Principle of Least Action



$$S\left(\mathbf{q}(t), \dot{\mathbf{q}}(t)\right) = \int_{t_1}^{t_2} \overbrace{T\left(\mathbf{q}(t), \dot{\mathbf{q}}(t)\right) - V\left(\mathbf{q}(t), \dot{\mathbf{q}}(t)\right)}^{\text{Lagrangian}} dt$$

The Principle of Least Action



Minimize by finding a flat spot

Hunt for a flat spot by perturbing the trajectory and seeing if S changes.

Perturbation

$$S\left(\mathbf{q}+\delta\mathbf{q},\dot{\mathbf{q}}+\delta\dot{\mathbf{q}}\right) = S\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right)$$

The Calculus of Variations

$$S\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right)=\int_{t_1}^{t_2}L\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right)dt$$

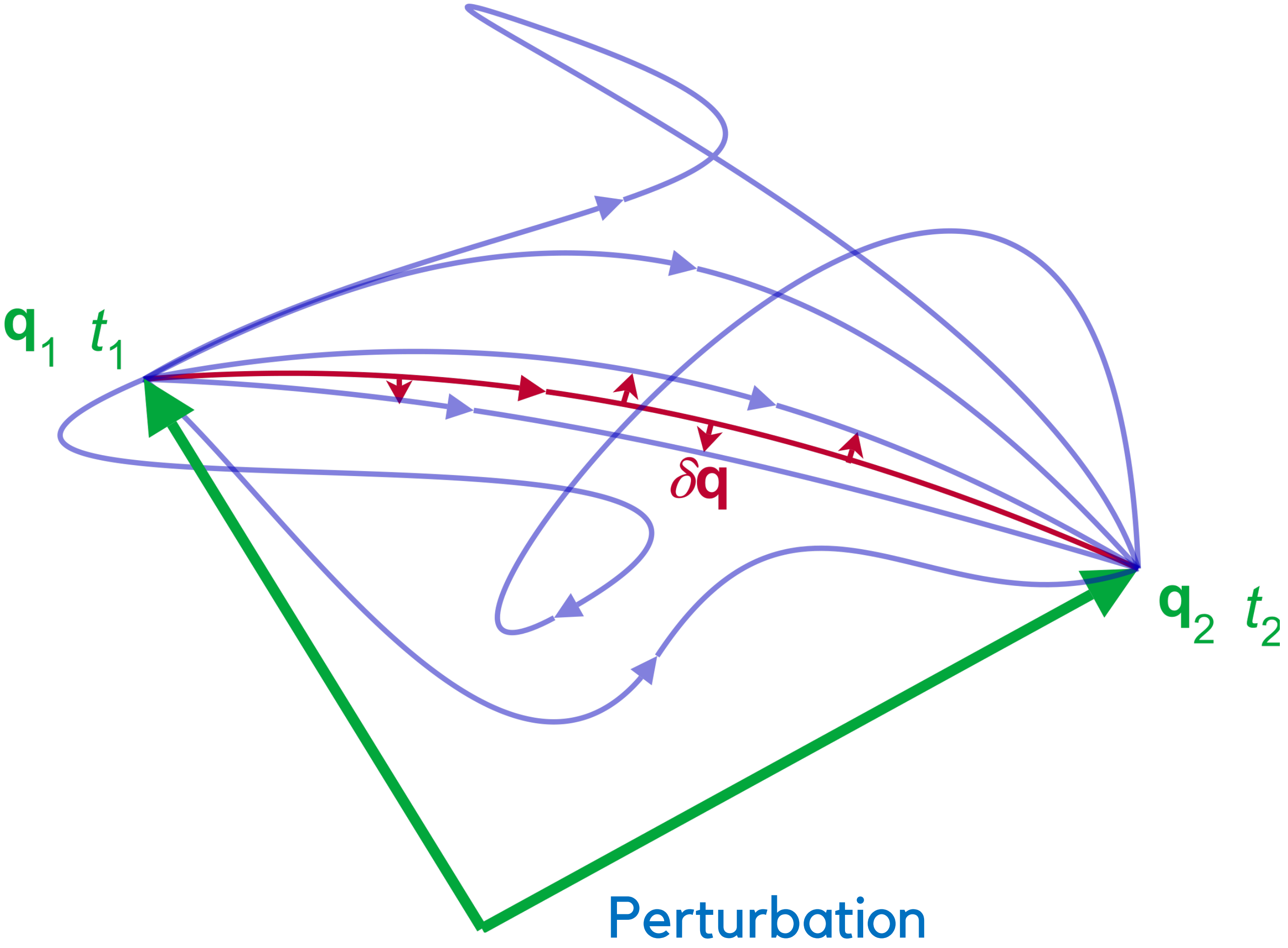
$$S\left(\mathbf{q}+\delta\mathbf{q},\dot{\mathbf{q}}+\delta\dot{\mathbf{q}}\right)=\int_{t_1}^{t_2}L\left(\mathbf{q}+\delta\mathbf{q},\dot{\mathbf{q}}+\delta\dot{\mathbf{q}}\right)dt$$

Apply Taylor Expansion

$$\approx\underbrace{\int_{t_1}^{t_2}L\left(\mathbf{q},\dot{\mathbf{q}}\right)dt}_{S\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right)}+\underbrace{\int_{t_1}^{t_2}\frac{\partial L}{\partial\mathbf{q}}\delta\mathbf{q}+\frac{\partial L}{\partial\dot{\mathbf{q}}}\delta\dot{\mathbf{q}}\,dt}_{\delta S\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right)}$$

First Variation

The Principle of Least Action

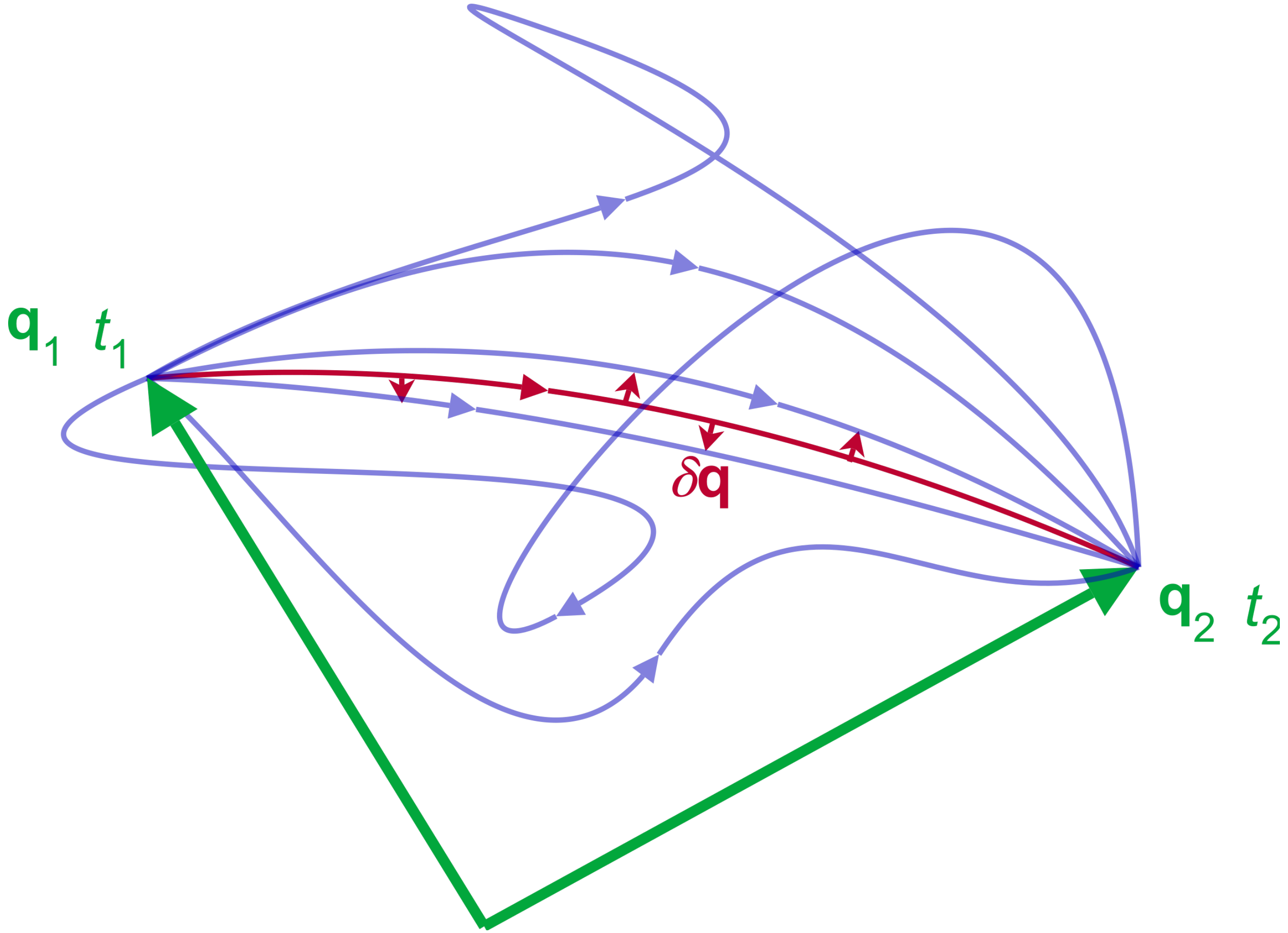


Minimize by finding a flat spot

Hunt for a flat spot by perturbing the trajectory and seeing if S changes.

$$S\left(\mathbf{q}+\delta\mathbf{q},\dot{\mathbf{q}}+\delta\dot{\mathbf{q}}\right) = S\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right)$$

The Principle of Least Action



Minimize by finding a flat spot

Hunt for a flat spot by perturbing the trajectory and seeing if S changes.

$$\delta S\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right) = 0$$

Back to the Calculus of Variations

$$\int_{t_1}^{t_2} \frac{\partial L}{\partial \mathbf{q}}\delta\mathbf{q} + \underbrace{\frac{\partial L}{\partial \dot{\mathbf{q}}}\delta\dot{\mathbf{q}}}_{\text{Apply Integration by Parts}}\, dt = 0$$

$$\int_{t_1}^{t_2} \frac{\partial L}{\partial \mathbf{q}}\delta\mathbf{q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}}\delta\mathbf{q}\, dt + \underbrace{\left.\frac{\partial L}{\partial \dot{\mathbf{q}}}\delta\mathbf{q}\right|_{t_0}^{t_1}}_{\text{Uh Oh, Boundary Conditions}} = 0$$

DON'T PANIC: Remember that **you know the end points**, so the variation there is 0.

Back to the Calculus of Variations

$$\int_{t_1}^{t_2} \frac{\partial L}{\partial \mathbf{q}}\delta\mathbf{q} + \underbrace{\frac{\partial L}{\partial \dot{\mathbf{q}}}\delta\dot{\mathbf{q}}}_{\text{Apply Integration by Parts}}\, dt = 0$$

$$\int_{t_1}^{t_2} \frac{\partial L}{\partial \mathbf{q}}\delta\mathbf{q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}}\delta\mathbf{q}\, dt + \underbrace{\cancel{\frac{\partial L}{\partial \dot{\mathbf{q}}}\delta\mathbf{q}\Big|_{t_1}^{t_2}}}_{\text{Uh Oh, Boundary Conditions}} = 0$$

DON'T PANIC: Remember that **you know the end points**, so the variation there is 0.

Back to the Calculus of Variations

$$\int_{t_1}^{t_2} \frac{\partial L}{\partial \mathbf{q}}\delta\mathbf{q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}}\delta\mathbf{q}\; dt = 0$$

A little bit o' factoring

$$\int_{t_1}^{t_2} \left(\frac{\partial L}{\partial \mathbf{q}} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}}\right)\delta\mathbf{q}\; dt = 0$$

Say the magic words -- "If δq is an arbitrary variation then the integrand must always be zero"

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}} = \frac{\partial L}{\partial \mathbf{q}}$$

Euler-Lagrange Equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}} = -\frac{\partial L}{\partial \mathbf{q}}$$

Why do we care ?

Unifying principle!

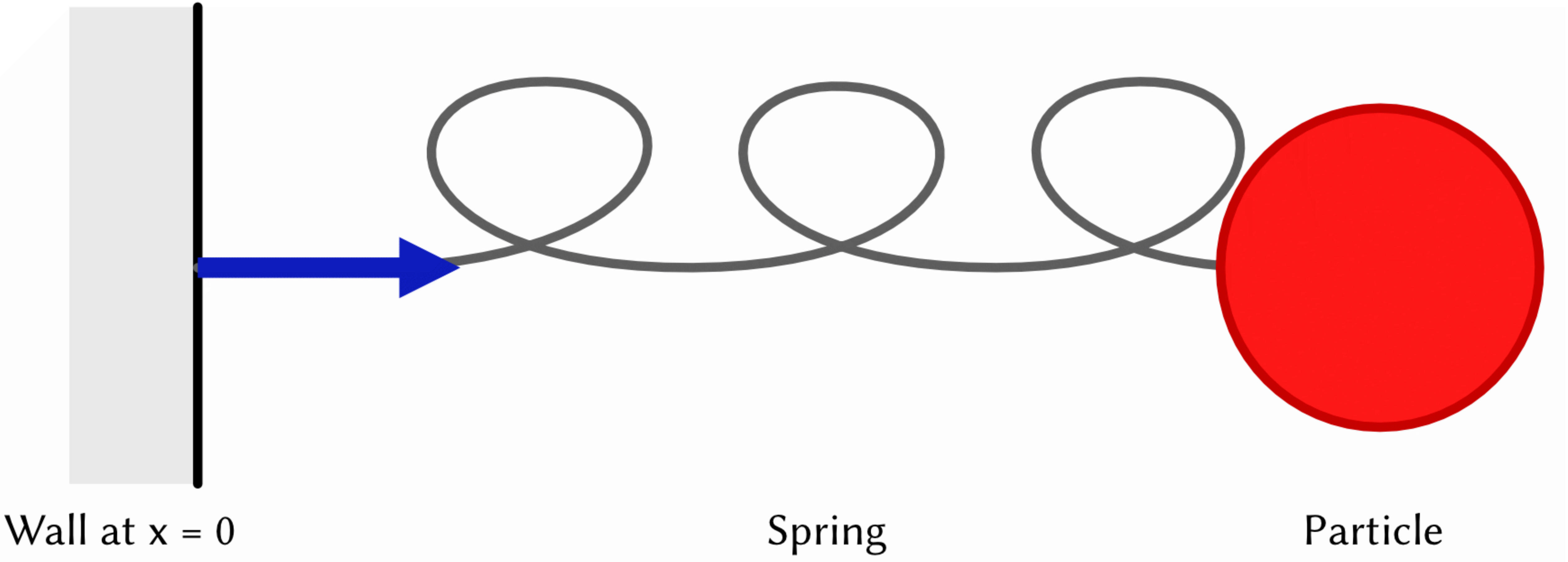
Can derive equations of motion for *more* than just particles
(*remember Rigid Body Drake*)

Deformable Objects

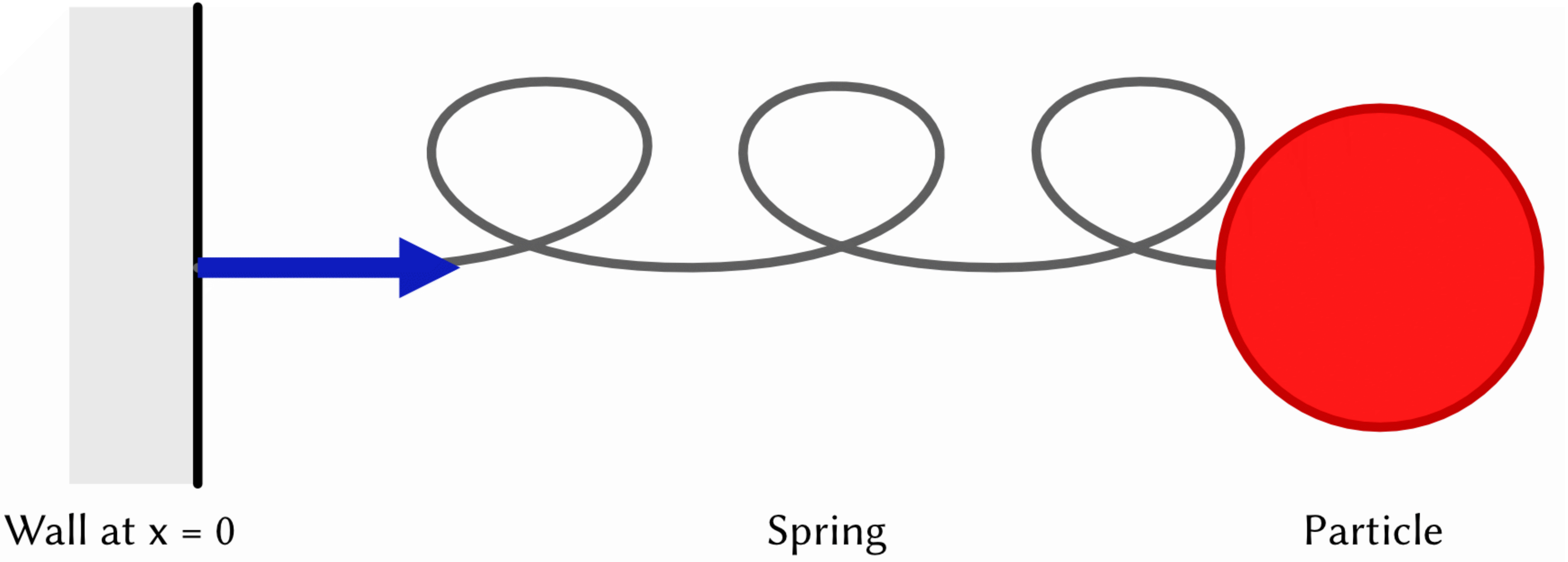
Fluids

Rigid Bodies and More !

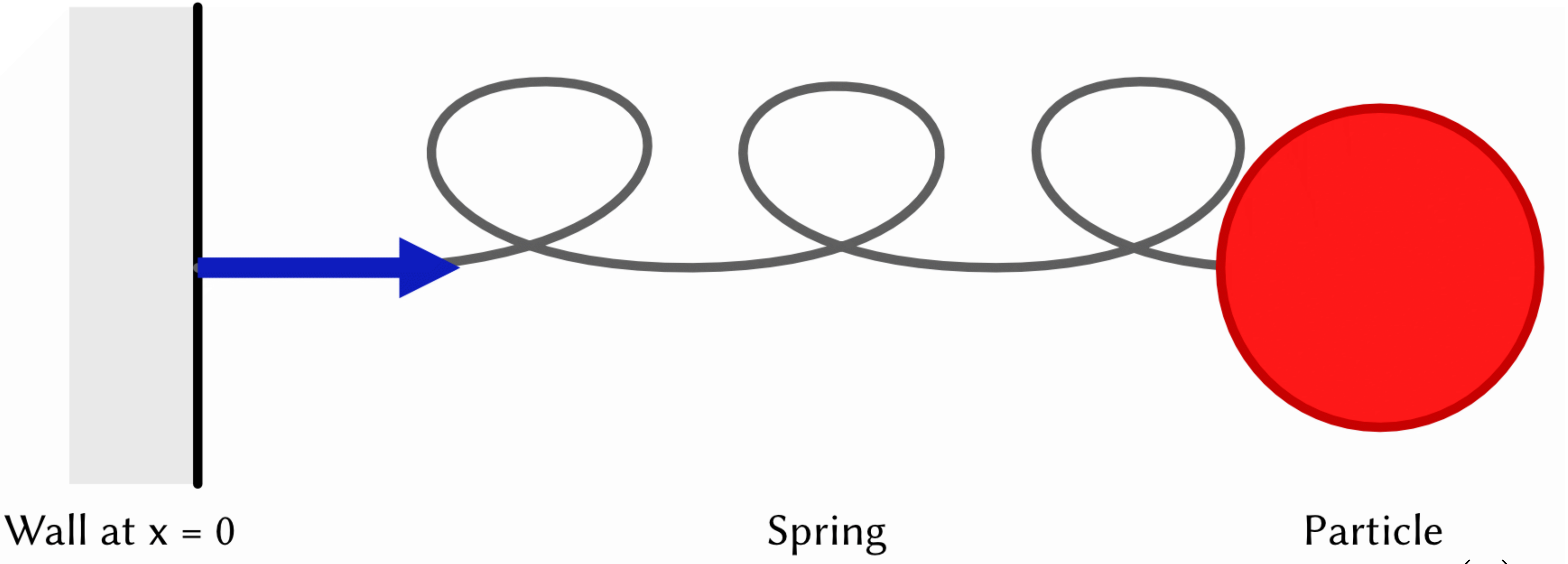
Mass-Spring Systems in 1D



Choosing Generalized Coordinates



Generalized Coordinates for Mass Spring System



$$q = x(t)$$
$$\dot{q} = v(t)$$

Kinetic Energy for Mass-Spring System

Kinetic Energy in 1D $$\frac{1}{2}mv^2$$

Done ☺

Potential Energy from a Spring

Hooke's Law – force is linearly proportional to stretch in spring

$$f = -kx$$

Potential energy is the negative of the mechanical work

$$W = \int \underbrace{-kx\left(t\right)}_{\text{force}} \overbrace{v\left(t\right)dt}^{\text{displacement}} = \int -kx\left(t\right)\ dx = -\frac{1}{2}kx^2$$

$$V = \frac{1}{2}kx^2 \quad \text{potential energy}$$

Stuff Everything into the Euler Lagrange Equations

$$L = \frac{1}{2}m\dot{q}^2 - \frac{1}{2}kq^2$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = \frac{d}{dt}(m\dot{q})$$

$$\frac{\partial L}{\partial q} = -kq$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}} = \frac{\partial L}{\partial \mathbf{q}}$$

$$\frac{d}{dt}(m\dot{q}) = -kq$$

$$\underbrace{m\ddot{q} = -kq}_{\text{equations of motion}}$$

Next Video

$$m\ddot{q} = -kq$$

How do we solve this ?