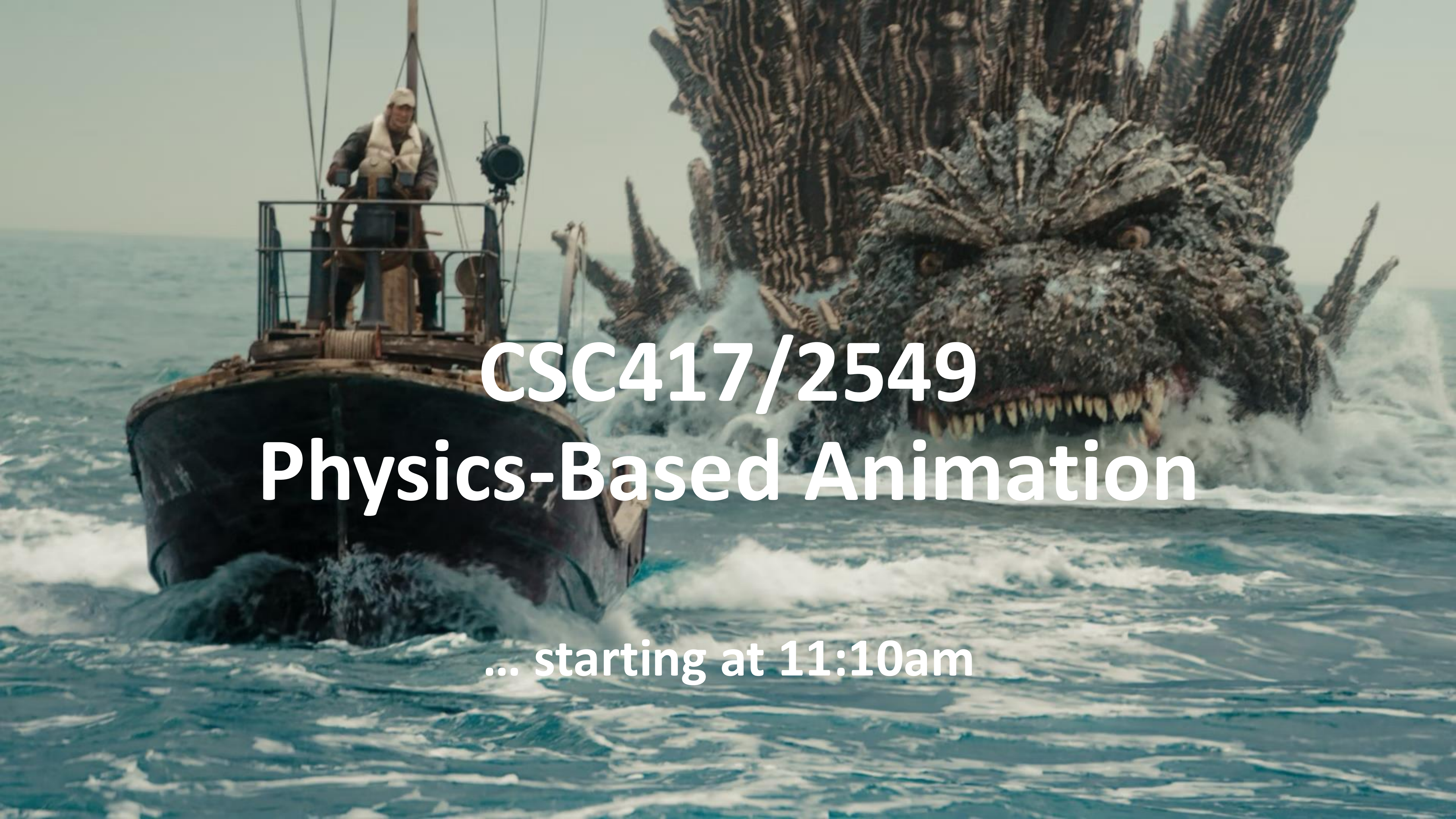
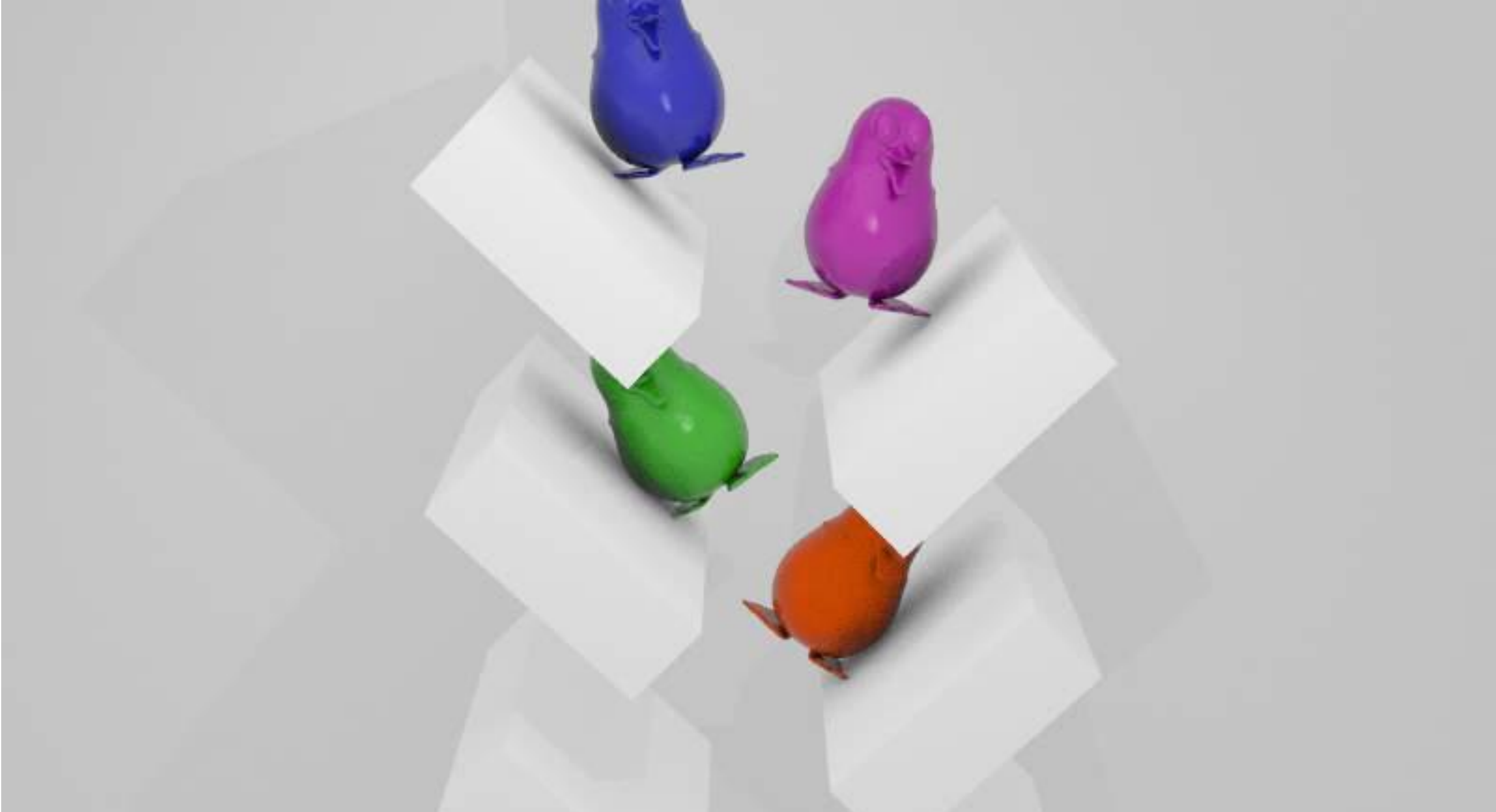
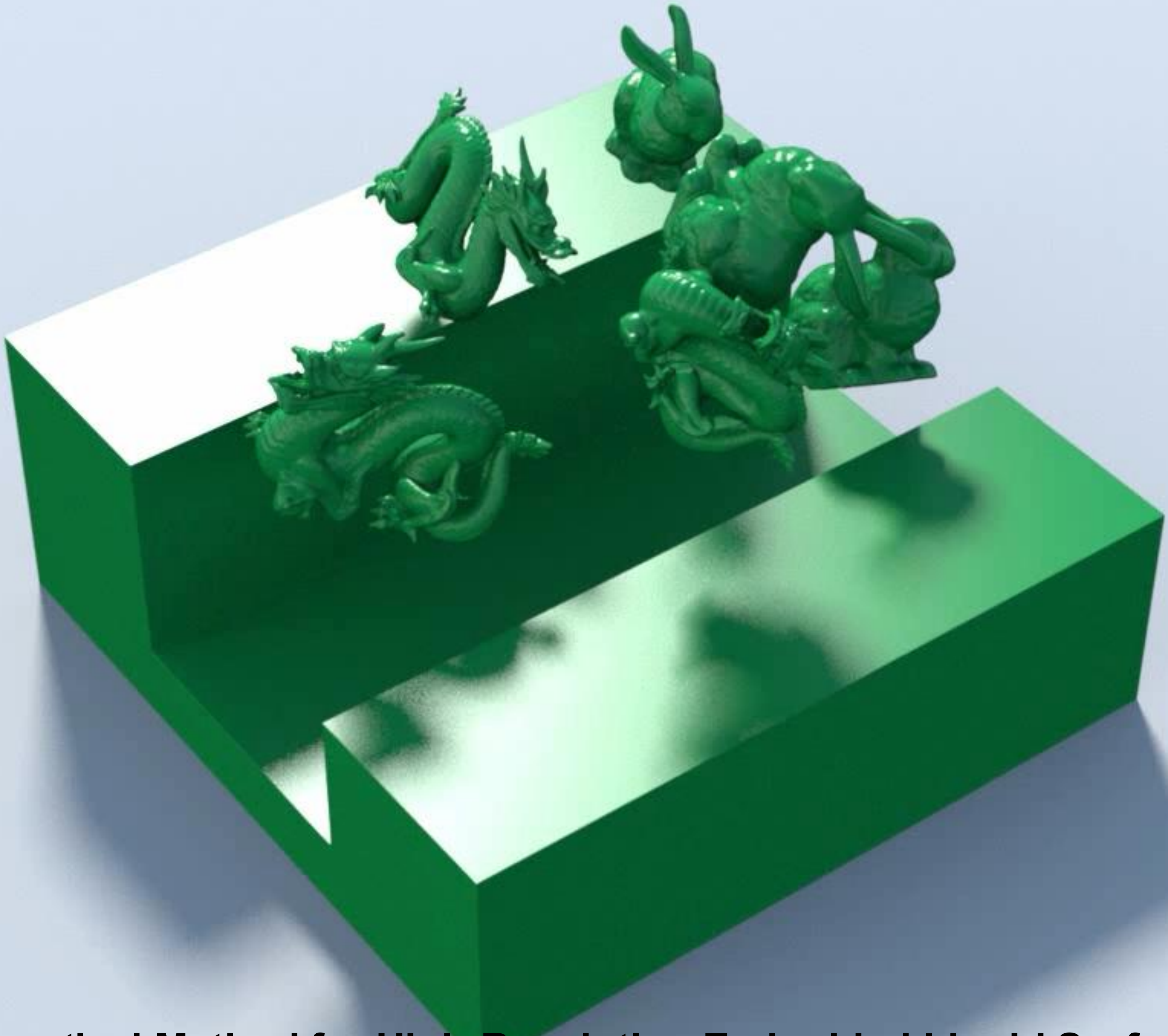


CSC417/2549
Physics-Based Animation

… starting at 11:10am

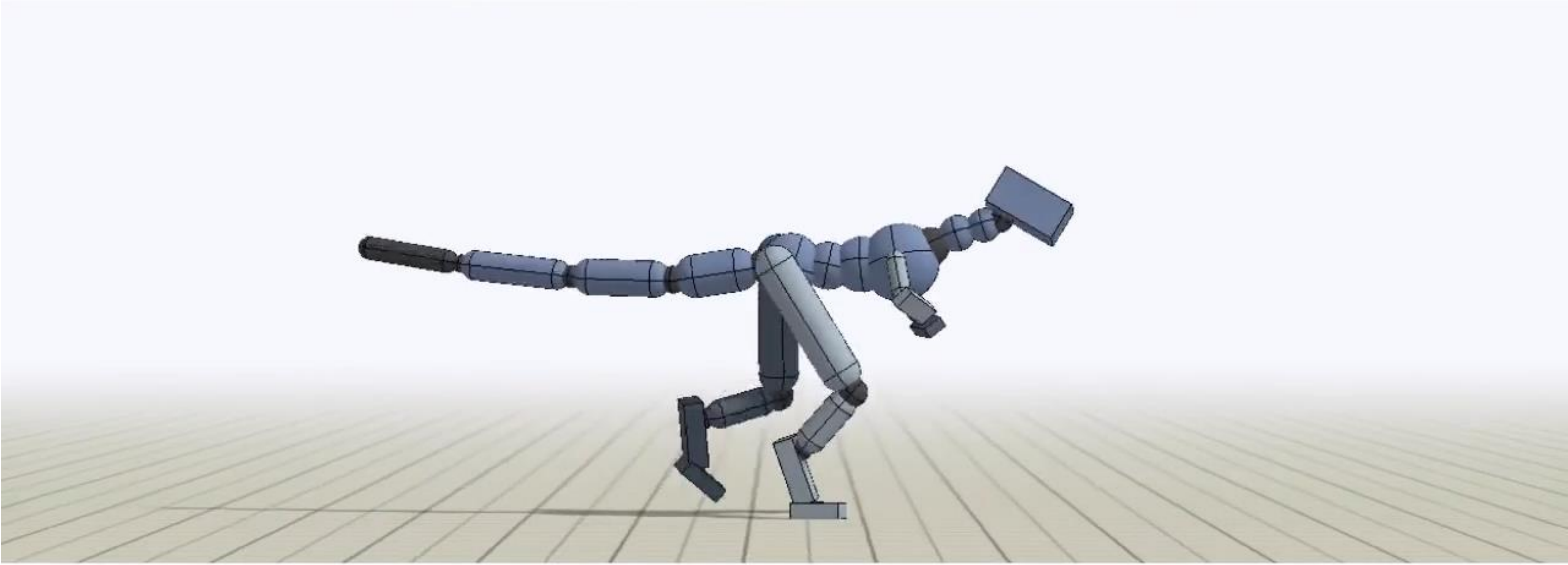


Eulerian-on-Lagrangian Simulation | with Fan, Litven and Pai



A Practical Method for High-Resolution Embedded Liquid Surfaces | Goldade et al

T-Rex: Walk



Simulated Character

DeepMimic: Example-Guided Deep Reinforcement Learning of Physics-Based Character Skills | Peng et al.

The Tools of the Trade

Linear Algebra

Multivariate Calculus

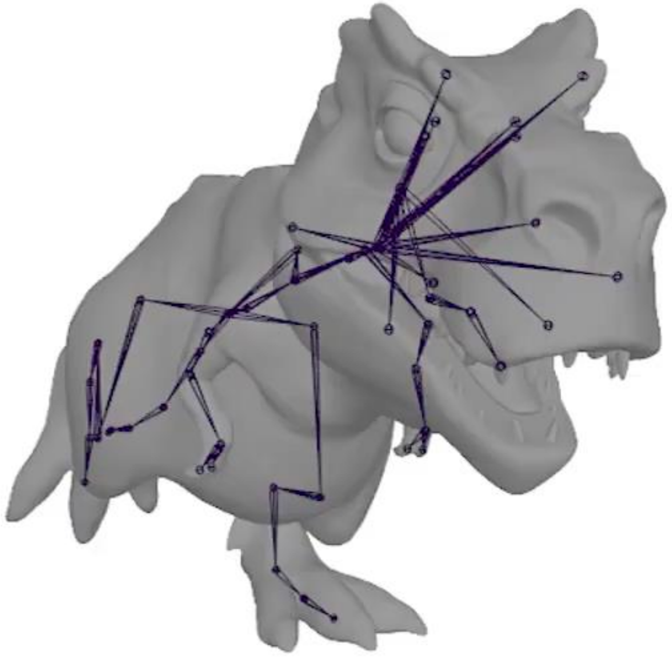
Calculus of Variations

Numerical Methods for Ordinary Differential Equations

Numerical Methods for Partial Differential Equations

Optimization

DON'T PANIC



Input

Output

Administrivia

Course web site (includes course information sheet):

https://github.com/dilevin/CSC417-physics-based-animation

Instructor:

Prof. David I.W. Levin diwlevin@cs.toronto.edu

TAs:

Shukui Chen

Jonathan Panuelos

João Pedro Vasconcelos Teixeira

TA + Instructor Email: csc417tas@cs.toronto.edu

Lectures

Wednesday 11:00-13:00 ES B142

Tutorials

Monday 11:00-noon (starts September 8th) **I will run the tutorials**

Administrivia

Discussion Board

https://piazza.com/utoronto.ca/fall2025/csc417

Assignments will be submitted via MarkUs

Coming Soon

Academic Honesty section of webpage is required reading for the course

Highlights: You can use CoPilot, ChatGPT, Cursor and similar AI tools on assignments

All New Course This Year

Dates	Topic	Assignments
9/3/2025	Introduction	
9/10/2025	Deformation and Finite Element Method	
9/17/2025	From Energy to Motion	
9/24/2025	Cloth Simulation	Release A1: Finite Element Methods
10/1/2025	Rigid Bodies / Affine Body Dynamics	
10/8/2025	Collisions	
10/15/2025	Intro to Fluid Simulation	Assignment 1 Due, Release A2: Affine Body Dynamics
10/22/2025	Material Point Method	
11/5/2025	Reduced-Order Models	Assignment 2 Due, Release A3: Fluids
11/11/2025	***Drop Deadline***	
11/12/2025	Fast Physics Solvers	Assignment 3 Due, Release A4: Reduced/Fast Methods
11/19/2025	Beyond Elasticity	
11/26/2025	***Final Exam***	Assignment 4 Due

Grading

%	Item
50%	Assignments
30%	In-Tutorial Quizzes
20%	Final Exam (must get >= 50% to pass course)

Details on quizzes are coming soon

Lateness Policy

Please read, this course has an involved late policy aimed at giving you maximum flexibility in scheduling your semester

Assignments are ***due by 11:59pm*** on the three due dates below.

Every student will recieve 21 "late days" that will be automatically applied before the late penalty begins to accumulate. You only have to inform the instructor if you **DO NOT** wish to use your late days for a particular assignment.

After late days have been exhausted, assignemnts accrue a penalty at the following rate: 0.007% off for every minute late.

Further extensions can only be issued by the instructor.

AI Policy

You can freely use AI tools such as CoPilot, ChatGPT or Cursor for your assignments but not for quizzes or the final exam.


Let’s Goooooo!

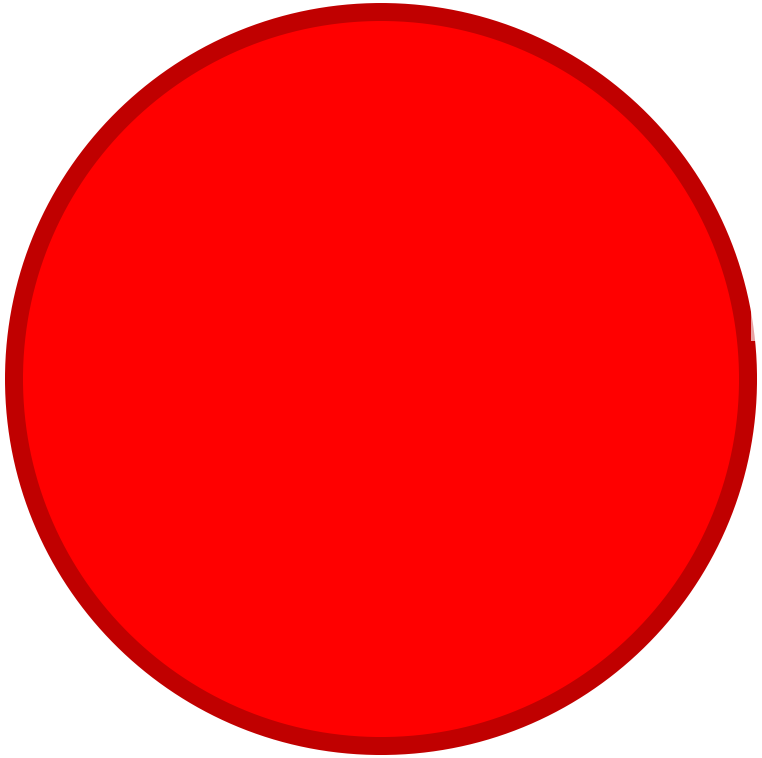
Newton’s Laws

1. Every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force

2. The force acting on an object is equal to the time rate-of-change of the momentum

3. For every action there is an equal and opposite reaction

Example Physical System


Particle

Position in space (m) $\mathbf{x}(t)$

Velocity in space (m/s) $\mathbf{v}(t) = \frac{d\mathbf{x}}{dt}(t)$

Acceleration in space (m/s²) $\mathbf{a}(t) = \frac{d^2\mathbf{x}}{dt^2}(t)$

Mass (kg) m

Newton's Laws

1. Every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force

2. The **force** acting on an object is equal to the **time rate-of-change of the momentum**

3. For every action there is an equal and opposite reaction

Newton's Laws

momentum $\underbrace{m}_{\text{mass}}\overbrace{\mathbf{v}}^{\text{velocity}}$

time rate-of-change of the momentum **force**

$$\frac{d}{dt}(m\mathbf{v}) = \mathbf{f}$$

for constant mass

$$m\overbrace{\frac{d\mathbf{v}}{dt}}^{\text{acceleration}} = \mathbf{f}$$

Vectorial Mechanics

Newton's Laws

momentum $m\mathbf{v}$

velocity

mass

time rate-of-change of the momentum force

$$\frac{d}{dt}(m\mathbf{v}) = \mathbf{f}$$

for constant mass

acceleration

$$m\frac{d\mathbf{v}}{dt} = \mathbf{f}$$

Variational Mechanics

or *Analytical Mechanics*

Based on two fundamental energies rather than two vectorial quantities

Kinetic and Potential Energy

Kinetic Energy: Energy due to motion

Potential Energy: Energy “held within” an object due to its position, internal stresses, electrical charge etc …

Potential energy has the *potential* to become kinetic energy

Kinetic and Potential Energy

Potential Energy from Gravity

$$m \cdot g \cdot h$$

acceleration due to gravity

Kinetic and Potential Energy

Potential Energy from Gravity

$$m \cdot g \cdot h$$

height above ground

Variational Mechanics

Also called "Analytical Mechanics"

Based on two fundamental energies rather than two vectorial quantities

Motion defined using a variational principle

$$\underbrace{e}_{\text{functional}}\Big(\overbrace{\mathbf{f}(t), \dot{\mathbf{f}}(t), \ldots}^{\text{functions of time and derivatives}}\Big) \rightarrow \underbrace{\mathbb{R}}_{\text{real numbers}}$$

Generalized Coordinates

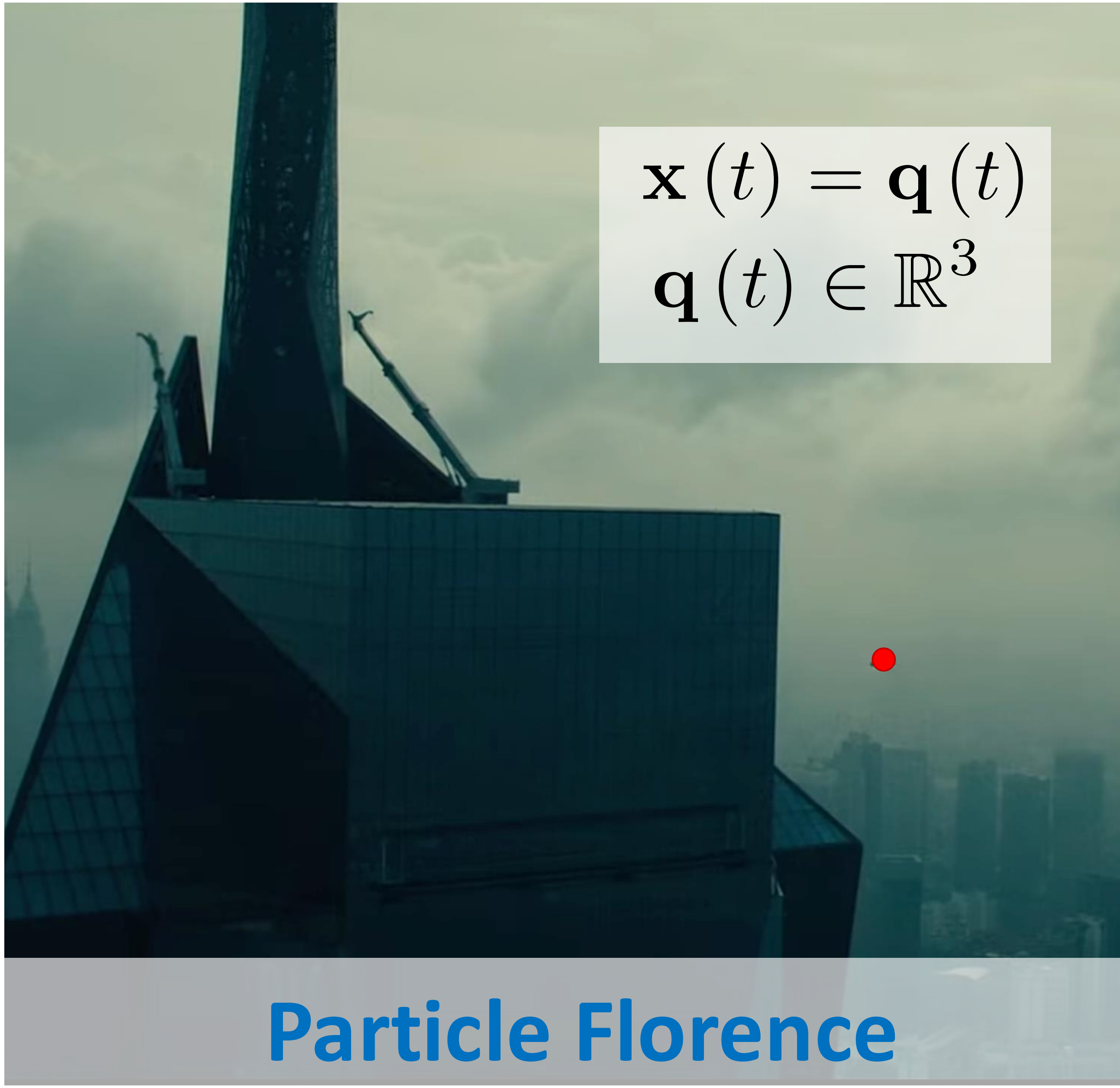
$$\mathbf{x}(t) = \mathbf{f}(\underbrace{\mathbf{q}(t)}_{\text{generalized coordinates}})$$

$$\frac{d\mathbf{x}}{dt}(t) = \overbrace{\frac{d\mathbf{f}}{d\mathbf{q}}}^{\text{Jacobian (lots of things are going to get called Jacobians)}}\underbrace{\dot{\mathbf{q}}(t)}_{\text{generalized velocity}}$$

Generalized Coordinates

$$\mathbf{x}(t) = \mathbf{f}(\underbrace{\mathbf{q}(t)}_{\text{generalized coordinates}})$$

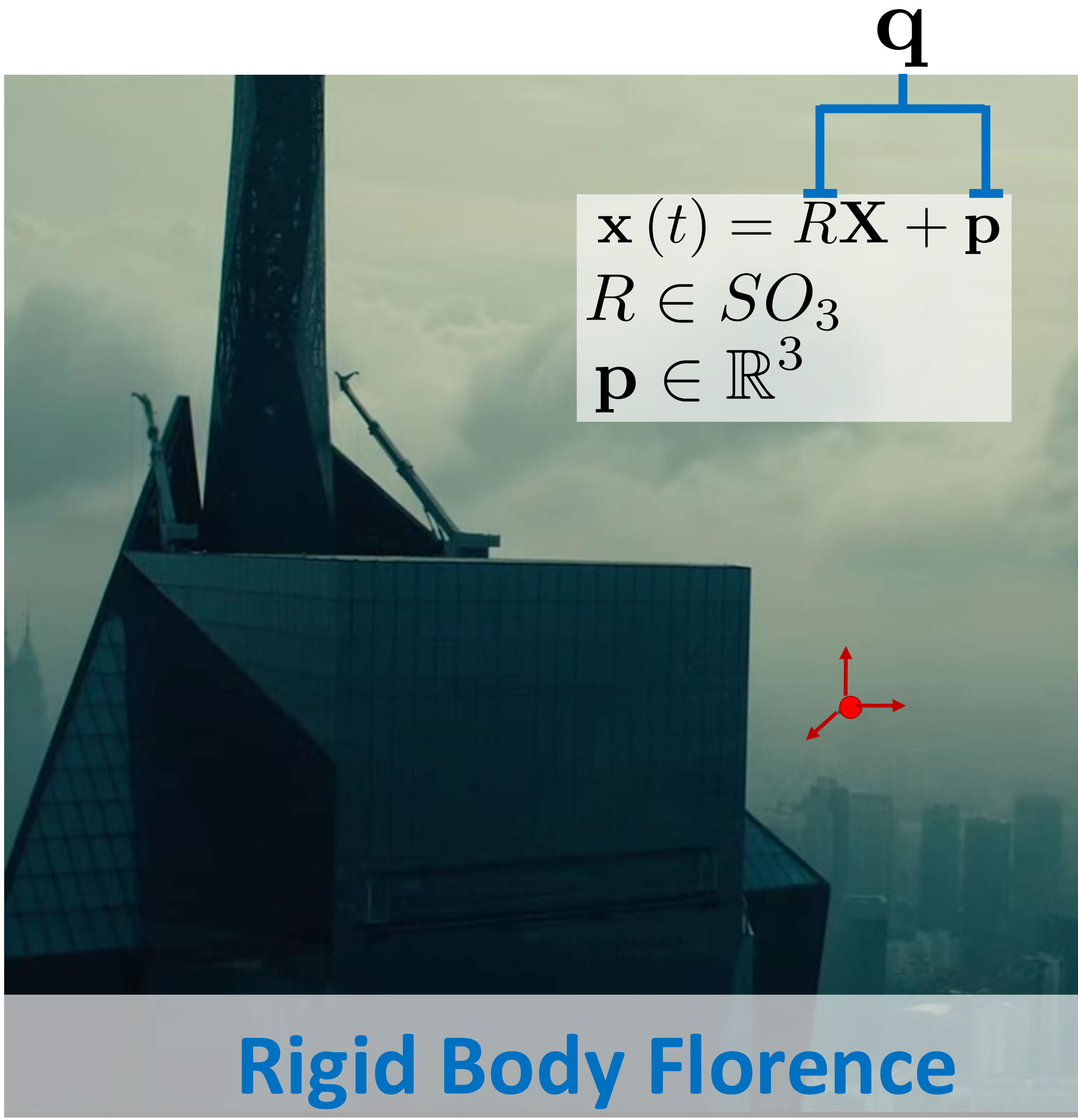
$$\frac{d\mathbf{x}}{dt}(t) = \overbrace{\frac{d\mathbf{f}}{d\mathbf{q}}}^{\text{Jacobian}} \underbrace{\dot{\mathbf{q}}(t)}_{\text{generalized velocity}}$$



Generalized Coordinates

$$\mathbf{x}(t) = \mathbf{f}(\underbrace{\mathbf{q}(t)}_{\text{generalized coordinates}}))$$

$$\frac{d\mathbf{x}}{dt}(t) = \overbrace{\frac{d\mathbf{f}}{d\mathbf{q}}}^{\text{Jacobian}} \underbrace{\dot{\mathbf{q}}(t)}_{\text{generalized velocity}}$$

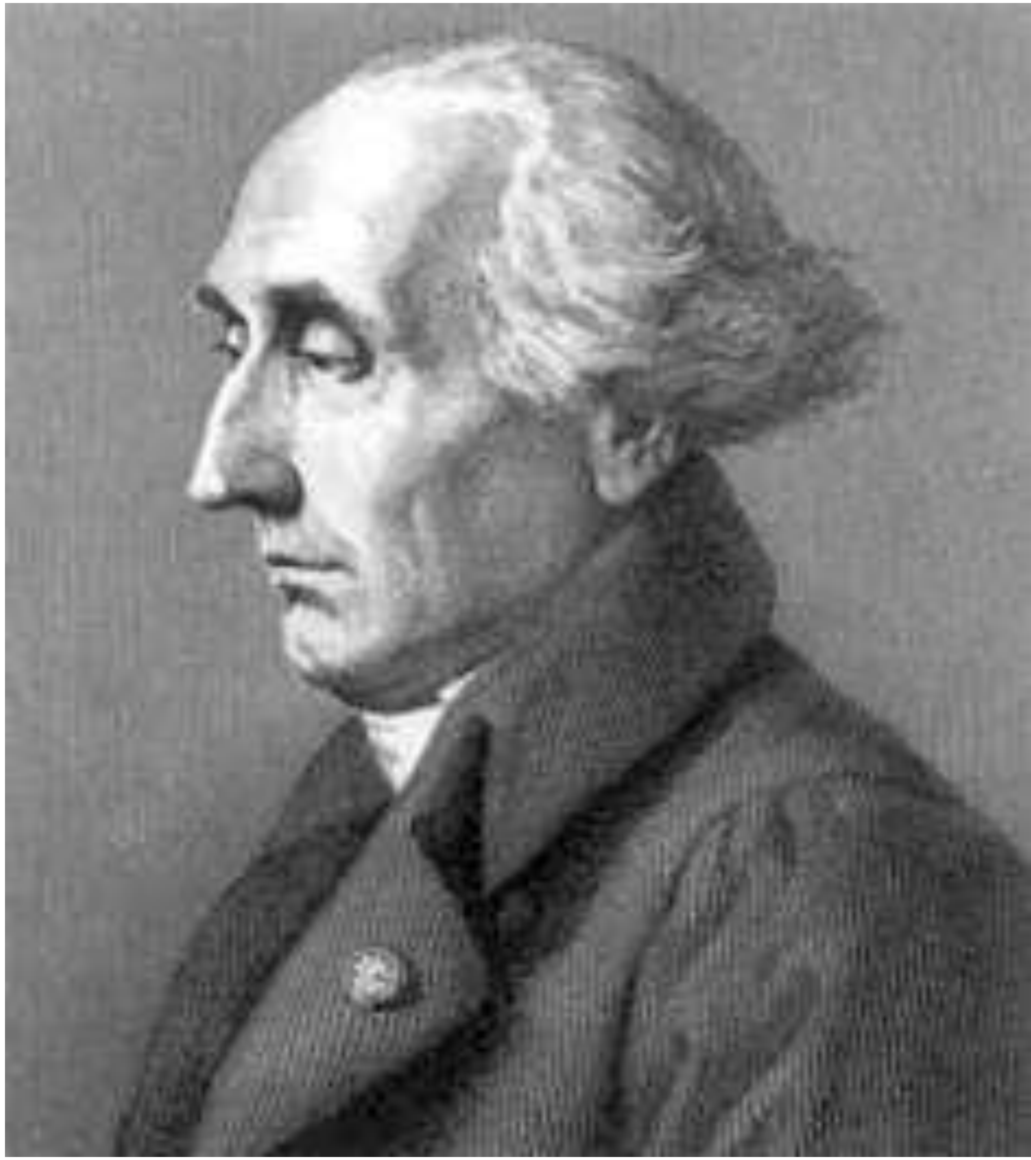


The Lagrangian

Potential Energy

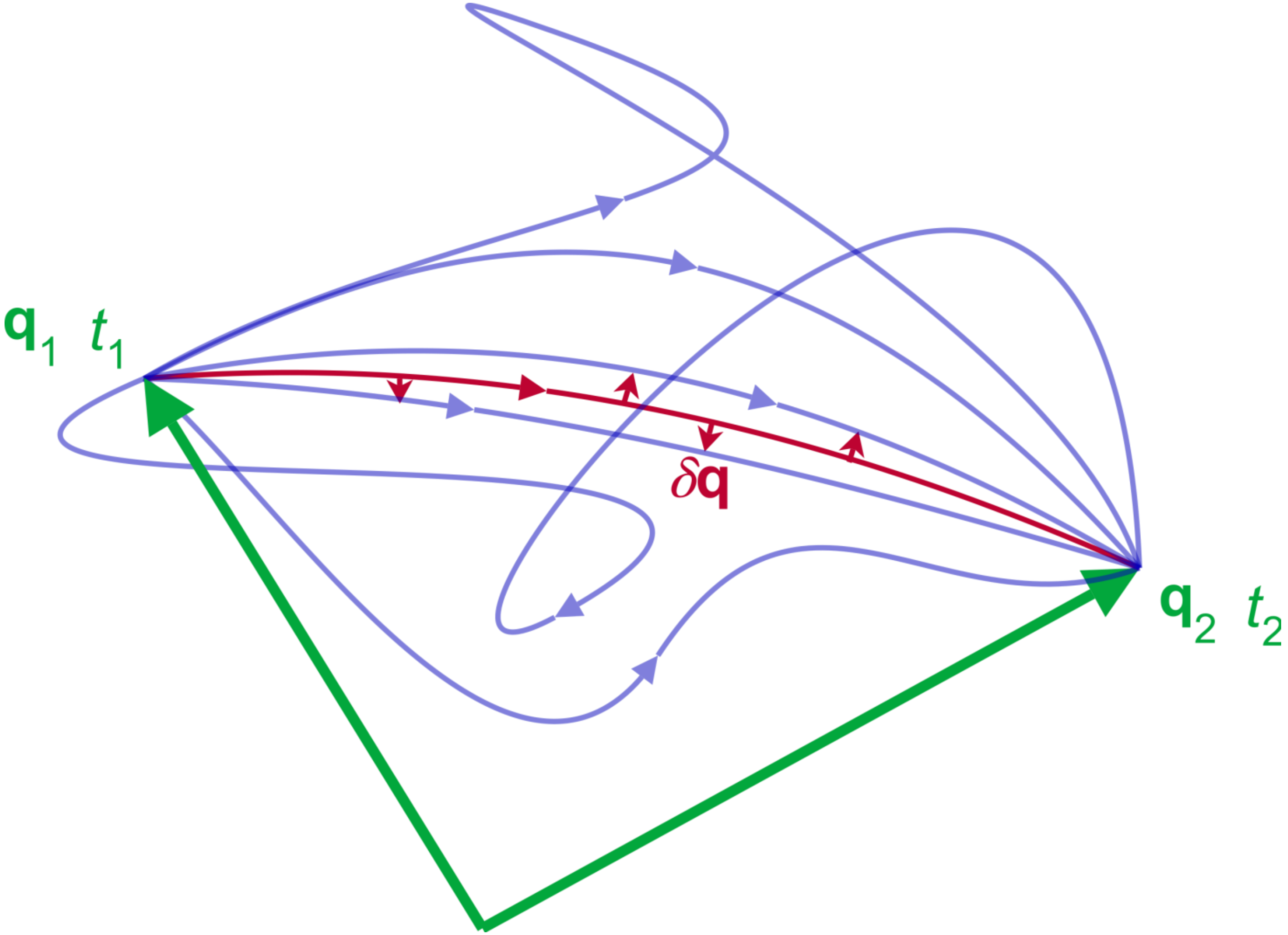
$$L = T - V$$

Kinetic Energy



Joseph-Louis Lagrange
(was pretty good at math)

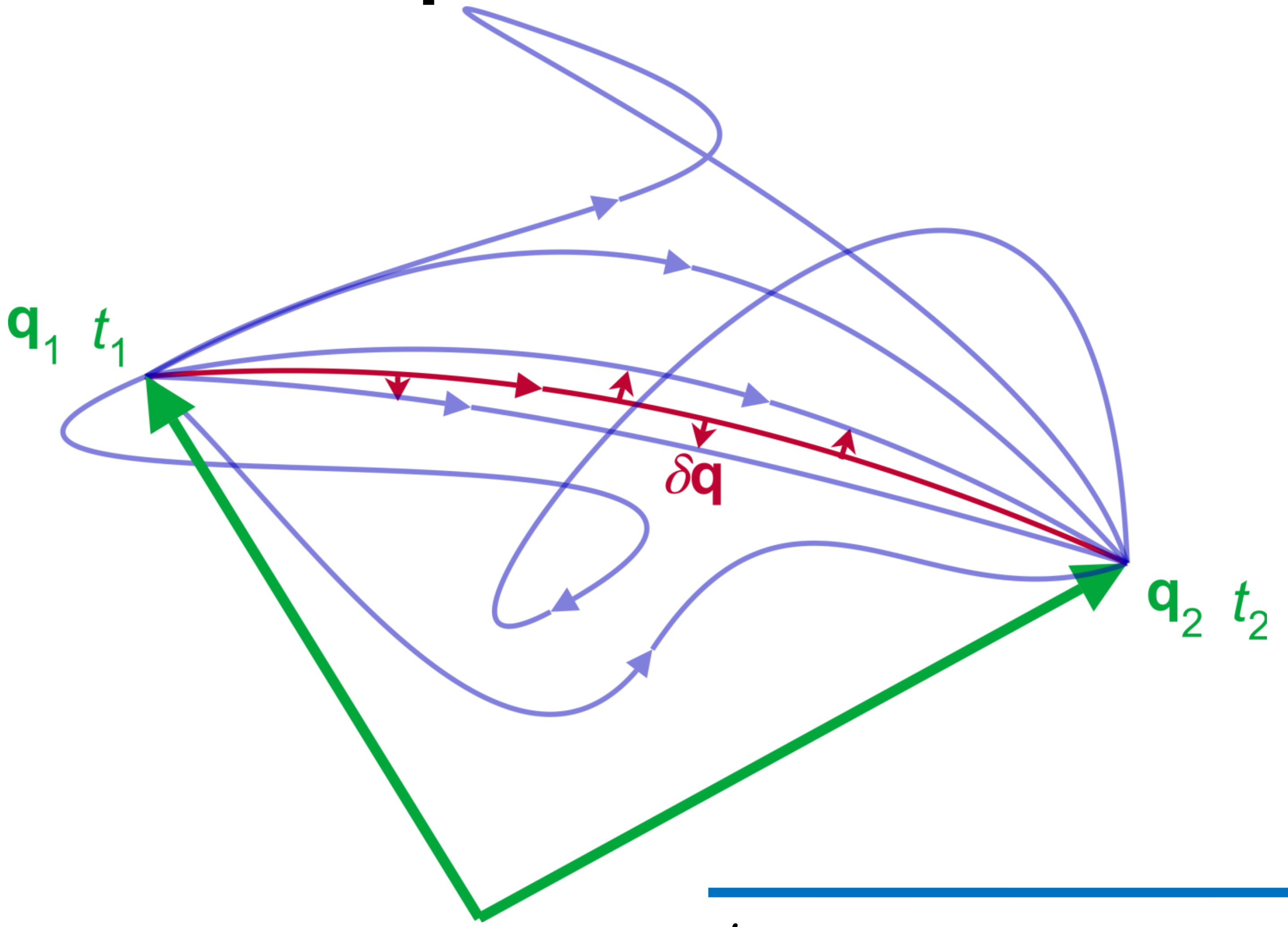
The Principle of Least Action



Assume you know the end points, find the path between them by finding a stationary point of the **ACTION**

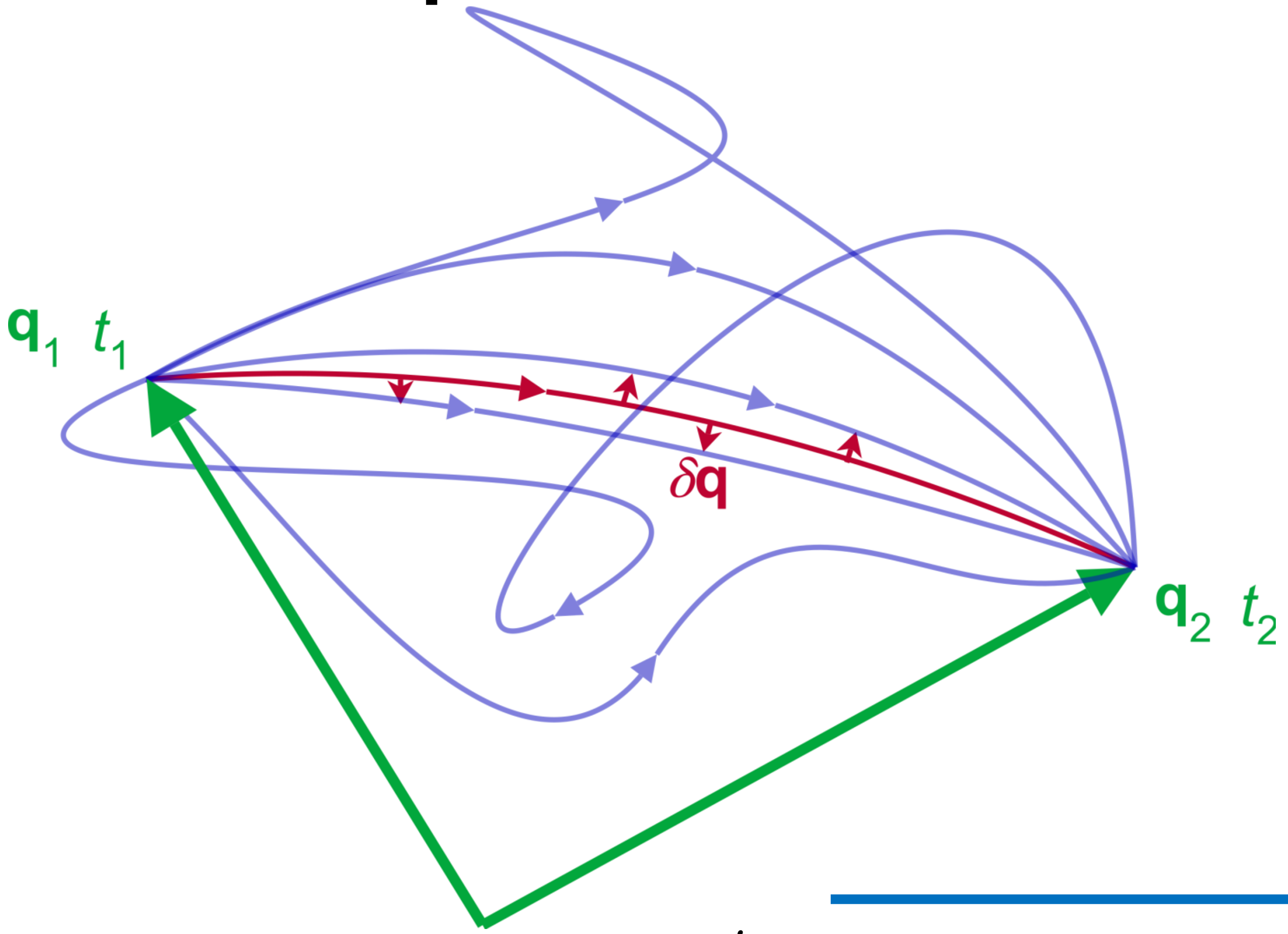
Gottfried Wilhelm Leibniz
(was pretty good at math)

The Principle of Least Action



$$S\left(\mathbf{q}(t),\dot{\mathbf{q}}(t)\right)=\overbrace{\int_{t_1}^{t_2} T\left(\mathbf{q}(t),\dot{\mathbf{q}}(t)\right)-V\left(\mathbf{q}(t),\dot{\mathbf{q}}(t)\right)dt}^{\text{functional}}$$

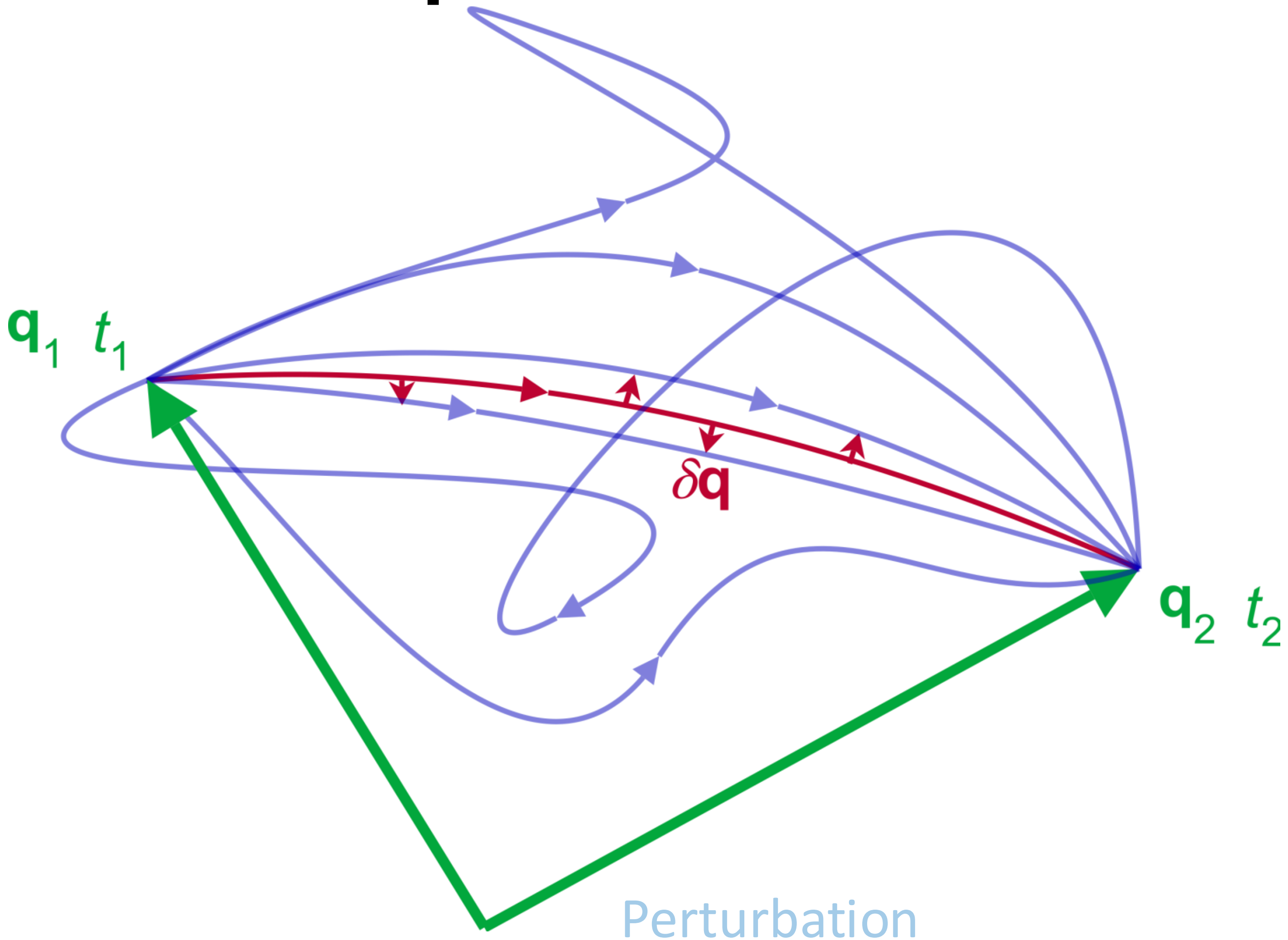
The Principle of Least Action



Lagrangian

$$S\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right)=\int_{t_1}^{t_2}\underbrace{T\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right)-V\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right)}\,dt$$

The Principle of Least Action



Minimize by finding a flat spot

Hunt for a flat spot by perturbing the trajectory and seeing if S changes.

$$S\left(\mathbf{q}+\delta\mathbf{q}, \dot{\mathbf{q}}+\overbrace{\delta\dot{\mathbf{q}}}^{\text{Perturbation}}\right) \quad = \quad S\left(\mathbf{q}\left(t\right), \dot{\mathbf{q}}\left(t\right)\right)$$

The Calculus of Variations

$$S\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right)=\int_{t_1}^{t_2}L\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right)dt$$

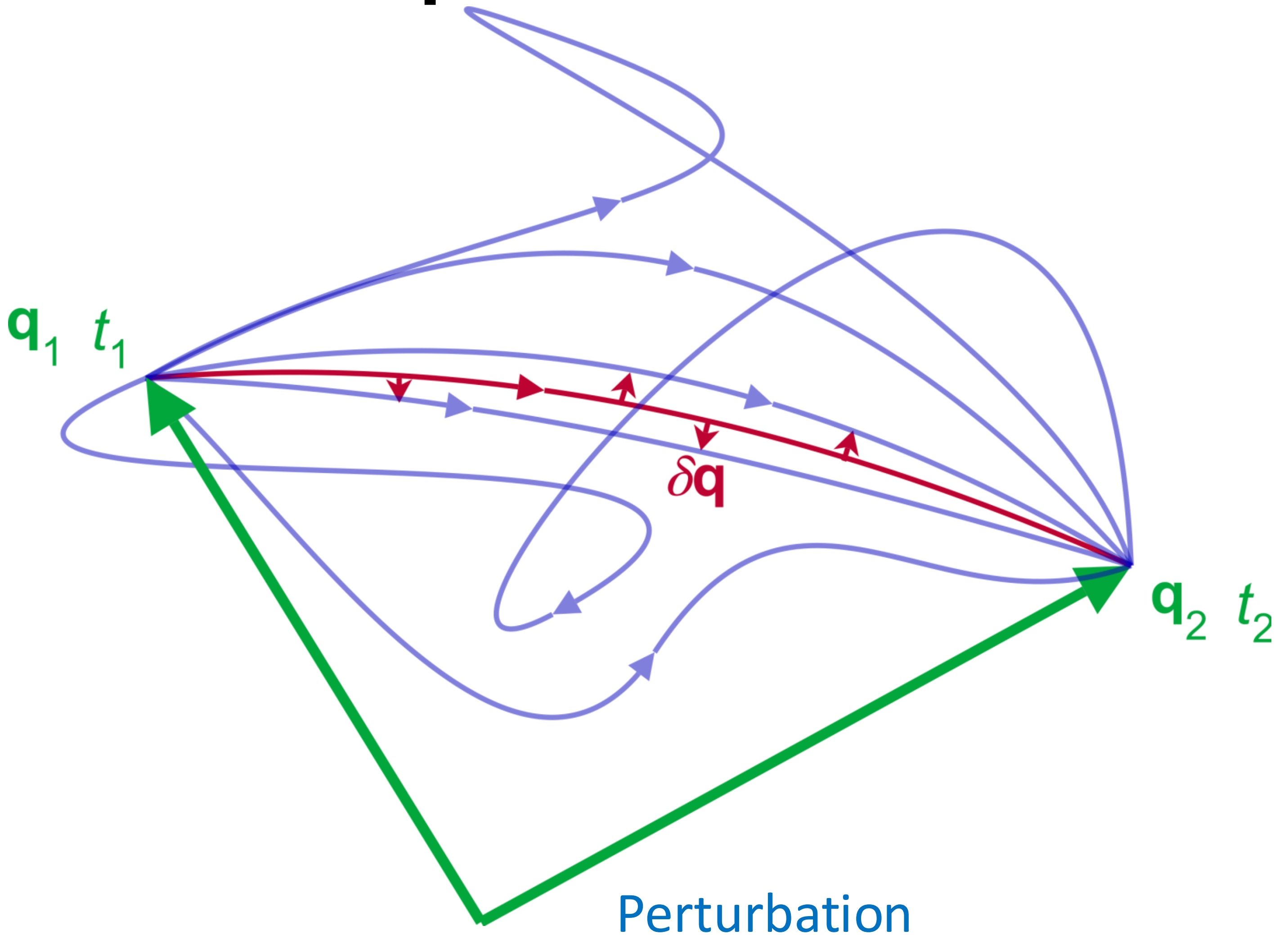
$$S\left(\mathbf{q}+\delta\mathbf{q},\dot{\mathbf{q}}+\delta\dot{\mathbf{q}}\right)=\int_{t_1}^{t_2}L\left(\mathbf{q}+\delta\mathbf{q},\dot{\mathbf{q}}+\delta\dot{\mathbf{q}}\right)dt$$

Apply Taylor Expansion

$$\approx\underbrace{\int_{t_1}^{t_2}L\left(\mathbf{q},\dot{\mathbf{q}}\right)dt}_{S\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right)}+\underbrace{\int_{t_1}^{t_2}\frac{\partial L}{\partial\mathbf{q}}\delta\mathbf{q}+\frac{\partial L}{\partial\dot{\mathbf{q}}}\delta\dot{\mathbf{q}}\,dt}_{\delta S\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right)}$$

First Variation

The Principle of Least Action



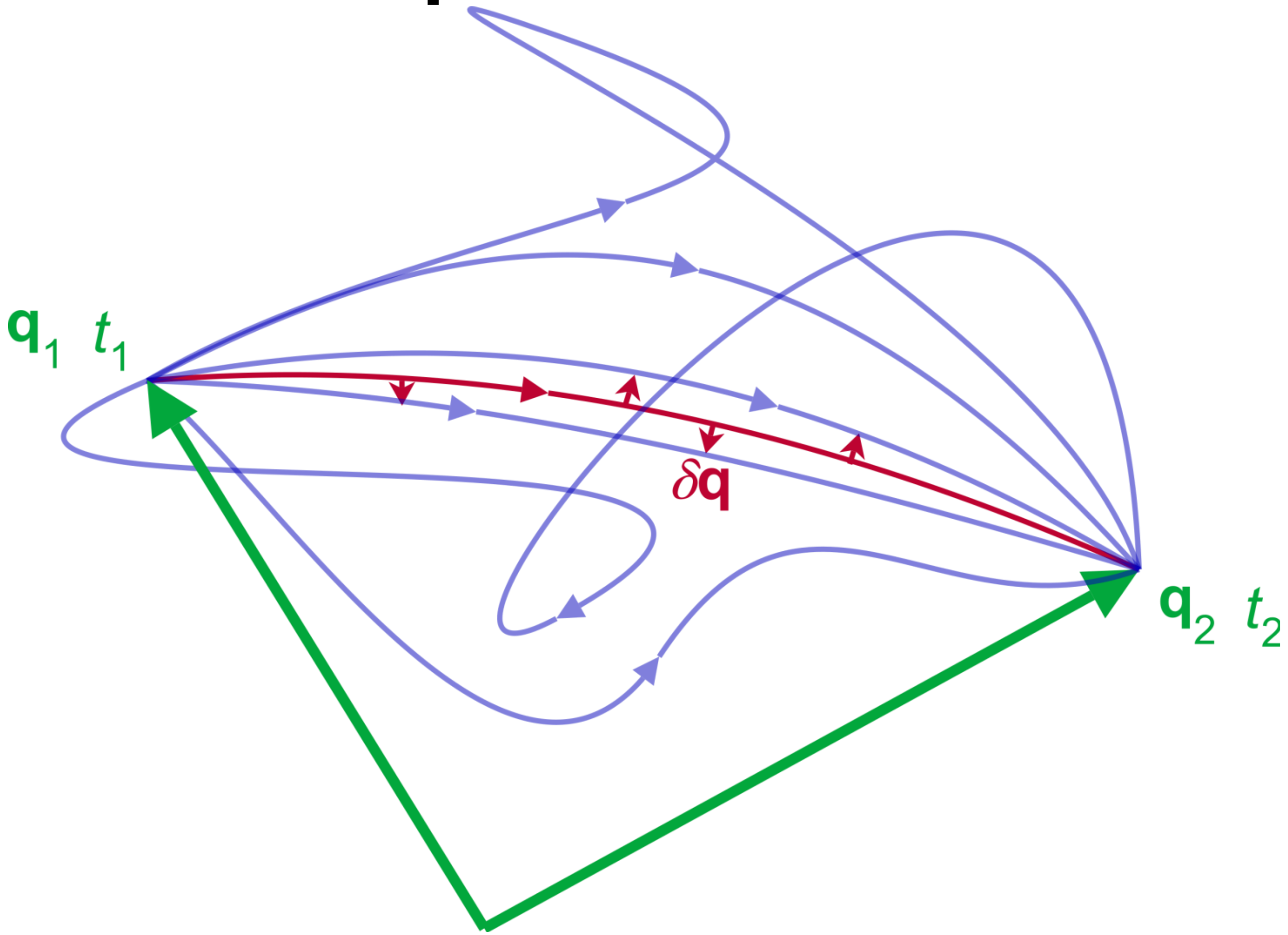
Minimize by finding a flat spot

Hunt for a flat spot by perturbing the trajectory and seeing if S changes.

Perturbation

$$S\left(\mathbf{q}+\delta\mathbf{q},\dot{\mathbf{q}}+\delta\dot{\mathbf{q}}\right) = S\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right)$$

The Principle of Least Action



Minimize by finding a flat spot

Hunt for a flat spot by perturbing the trajectory and seeing if S changes.

$$\delta S\left(\mathbf{q}\left(t\right),\dot{\mathbf{q}}\left(t\right)\right) \qquad = \qquad 0$$

Back to the Calculus of Variations

$$\int_{t_1}^{t_2} \frac{\partial L}{\partial \mathbf{q}} \delta \mathbf{q} + \underbrace{\frac{\partial L}{\partial \dot{\mathbf{q}}} \delta \dot{\mathbf{q}}}_{} \, dt = 0$$

Apply Integration by Parts

$$\int_{t_1}^{t_2} \frac{\partial L}{\partial \mathbf{q}} \delta \mathbf{q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{\mathbf{q}}} \delta \mathbf{q} \, dt + \underbrace{\frac{\partial L}{\partial \dot{\mathbf{q}}} \delta \mathbf{q} \Big|_{t_0}^{t_1}}_{} = 0$$

Uh Oh, Boundary Conditions

DON'T PANIC: Remember that **you know the end points**, so the variation there is 0.

Back to the Calculus of Variations

$$\int_{t_1}^{t_2} \frac{\partial L}{\partial \mathbf{q}} \delta \mathbf{q} + \underbrace{\frac{\partial L}{\partial \dot{\mathbf{q}}} \delta \dot{\mathbf{q}}}_{\text{Apply Integration by Parts}} \, dt = 0$$

$$\int_{t_1}^{t_2} \frac{\partial L}{\partial \mathbf{q}} \delta \mathbf{q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{\mathbf{q}}} \delta \mathbf{q} \, dt + \underbrace{\cancel{\left. \frac{\partial L}{\partial \dot{\mathbf{q}}} \delta \mathbf{q} \right|_{t_1}^{t_2}}}_{\text{Uh Oh, Boundary Conditions}} = 0$$

DON'T PANIC: Remember that **you know the end points**, so the variation there is 0.

Back to the Calculus of Variations

$$\int_{t_1}^{t_2} \frac{\partial L}{\partial \mathbf{q}}\delta\mathbf{q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}}\delta\mathbf{q}\; dt = 0$$

A little bit o' factoring

$$\int_{t_1}^{t_2} \left(\frac{\partial L}{\partial \mathbf{q}} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}}\right)\delta\mathbf{q}\; dt = 0$$

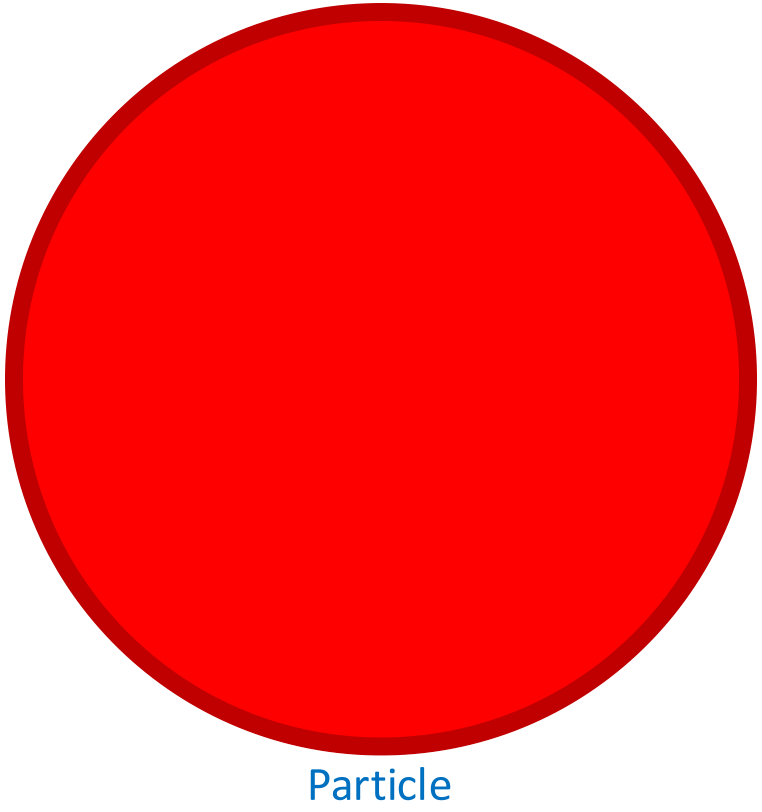
Say the magic words -- "If $\delta\mathbf{q}$ is an arbitrary variation then the integrand must always be zero"

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}} = \frac{\partial L}{\partial \mathbf{q}}$$

Euler-Lagrange Equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}} = \frac{\partial L}{\partial \mathbf{q}}$$

Example Physical System



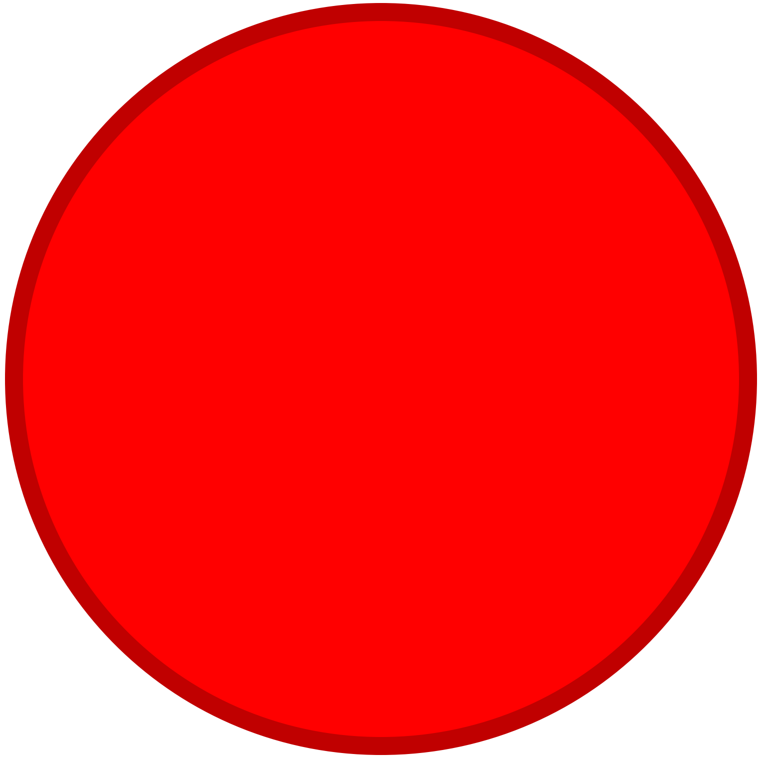
Position in space (m) $\mathbf{x}(t)$

Velocity in space (m/s) $\mathbf{v}(t) = \frac{d\mathbf{x}}{dt}(t)$

Acceleration in space (m/s^2) $\mathbf{a}(t) = \frac{d^2\mathbf{x}}{dt^2}(t)$

Mass (kg) m

Example Physical System



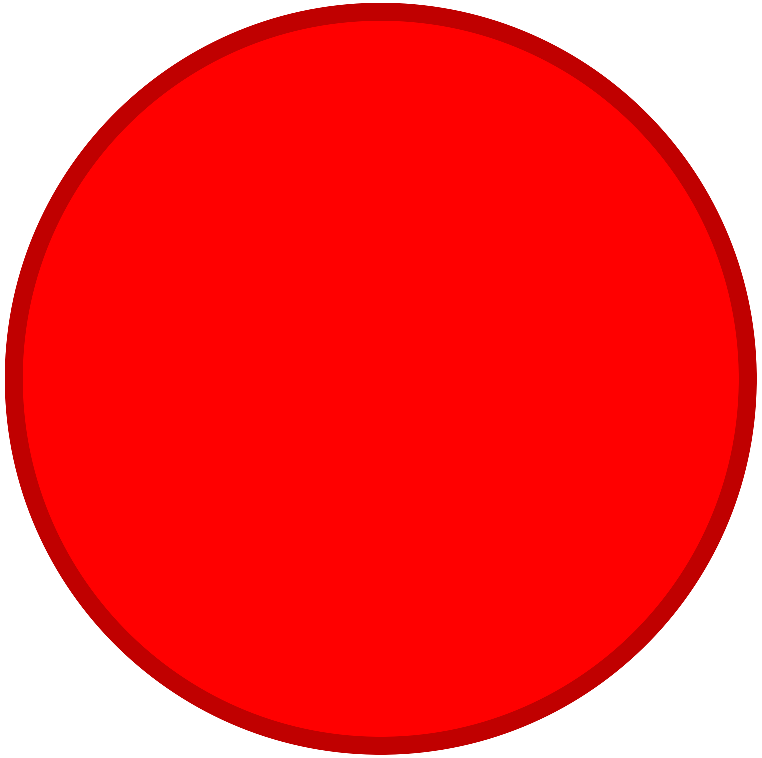
Position in space (m) $\mathbf{q}(t) = \mathbf{x}\left(t\right)$

Velocity in space (m/s) $\dot{\mathbf{q}}(t) = \mathbf{v}\left(t\right) = \frac{d\mathbf{x}}{dt}\left(t\right)$

Acceleration (m/s²) $\ddot{\mathbf{q}}(t) = \mathbf{a}\left(t\right) = \frac{d^2\mathbf{x}}{dt^2}\left(t\right)$

Mass (kg) m

Energies



Particle

Kinetic Energy = $\frac{1}{2} m \dot{\mathbf{q}}^T \dot{\mathbf{q}}$

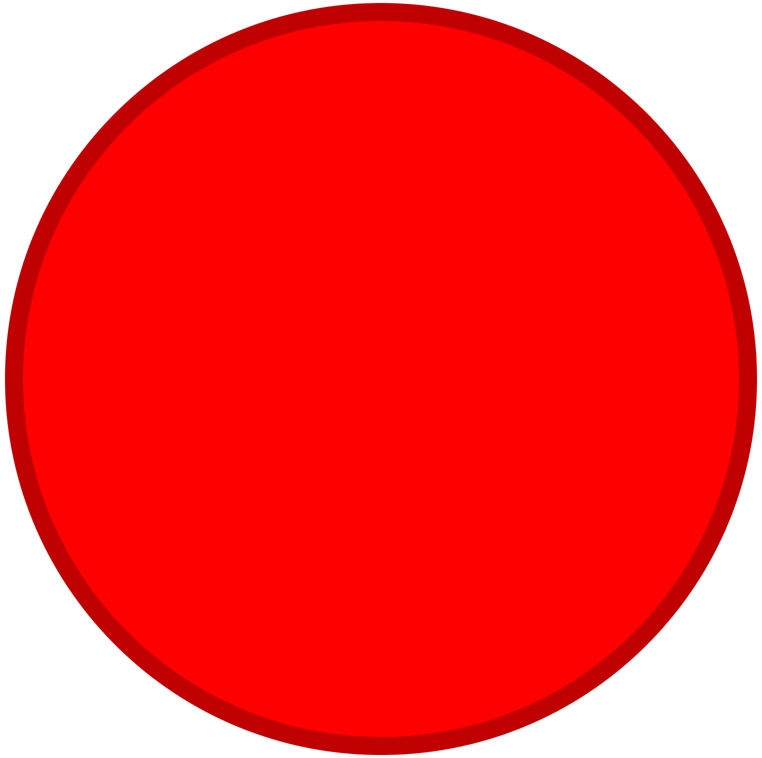
Potential Energy = $-m \mathbf{q}^T \underbrace{\mathbf{g}}_{\text{Gravitational Acceleration}}$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}} = \frac{\partial L}{\partial \mathbf{q}}$$

$$\frac{d}{dt} m\dot{\mathbf{q}} \qquad m\mathbf{g}$$

$$m\ddot{\mathbf{q}}$$

Equations of Motion

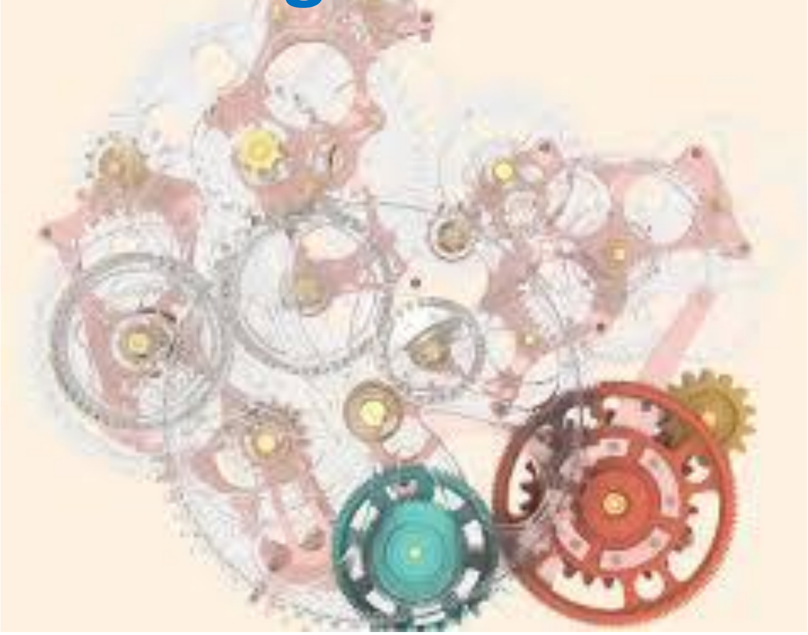


Particle

$$m\ddot{\mathbf{q}} = m\mathbf{g}$$

Newton's Second Law for Particle Under Gravity

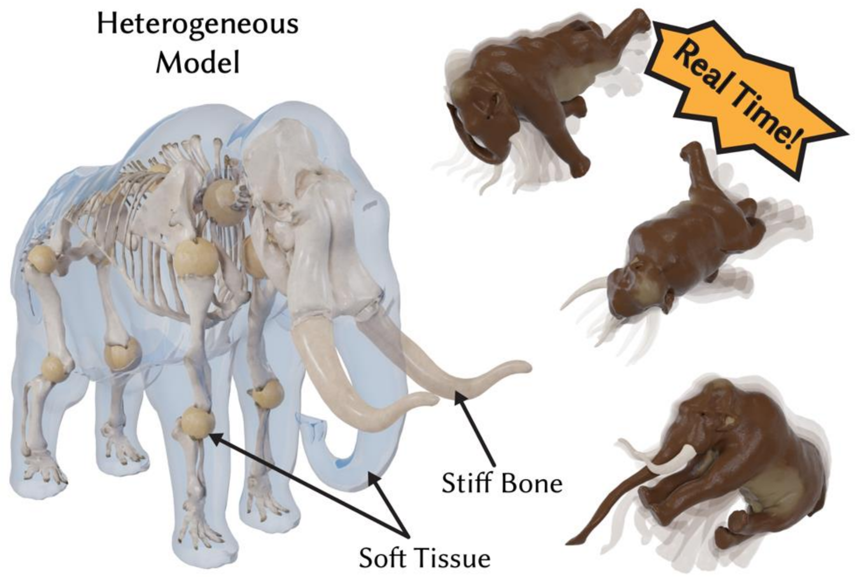
Rigid Bodies

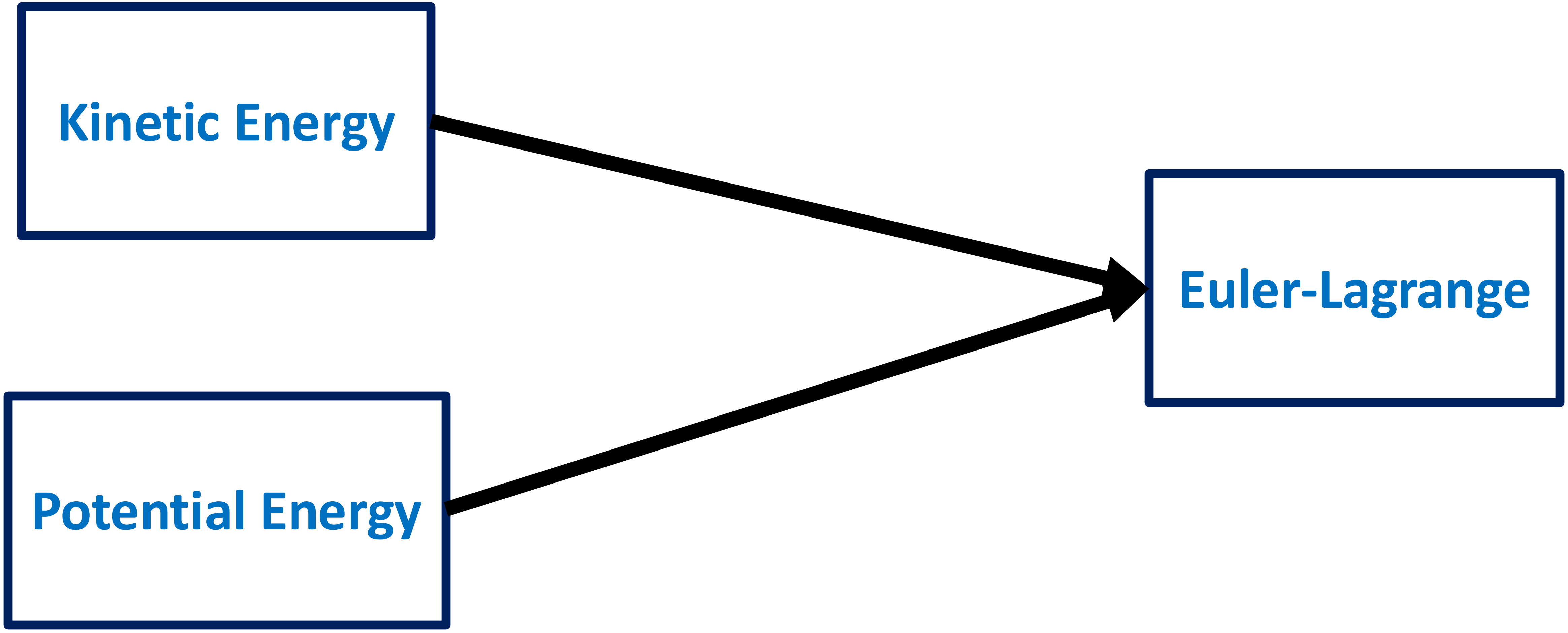


Cloth

Volumes

Reduced-Order

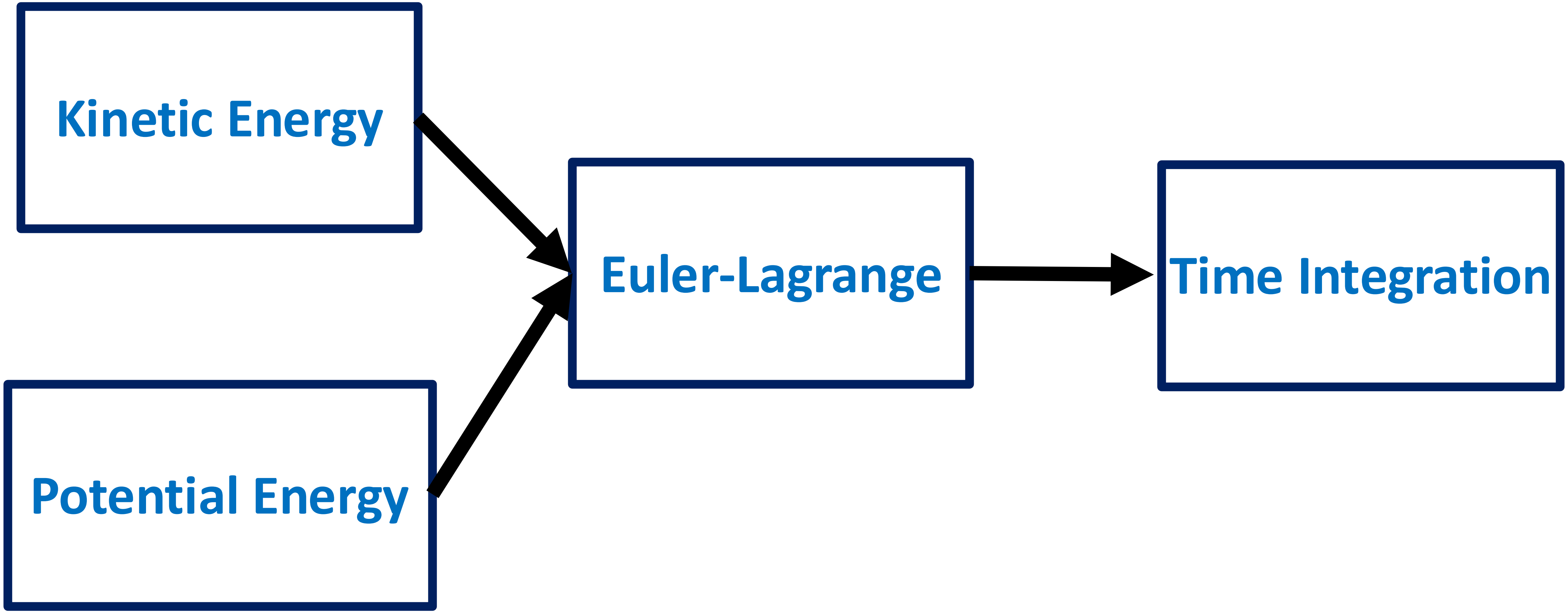



Kinetic Energy
Potential Energy
Euler-Lagrange

Correct Equation

$$m\ddot{\mathbf{q}} = m\mathbf{g}$$

How does this become an animation?


Kinetic Energy
Potential Energy
Euler-Lagrange
Time Integration

skeleton

Image courtesy of Weta Digital

What's Coming Up

Today

First Tutorial And Quiz Next Monday

Next Lecture

Deformation and the Finite Element Method

The End ☺

Questions ?