

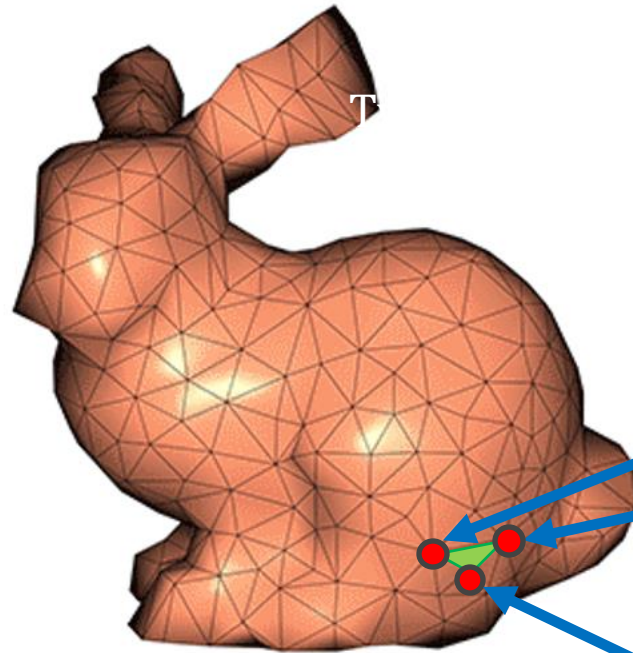
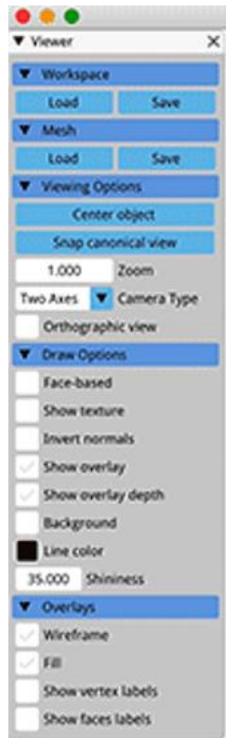


CSC417 Physics-Based Animation

36M verts, 124M tets
avg frame time: 7.2s
max: 7.8s

Vertex Block Descent|Chen et al

Last Week: Reduced-Order Methods



$$\begin{pmatrix} \mathbf{x}_0 \\ \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_n \end{pmatrix} : \mathbf{q}(0) + \mathbf{U}\mathbf{r}$$

|
Generalized
Coordinates

Questions from Previous Lecture ?

This Week: Fast Simulation via Coordinate Descent



48M verts, 151M tets
avg frame time: 14.4s, max: 15.6s

We Solve This Every Time Step

$$E(\mathbf{q}^{i+1}) = \frac{1}{2} (\mathbf{q}^{i+1} - \tilde{\mathbf{q}}^i)^T M (\mathbf{q}^{i+1} - \tilde{\mathbf{q}}^i) + h^2 V(\mathbf{q}^{i+1})$$

Gradient of what equals this ? Let's guess, then check

Newton's Method

Choose an initial guess

$$i = 0$$

$$\mathbf{v}^0 = \text{something}$$

Check for convergence

$$\left\| \frac{\partial E}{\partial \mathbf{v}} \Big|_{\mathbf{v}^i} \right\| < \text{tol}$$

Choose search direction

$$\mathbf{H}^i \mathbf{d} = -\mathbf{g}^i \quad \text{Solve linear system to get } \mathbf{d}$$

Choose α using line search

Use search direction to update current guess

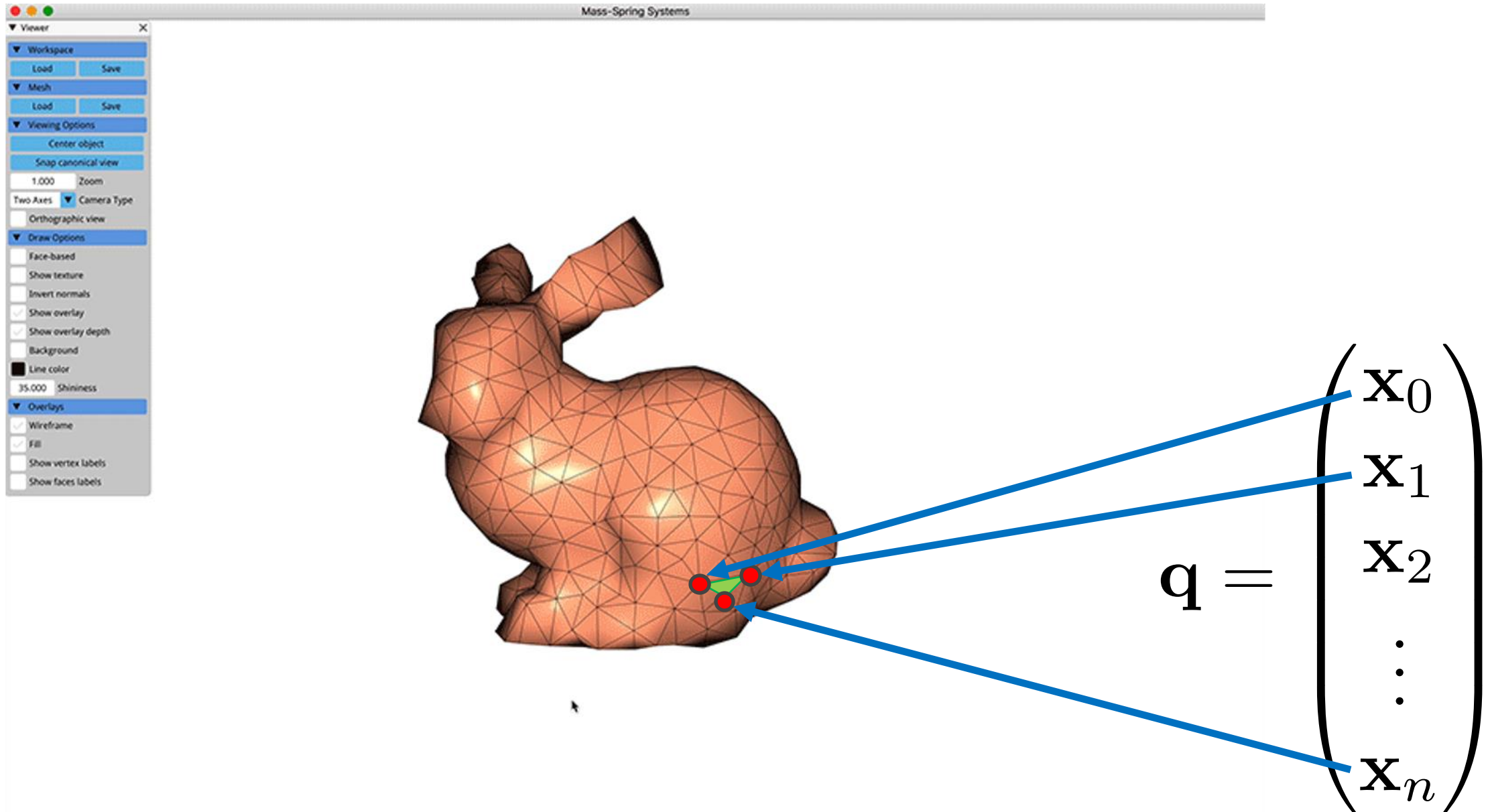
$$\mathbf{v}^{i+1} = \mathbf{v}^i + \alpha \mathbf{d}$$

$$i = i + 1$$

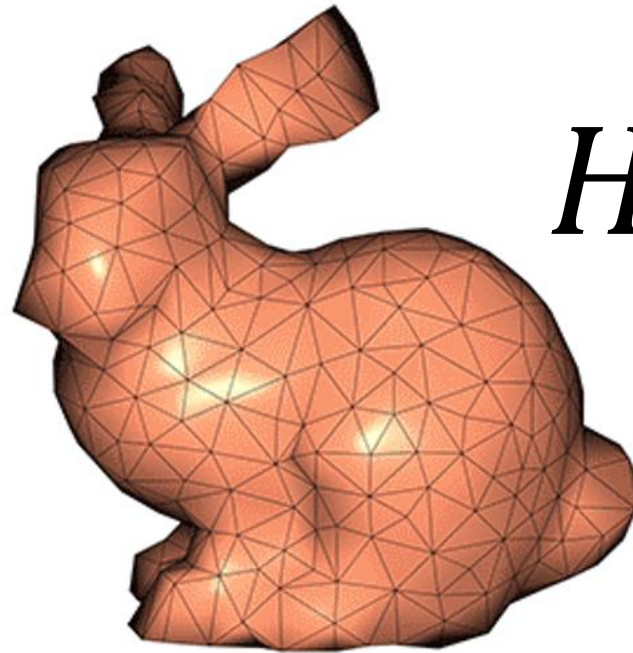
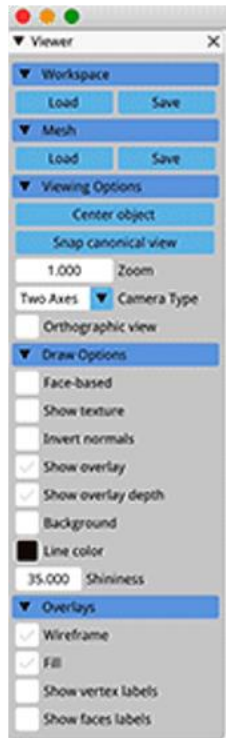
Repeat until converged



Spatial Discretization -- Finite Elements



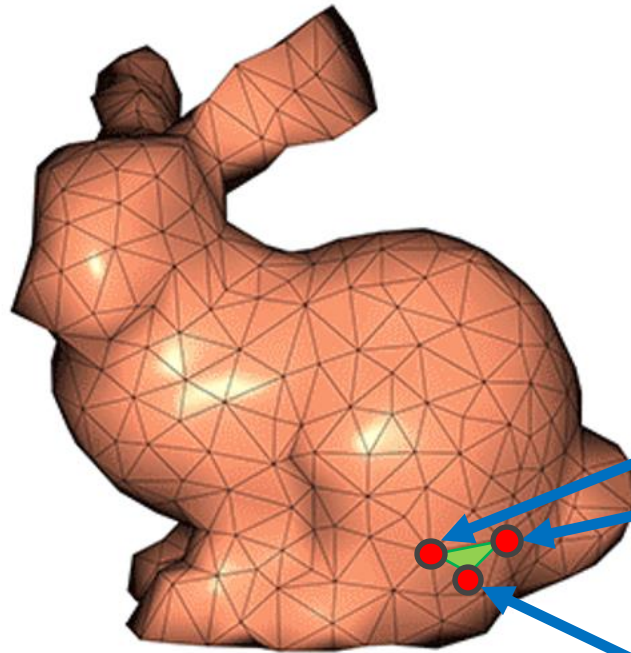
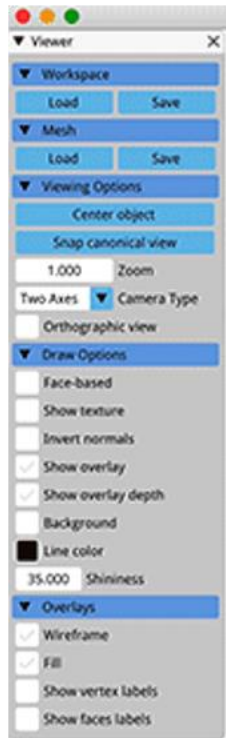
Assembly still visits every element ☹



$$H = \sum_t E_i^T H_i E_i$$

Per-tetrahedron Hessian

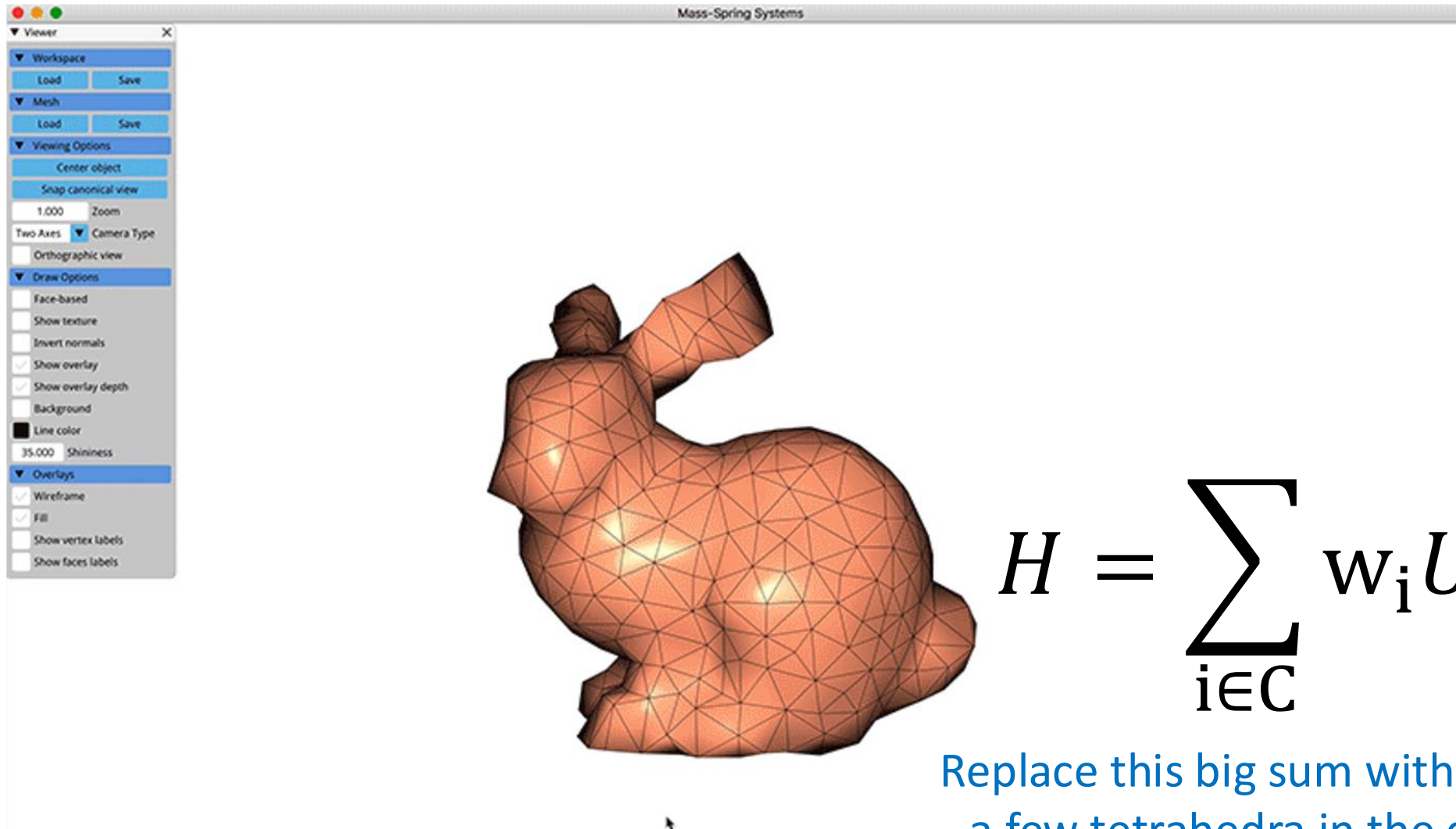
Last Week: Reduced-Order Methods



$$\begin{pmatrix} \mathbf{x}_0 \\ \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_n \end{pmatrix} : \mathbf{q}(0) + \mathbf{U}\mathbf{r}$$

|
Generalized
Coordinates

Optimal Quadrature



Another way to go fast

1. Avoid reducing the simulation space
2. Split up problem in a way that allows us to exploit parallelism and avoid building large matrices

Re-thinking Energy Minimization

$$E(\mathbf{q}^{i+1}) = \underbrace{\frac{1}{2} (\mathbf{q}^{i+1} - \tilde{\mathbf{q}}^i)^T M (\mathbf{q}^{i+1} - \tilde{\mathbf{q}}^i)}_{\text{Global "Inertial" Energy}} + \underbrace{h^2 V(\mathbf{q}^{i+1})}_{\text{Global Potential Energy}}$$

Newton's Method

Choose an initial guess

$$i = 0$$

$$\mathbf{v}^0 = \text{something}$$

Check for convergence

$$\left\| \frac{\partial E}{\partial \mathbf{v}} \Big|_{\mathbf{v}^i} \right\| < \text{tol}$$

Choose search direction

$$\mathbf{H}^i \mathbf{d} = -\mathbf{g}^i$$

Large Global Sparse System

Choose α using line search

Use search direction to update current guess

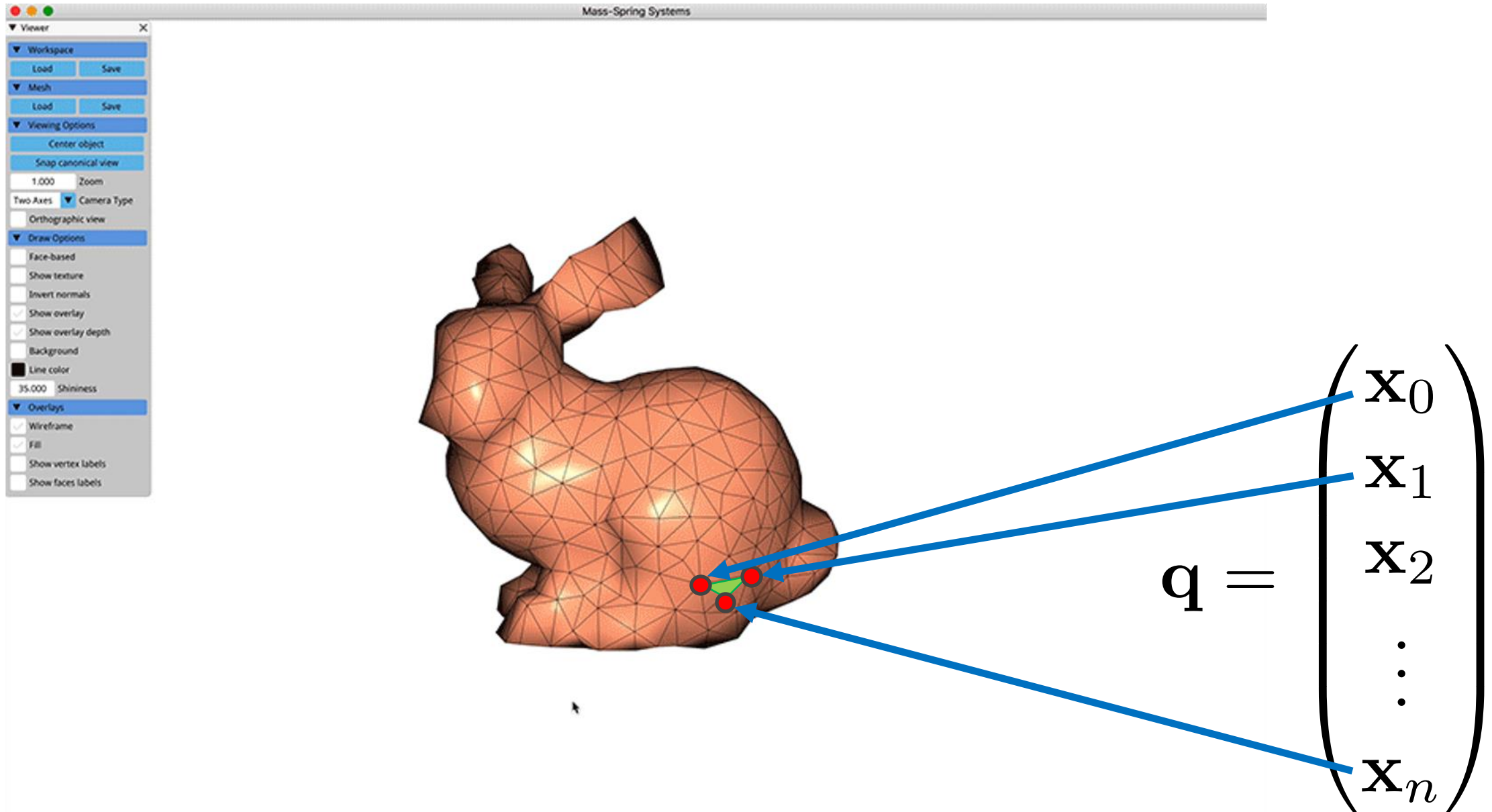
$$\mathbf{v}^{i+1} = \mathbf{v}^i + \alpha \mathbf{d}$$

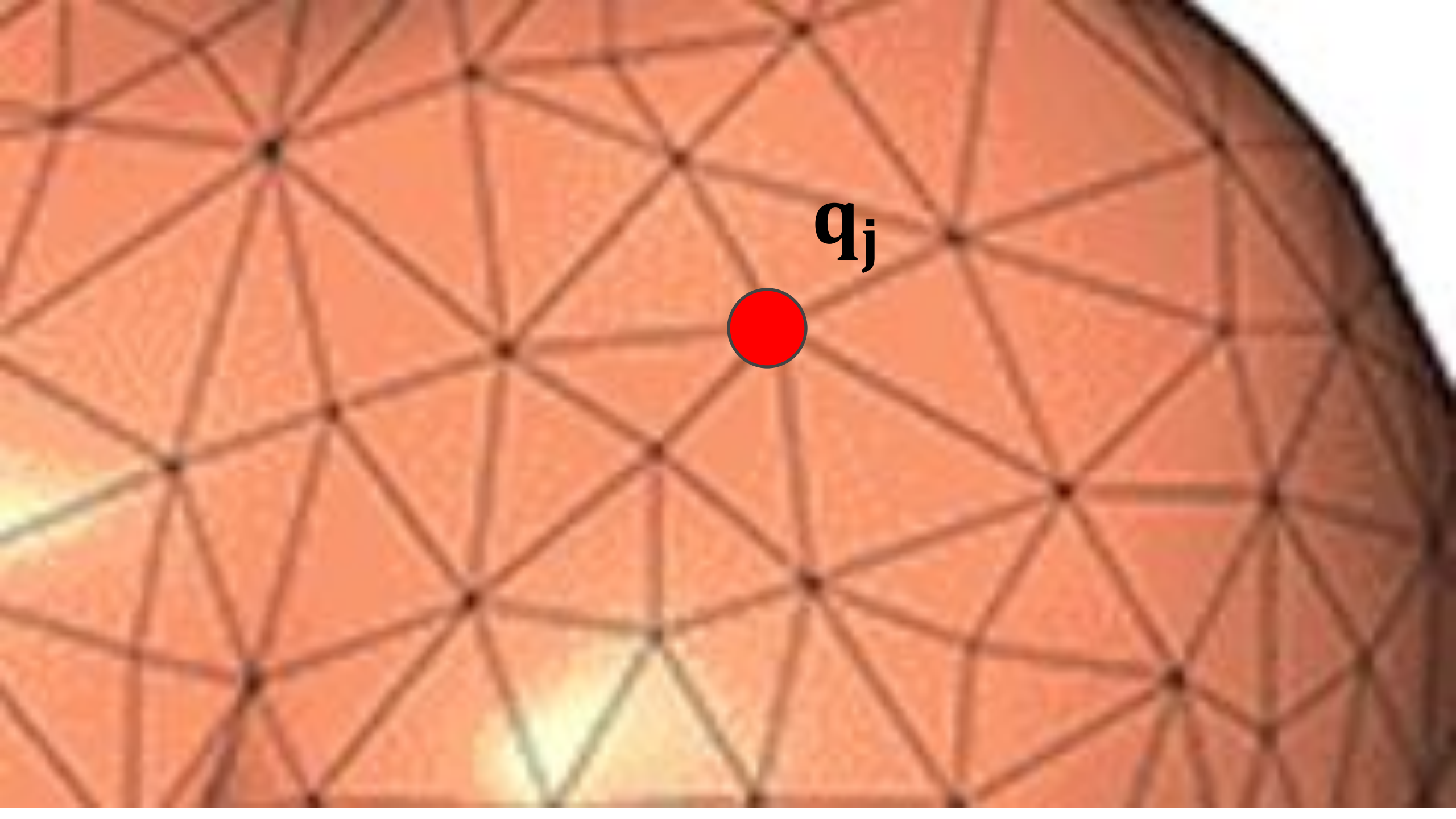
$$i = i + 1$$

Repeat until converged



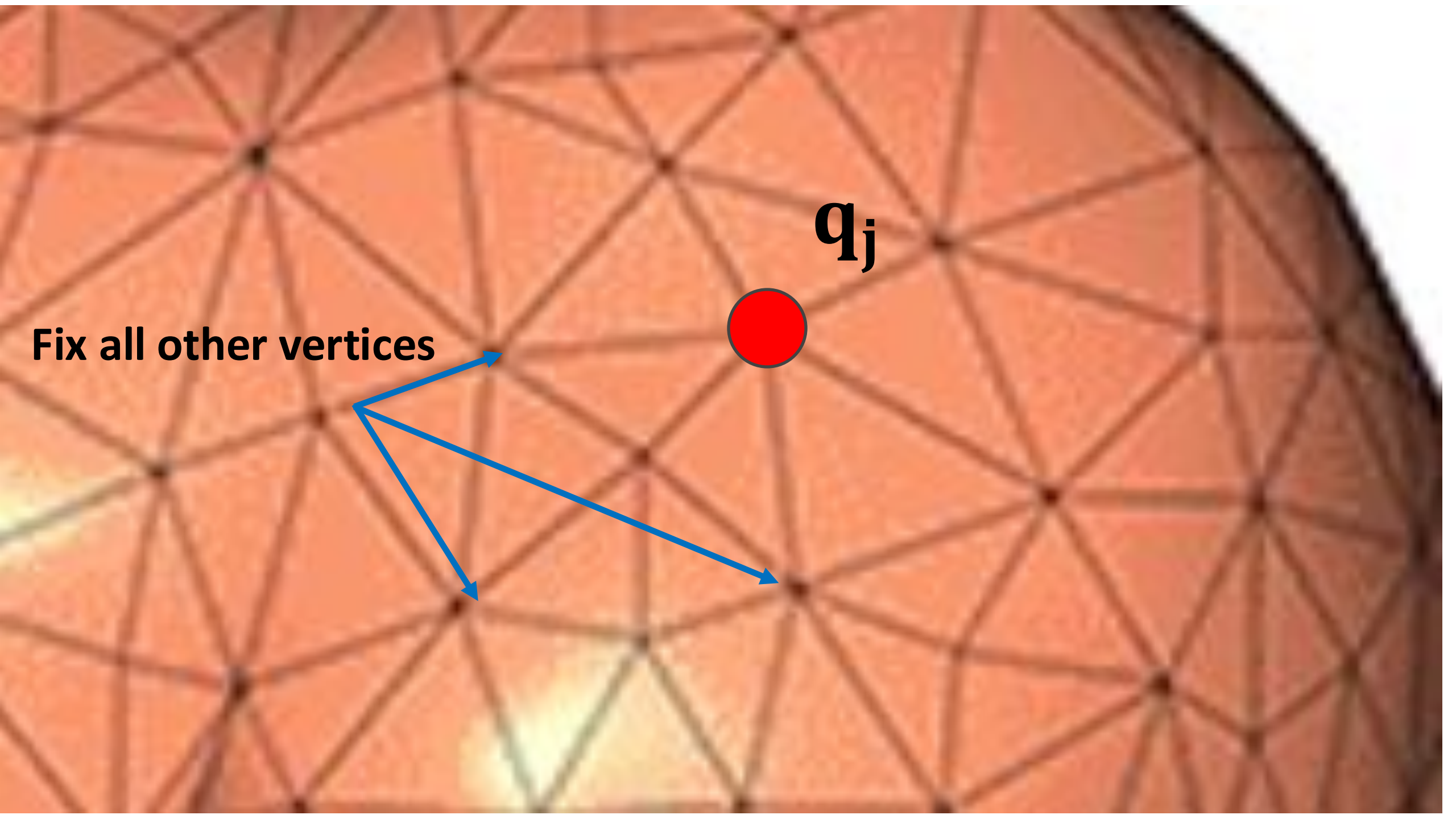
Coordinate Descent





What happens if we just minimize wrt \mathbf{q}_j ?

$$E(\mathbf{q}^{i+1}) = \underbrace{\frac{1}{2} (\mathbf{q}^{i+1} - \tilde{\mathbf{q}}^i)^T M (\mathbf{q}^{i+1} - \tilde{\mathbf{q}}^i)}_{\text{Global "Inertial" Energy}} + \underbrace{h^2 V(\mathbf{q}^{i+1})}_{\text{Global Potential Energy}}$$



q_j

Fix all other vertices

What happens if we just minimize wrt \mathbf{q}_j ?

$$E(\mathbf{q}^{i+1}) = \underbrace{\frac{1}{2} (\mathbf{q}^{i+1} - \tilde{\mathbf{q}}^i)^T M (\mathbf{q}^{i+1} - \tilde{\mathbf{q}}^i)}_{\text{Global "Inertial" Energy}} + \underbrace{h^2 V(\mathbf{q}^{i+1})}_{\text{Global Potential Energy}}$$

Inertial Energy Structure

$$\frac{1}{2} (\mathbf{q}_0^{i+1} - \tilde{\mathbf{q}}_0^i, \mathbf{q}_1^{i+1} - \tilde{\mathbf{q}}_1^i, \dots)$$

	Vertex 0	Vertex 1	Vertex 2
Vertex 0	M_{00}	M_{01}	M_{02}
Vertex 1	...	M_{11}	M_{12}
Vertex 2	M_{22}

$$\begin{pmatrix} \mathbf{q}_0^{i+1} - \tilde{\mathbf{q}}_0^i \\ \mathbf{q}_1^{i+1} - \tilde{\mathbf{q}}_1^i \\ \dots \end{pmatrix}$$

Inertial Energy Structure

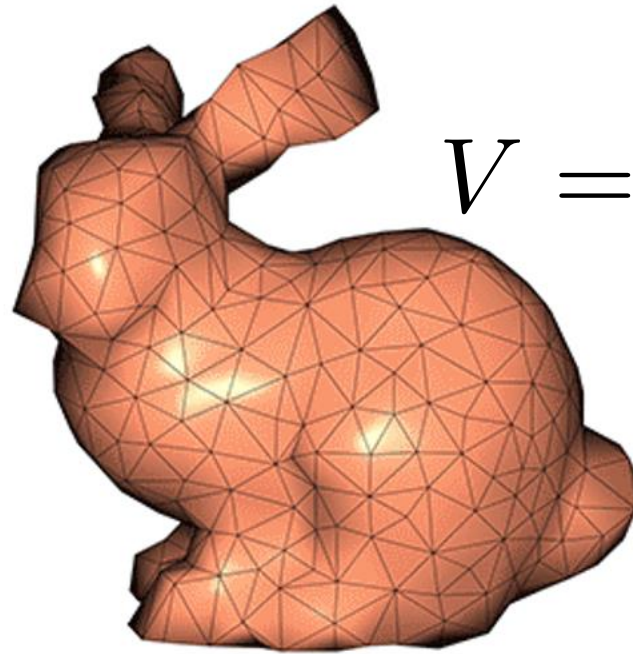
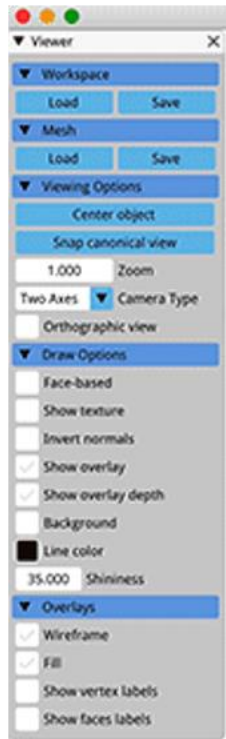
$$\frac{1}{2} (\mathbf{q}_0^{i+1} - \tilde{\mathbf{q}}_0^i)^T M_{00} (\mathbf{q}_0^{i+1} - \tilde{\mathbf{q}}_0^i)$$

$$+ (\mathbf{q}_1^{i+1} - \tilde{\mathbf{q}}_1^i)^T M_{01} (\mathbf{q}_0^{i+1} - \tilde{\mathbf{q}}_0^i)$$

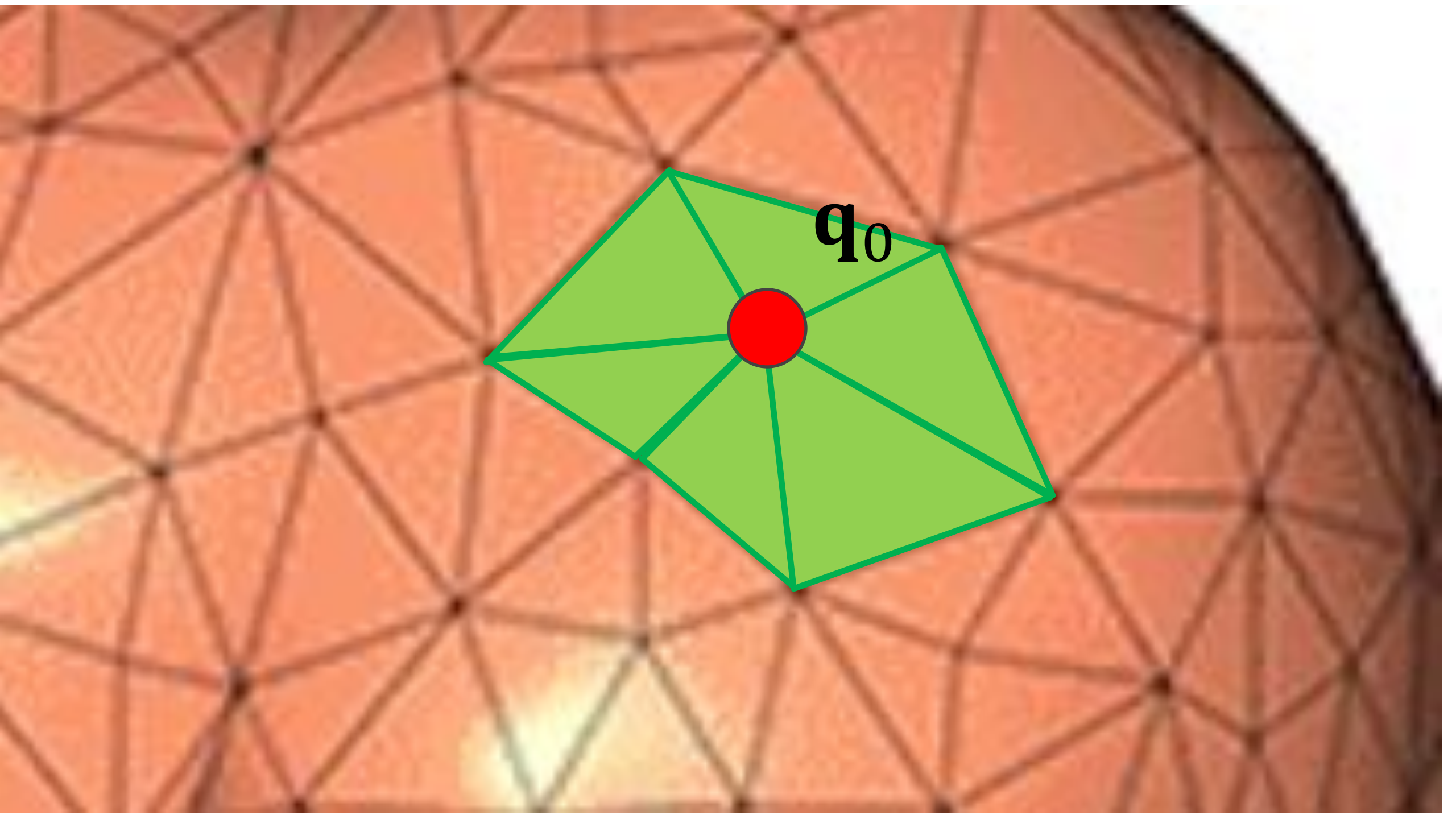
$$+ (\mathbf{q}_2^{i+1} - \tilde{\mathbf{q}}_2^i)^T M_{02} (\mathbf{q}_0^{i+1} - \tilde{\mathbf{q}}_0^i)$$

+ Bunch of terms that don't depend on \mathbf{q}_0

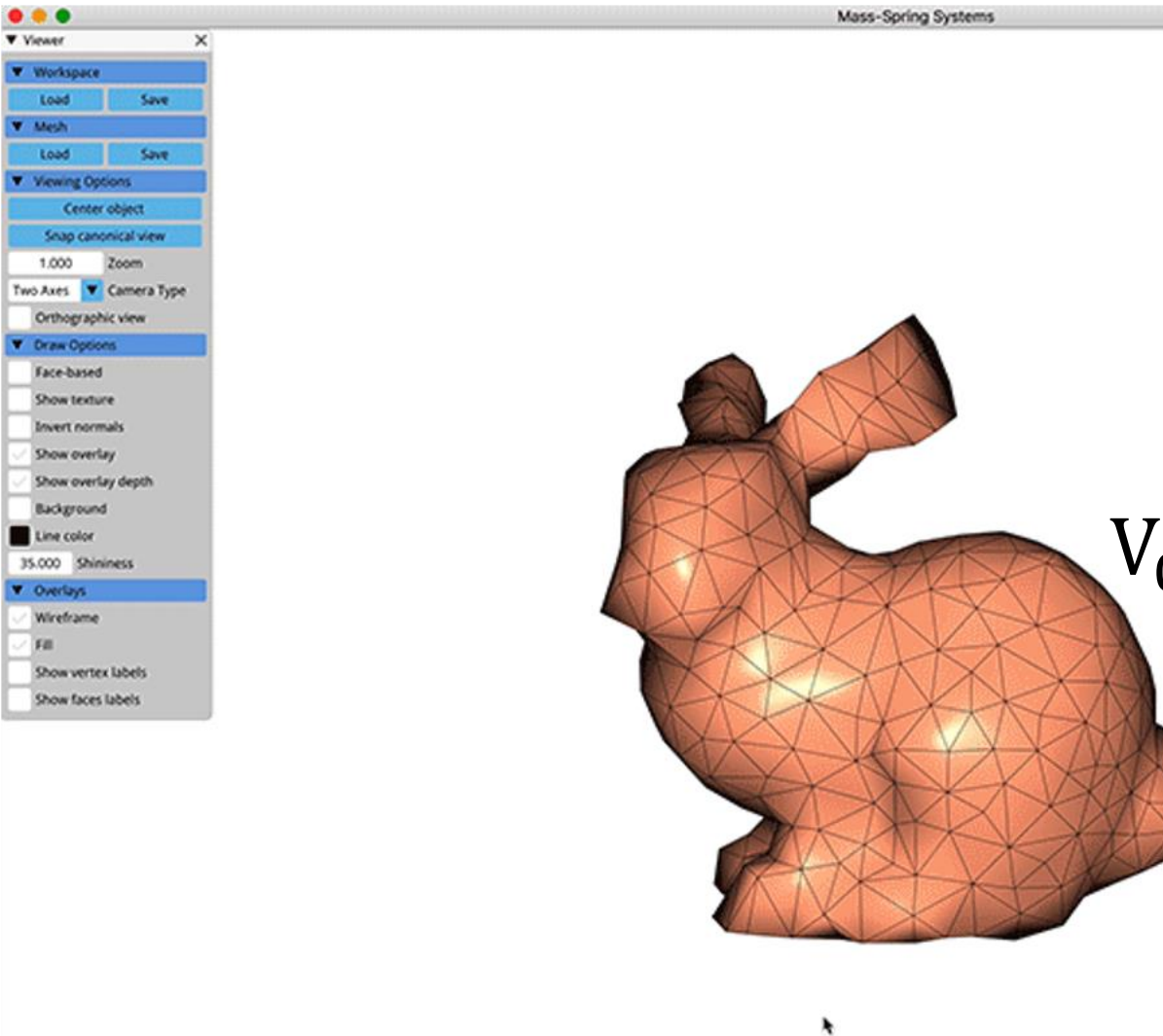
Potential Energy Structure (Way back in Lectures 2 and 3)



$$V = \sum_{j=0}^{m-1} \text{vol}_j \cdot \psi \left(F_j \left(E_j \mathbf{q} \right) \right)$$



Potential Energy Structure (Way back in Lectures 2 and 3)



$$V_0(\mathbf{q}_0^{\mathbf{i}+1}) = \sum_{\mathbf{i} \in \mathcal{N}} \underset{\text{Tetrahedra containing } \mathbf{q}_0}{\text{vol}_i} \psi(\underset{\text{Other vertices of the tetrahedron}}{F_i(\mathbf{q}_0^{\mathbf{i}+1}, \dots)})$$

For a single vertex

$$E(\mathbf{q}_j^{i+1}) = \frac{1}{2} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i)^T M_{jj} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + \mathbf{p}_j^T (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + h^2 V_j(\mathbf{q}_j^{i+1})$$



Momentum from other degrees-of-freedom

Inertial Energy Structure

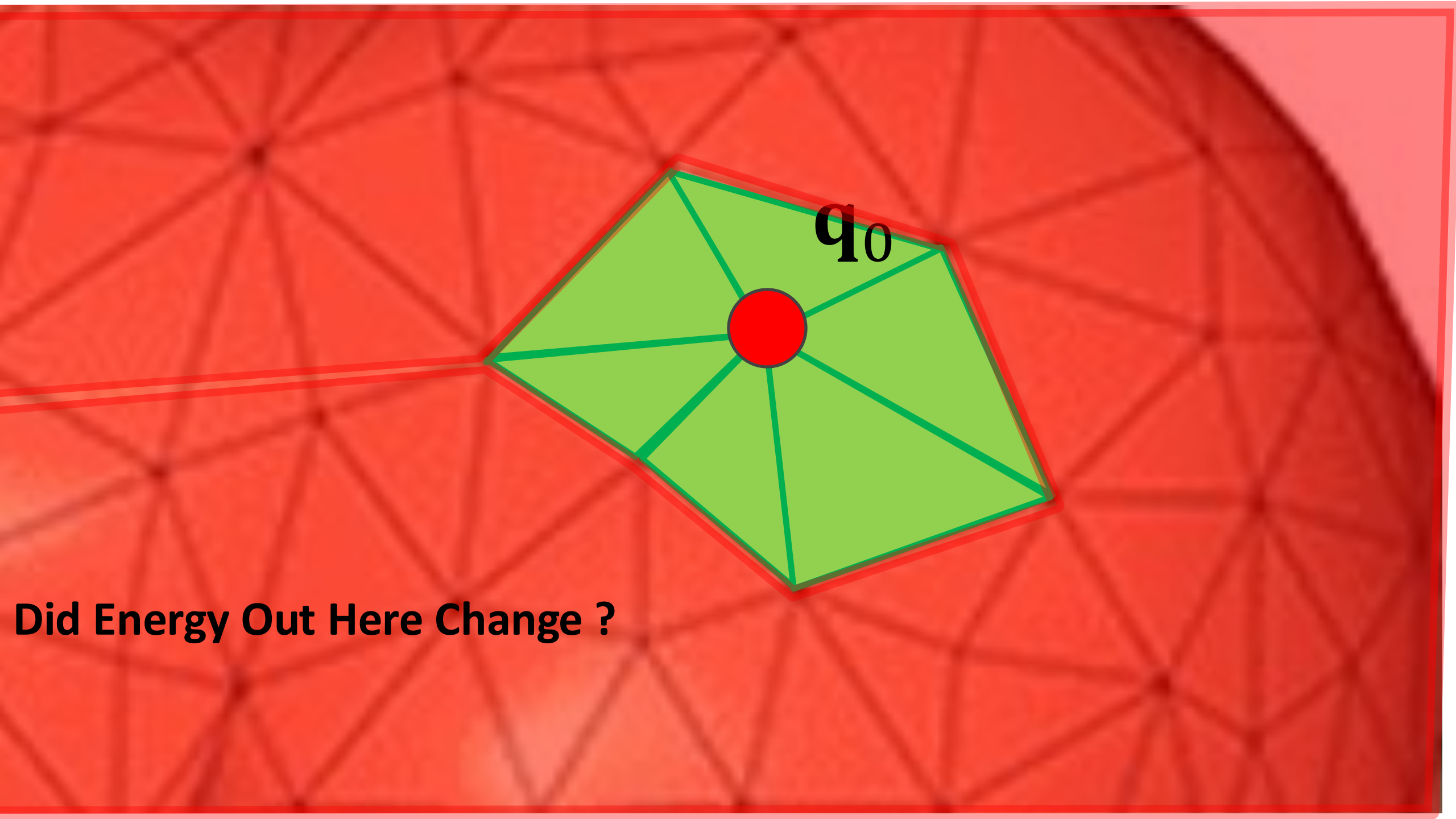
$$\mathbf{p}_0 = \begin{matrix} \text{Vertex 0} \end{matrix} \left[\begin{matrix} \text{Vertex 1} & \text{Vertex 2} \\ M_{01} & M_{02} \end{matrix} \right] \begin{pmatrix} \mathbf{q}_1^{i+1} - \tilde{\mathbf{q}}_1^i \\ \mathbf{q}_2^{i+1} - \tilde{\mathbf{q}}_2^i \\ \dots \end{pmatrix}$$

For a single vertex

$$E(\mathbf{q}_j^{i+1}) = \frac{1}{2} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i)^T M_{jj} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + \mathbf{p}_j^T (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + h^2 V_j(\mathbf{q}_j^{i+1})$$

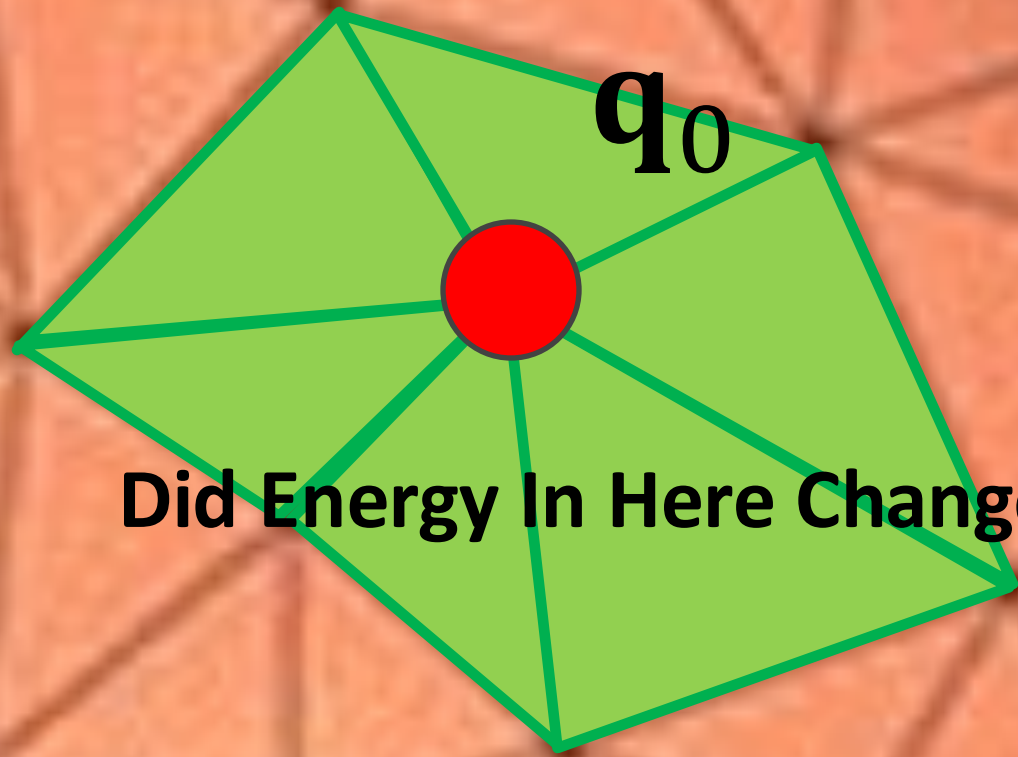
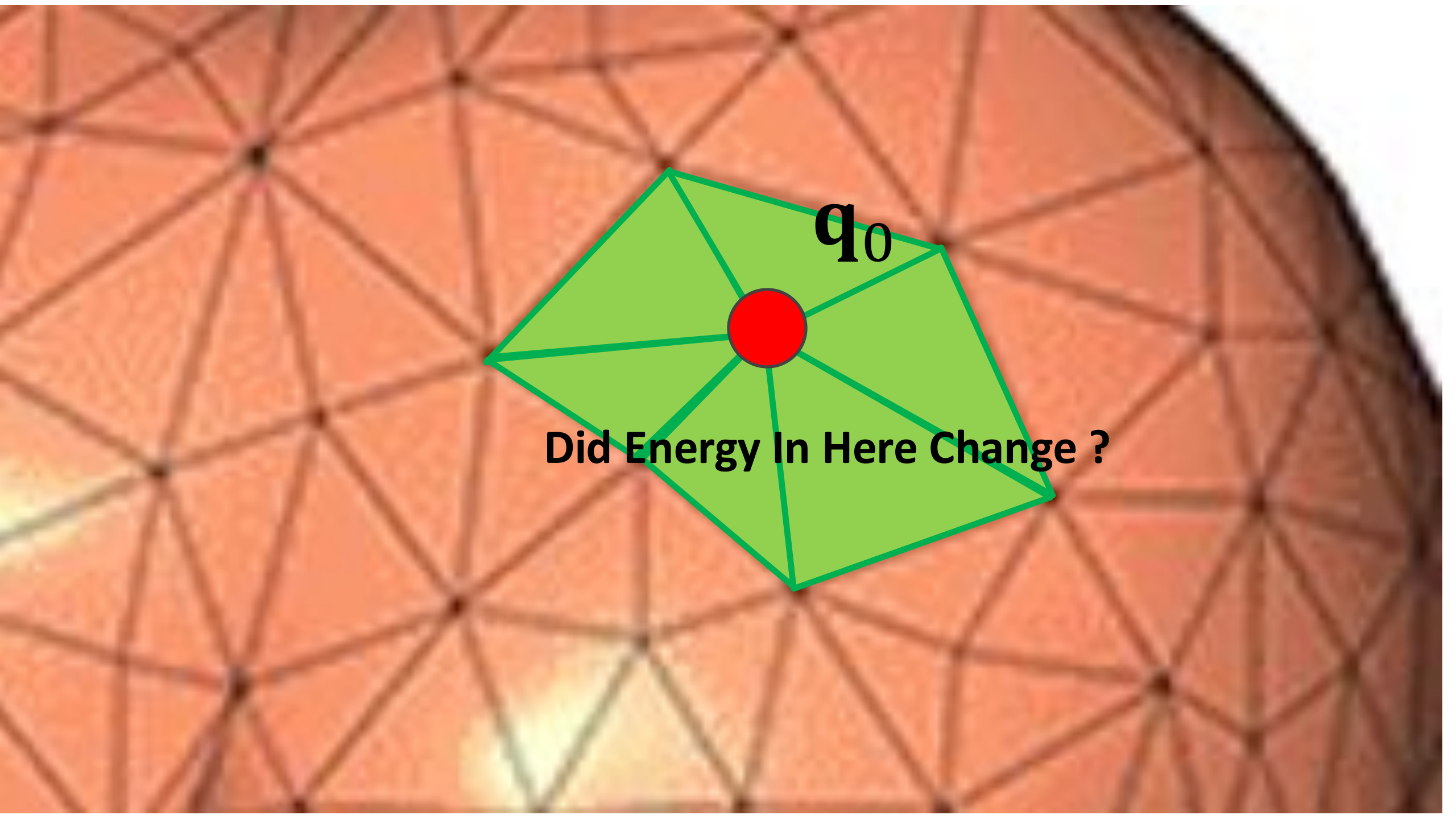


Momentum from other degrees-of-freedom



q_0

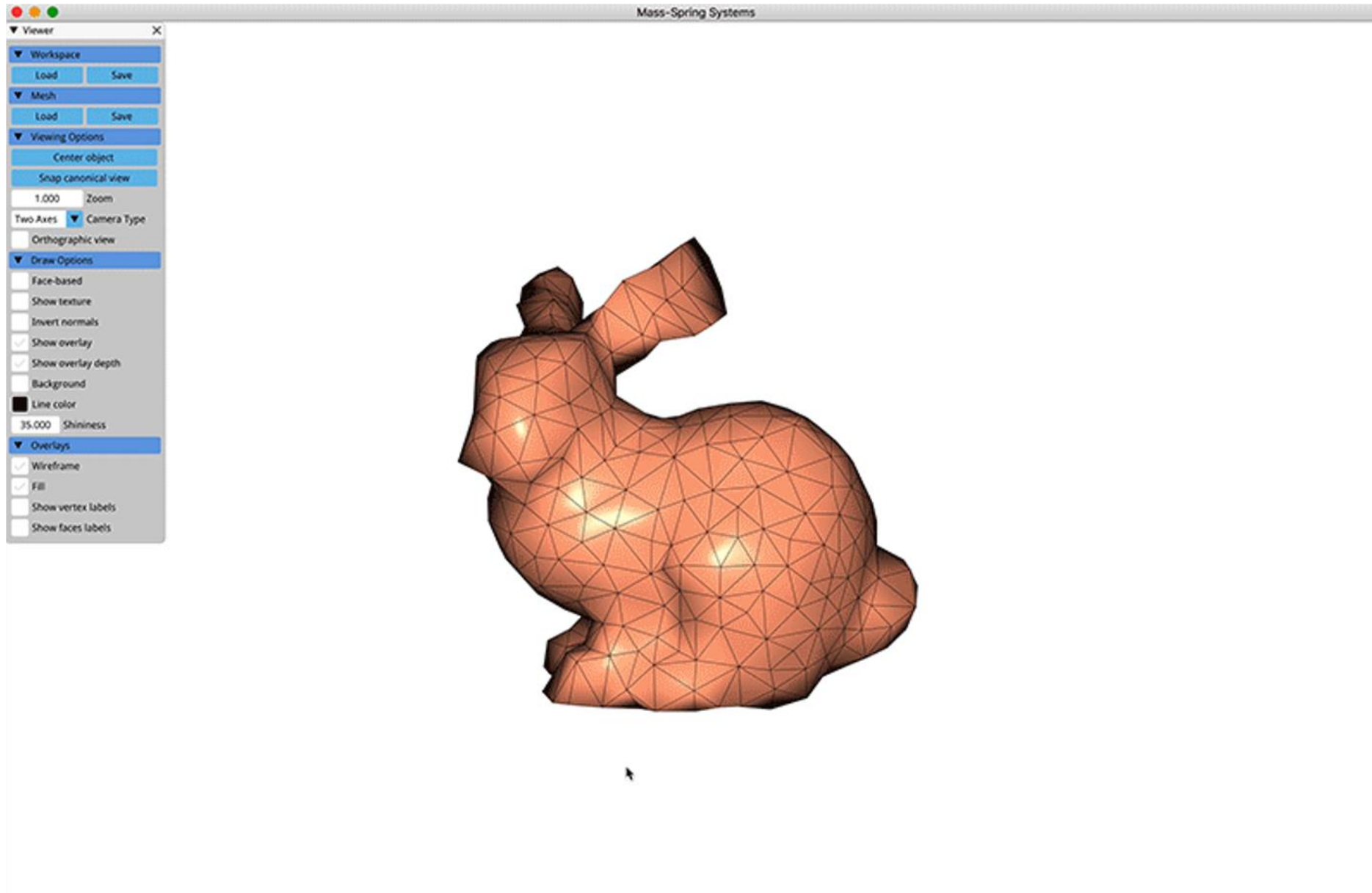
Did Energy Out Here Change ?



q_0

Did Energy In Here Change ?

What happened to total energy then ?



So this suggests a new minimization strategy

Repeat for awhile

For each vertex, j

Minimize

$$E(\mathbf{q}_j^{i+1}) = \frac{1}{2} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i)^T M_{jj} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + \mathbf{p}_j^T (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + h^2 V_j(\mathbf{q}_j^{i+1})$$

Why is this fast ?

Repeat for awhile

For each vertex, j

Minimize

$$E(\mathbf{q}_j^{i+1}) = \frac{1}{2} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i)^T M_{jj} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + \mathbf{p}_j^T (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + h^2 V_j(\mathbf{q}_j^{i+1})$$


Why is this fast ?

Why is this fast ?

Repeat for awhile

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Minimize

$$E(\mathbf{q}_j^{i+1}) = \frac{1}{2} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i)^T M_{jj} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + \mathbf{p}_j^T (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + h^2 V_j(\mathbf{q}_j^{i+1})$$


Fast to solve using Newton's Method

Newton's Method

Choose an initial guess

$$i = 0$$

$$\mathbf{v}^0 = \text{something}$$

Check for convergence

$$\left\| \frac{\partial E}{\partial \mathbf{v}} \Big|_{\mathbf{v}^i} \right\| < \text{tol}$$

Choose search direction

$$\mathbf{H}^i \mathbf{d} = -\mathbf{g}^i$$

Here's our main cost

Choose α using line search

Use search direction to update current guess

$$\mathbf{v}^{i+1} = \mathbf{v}^i + \alpha \mathbf{d}$$

$$i = i + 1$$

Repeat until converged

So this suggests a new minimization strategy

Repeat for awhile

For each vertex, j

Minimize

$$E(\mathbf{q}_j^{i+1}) = \frac{1}{2} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i)^T M_{jj} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + \mathbf{p}_j^T (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + h^2 V_j(\mathbf{q}_j^{i+1})$$

$$\mathbf{q}_j^{i+1} \in \mathcal{R}^3$$

$$\mathbf{g} = \frac{\partial E_j}{\partial \mathbf{q}_j^{i+1}} \in \mathcal{R}^3$$

$$\mathbf{H} = \frac{\partial^2 E_j}{\partial \mathbf{q}_j^{i+1} \partial \mathbf{q}_j^{i+1}} \in \mathcal{R}^{3 \times 3}$$

Newton's Method

Choose an initial guess

$$i = 0$$

$$\mathbf{v}^0 = \text{something}$$

Check for convergence

$$\left\| \frac{\partial E}{\partial \mathbf{v}} \Big|_{\mathbf{v}^i} \right\| < \text{tol}$$

Choose search direction

$$\mathbf{H}^i \mathbf{d} = -\mathbf{g}^i$$

Small, dense 3x3 linear system

Choose α using line search

Use search direction to update current guess

$$\mathbf{v}^{i+1} = \mathbf{v}^i + \alpha \mathbf{d}$$

$$i = i + 1$$

Repeat until converged

Why is this fast ?

Repeat for awhile

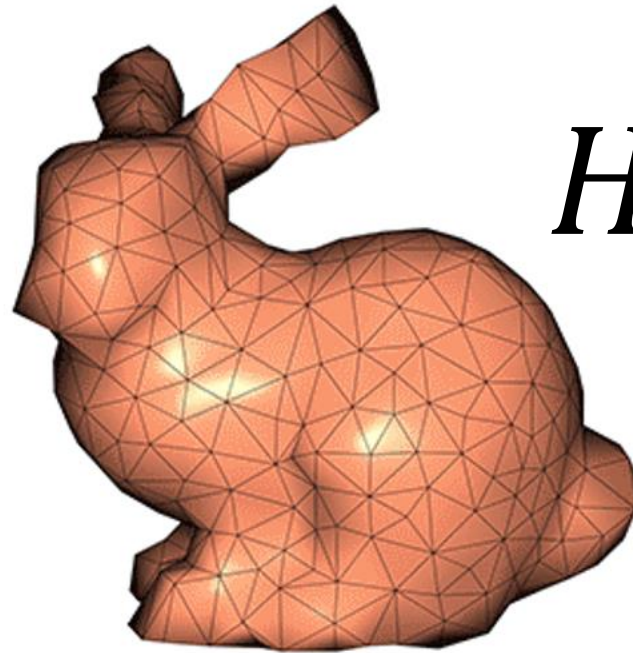
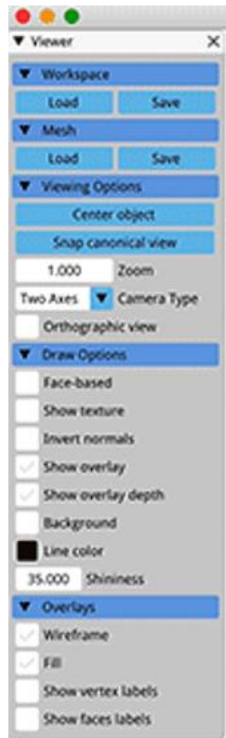
For each vertex, j

Minimize

$$E(\mathbf{q}_j^{i+1}) = \frac{1}{2} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i)^T M_{jj} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + \mathbf{p}_j^T (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + h^2 V_j(\mathbf{q}_j^{i+1})$$

Why is this fast ?

Assembly still visits every element ☹



$$H = \sum_t E_i^T H_i E_i$$

Per-tetrahedron Hessian

Why is this fast ?

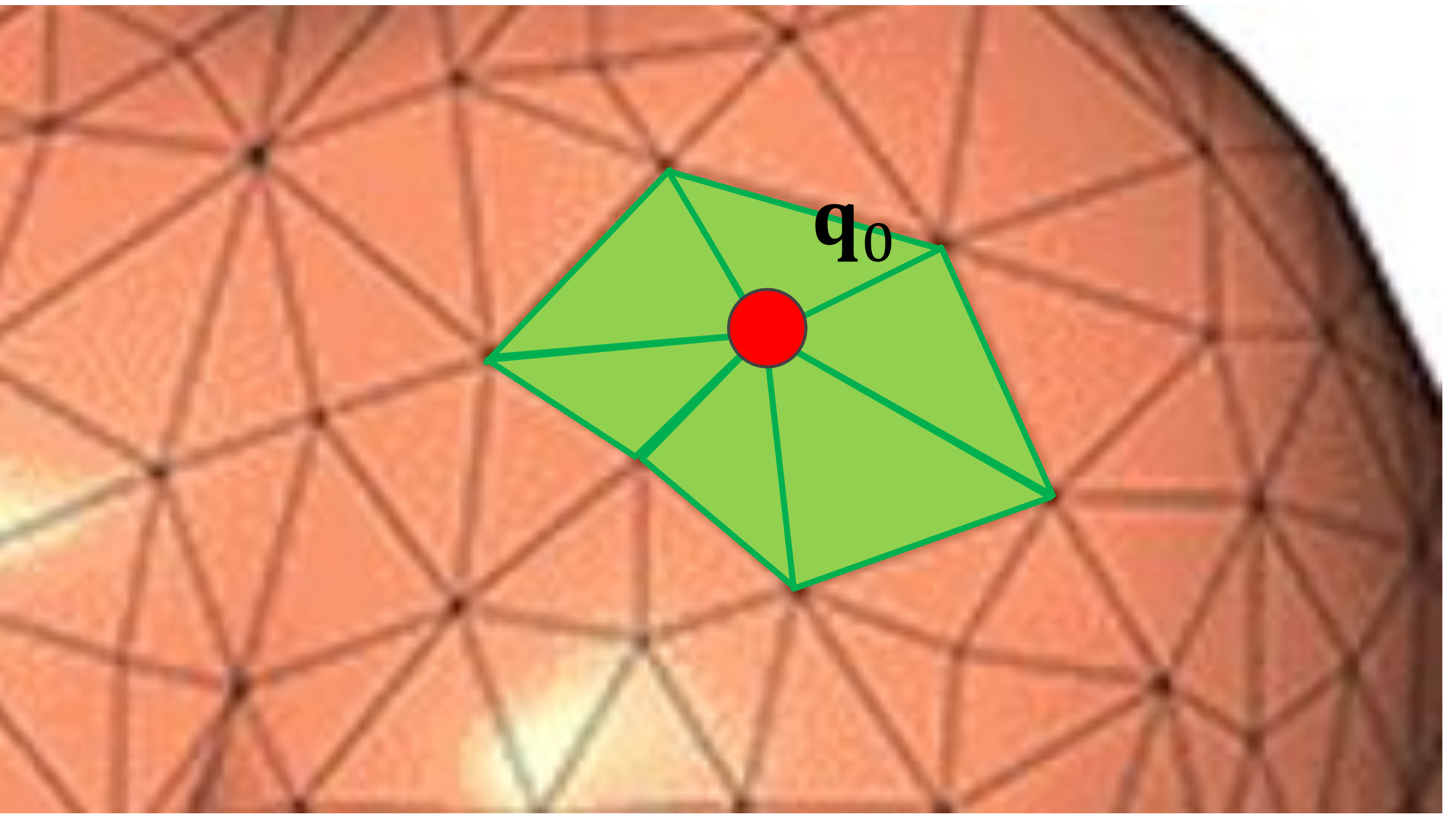
Repeat for awhile

For each vertex, j

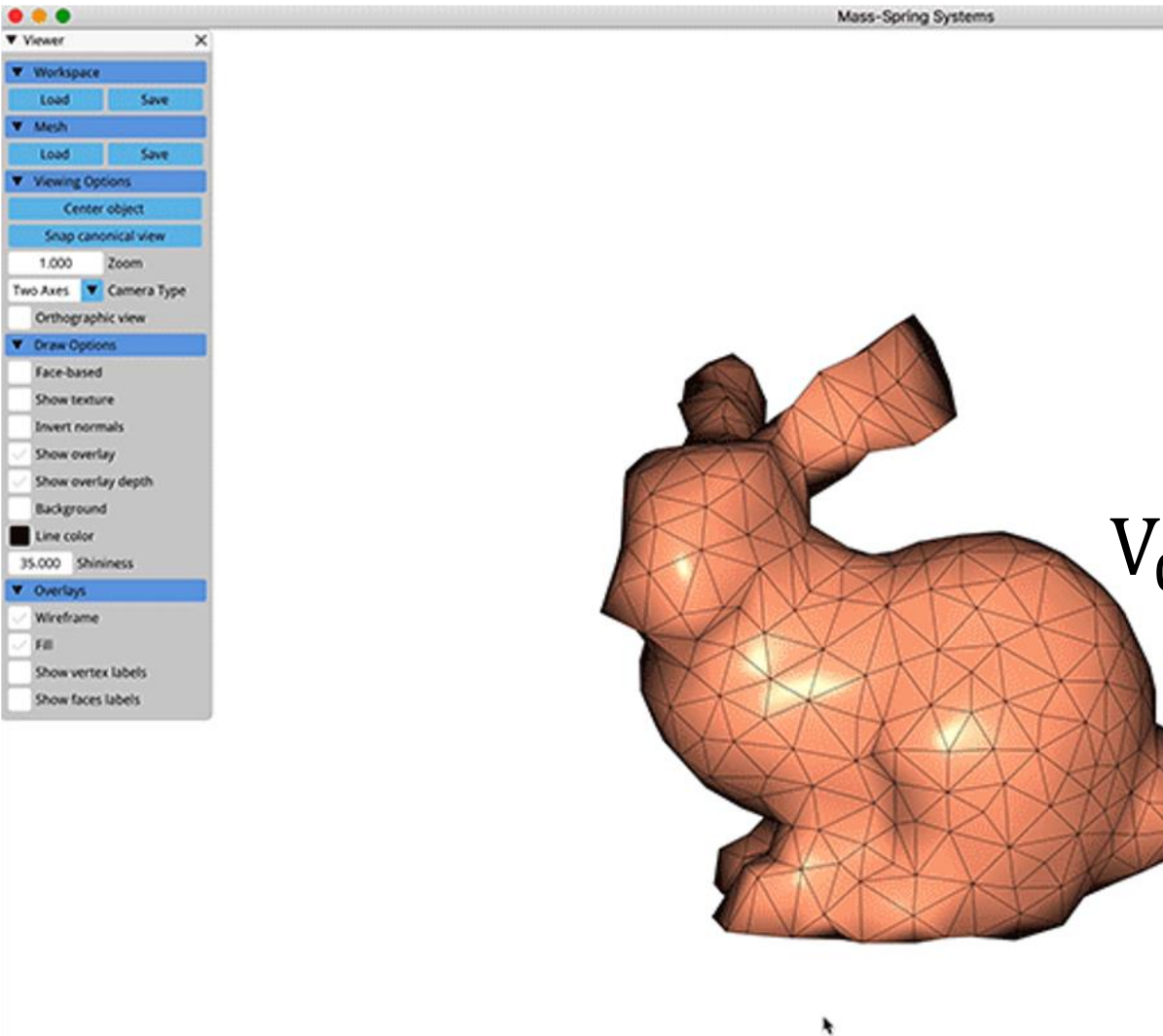
Minimize

$$E(\mathbf{q}_j^{i+1}) = \frac{1}{2} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i)^T M_{jj} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + \mathbf{p}_j^T (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + h^2 V_j(\mathbf{q}_j^{i+1})$$

Only depends on neighborhood of j



Potential Energy Structure (Way back in Lectures 2 and 3)



$$V_0(\mathbf{q}_0^{\mathbf{i}+1}) = \sum_{\mathbf{i} \in \mathcal{N}} \underset{\text{Tetrahedra containing } \mathbf{q}_0}{\text{vol}_i} \psi(\underset{\text{Other vertices of the tetrahedron}}{F_i(\mathbf{q}_0^{\mathbf{i}+1}, \dots)})$$

Why is this fast ?

Repeat for awhile

For each vertex, j

Minimize

$$E(\mathbf{q}_j^{i+1}) = \frac{1}{2} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i)^T M_{jj} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + \mathbf{p}_j^T (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + h^2 V_j(\mathbf{q}_j^{i+1})$$

Why is this fast ?

Parallelization

Repeat for awhile

For each vertex, j

Minimize

$$E(\mathbf{q}_j^{i+1}) = \frac{1}{2} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i)^T M_{jj} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + \mathbf{p}_j^T (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + h^2 V_j(\mathbf{q}_j^{i+1})$$

It's not obvious but we can do this in parallel

Parallelization

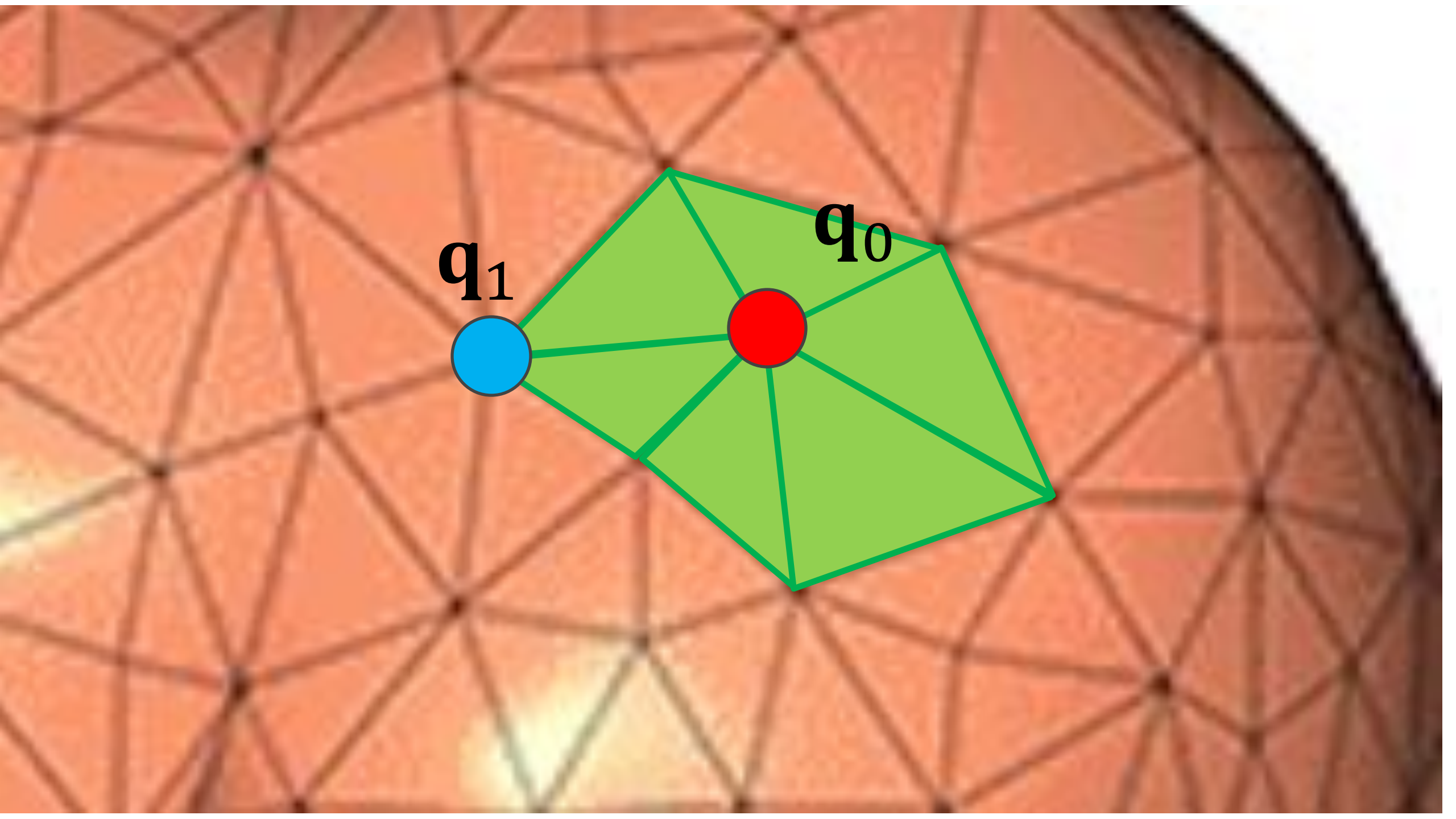
Repeat for awhile

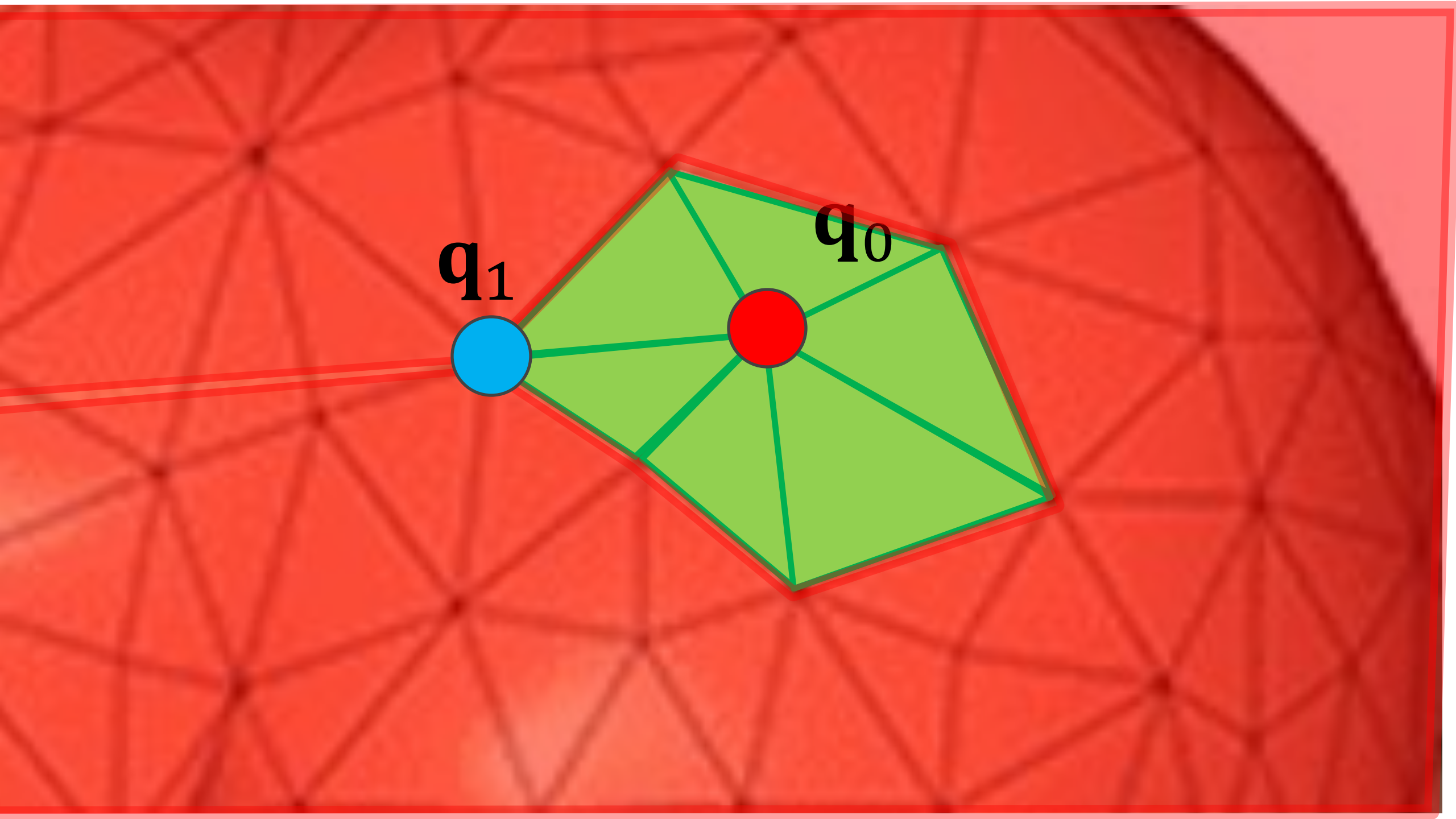
For each vertex, j **Parallelize this Loop !!!**

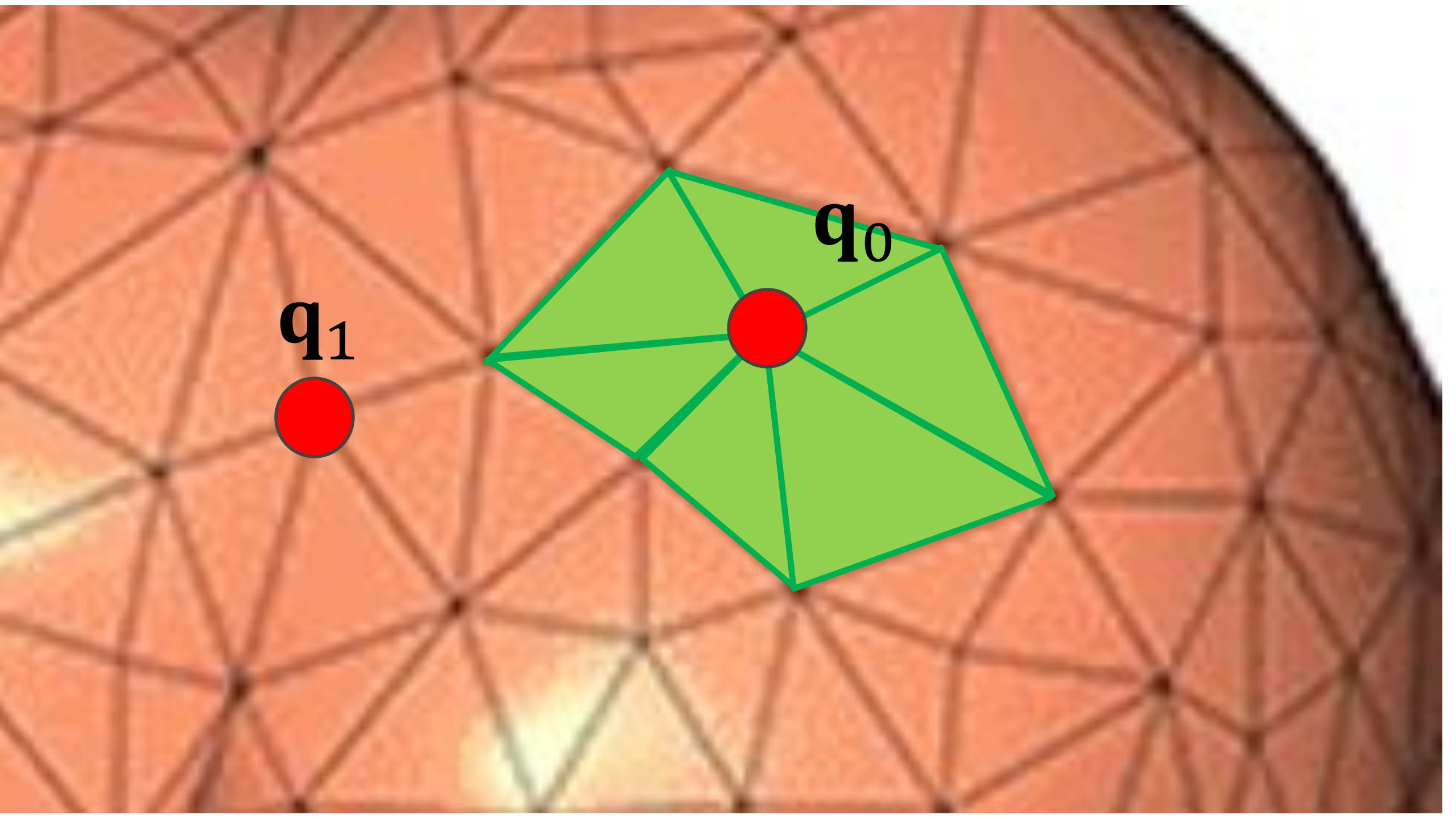
Minimize

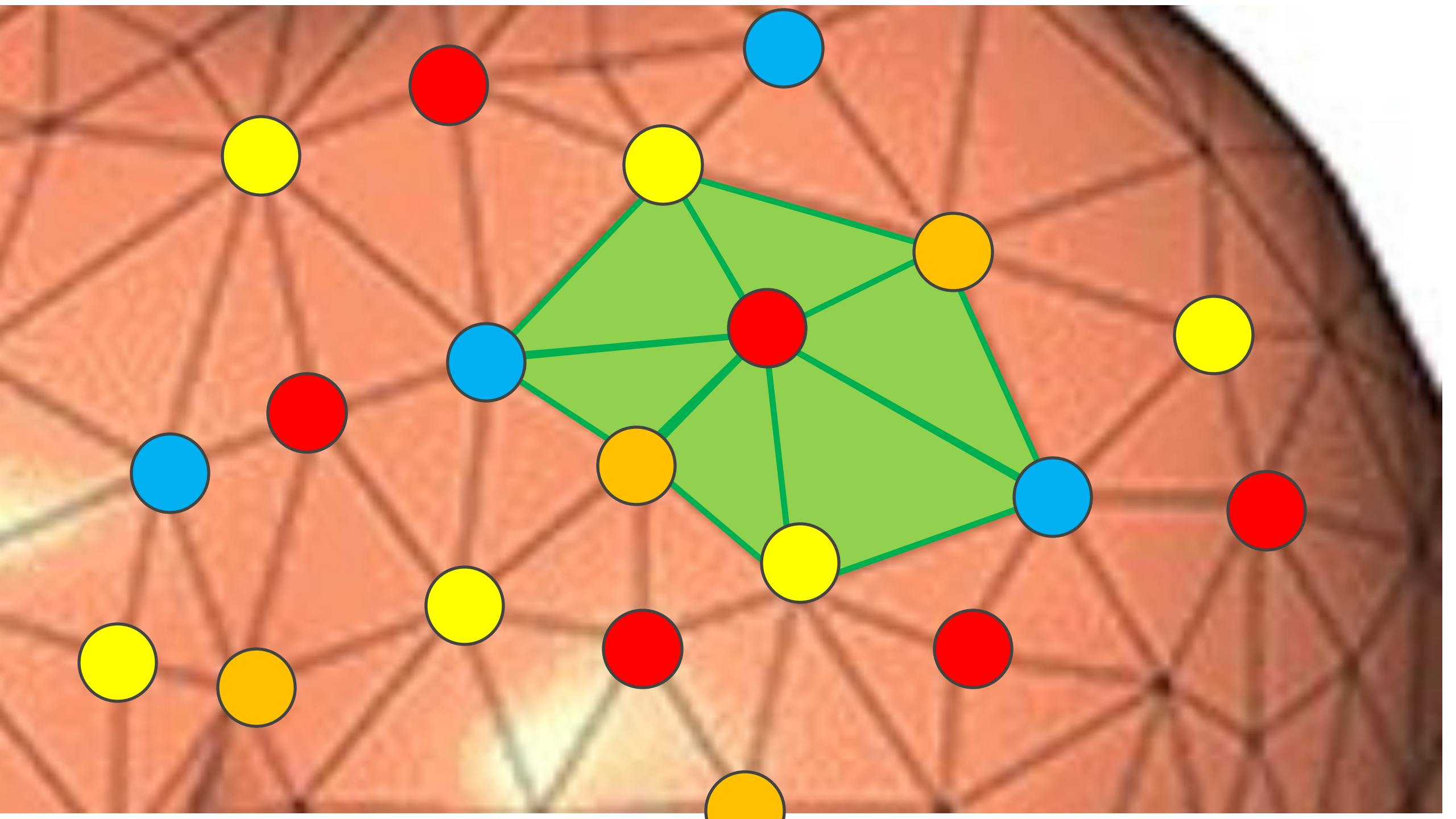
$$E(\mathbf{q}_j^{i+1}) = \frac{1}{2} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i)^T M_{jj} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + \mathbf{p}_j^T (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + h^2 V_j(\mathbf{q}_j^{i+1})$$

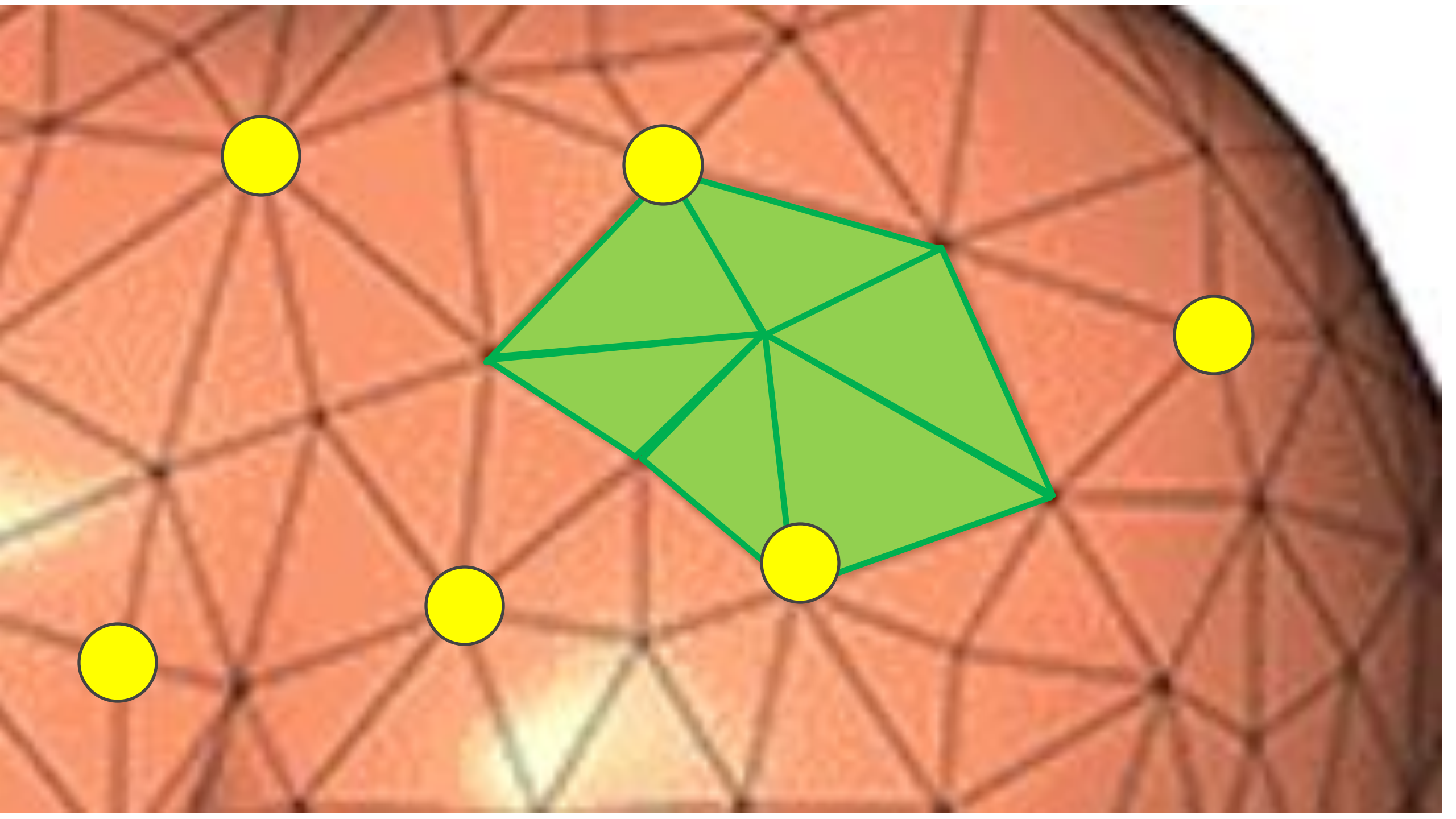
It's not obvious but we can do this in parallel





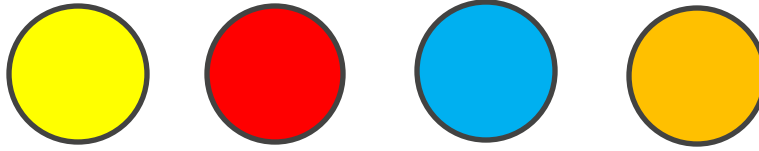






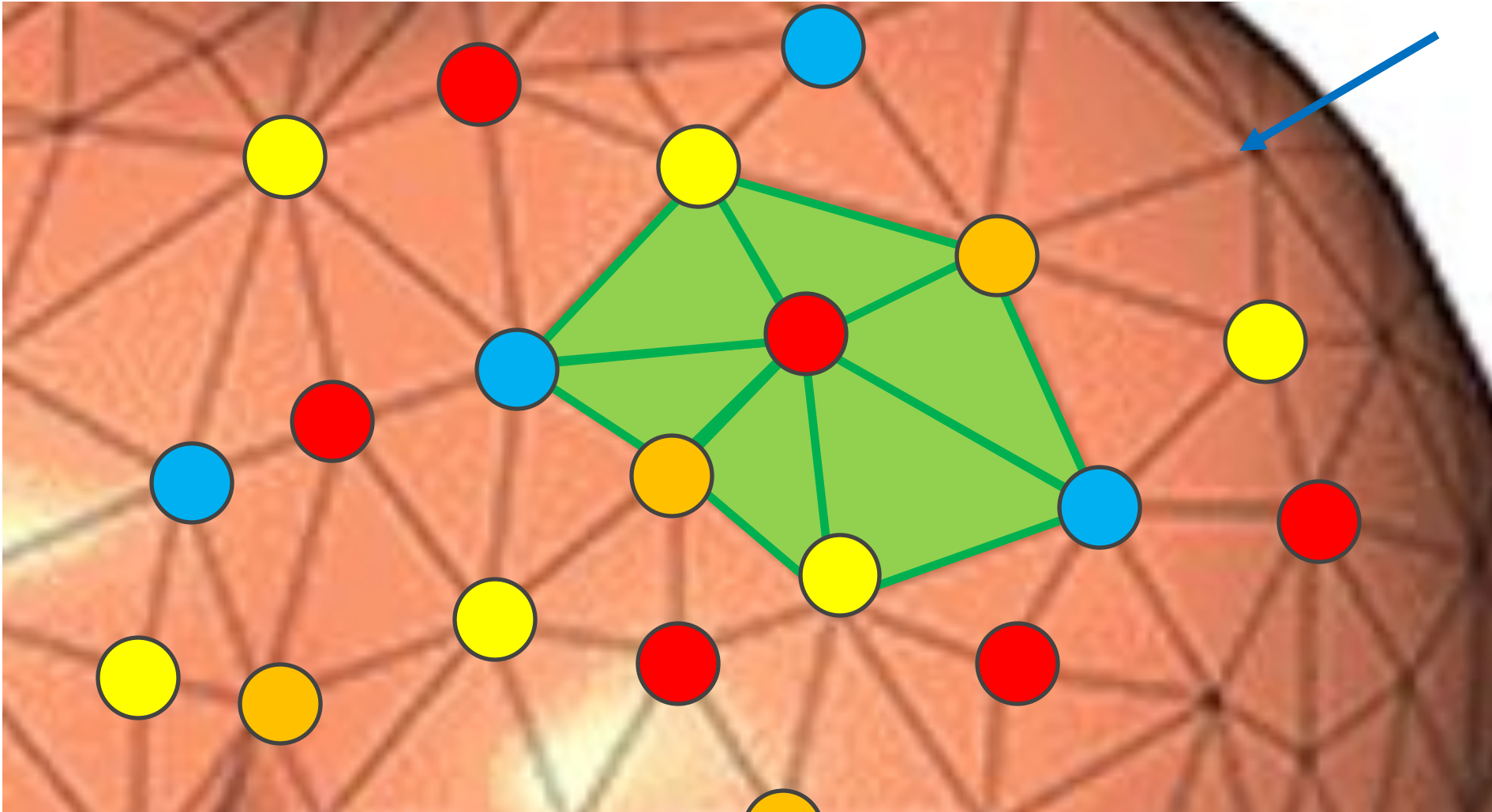
Greedy Graph Coloring

Pick a set of colors



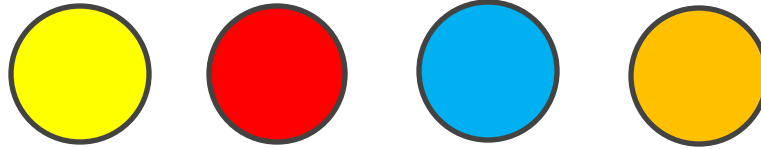
Order all vertices in the mesh

Assign vertex first feasible color



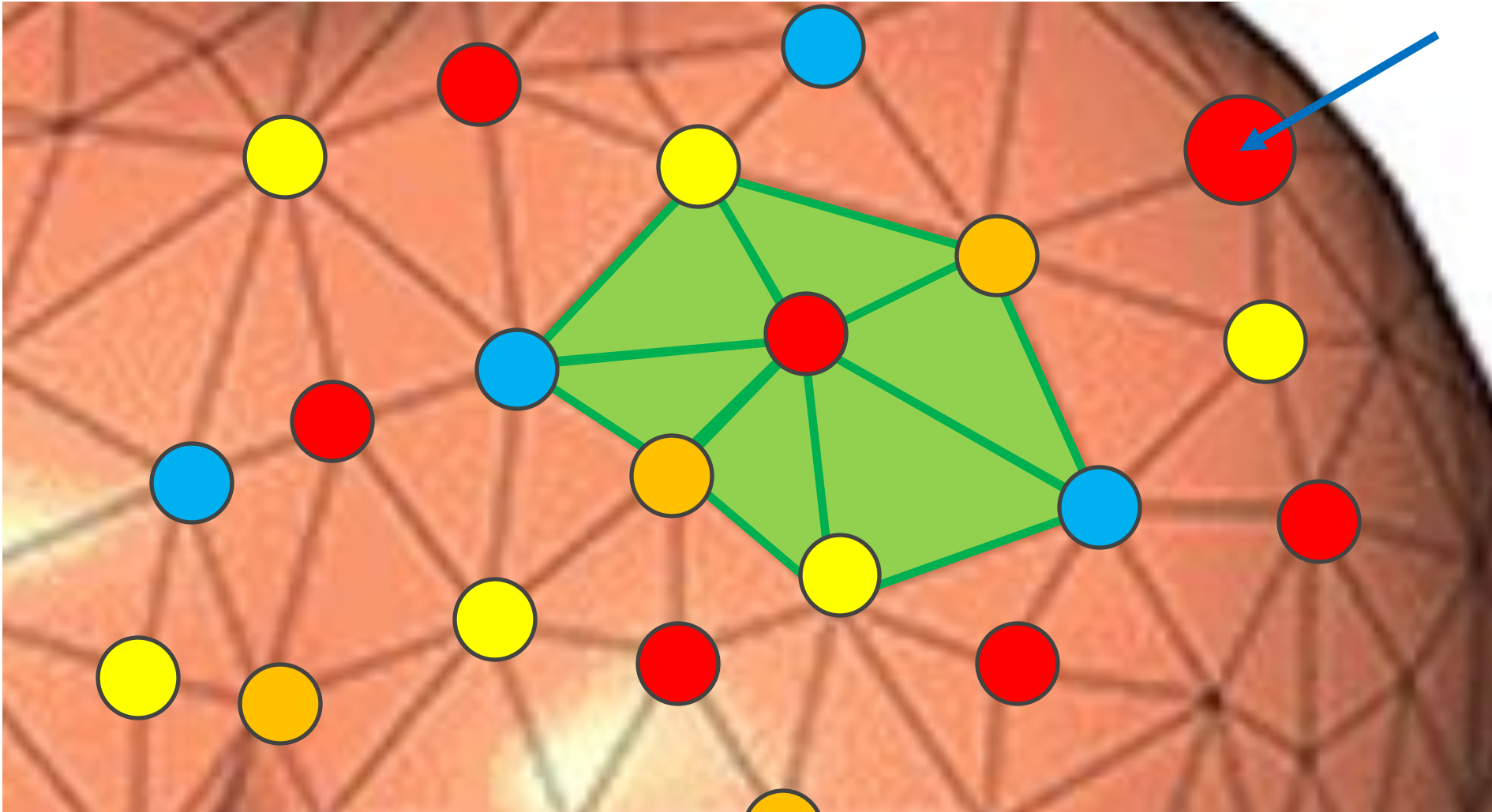
Greedy Graph Coloring

Pick a set of colors

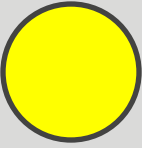
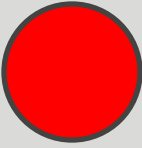
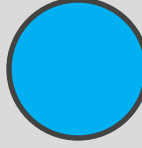
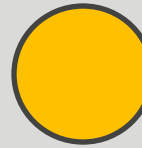


Order all vertices in the mesh

Assign vertex first feasible color



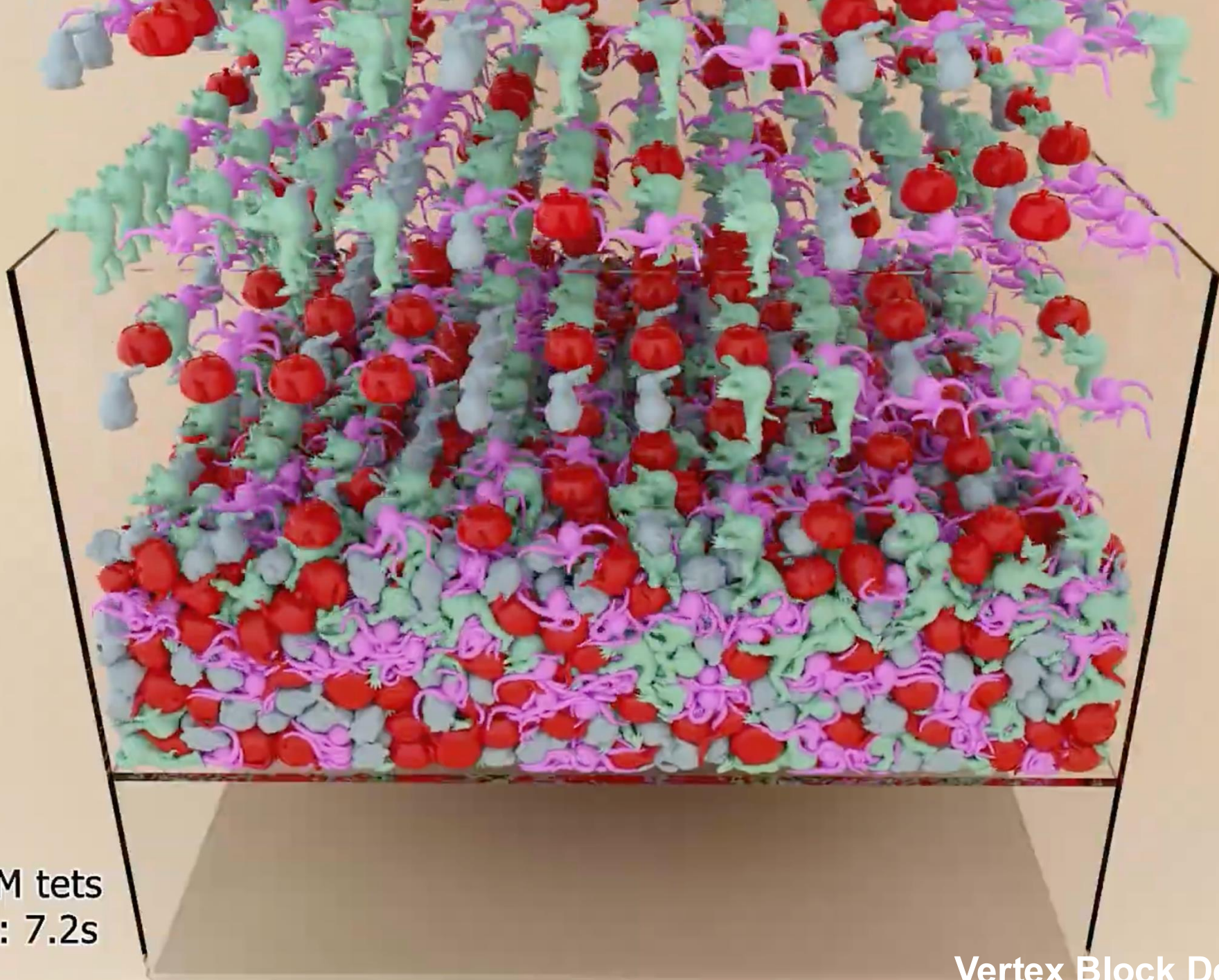
Parallelization

For each color,    

In parallel, minimize for all $j \in \text{color}$

$$E(\mathbf{q}_j^{i+1}) = \frac{1}{2} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i)^T M_{jj} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + \mathbf{p}_j^T (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + h^2 V_j(\mathbf{q}_j^{i+1})$$





36M verts, 124M tets
avg frame time: 7.2s
max: 7.8s

Vertex Block Descent|Chen et al

Pros and Cons

Pros

Very, very fast due to parallelized small solves

Cons

None ?

Pros and Cons

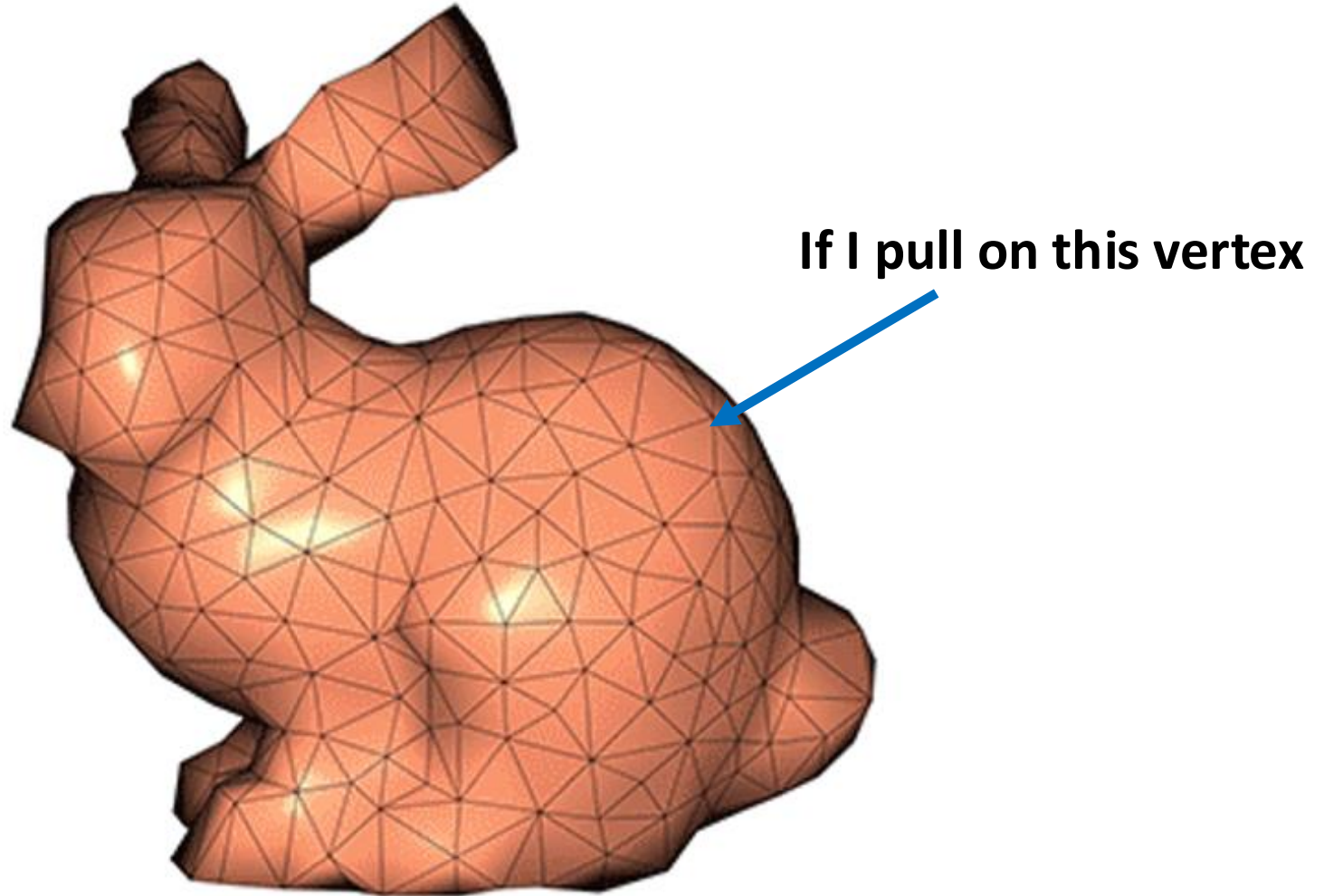
Pros

Very, very fast due to parallelized small solves

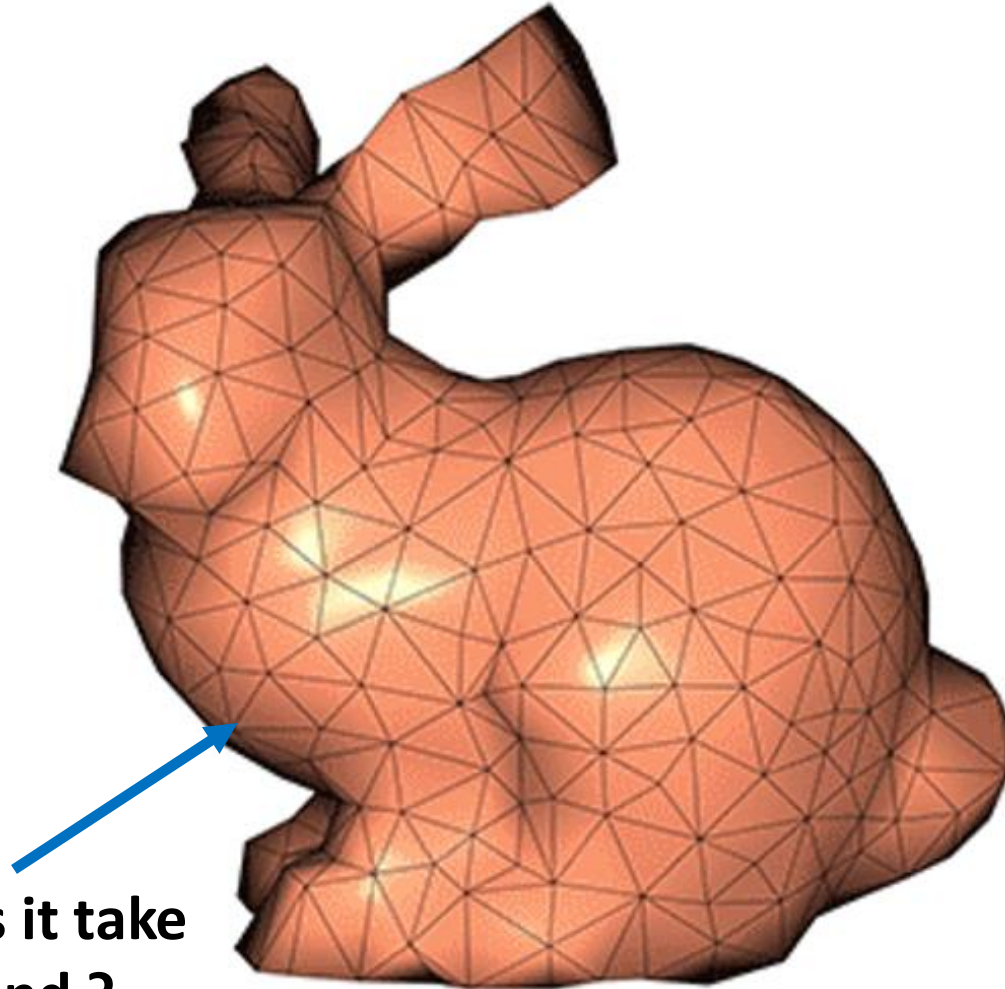
Cons

None ? (Sadly know)

Information Propagation

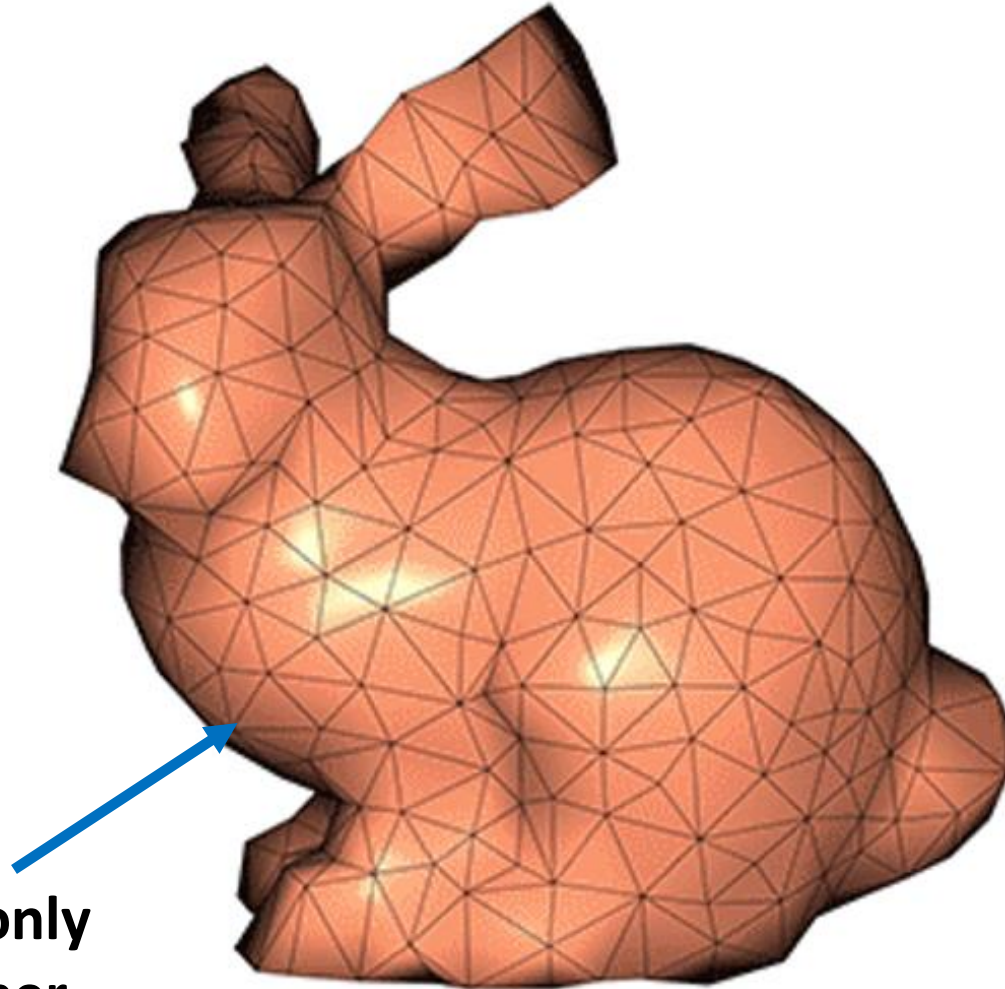


Information Propagation



How many iterations does it take
for this vertex to respond ?

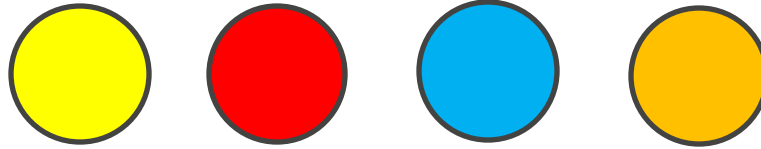
Information Propagation



A lot ! The deformation only travels along one edge per iteration

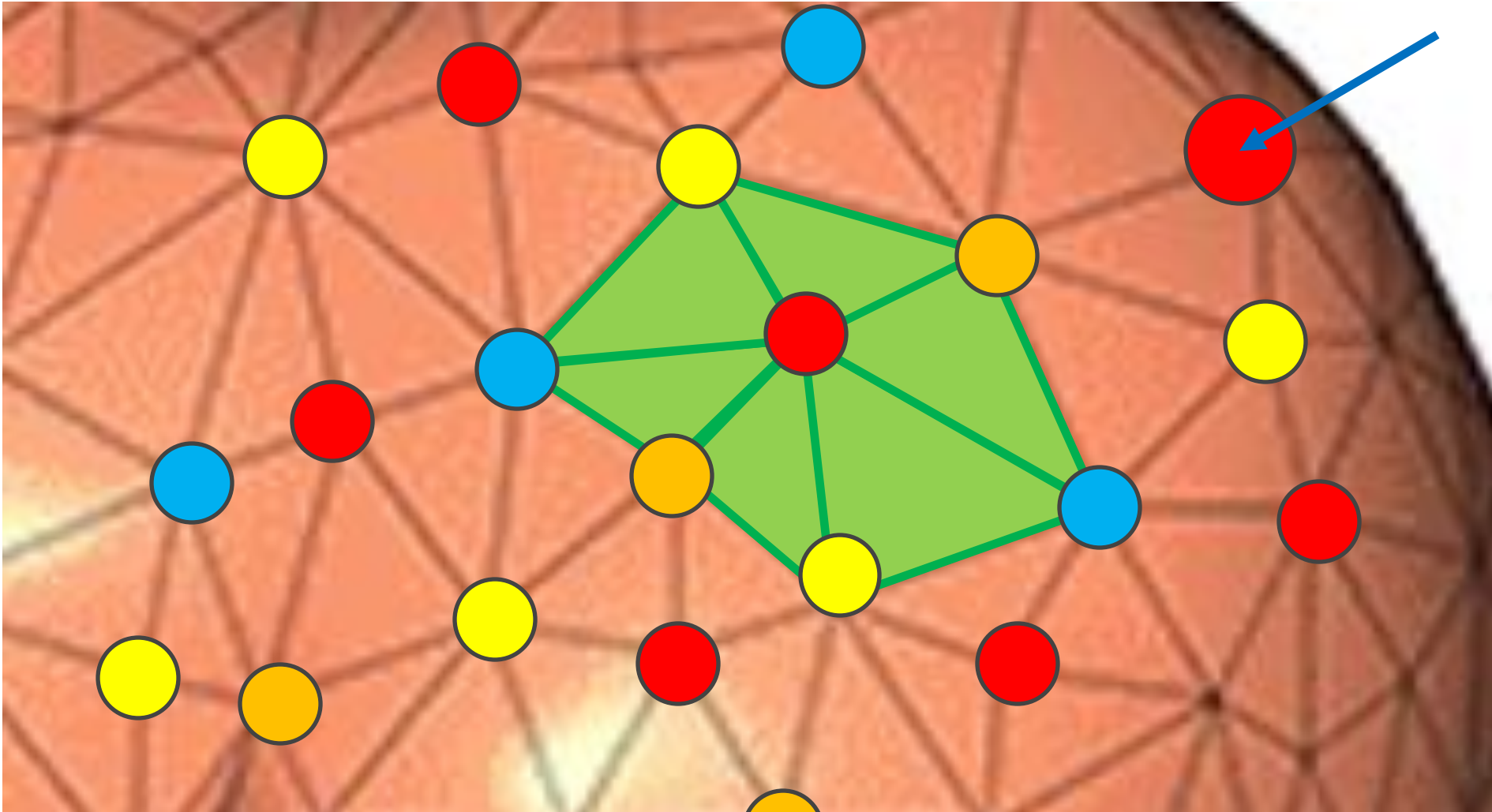
Greedy Graph Coloring

Pick a set of colors

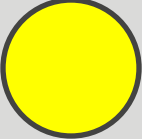

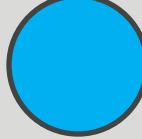
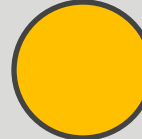


Order all vertices in the mesh

Assign vertex first feasible color



Parallelization

For each color,    

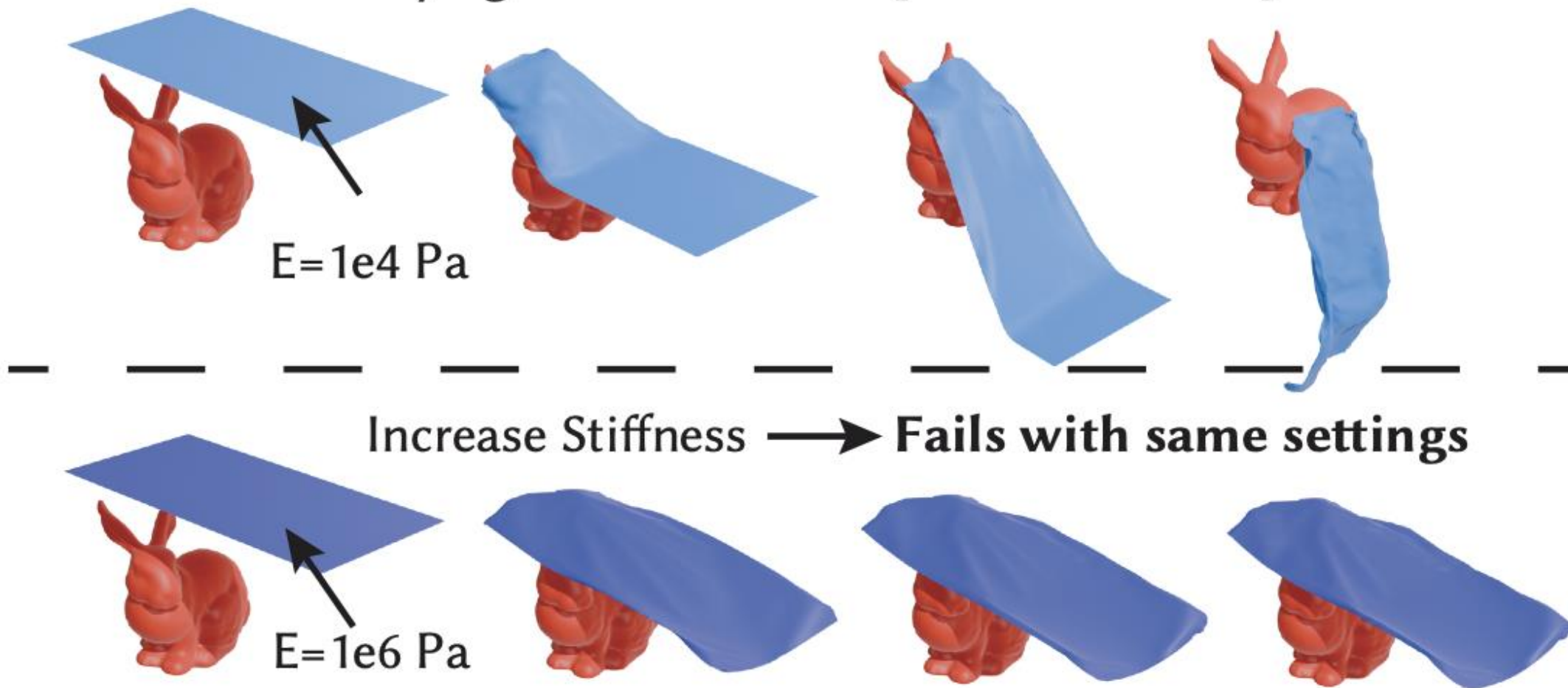
In parallel, minimize for all $j \in \text{color}$

$$E(\mathbf{q}_j^{i+1}) = \frac{1}{2} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i)^T M_{jj} (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + \mathbf{p}_j^T (\mathbf{q}_j^{i+1} - \tilde{\mathbf{q}}_j^i) + h^2 V_j(\mathbf{q}_j^{i+1})$$



Information Propagation

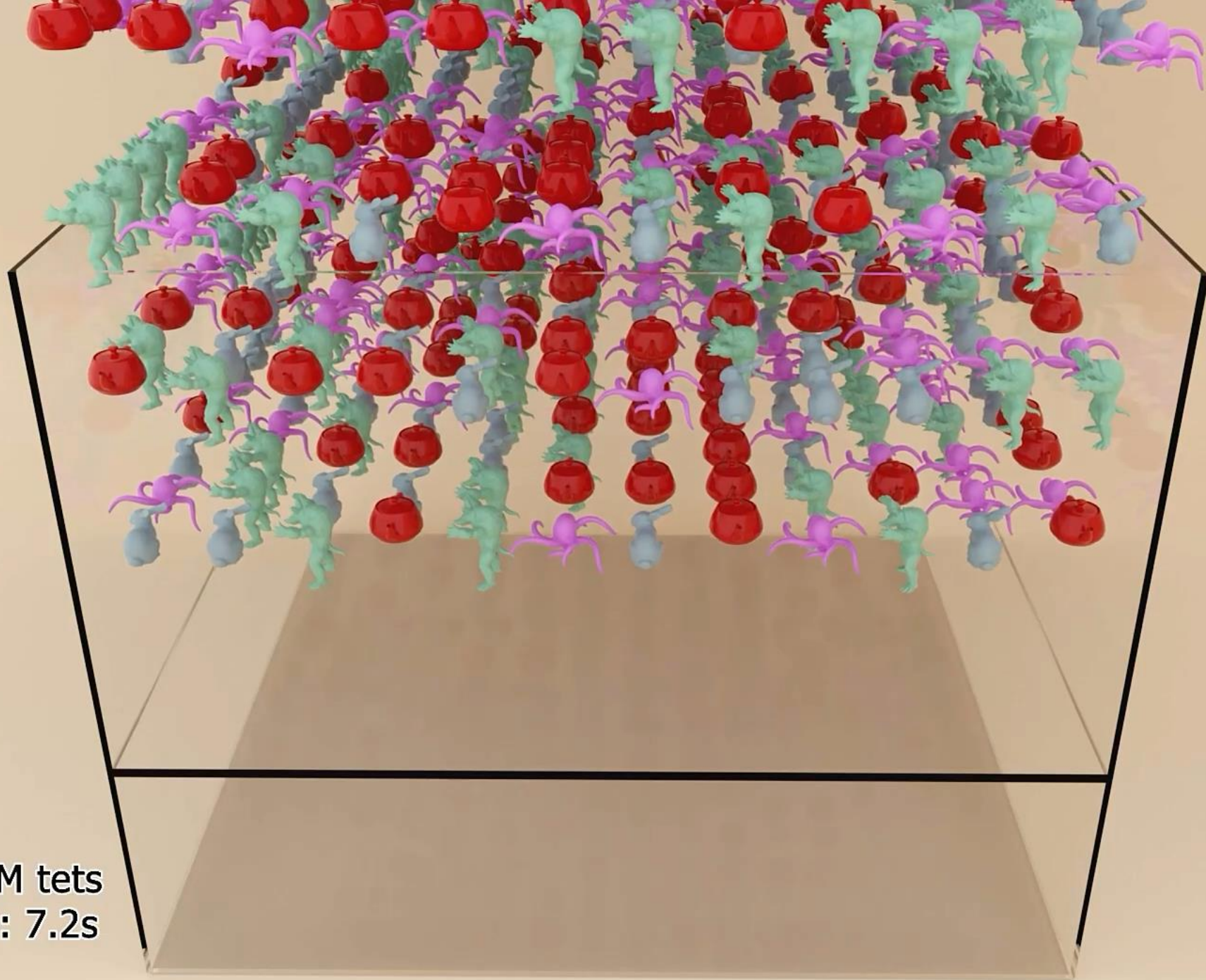
Varying stiffness in VBD [Chen et al. 2024]



Information Propagation

High resolution geometry and high stiffness exacerbates the issue

Not a deal breaker because you can tune your parameters and geometry to get reasonable output, but not a black box algorithm as pure Newton's method.



36M verts, 124M tets
avg frame time: 7.2s
max: 7.8s

Next Lecture: NO MORE STRUCTURED LECTURES

