

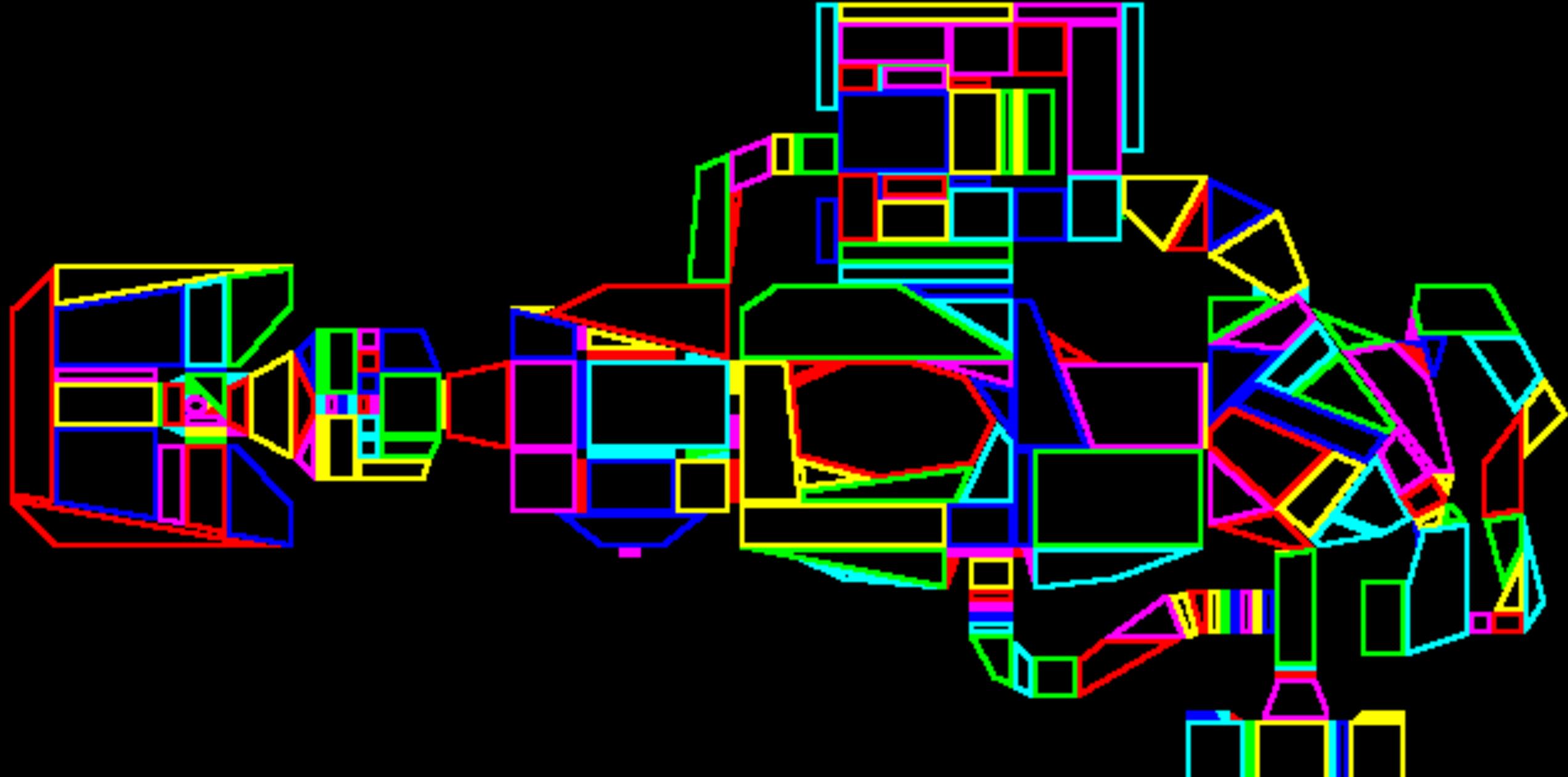
A Star Wars scene featuring Stormtroopers on a beach at sunset. In the foreground, a Stormtrooper stands on the right, holding a blaster. In the background, another Stormtrooper stands near some palm trees. A speeder bike flies through the sky above the horizon, leaving a trail of smoke. The sky is filled with warm, orange and yellow hues of a setting sun.

CSC418/2504 Computer Graphics

Rock

Some Slides/Images adapted from Marschner and Shirley

Today: Boundary Volume Hierarchies



Announcements

Assignment 1 grades soon

Assignment 3 is due on Friday

Assignment 4 is out soon

A4 requires OpenGL – *we officially support this on CDF only.*

Any Questions ?

Today: Boundary Volume Hierarchy

Common Geometric Queries on Graphics

Bounding Volumes

- Spheres

- Boxes

Object-Partitioning Hierarchies

- Sphere Trees

- AABB Trees

Space-Partitioning Hierarchies

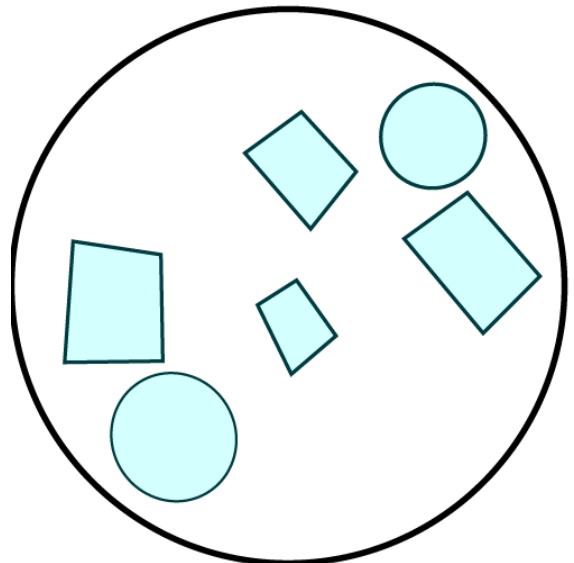
- Uniform Spatial Subdivision

- Axis-Aligned Spatial Subdivision

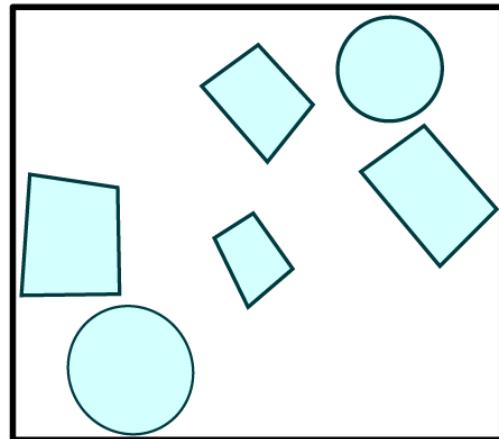
Bounding Volumes (BVs)

“Simple” geometry that fully encloses a **collection** of other geometry

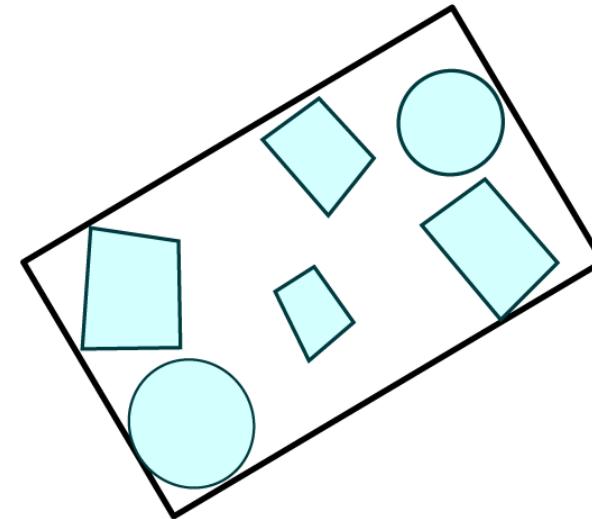
Sphere



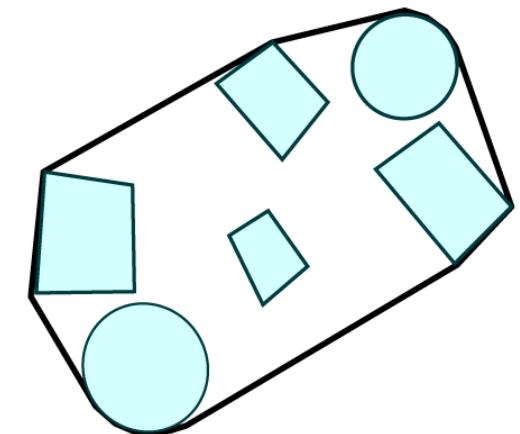
Object Oriented Bounding Box
(OOBB)



Axis-Aligned Bounding Box
(AABB)



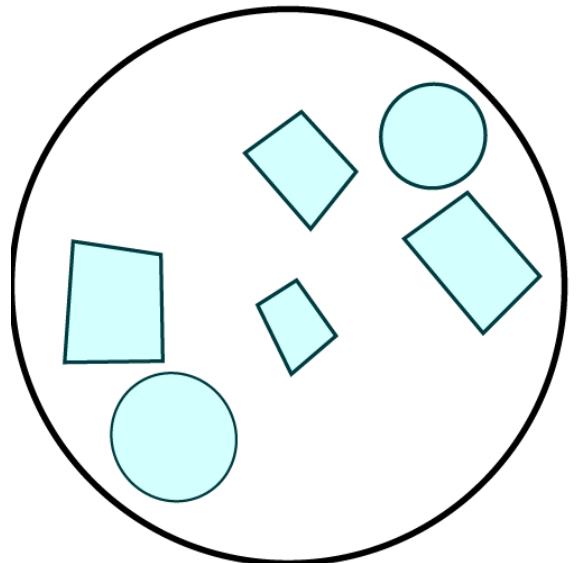
Convex Hull



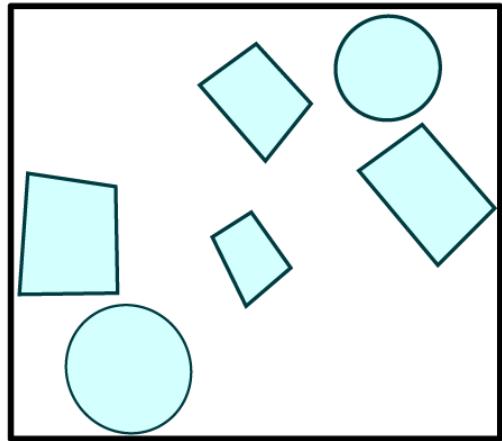
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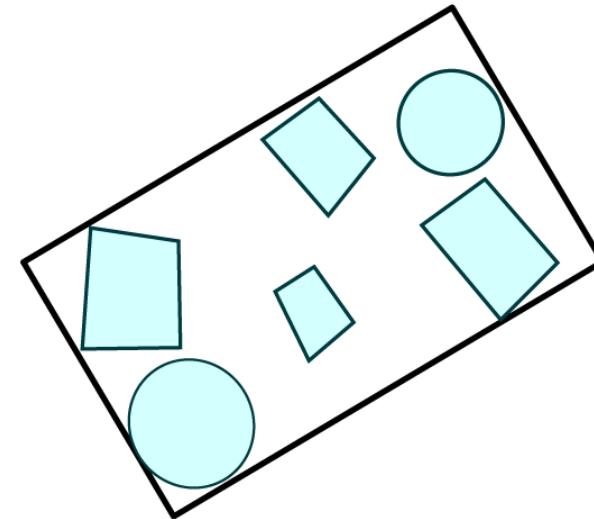
Sphere (A lot)



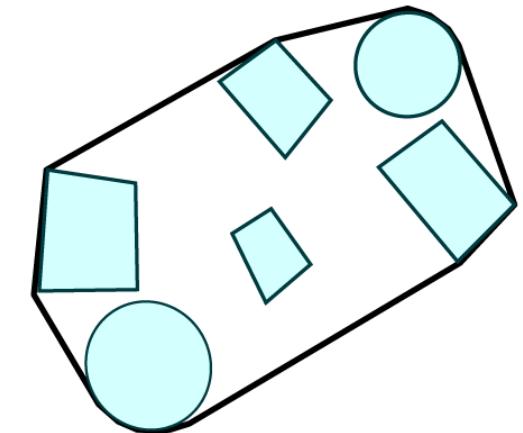
Object Oriented Bounding Box
(OOBB) (A little)



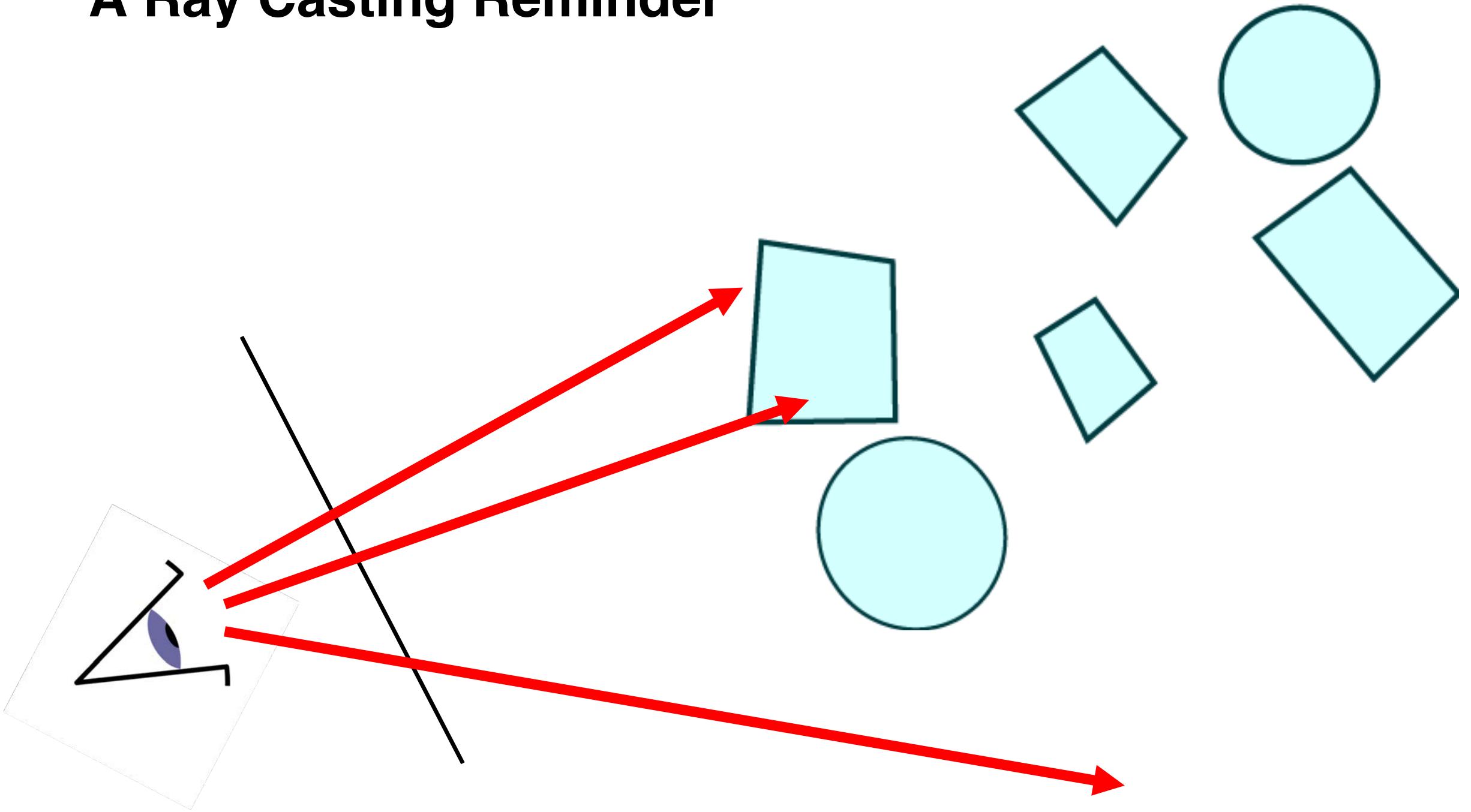
Axis-Aligned Bounding Box
(A lot)



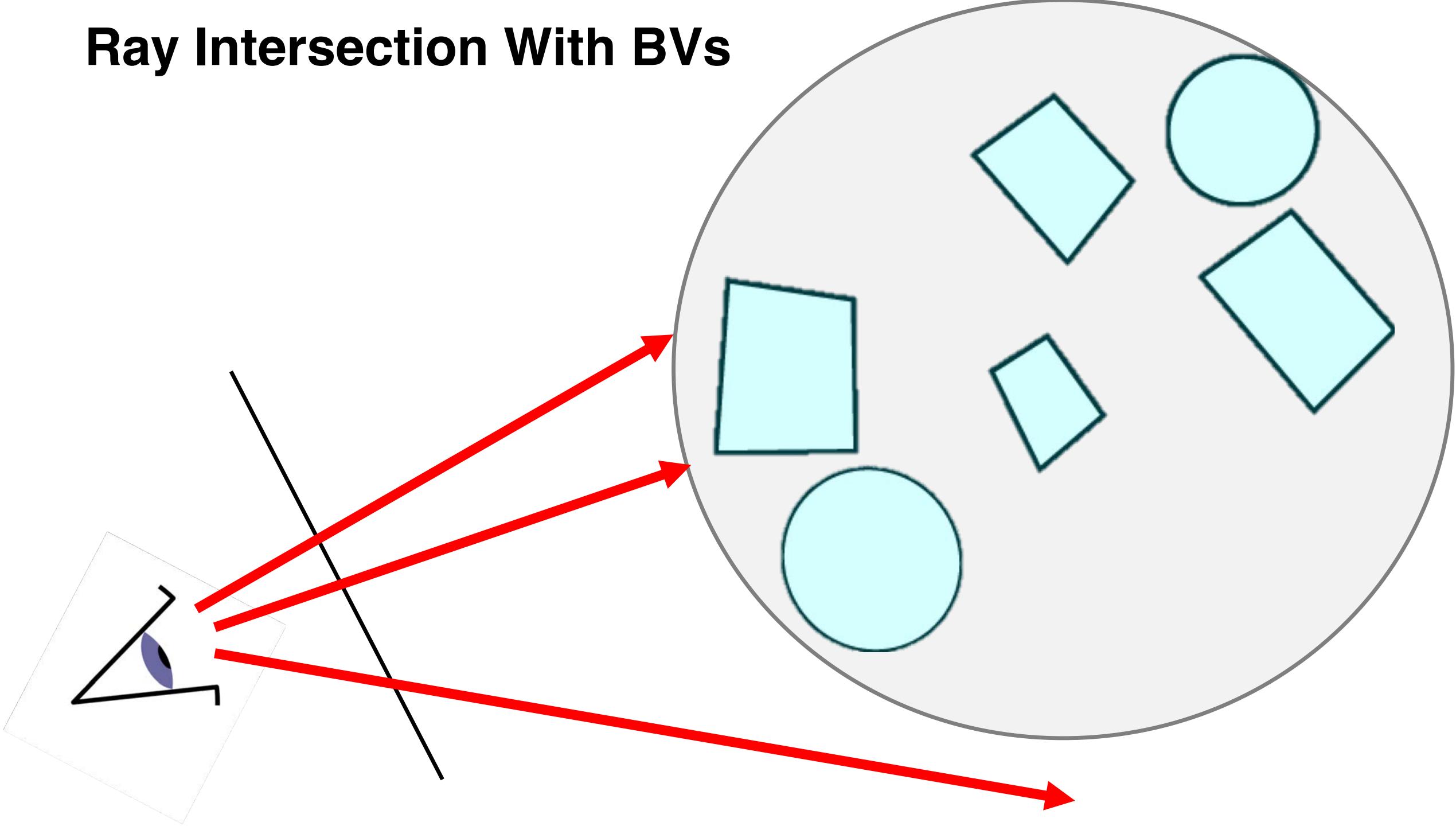
Convex Hull
(Nope)



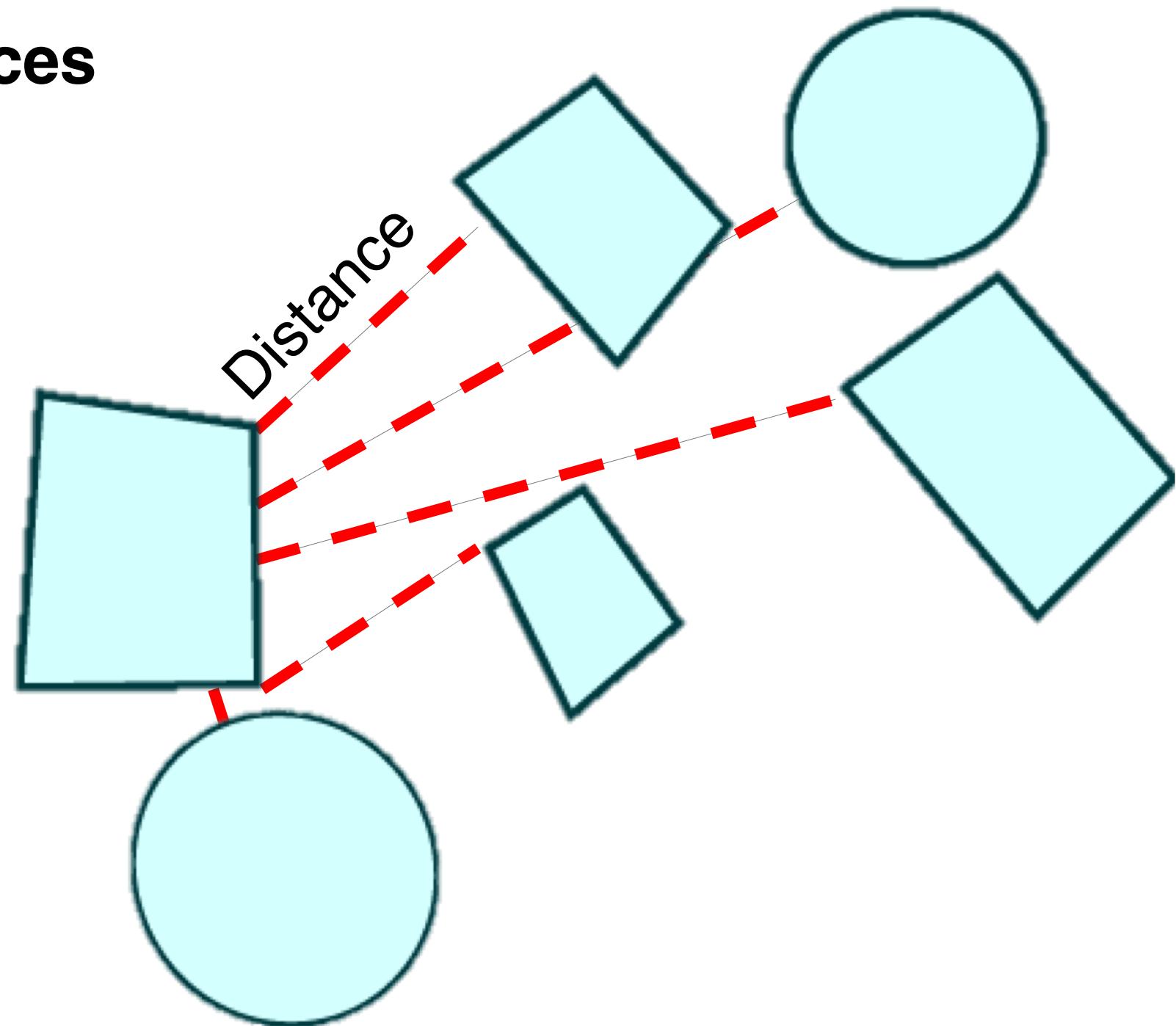
A Ray Casting Reminder



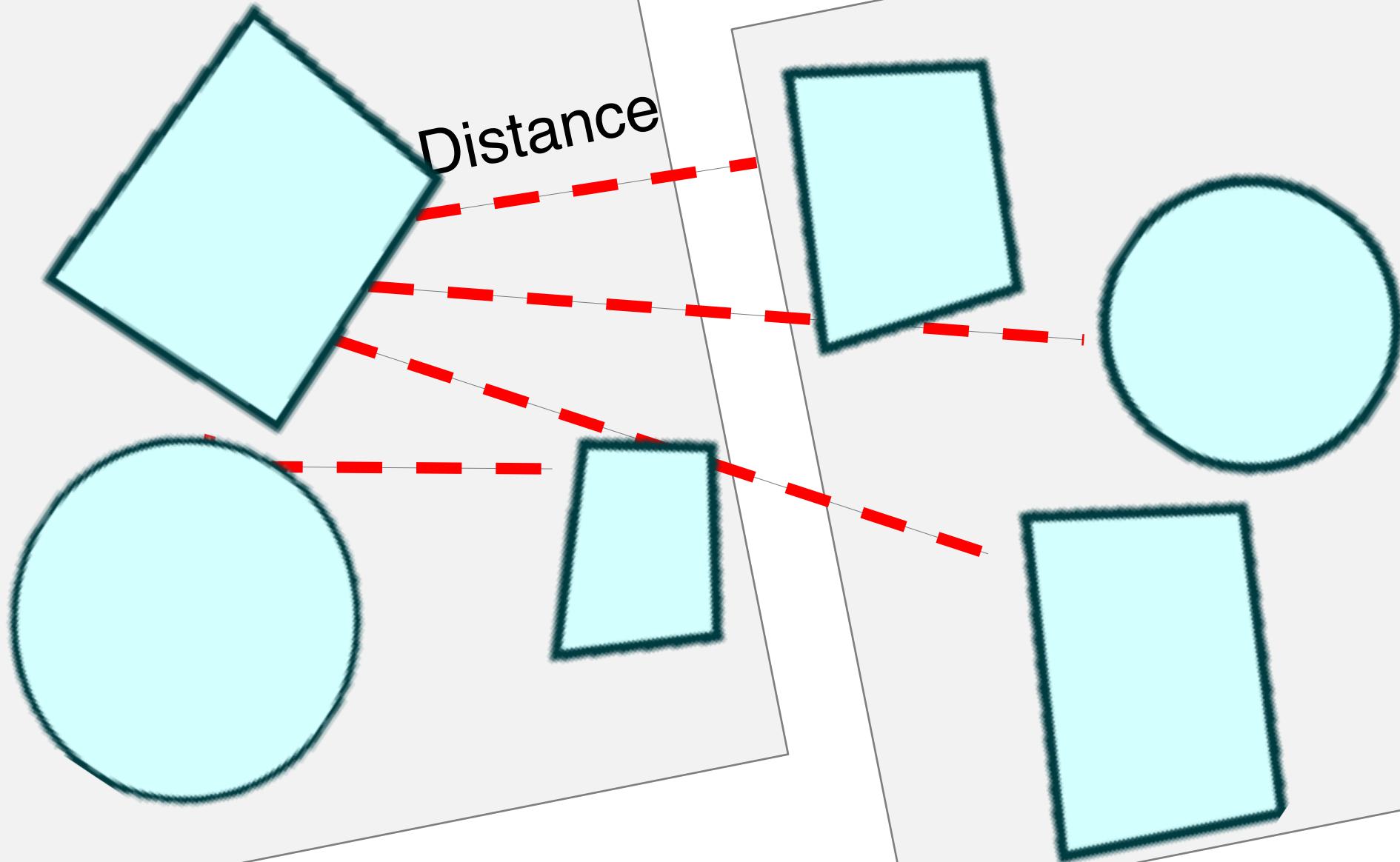
Ray Intersection With BVs



Closest Distances



Closest Distances with BVs

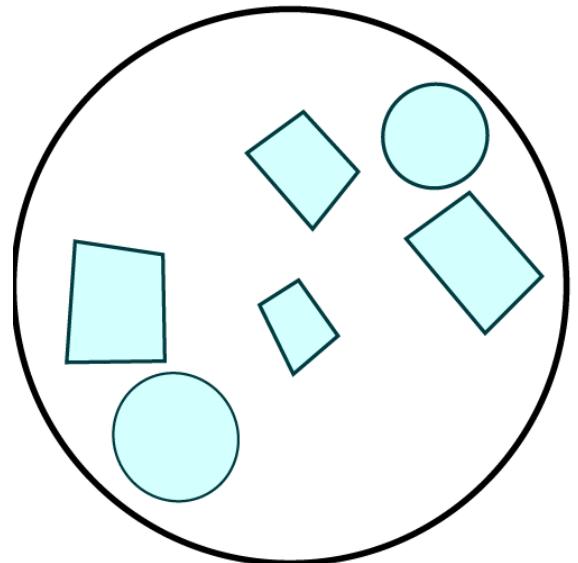


Bounding Volumes (BVs)

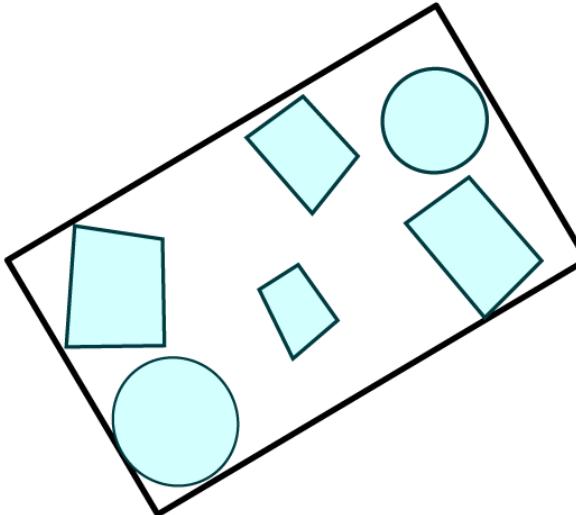
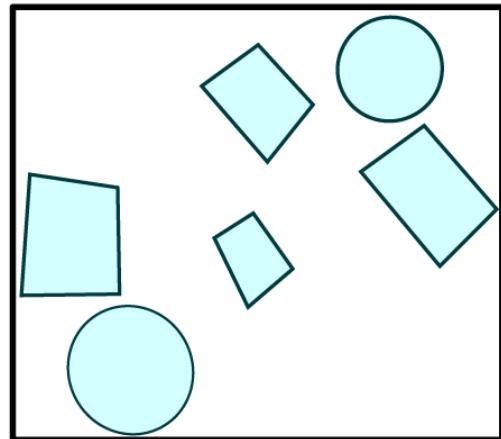
“Simple” geometry that fully encloses a **collection** of other geometry

Should fit geometry tightly

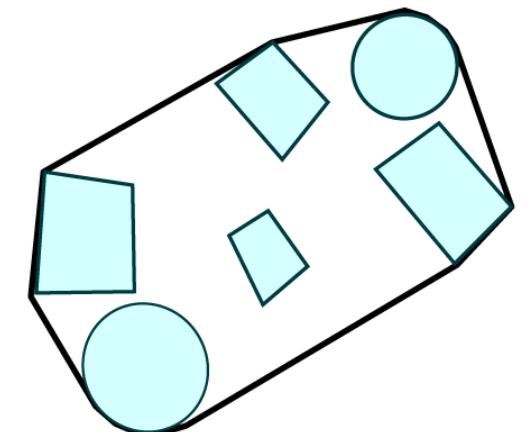
Sphere



Object Oriented Bounding Box
(OOBB)



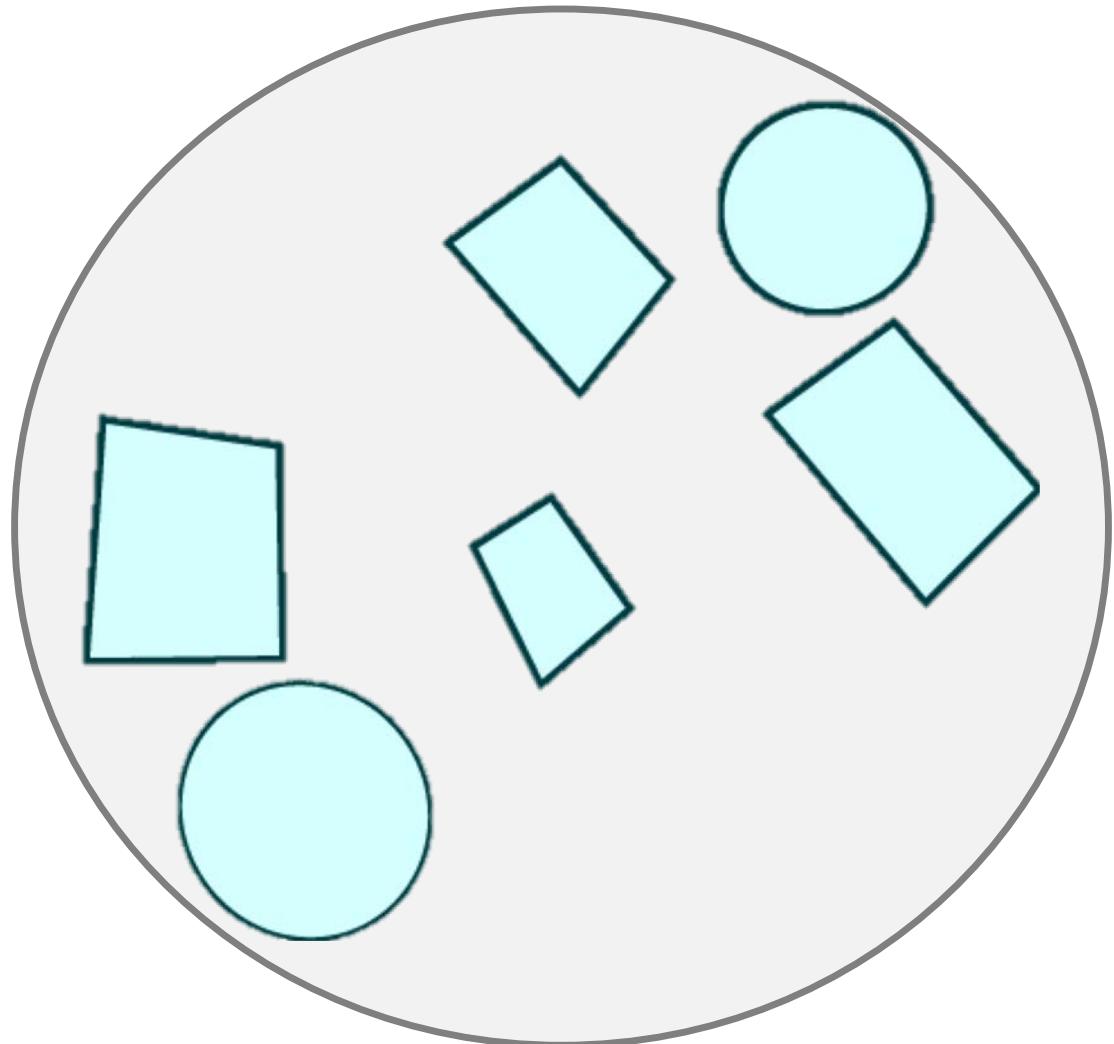
Axis-Aligned Bounding Box
(AABB)



Convex Hull

Building a Bounding Sphere

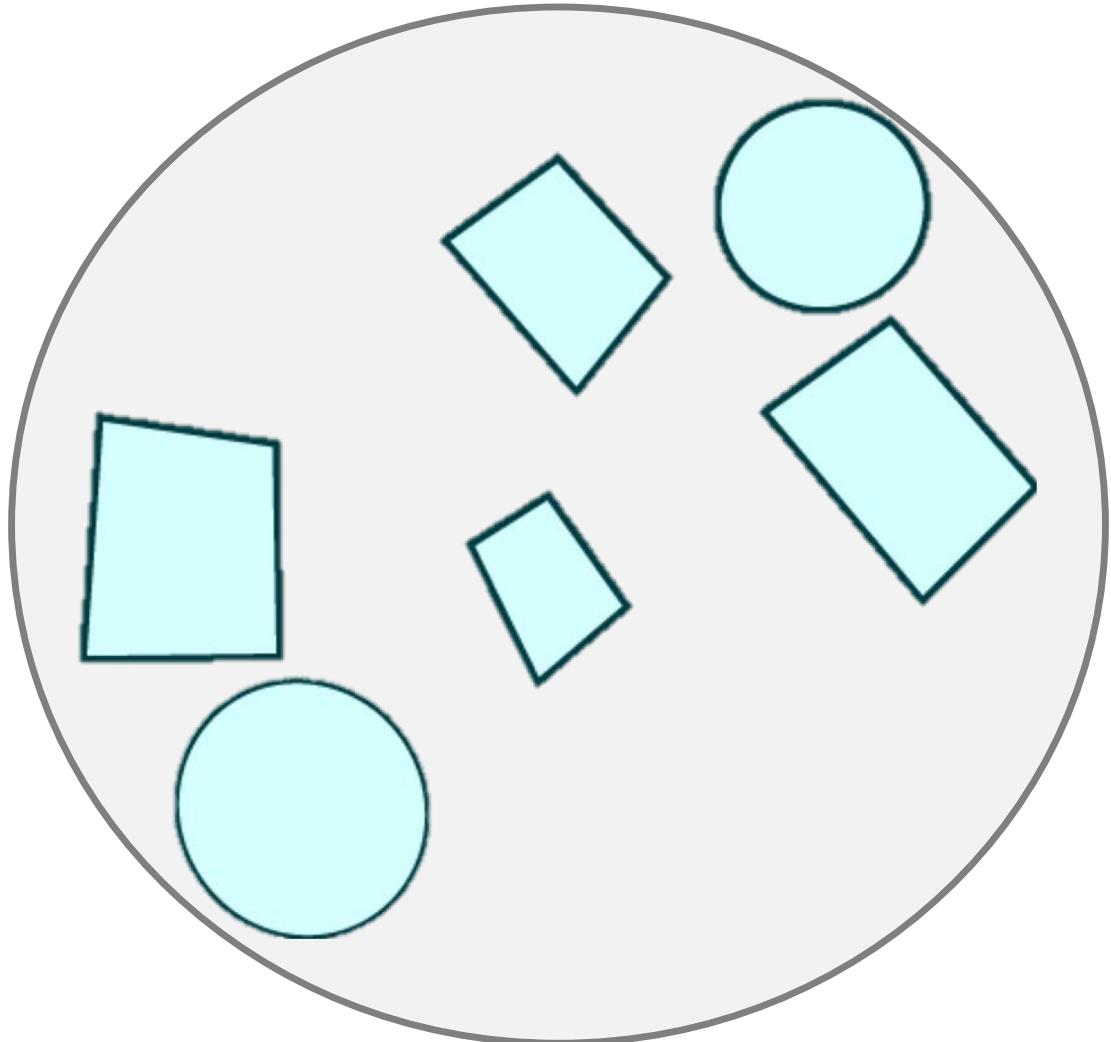
Parameters of a Sphere ?



Building a Bounding Sphere

Parameters of a Sphere:

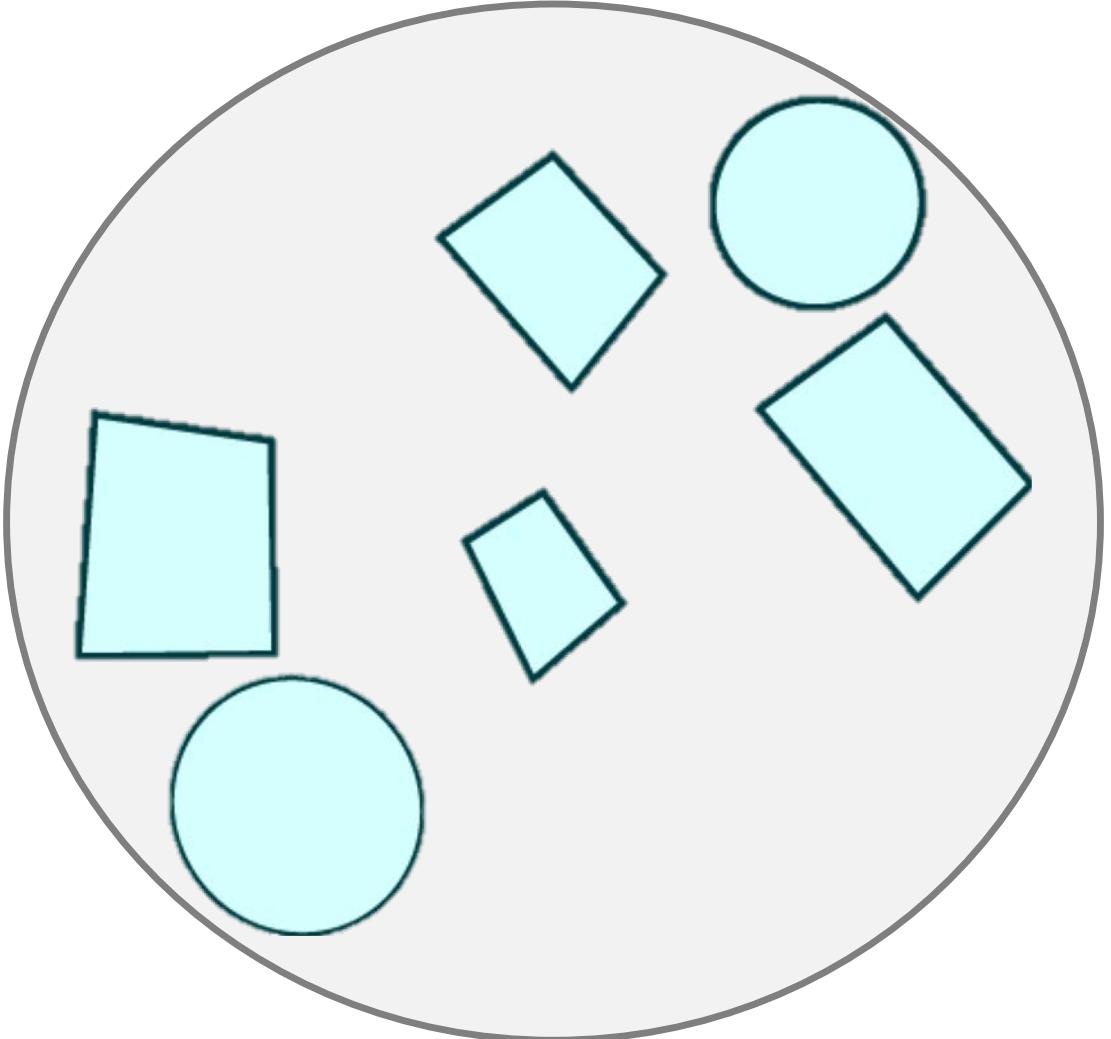
1. Center = ?
2. Radius = ?



Building a Bounding Sphere

Parameters of a Sphere:

1. Center = $\mathbf{c} = \frac{1}{n} \sum_{i=1}^n \mathbf{v}^i$
2. Radius = $r = \max (\mathbf{v}^i - \mathbf{c})$
 $\mathbf{v}^i \in \text{Vertices}$



Ray-Sphere Intersection



Ray-Sphere Intersection

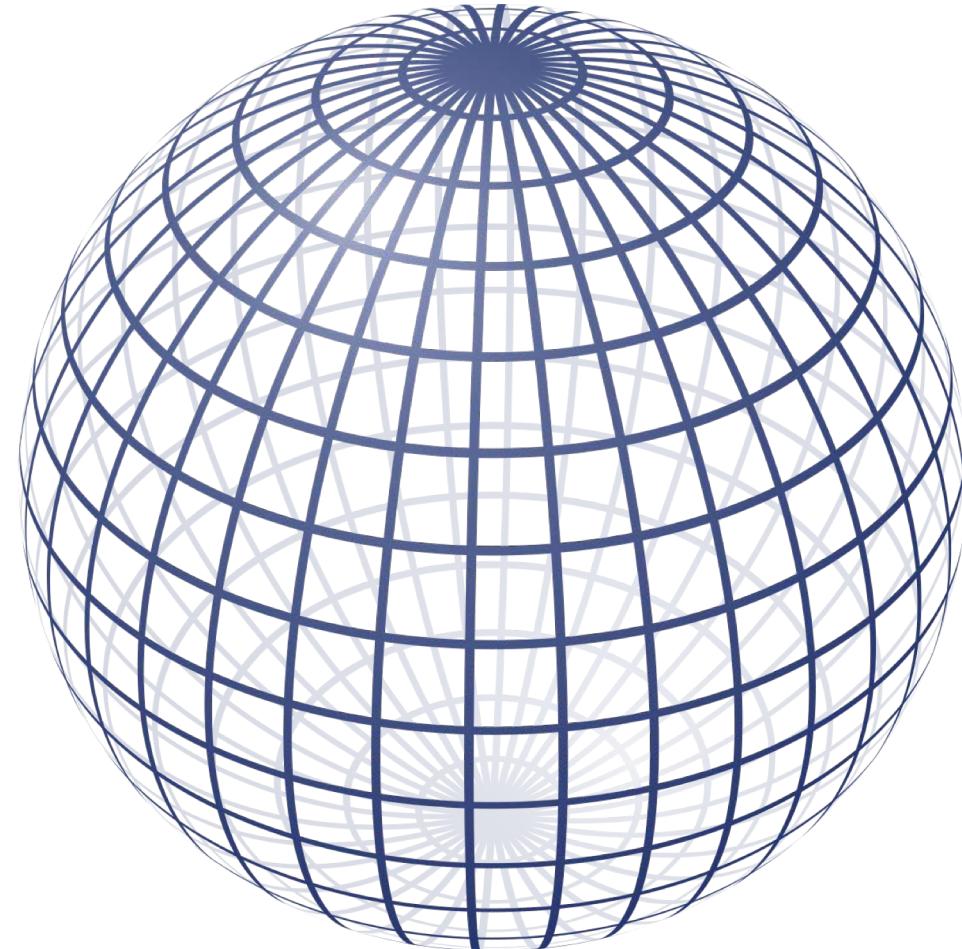
$$\mathbf{p}(t)^T \mathbf{p}(t) - r^2 = 0$$

$$a \cdot t^2 + b \cdot t + c = 0$$

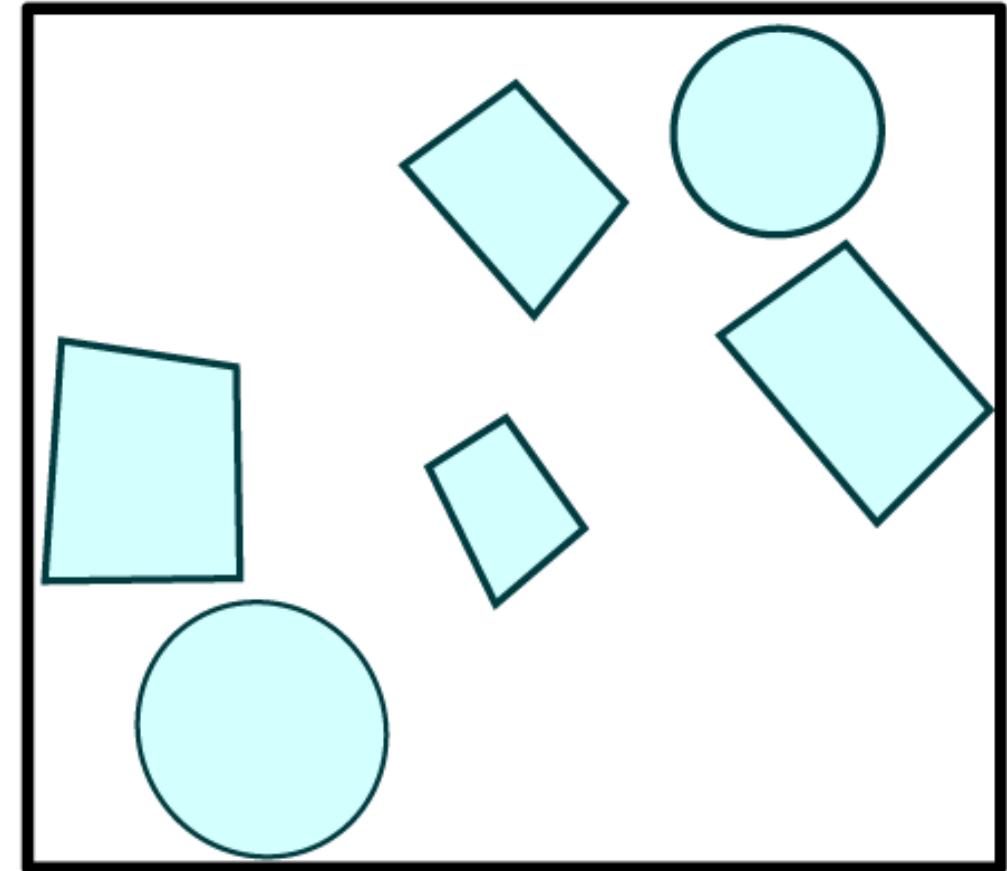
$$a = (\mathbf{s} - \mathbf{e})^T (\mathbf{s} - \mathbf{e})$$

$$b = 2\mathbf{e}^T (\mathbf{s} - \mathbf{e})$$

$$c = \mathbf{e}^T \mathbf{e} - r^2$$



Building and Axis-Aligned Bounding Box (AABB)



Building and Axis-Aligned Bounding Box (AABB)

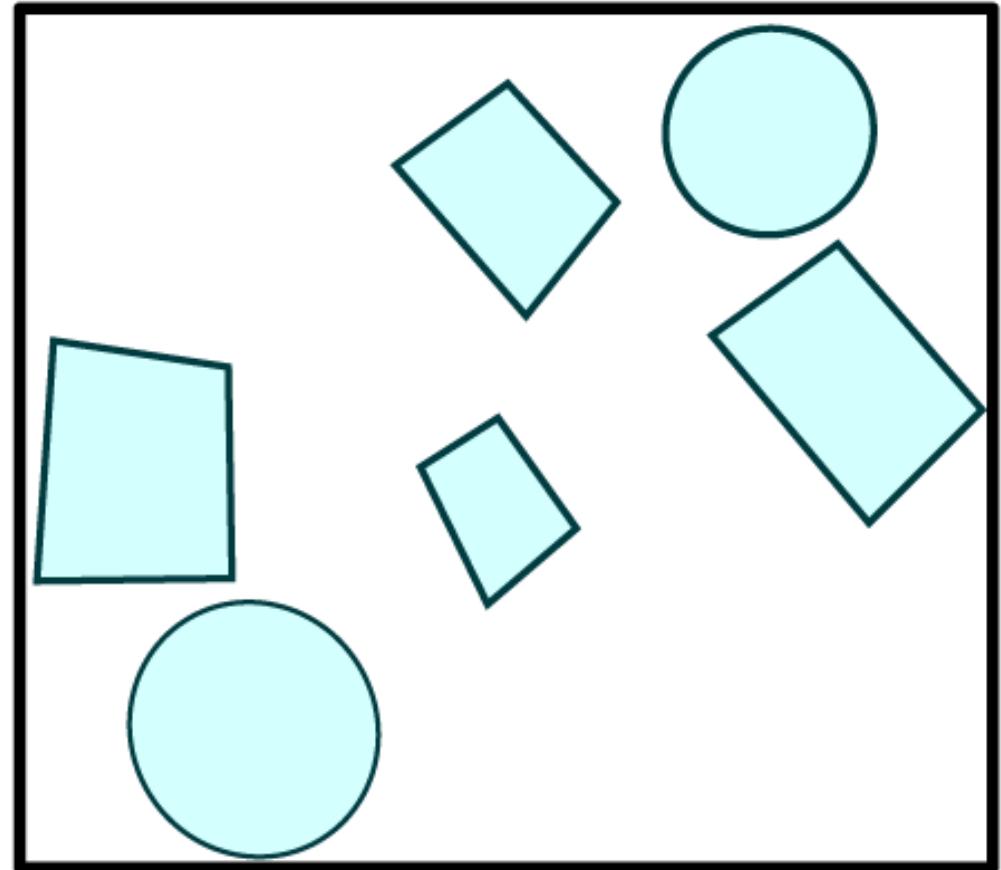
$$x_{min} = \min(v_x^i)$$

$$x_{max} = \max(v_x^i)$$

$$y_{min} = \min(v_y^i)$$

$$y_{max} = \max(v_y^i)$$

$\mathbf{v}^i \in \text{Vertices}$



Ray-AABB Intersection

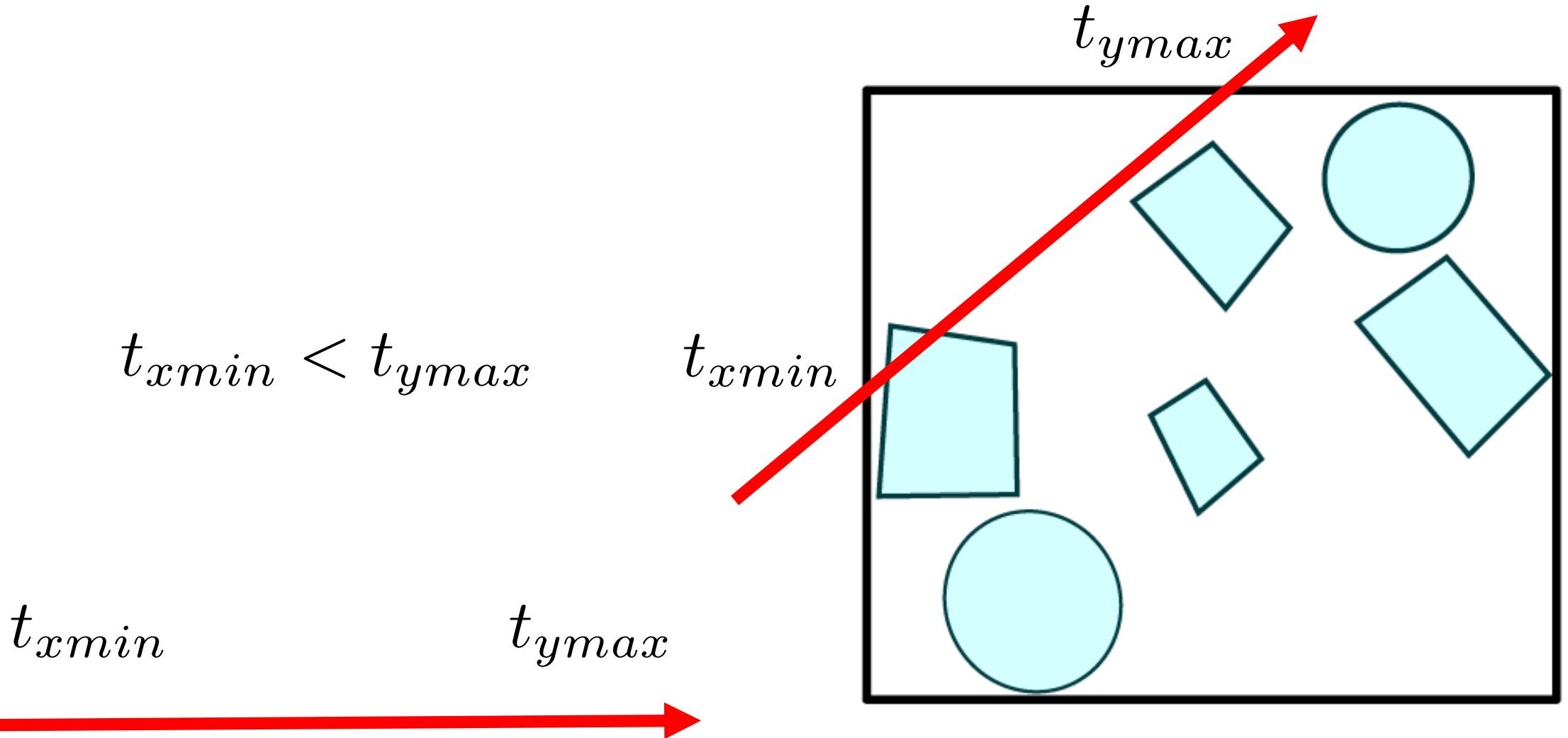
$$t_{\text{xmin}} = (x_{\text{min}} - x_e) / x_d$$

$$t_{\text{xmax}} = (x_{\text{max}} - x_e) / x_d$$

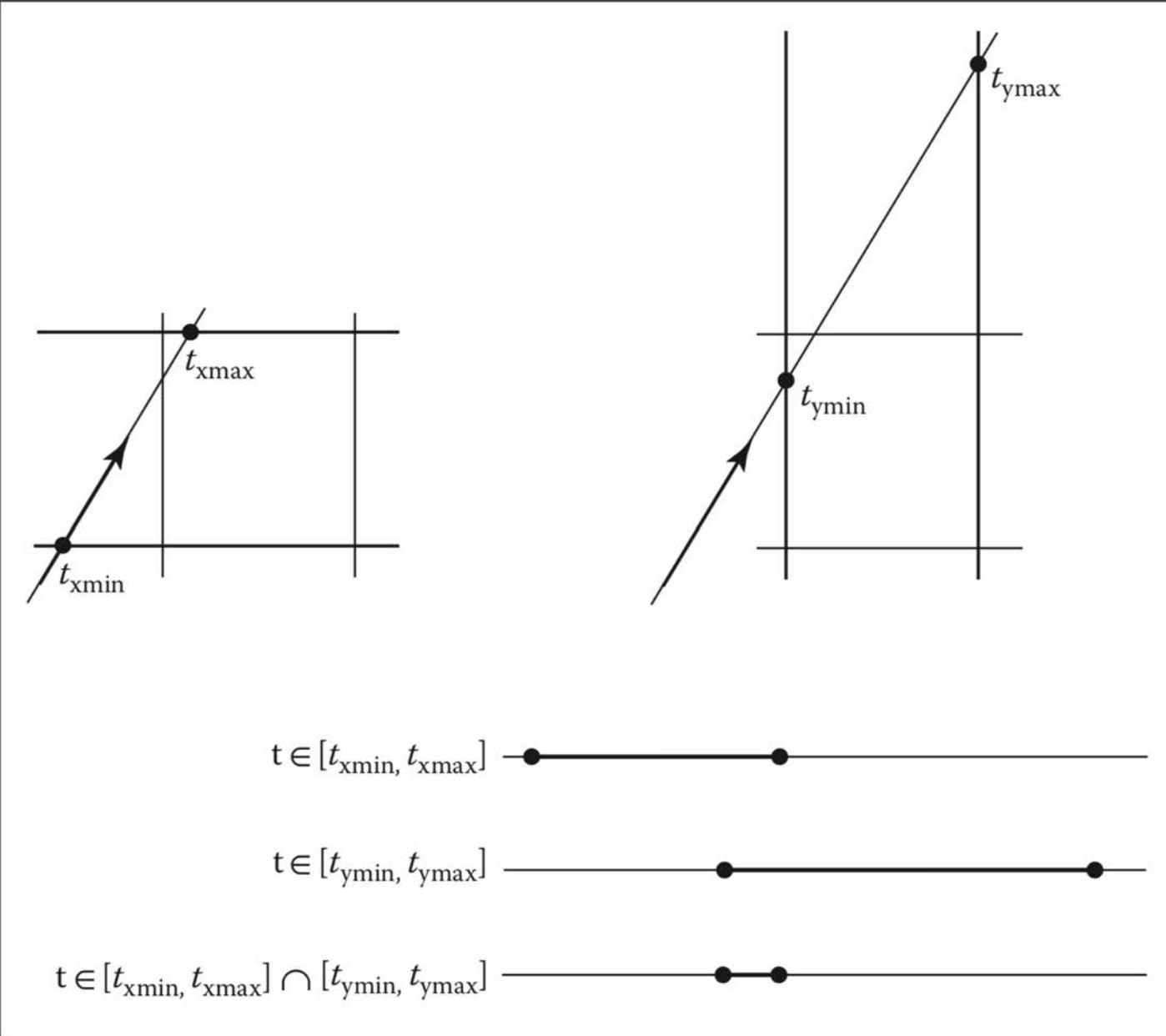
$$t_{\text{ymin}} = (y_{\text{min}} - y_e) / y_d$$

$$t_{\text{ymax}} = (y_{\text{max}} - y_e) / y_d$$

Ray-AABB Intersection



Ray-AABB Intersection



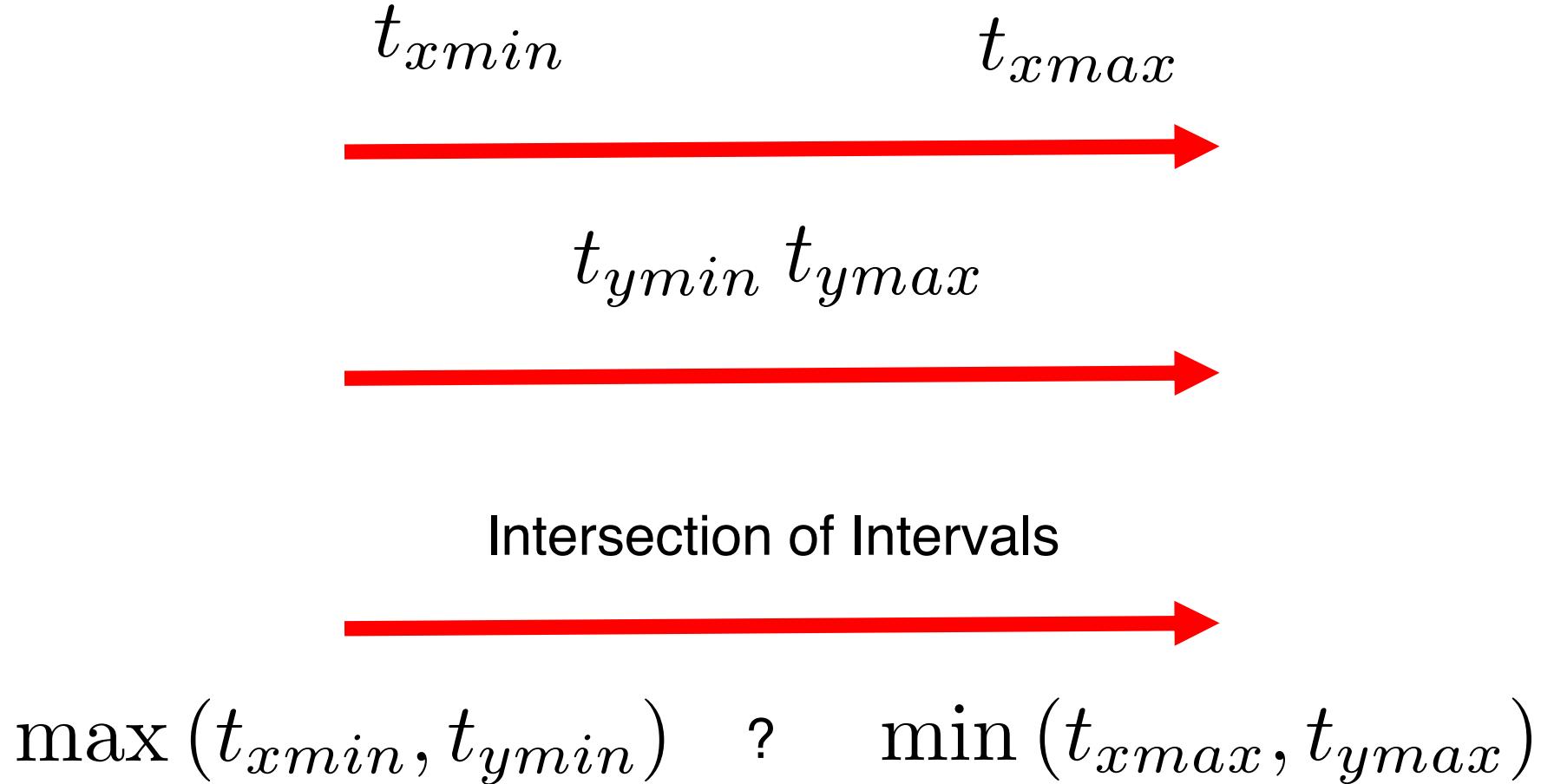
Ray-AABB Intersection



Intersection of Intervals ?



Ray-AABB Intersection

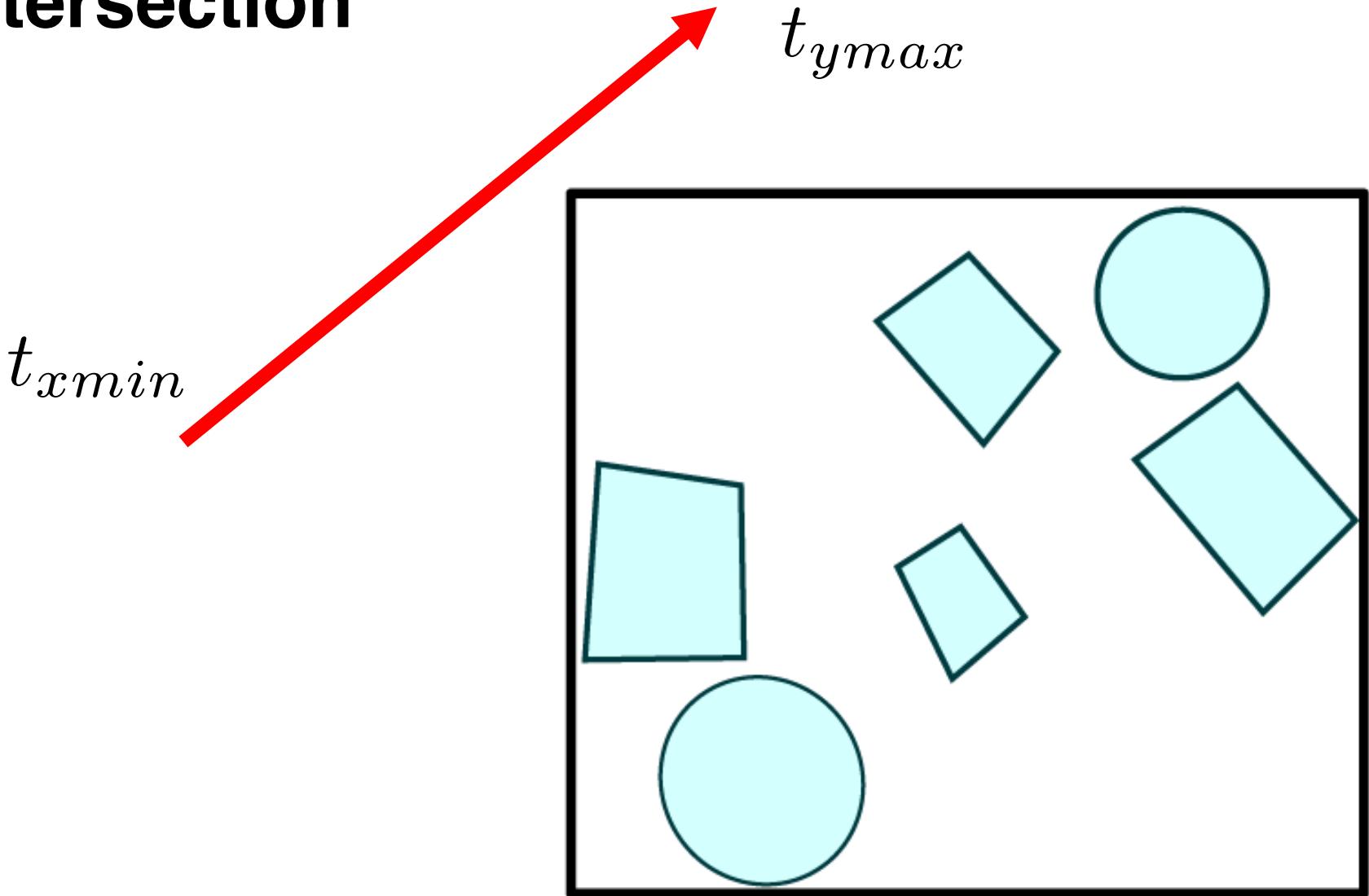


Ray-AABB Intersection



$$\max(t_{xmin}, t_{ymin}) < \min(t_{xmax}, t_{ymax})$$

Ray-AABB Intersection



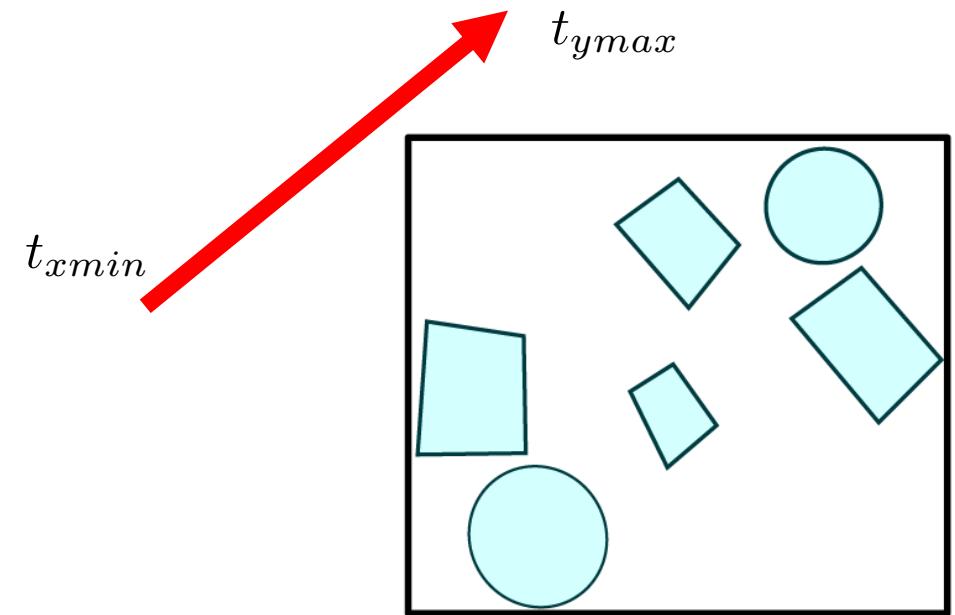
Ray-AABB Intersection



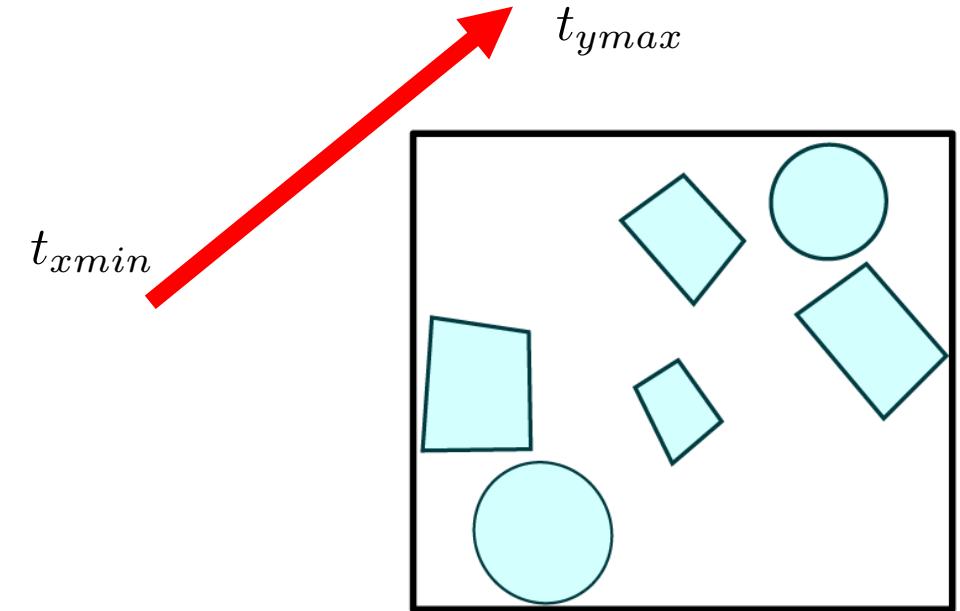
Intersection of Intervals



$$\max(t_{xmin}, t_{ymin}) \quad ? \quad \min(t_{xmax}, t_{ymax})$$



Ray-AABB Intersection



Intersection of Intervals



$$\max(t_{xmin}, t_{ymin}) > \min(t_{xmax}, t_{ymax})$$

Ray-AABB Intersection



Intersection of Intervals

A black rectangle represents an axis-aligned bounding box (AABB). Inside the box, there are several 3D objects: two spheres and four rectangular prisms (cubes). A red arrow points upwards from the text "Intersection of Intervals".

$$\max(t_{xmin}, t_{ymin}) > \min(t_{xmax}, t_{ymax}) \quad \text{--> CHECK}$$

Building and Axis-Aligned Bounding Box (AABB)

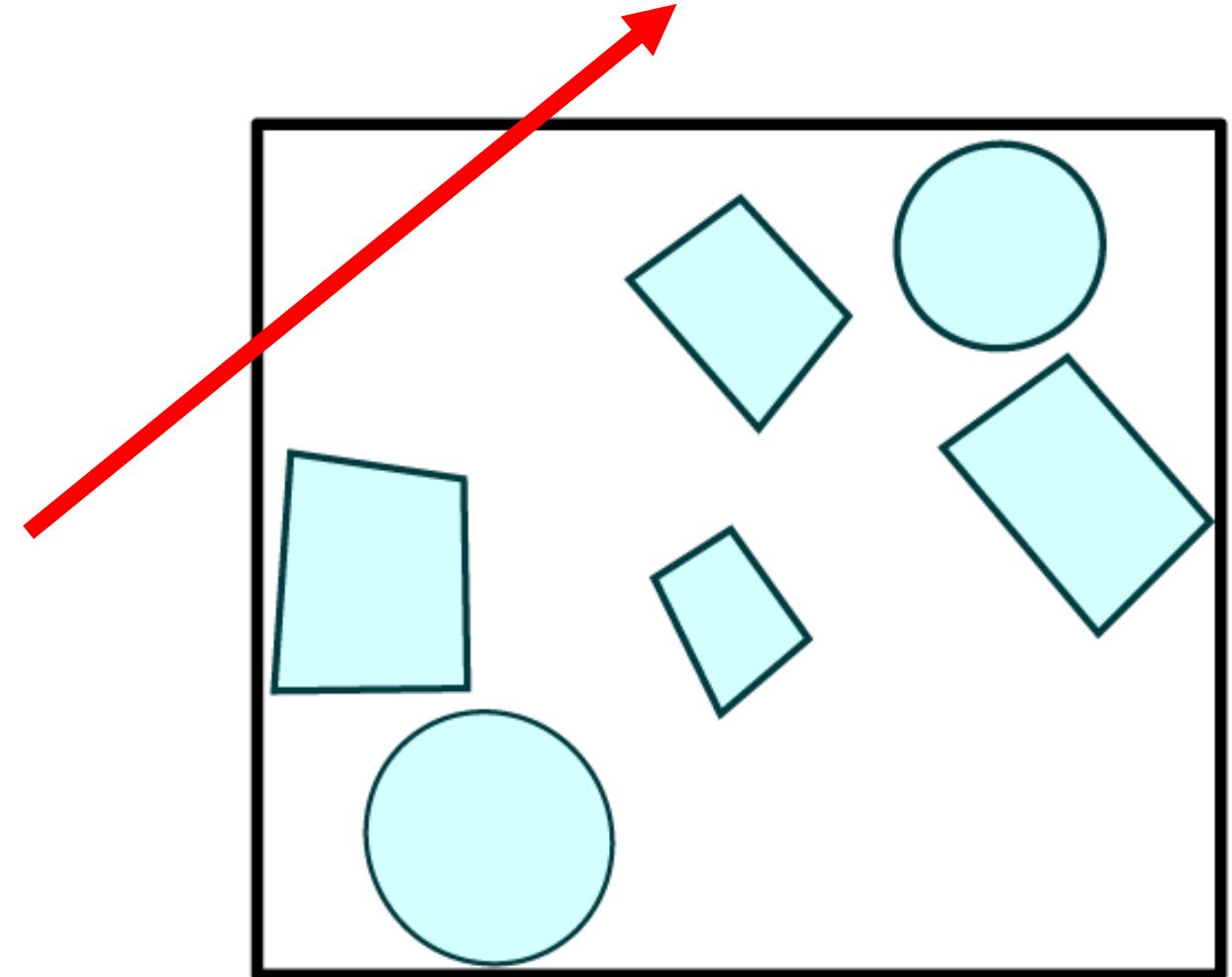
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Building and Axis-Aligned Bounding Box (AABB)

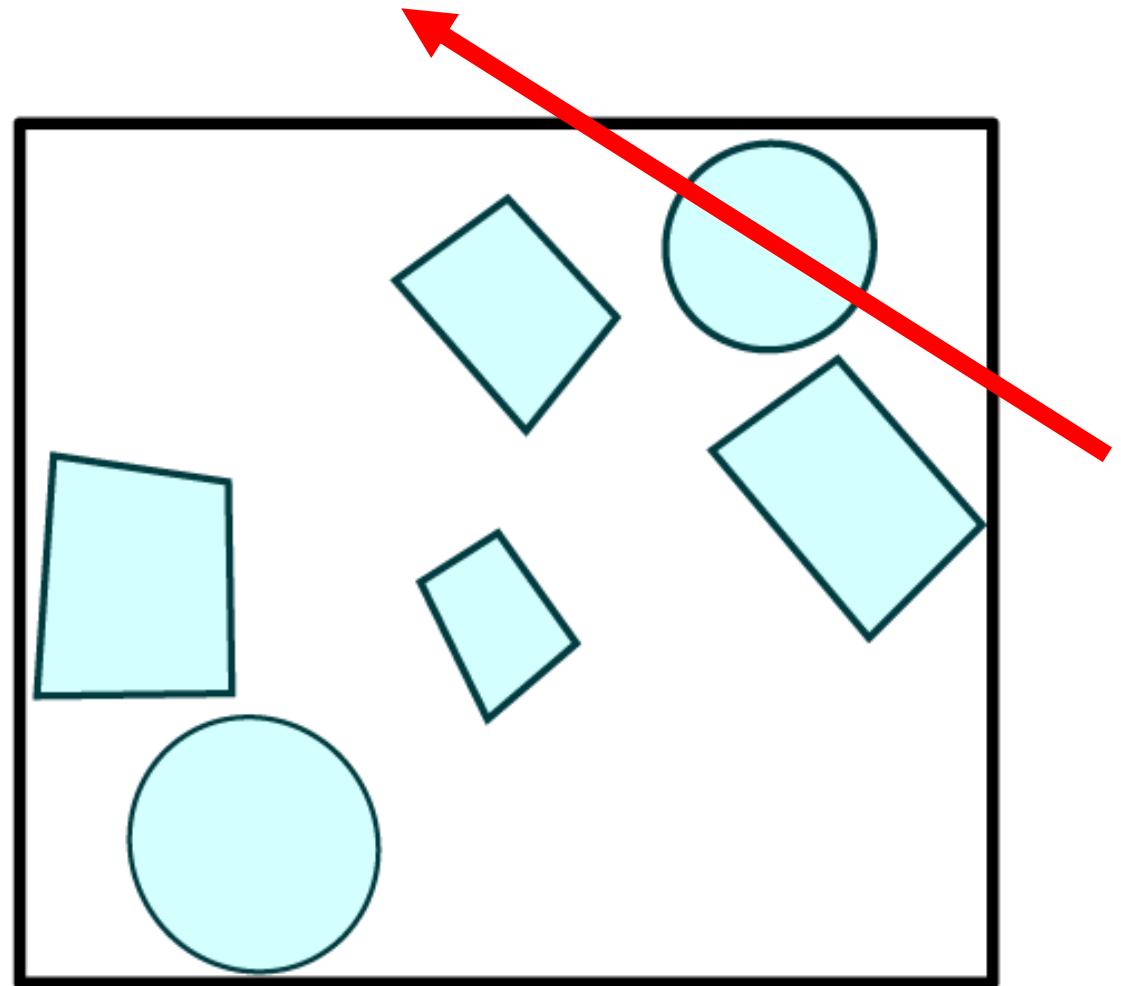
$$x_{min} = \min(v_x^i)$$

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Building and Axis-Aligned Bounding Box (AABB)

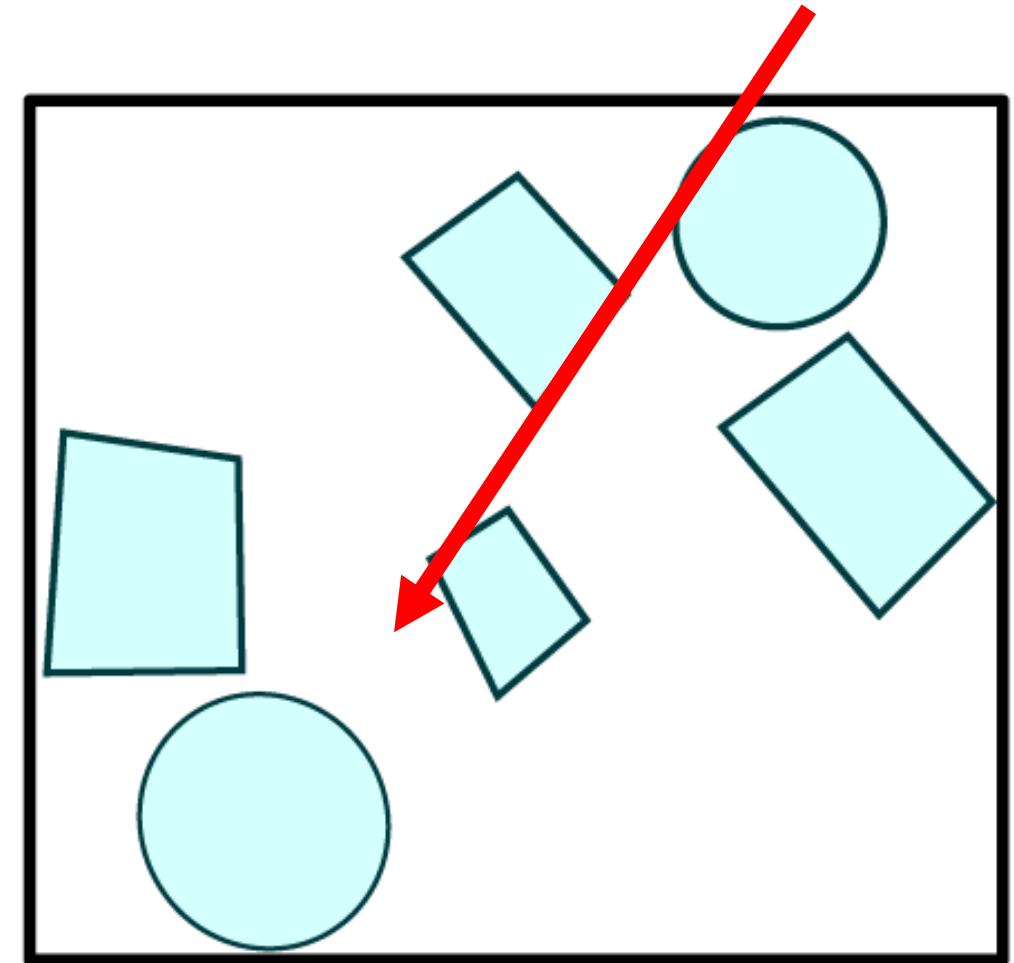
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$\mathbf{v}^i \in$ Vertices



Ray-AABB Intersection

if ($x_d \geq 0$) **then**

$$t_{\text{xmin}} = (x_{\text{min}} - x_e) / x_d$$

$$t_{\text{xmax}} = (x_{\text{max}} - x_e) / x_d$$

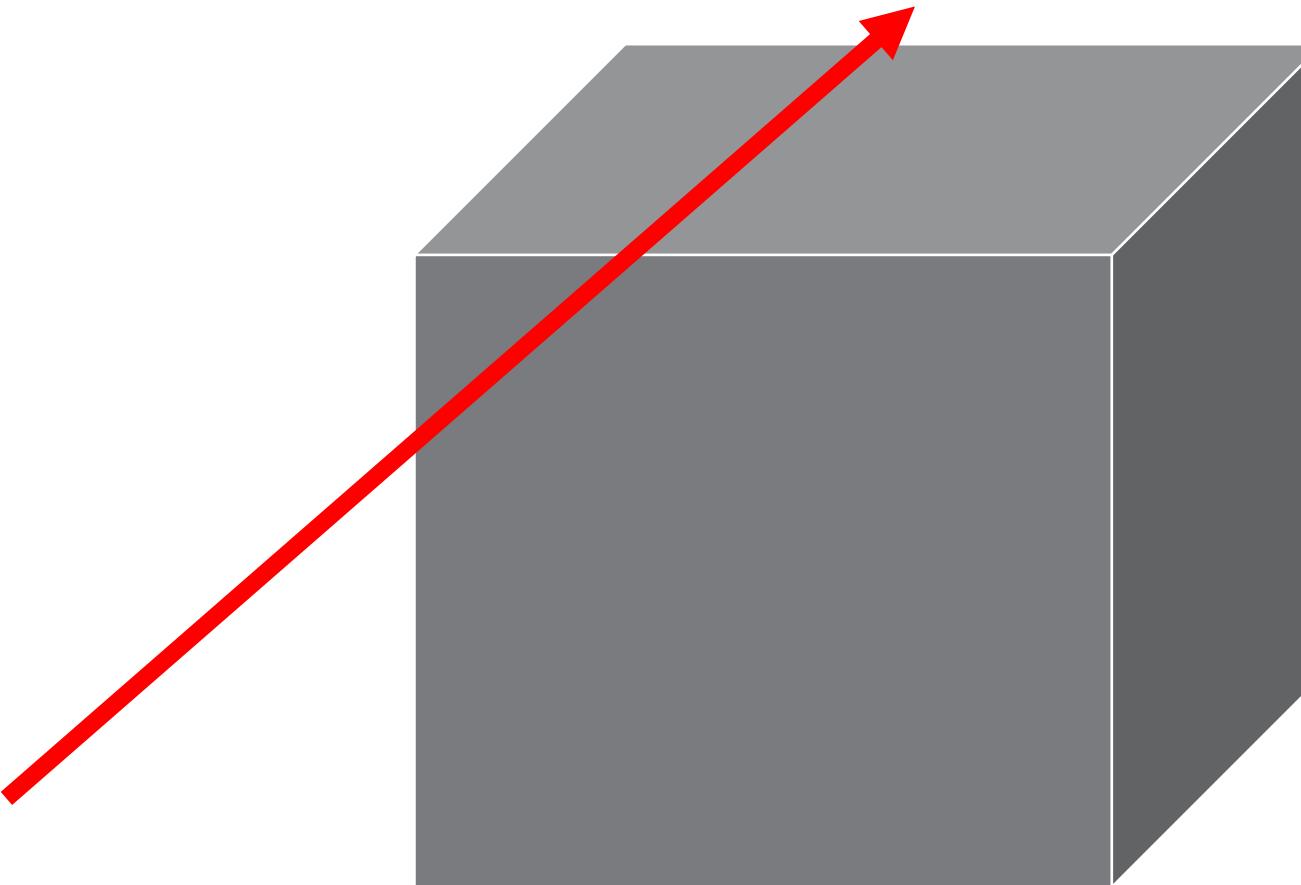
else

$$t_{\text{xmin}} = (x_{\text{max}} - x_e) / x_d$$

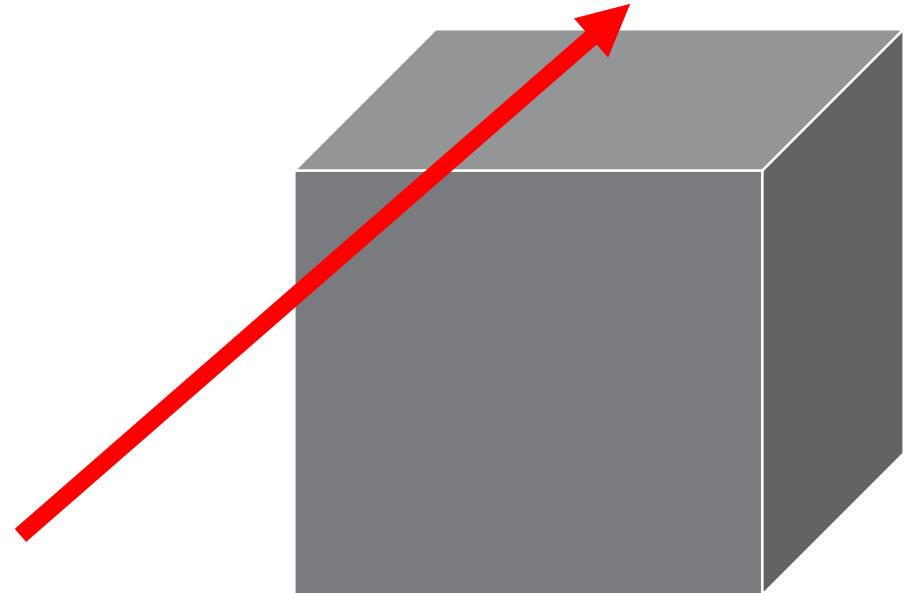
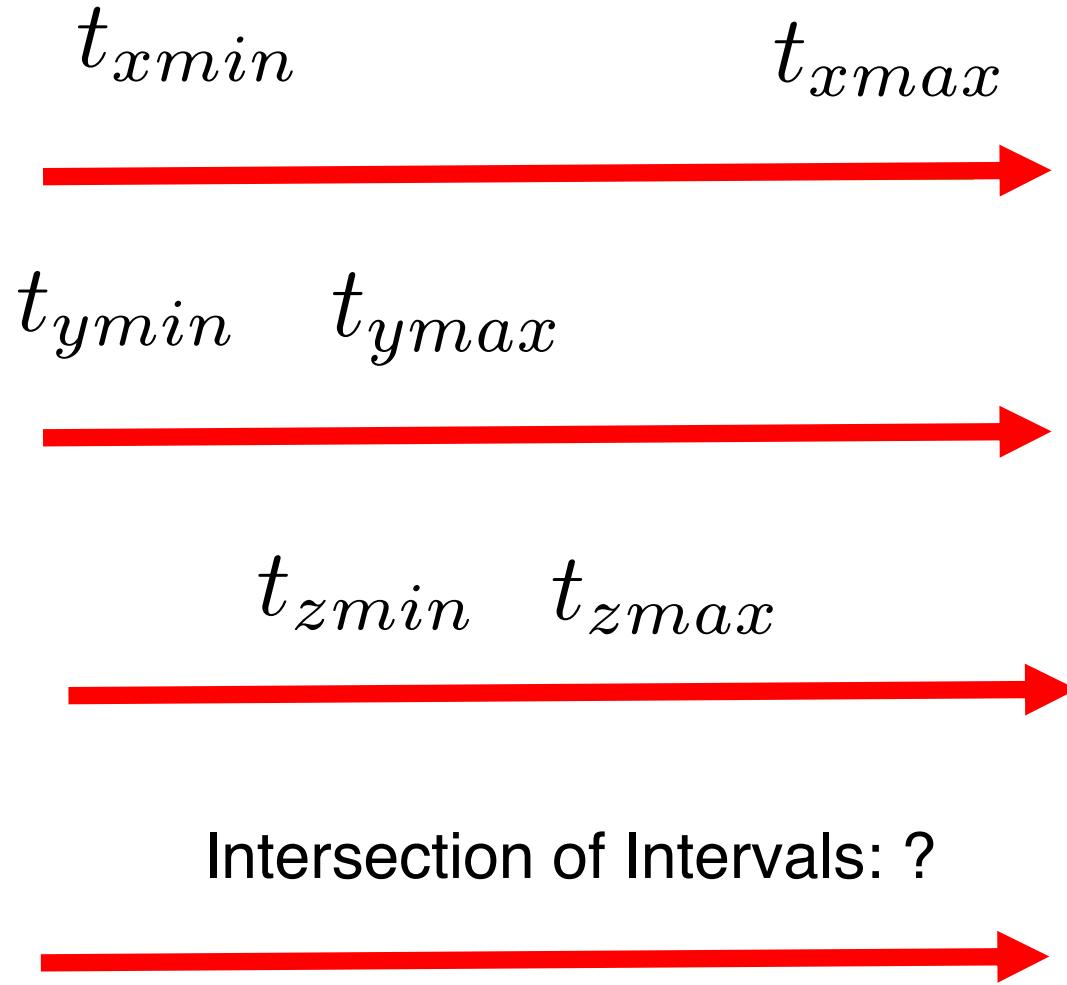
$$t_{\text{xmax}} = (x_{\text{min}} - x_e) / x_d$$

When does this fail ?

What happens in 3D ?



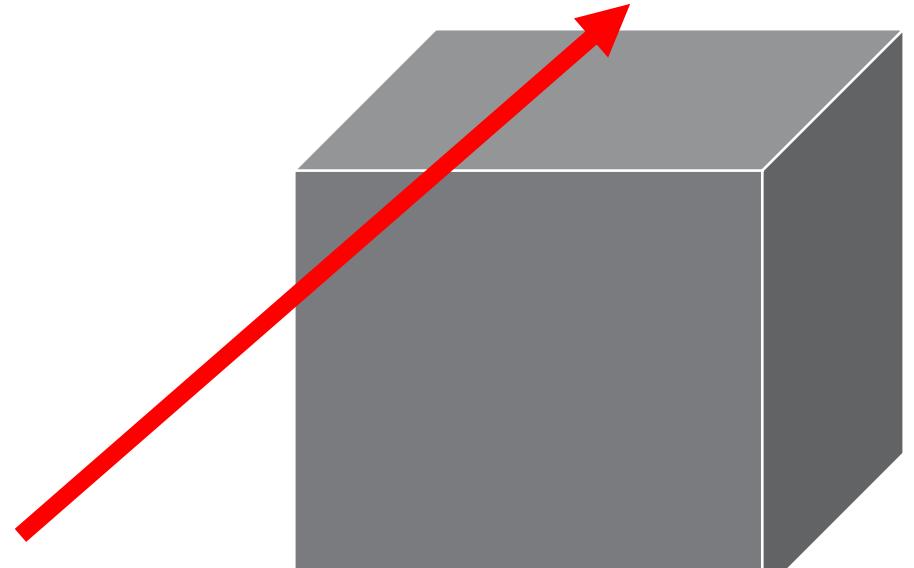
Ray-AABB Intersection



Intersection of Intervals: ?

Ray-AABB Intersection

$$\begin{array}{c} t_{xmin} \quad t_{xmax} \\ \hline \text{---} \rightarrow \\ t_{ymin} \quad t_{ymax} \\ \hline \text{---} \rightarrow \\ t_{zmin} \quad t_{zmax} \\ \hline \text{---} \rightarrow \\ \min(t_{xmax}, t_{ymax}, t_{zmax}) \\ \hline \text{---} \rightarrow \\ \max(t_{xmin}, t_{ymin}, t_{zmin}) \end{array}$$



Ray-AABB Intersection

if ($x_d \geq 0$) **then**

$$t_{\text{xmin}} = (x_{\text{min}} - x_e) / x_d$$

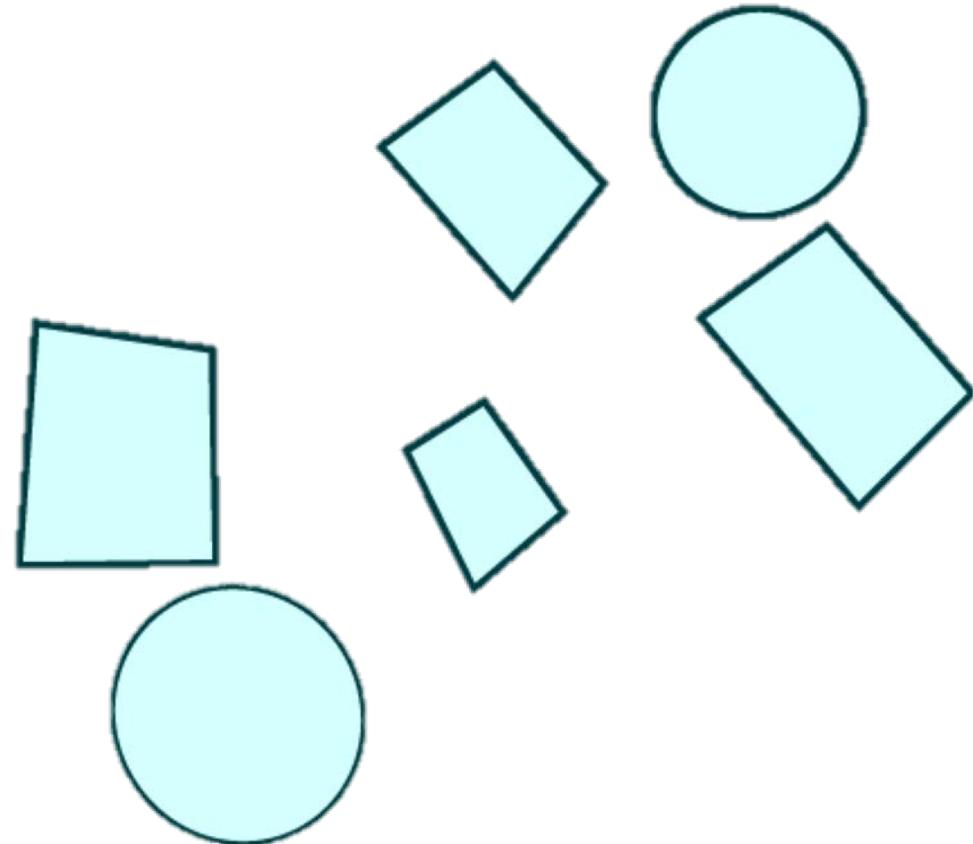
$$t_{\text{xmax}} = (x_{\text{max}} - x_e) / x_d$$

else

$$t_{\text{xmin}} = (x_{\text{max}} - x_e) / x_d$$

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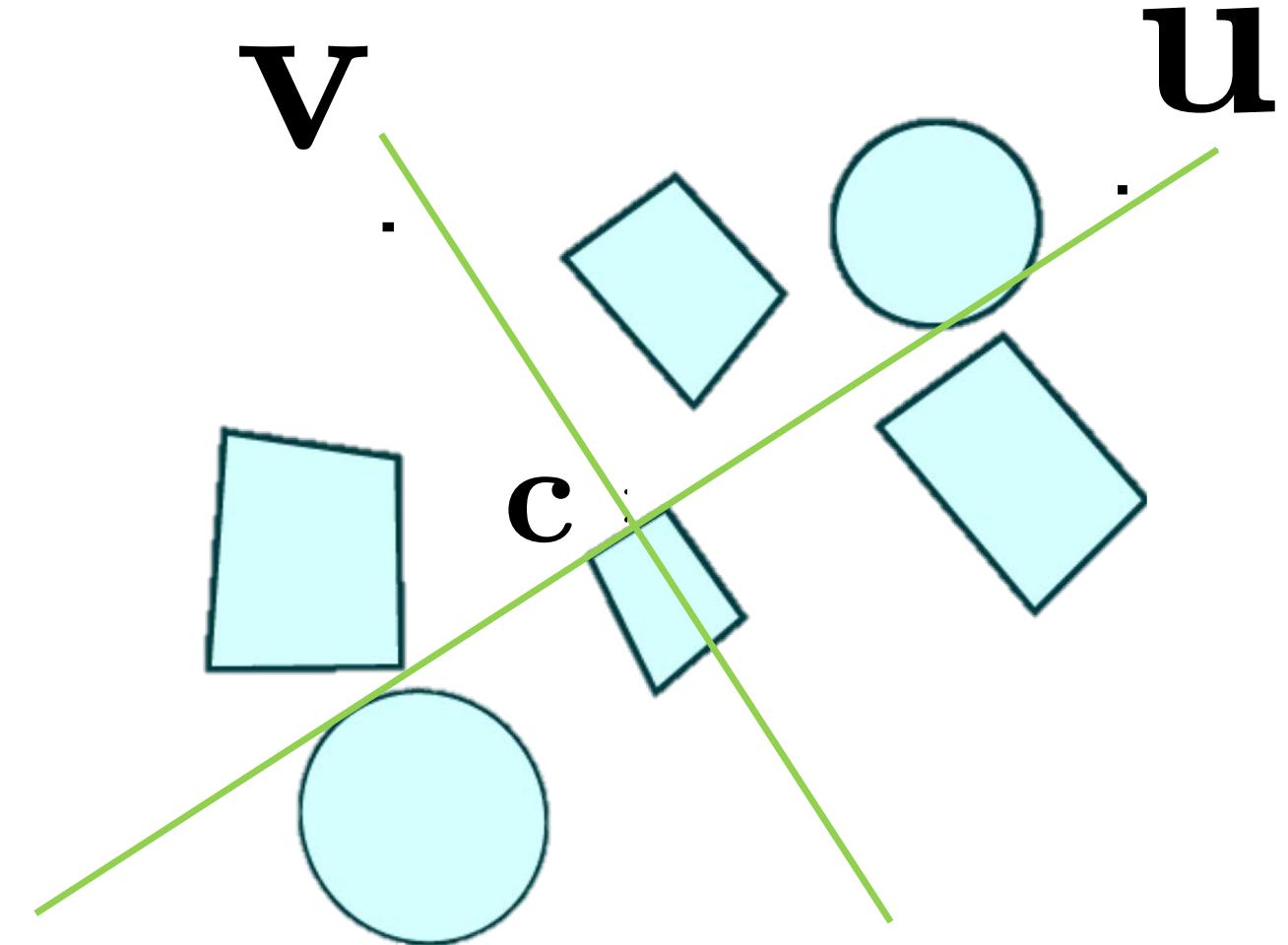
Building an Object-Oriented Bounding Box (OOBB)



Building an Object-Oriented Bounding Box (OOBB)

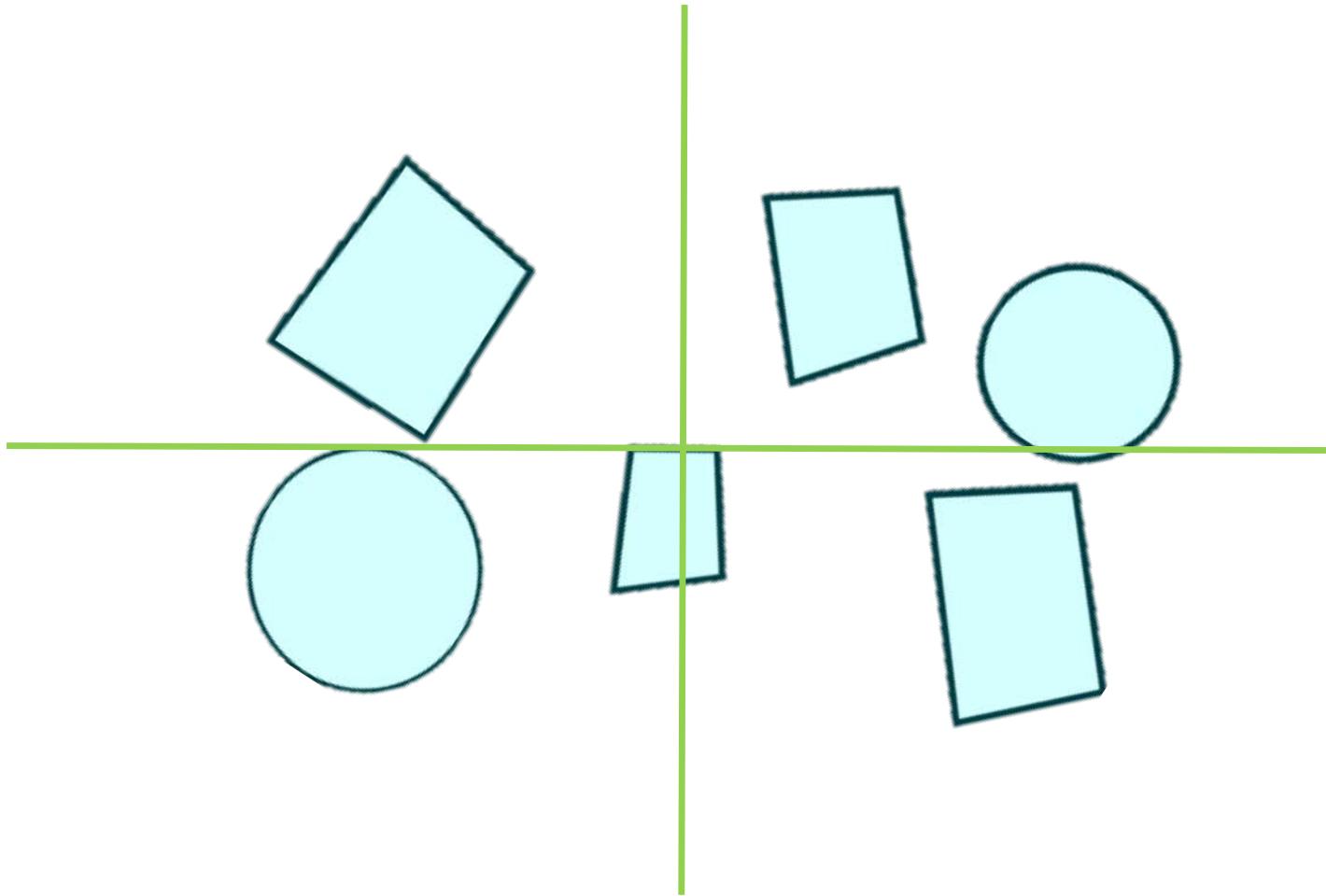
$$c = \frac{1}{n} \sum_{i=1}^n v^i$$

$$[u \quad v]$$



Find directions of maximum and minimum variance

Building an Object-Oriented Bounding Box (OOBB)



Build Rotation Matrix

Collision Query with Object-Oriented Bounding Box



Spatial Data Structures

Basic Idea – asymptotic improvement in spatial queries by subdividing

Two types of subdivisions – *object-based* and *spatial*

Spatial Data Structures

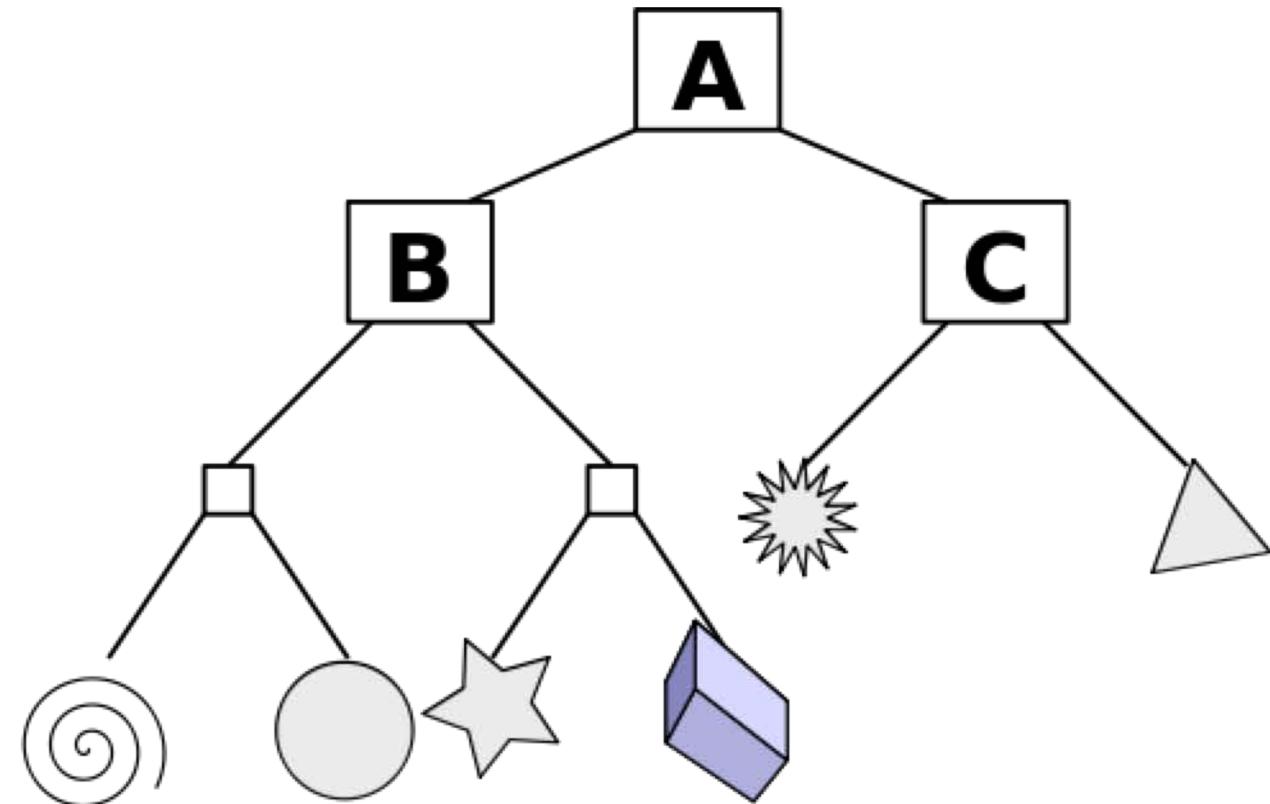
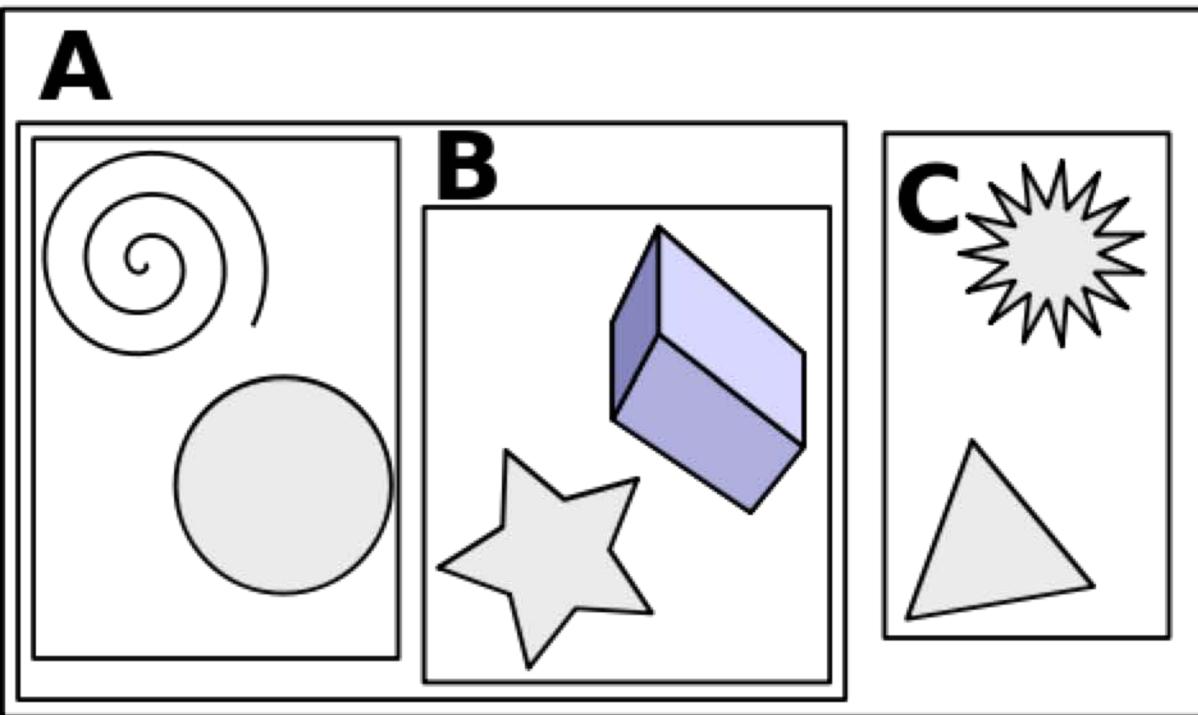
Basic Idea – asymptotic improvement in spatial queries by subdividing

Two types of subdivisions – ***object-based*** and ***spatial***

Our object-based data structures will be boundary volume hierarchies or BVHs.

BVHs are hierarchies of BVs represented by trees

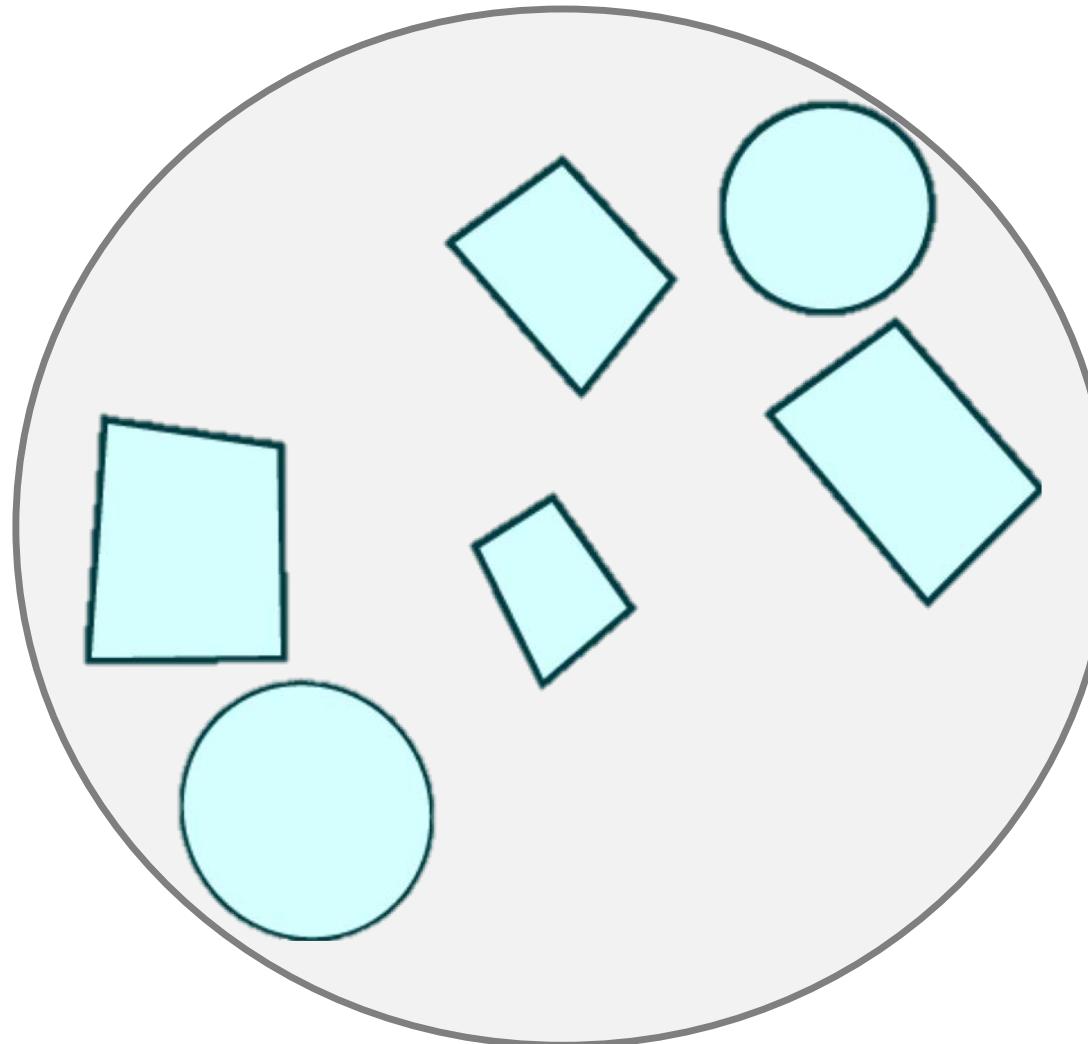
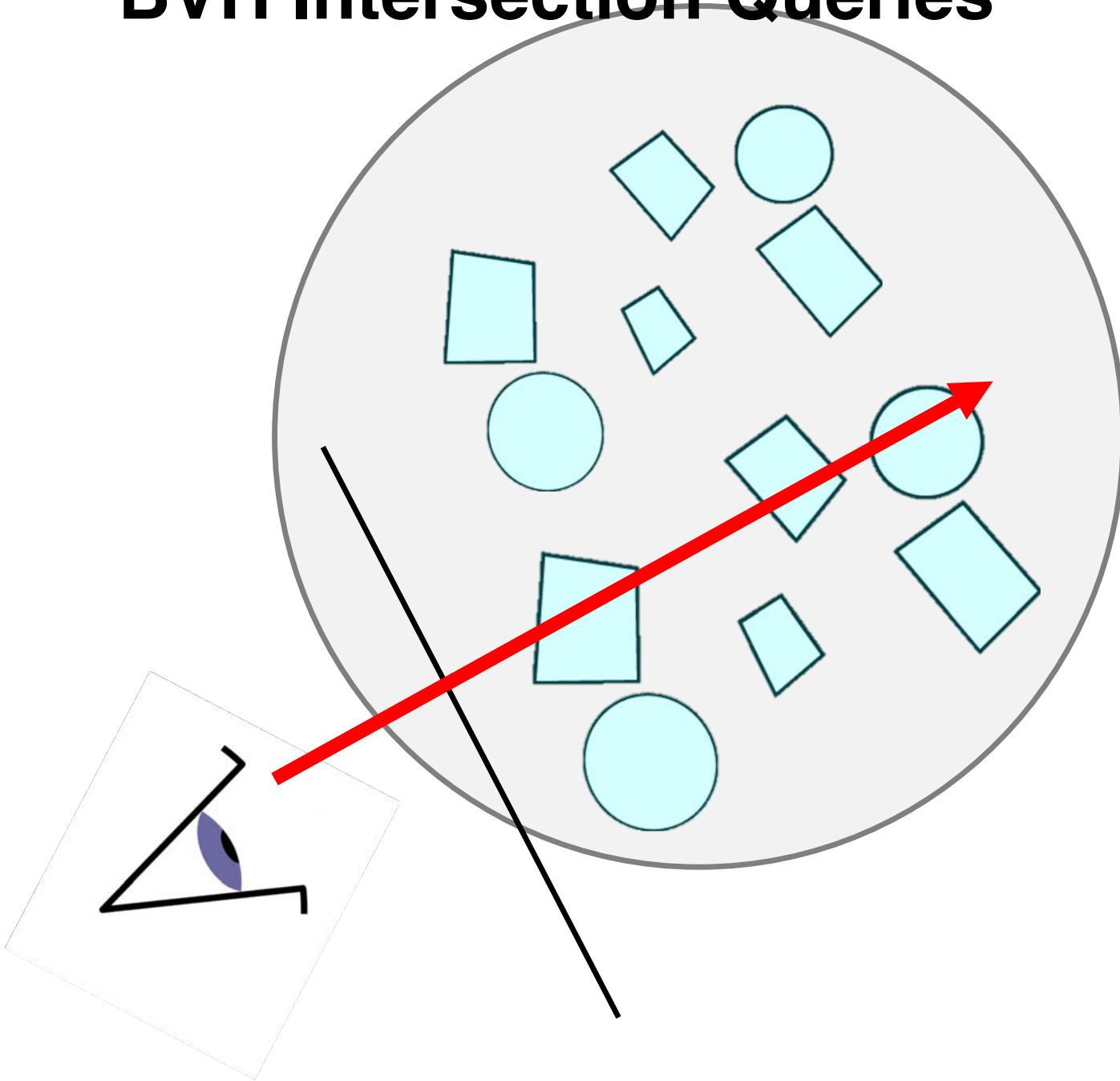
Boundary Volume Hierarchy



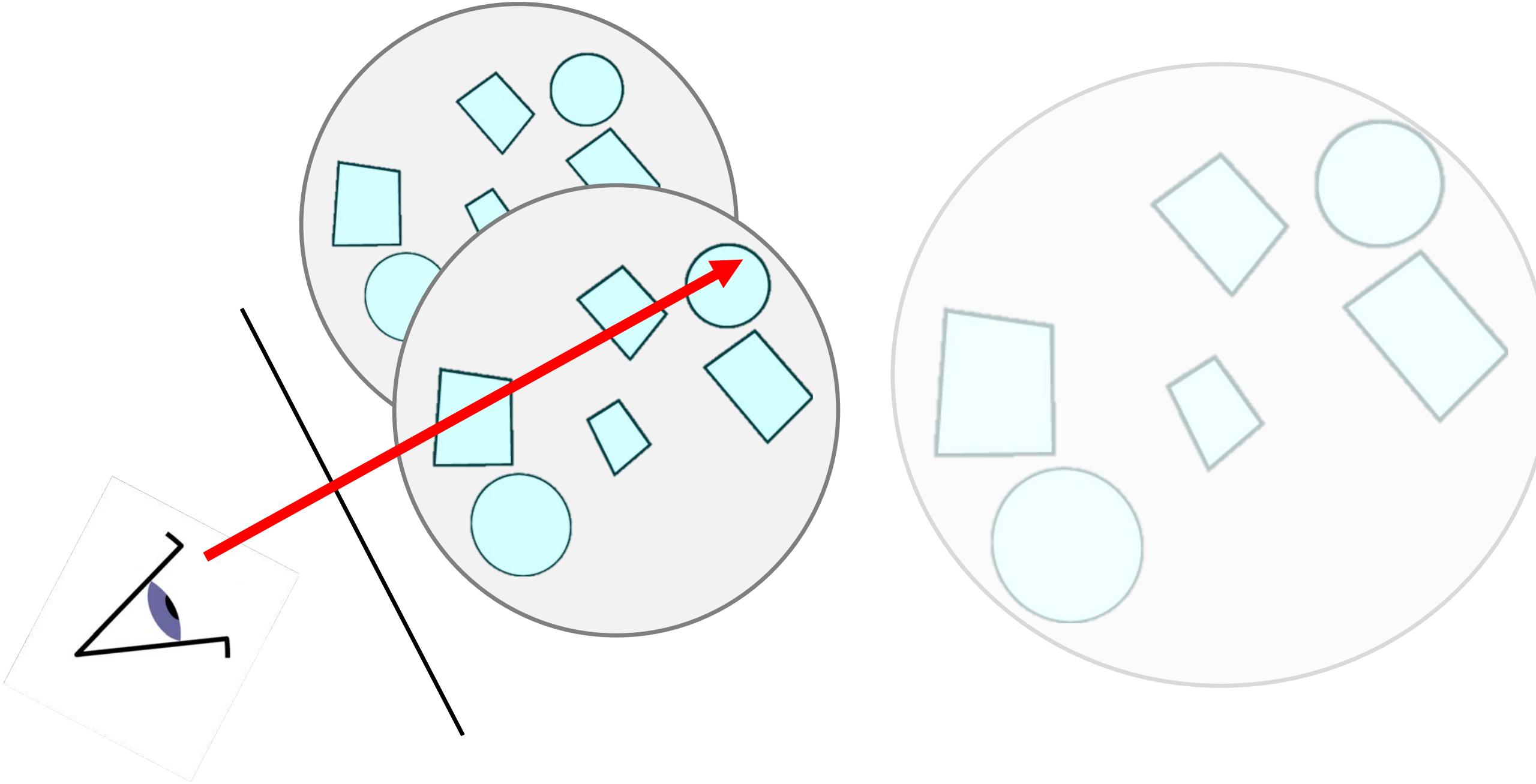
BVH Intersection Queries

```
intersect(bvNode, ray,t)
{
    if (bvNode== null || !bvNode.intersect(ray,t))
        return false;
    else
    {
        i1=intersect(bvNode.left, ray,t1); //check left BV
        i2=intersect(bvNode.right ray,t2); //check right BC
        if (i1 && i2) { t=min(t1,t2); return true; }
        if (i1) { t=t1; return true; }
        if (i2) { t=t2; return true; }
        return false;
    }
}
```

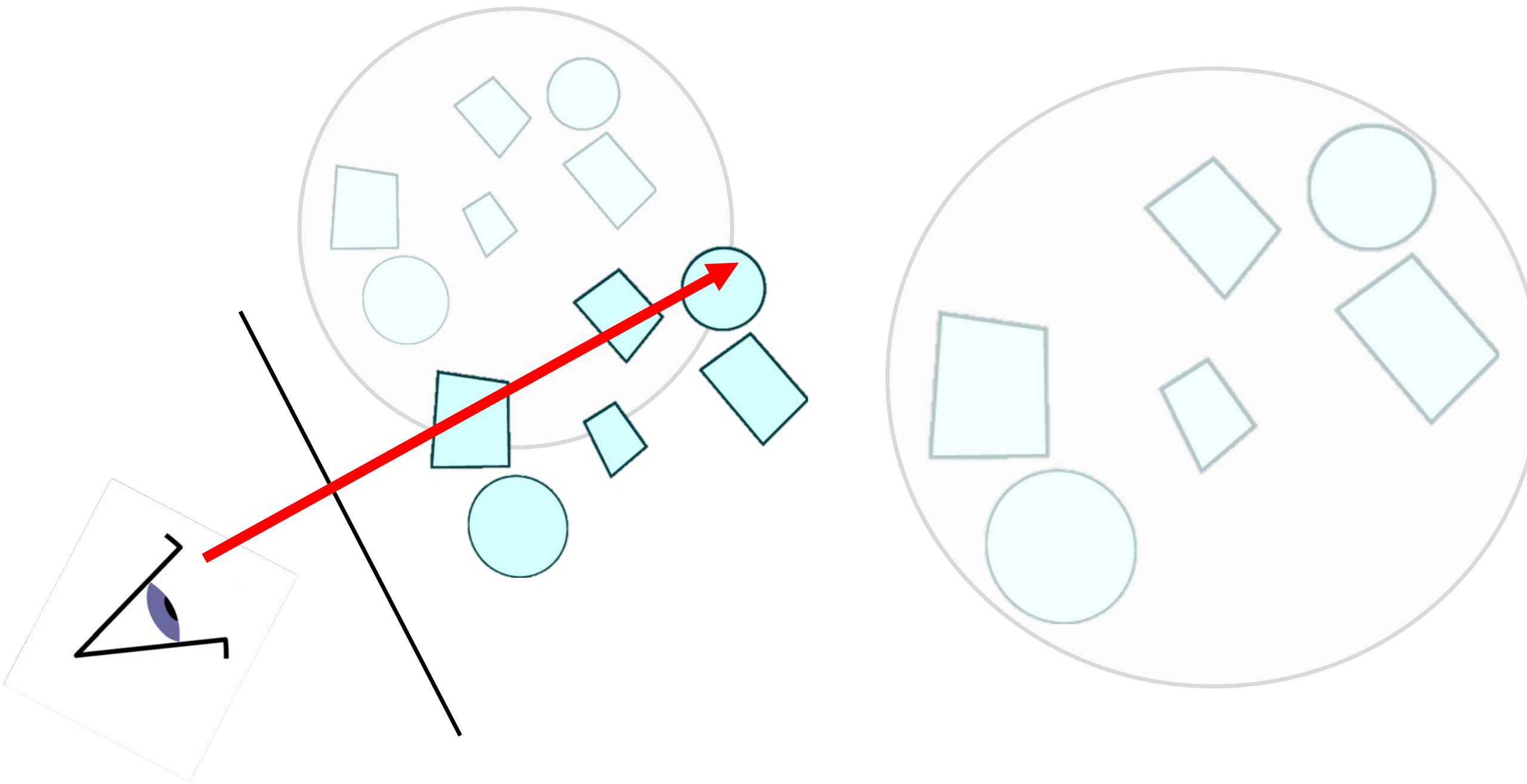
BVH Intersection Queries



BVH Intersection Queries



BVH Intersection Queries



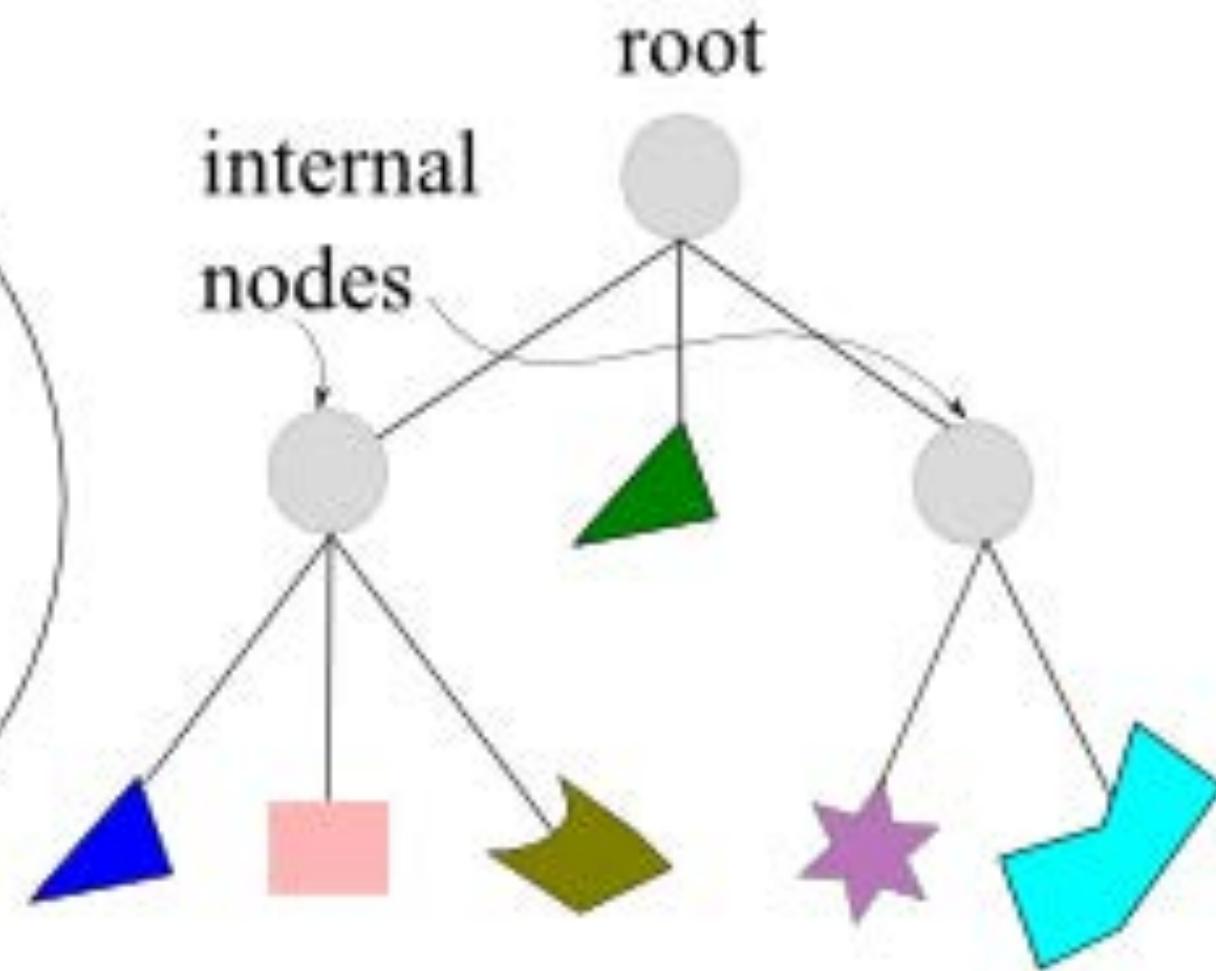
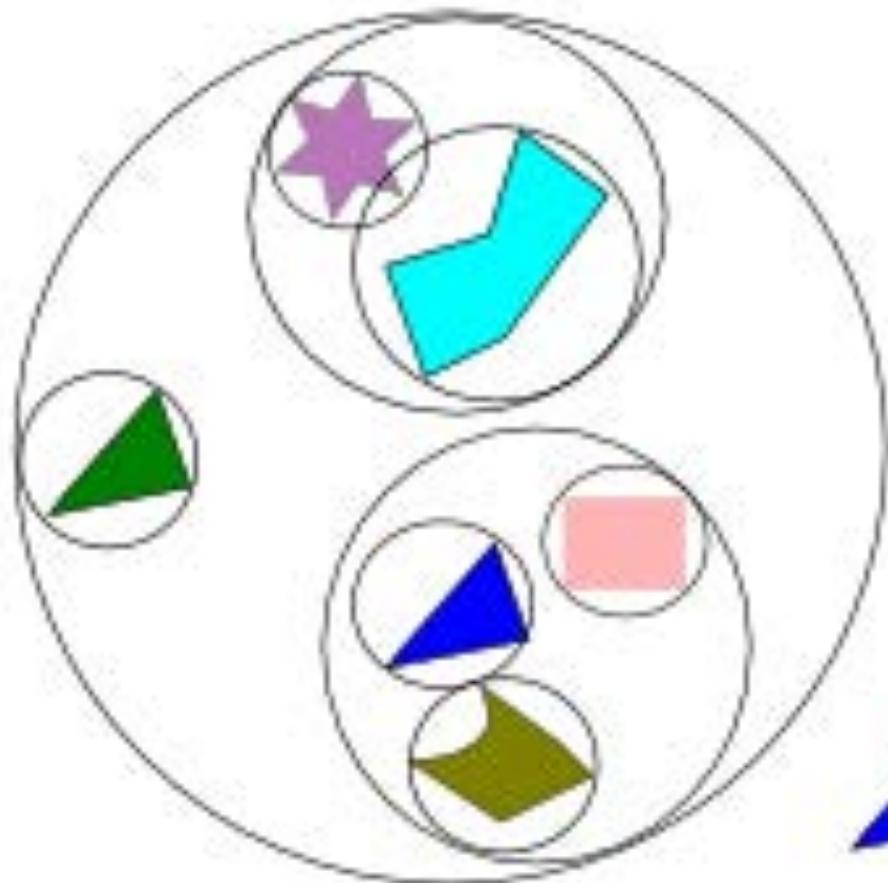
BVH Distance Queries

```
minDistance(bvNode, point, currentMin)
{
    d1=minDistance(bvNode.left, point, currentMin);
    d2=minDistance(bvNode.right, point, currentMin);

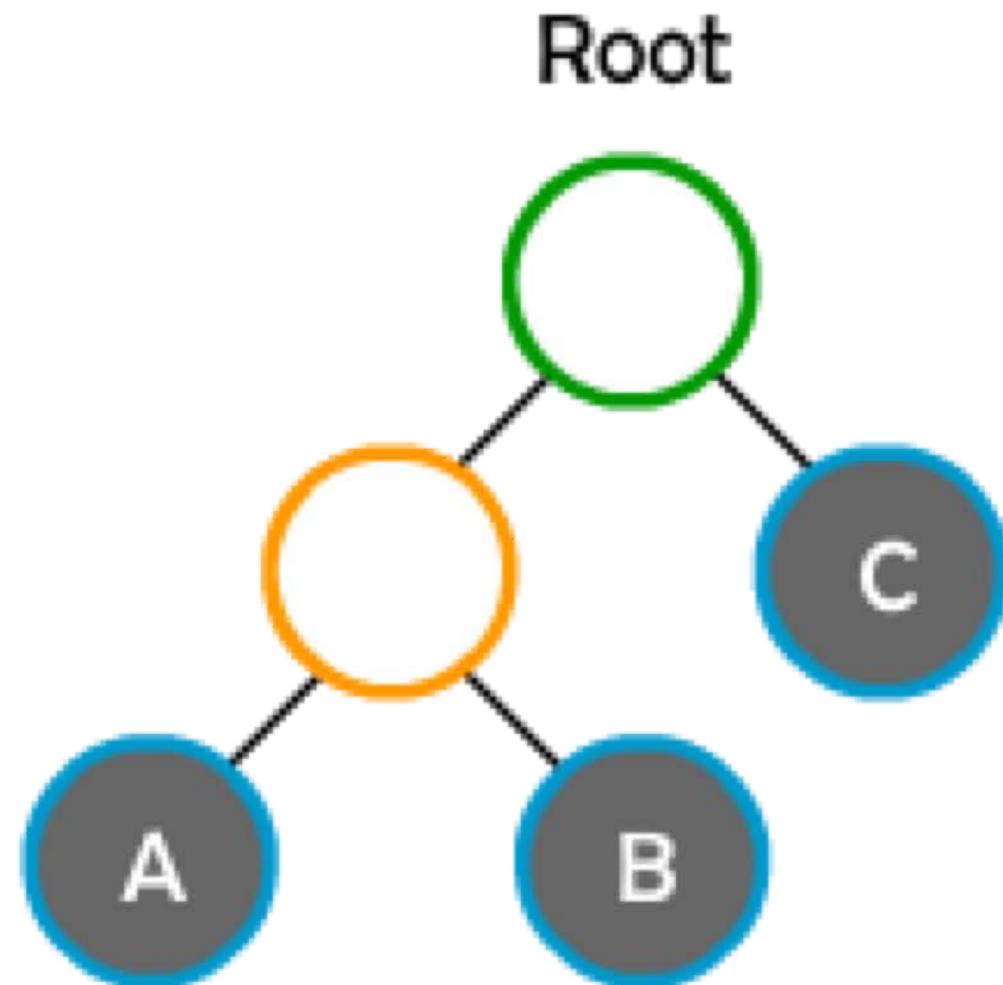
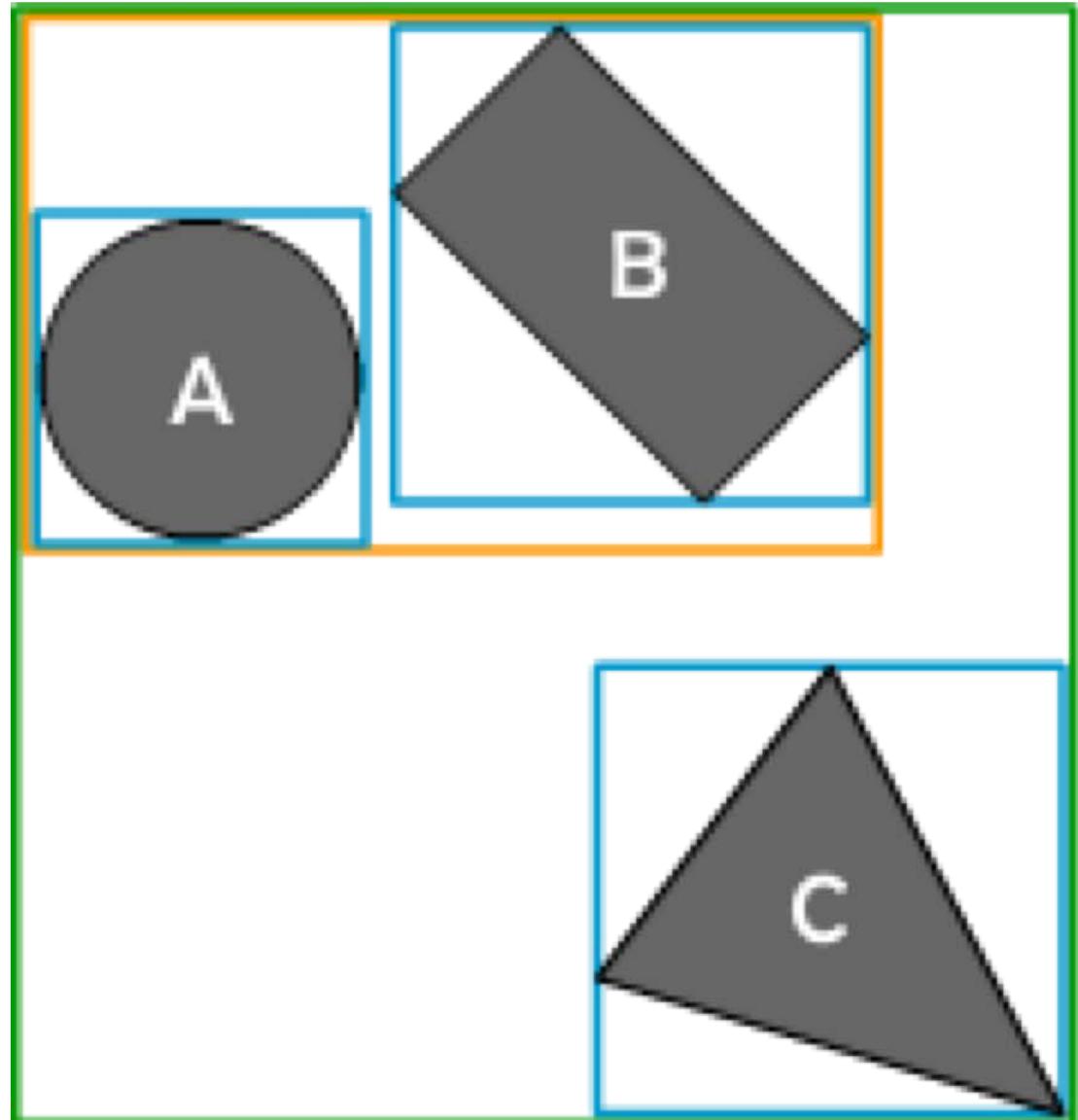
    if(min(d1,d2) > currentMin) {
        return currentMin
    }

    return min(d1,d2)
}
```

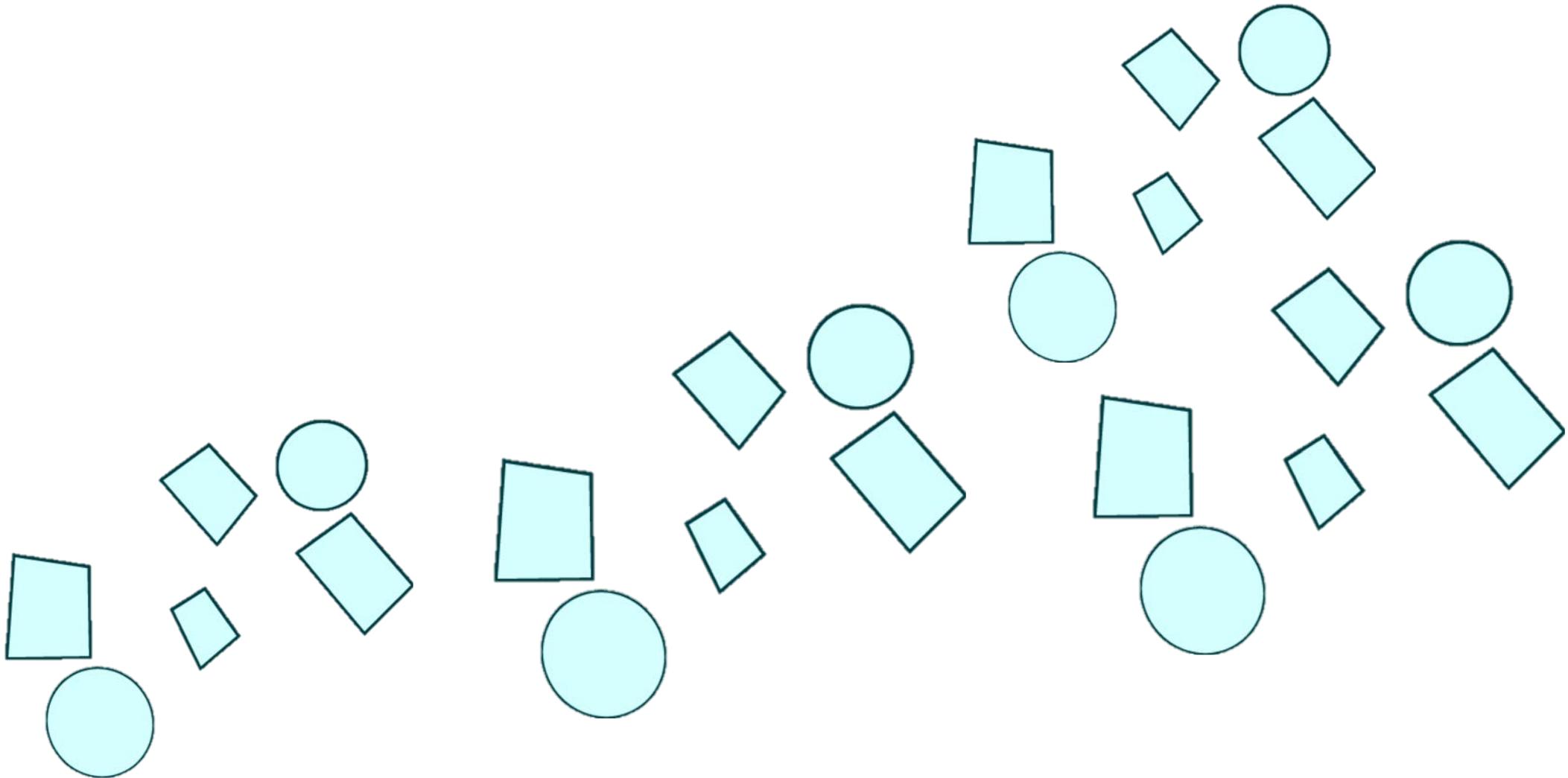
Sphere Trees



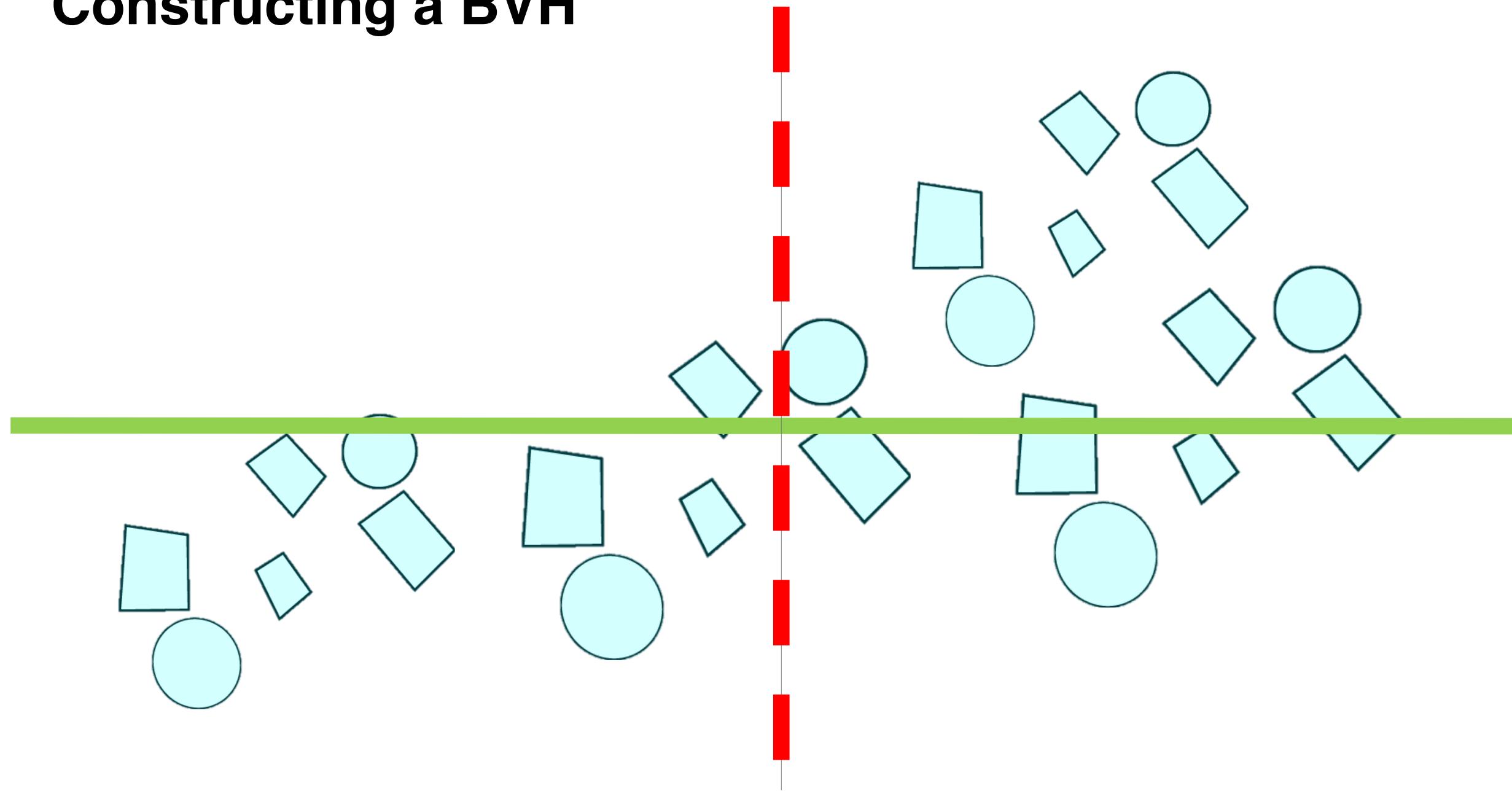
AABB Trees



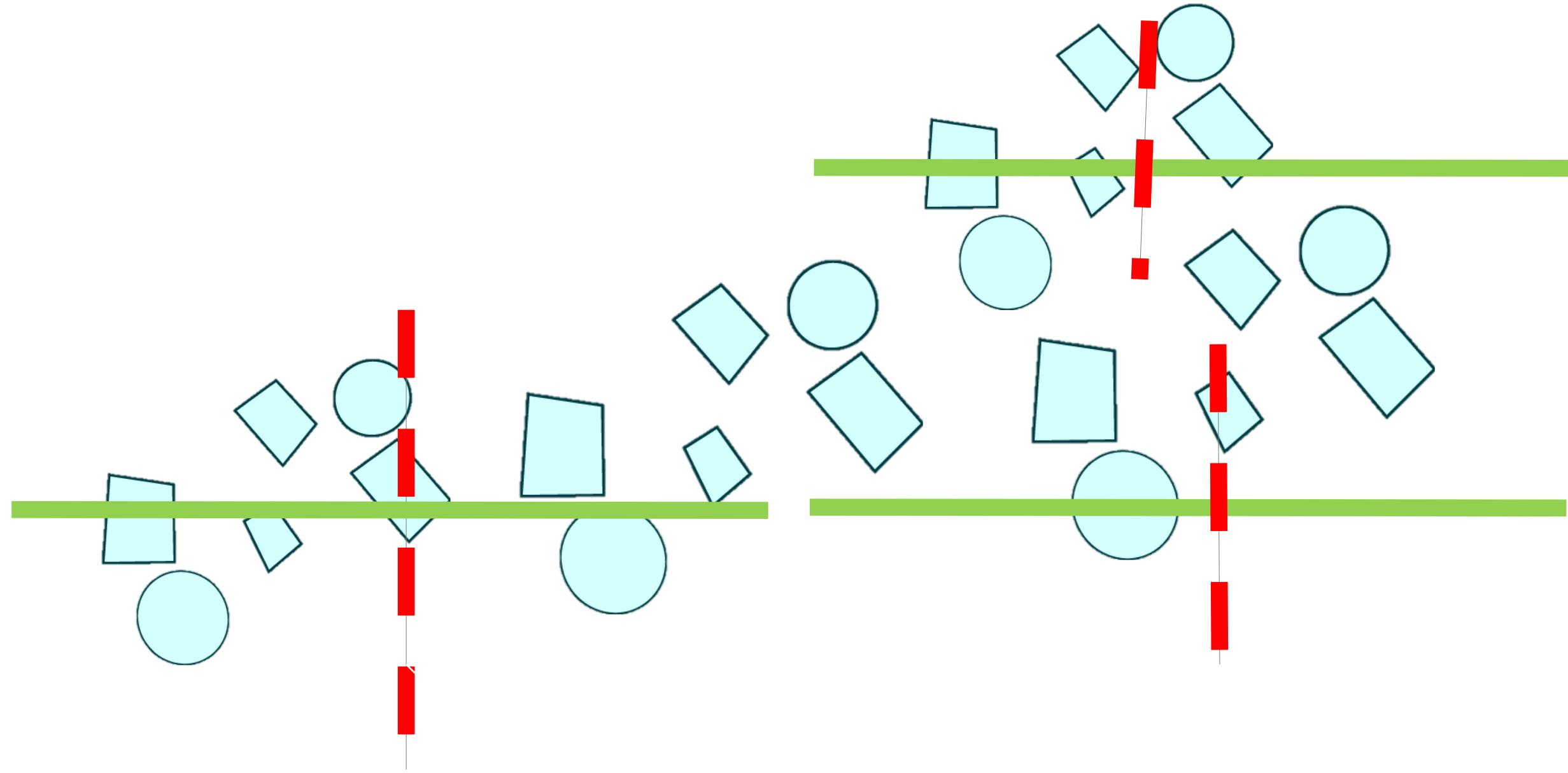
Constructing a BVH



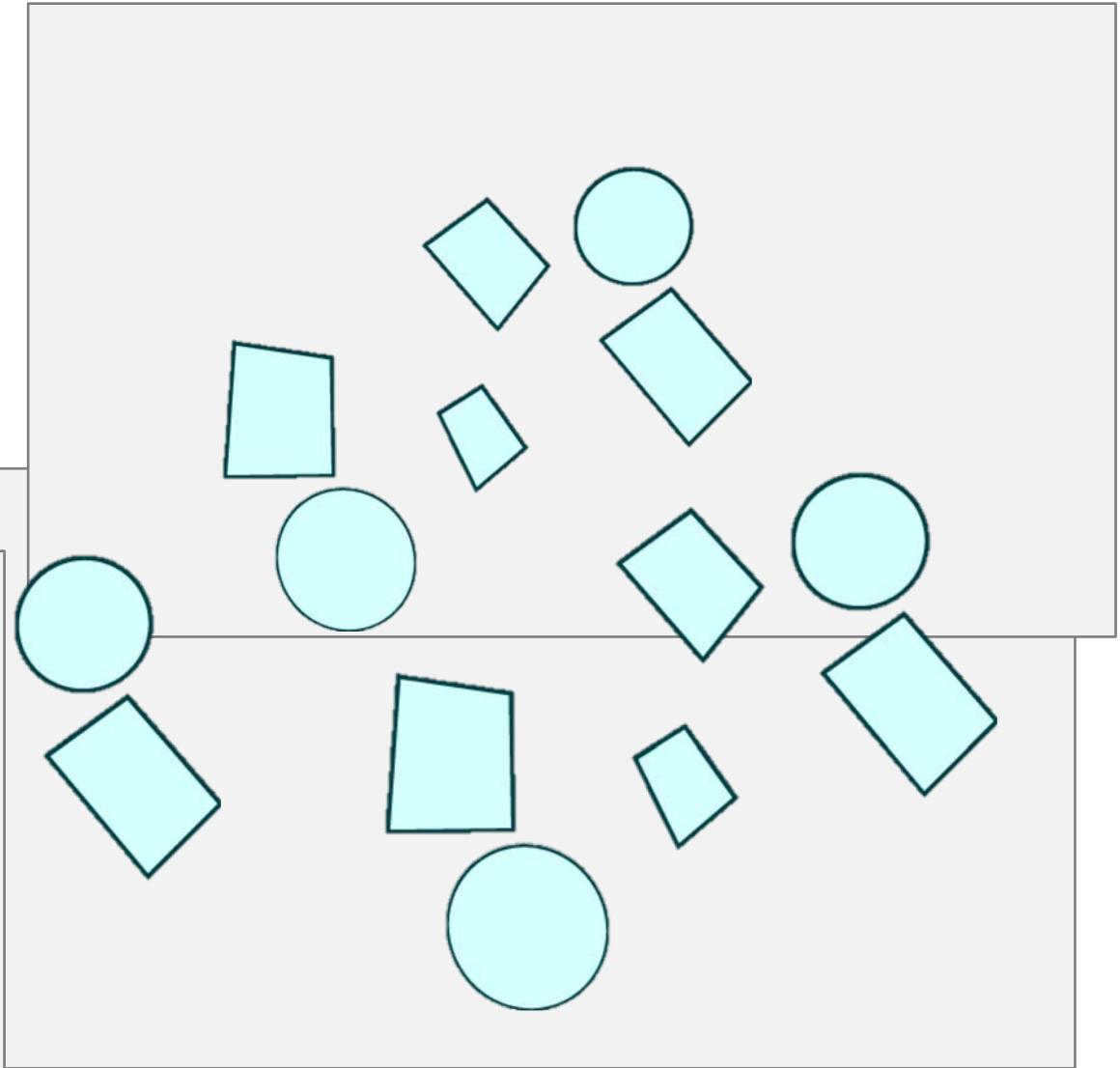
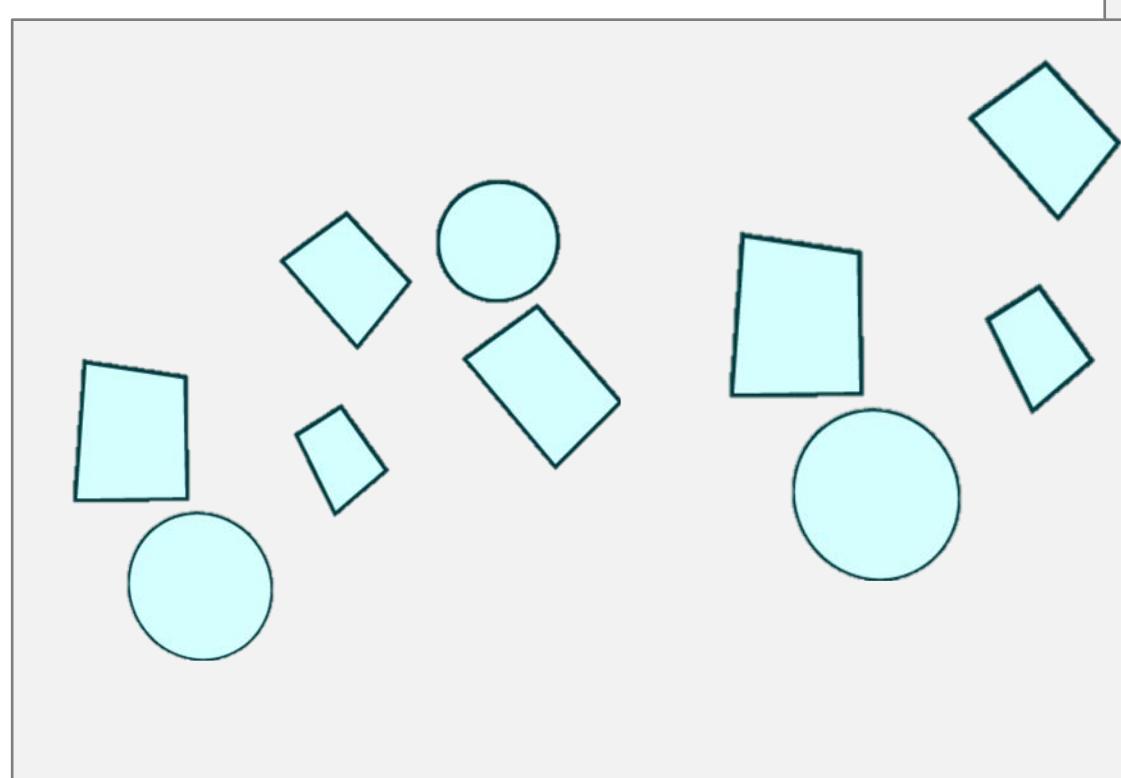
Constructing a BVH



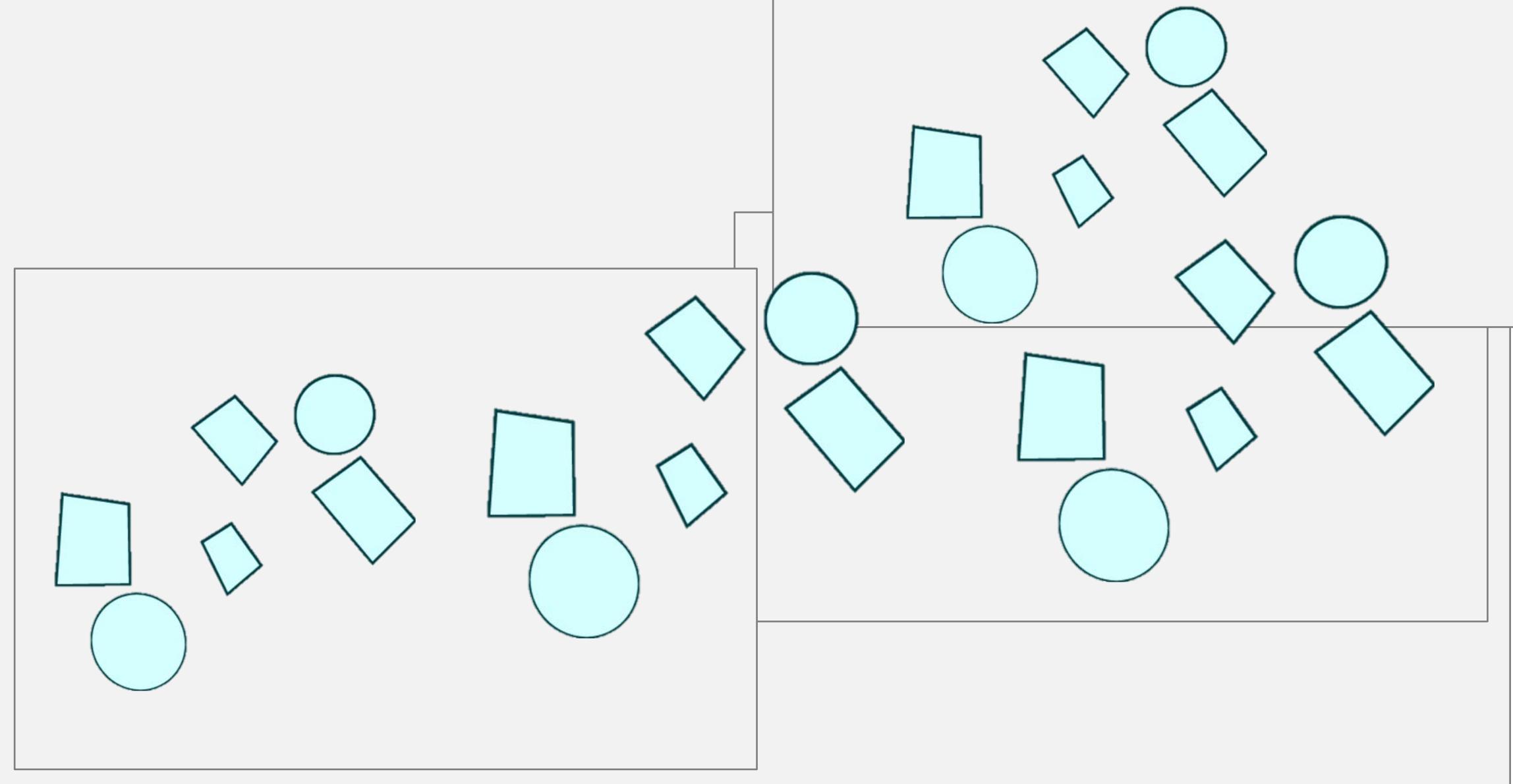
Constructing a BVH



Constructing a BVH



Constructing a BVH



Spatial Data Structures

Basic Idea – asymptotic improvement in spatial queries by subdividing

Two types of subdivisions – ***object-based*** and ***spatial***

Our object-based data structures will be boundary volume hierarchies or BVHs.

BVHs are hierarchies of BVs represented by trees

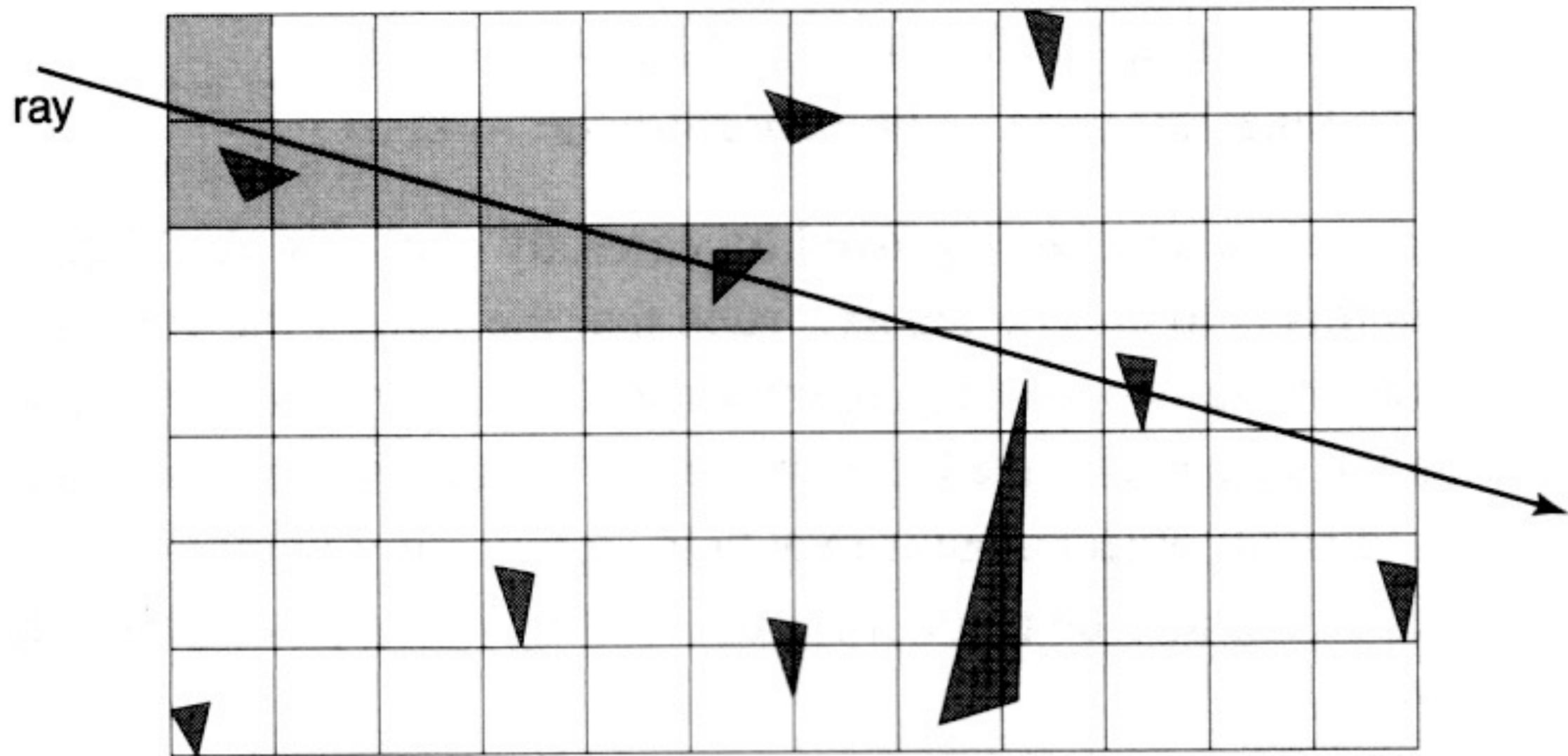
Spatial Data Structures

Basic Idea – asymptotic improvement in spatial queries by subdividing

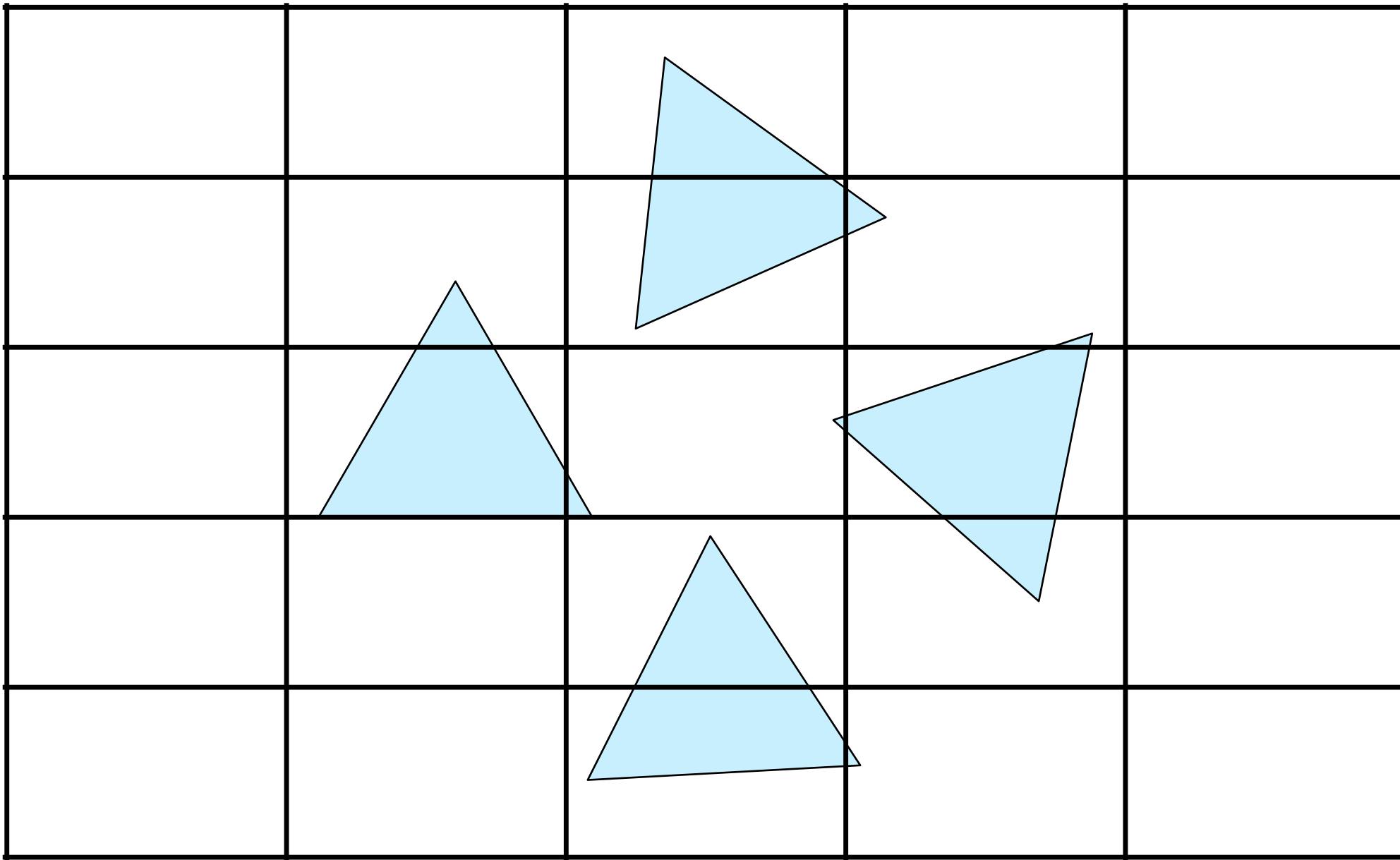
Two types of subdivisions – ***object-based*** and ***spatial***

Spatial subdivision divides space hierarchically and represents this as a tree.

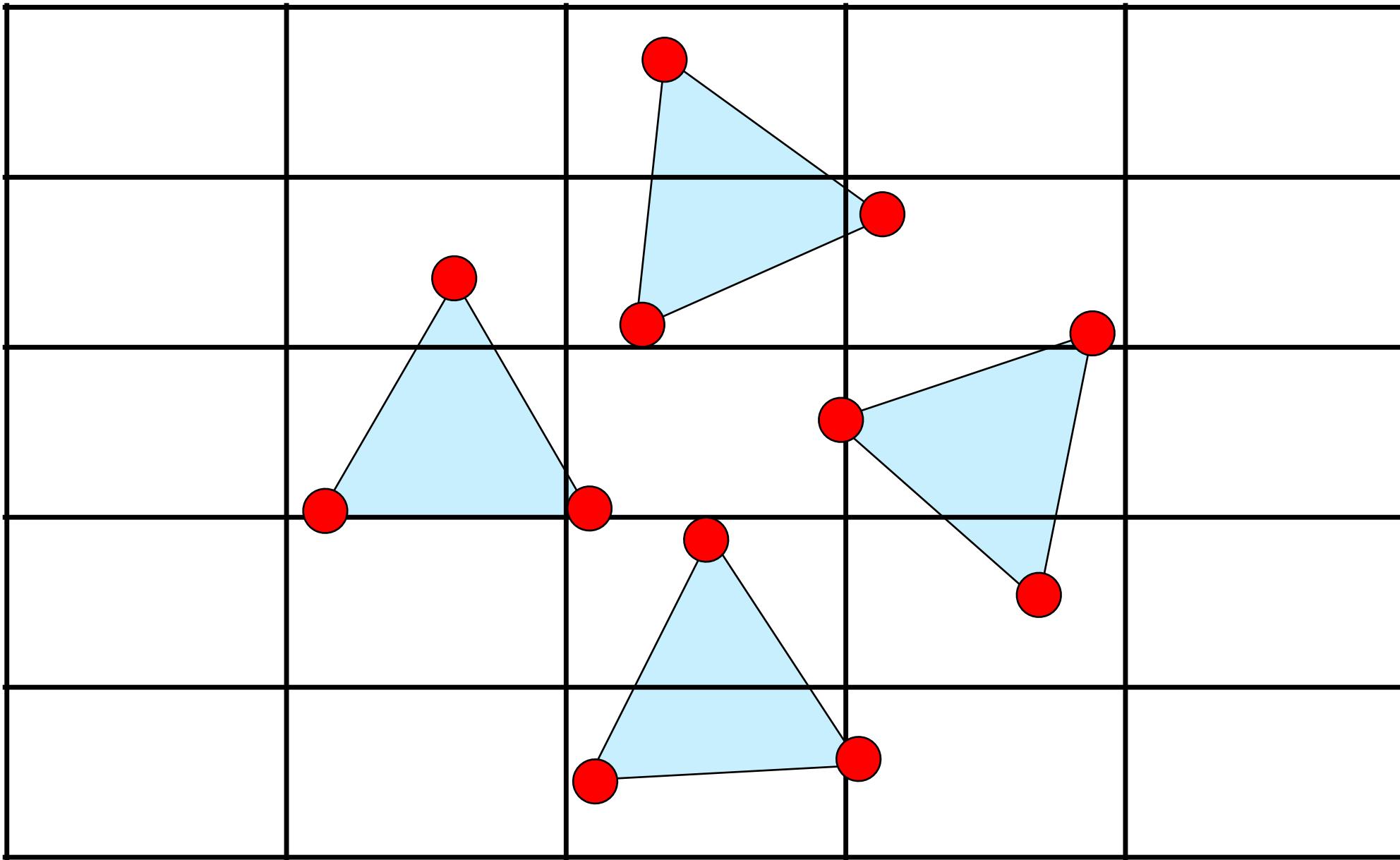
Axis-Aligned Spatial Subdivision (Uniform)



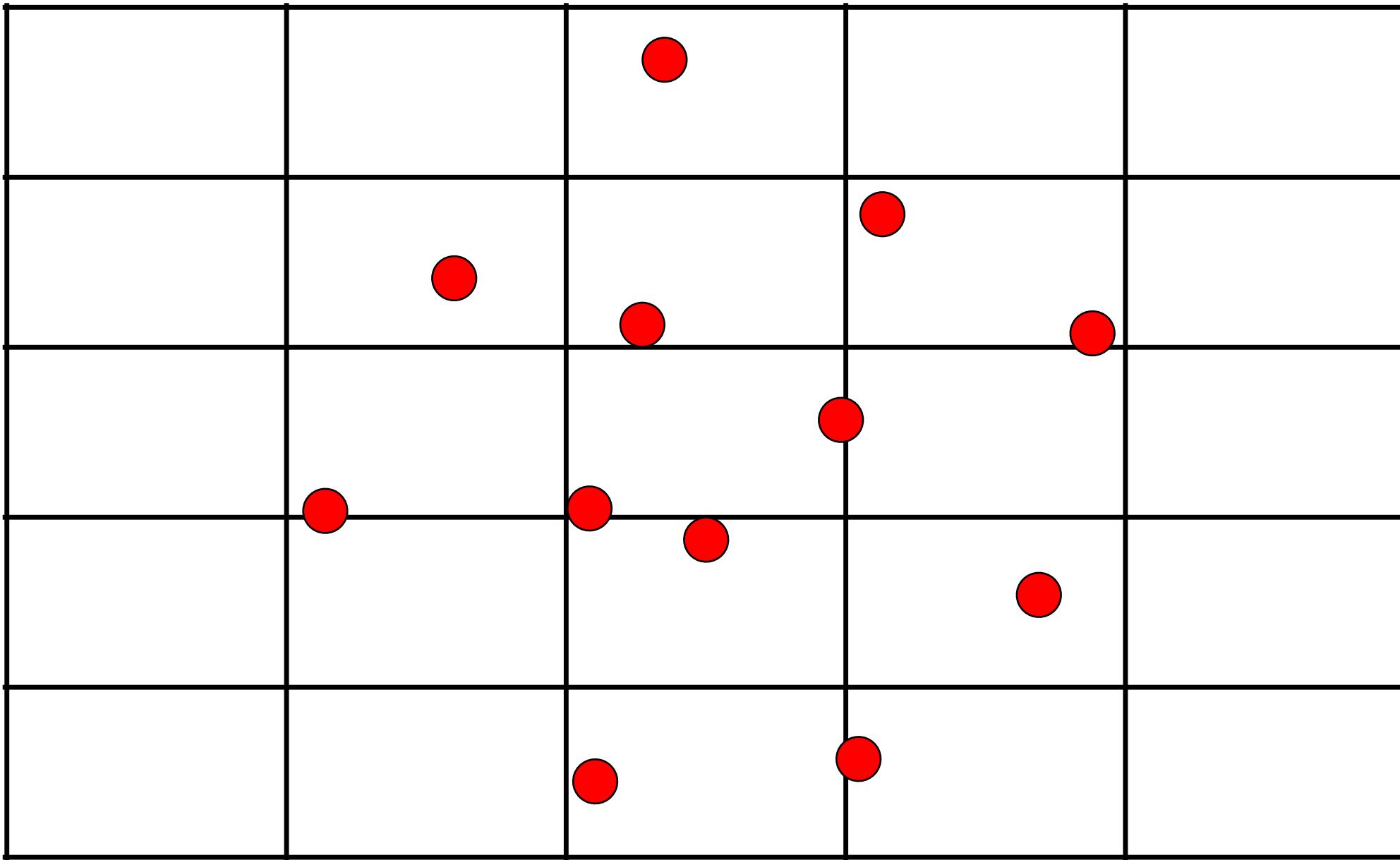
Construction



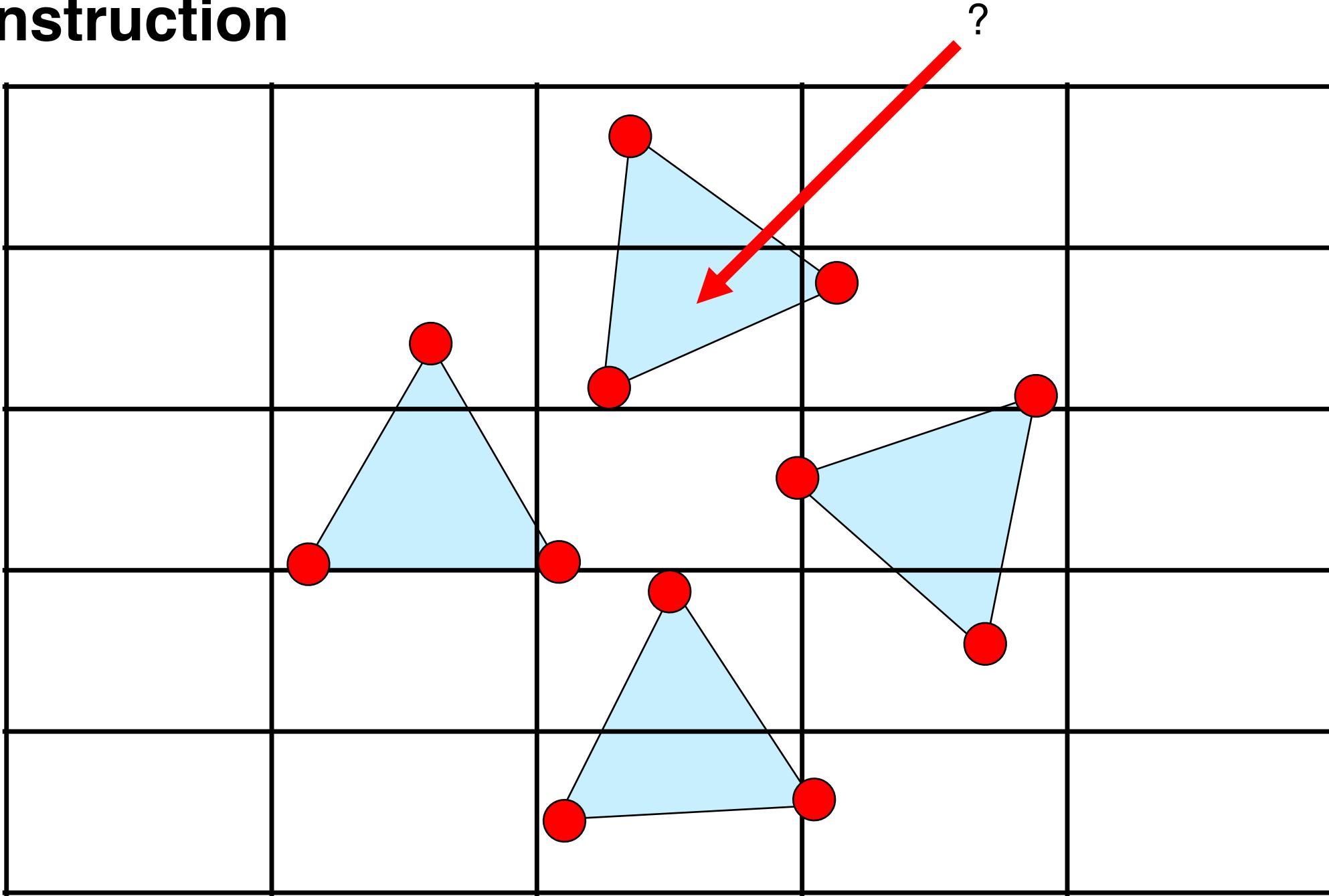
Construction



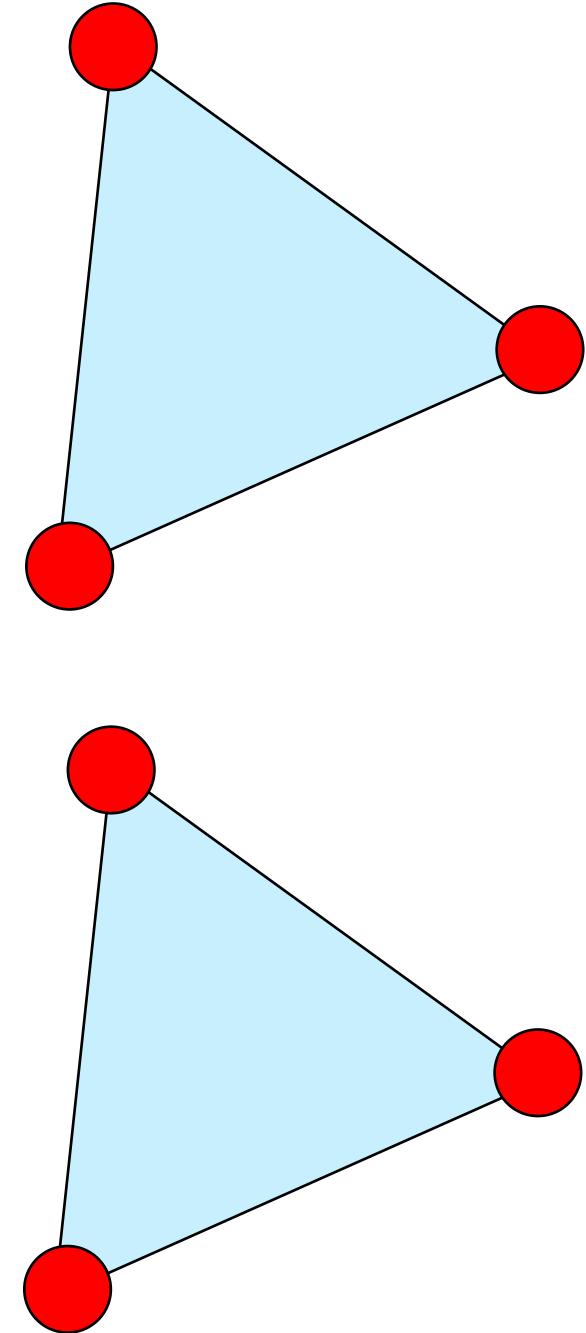
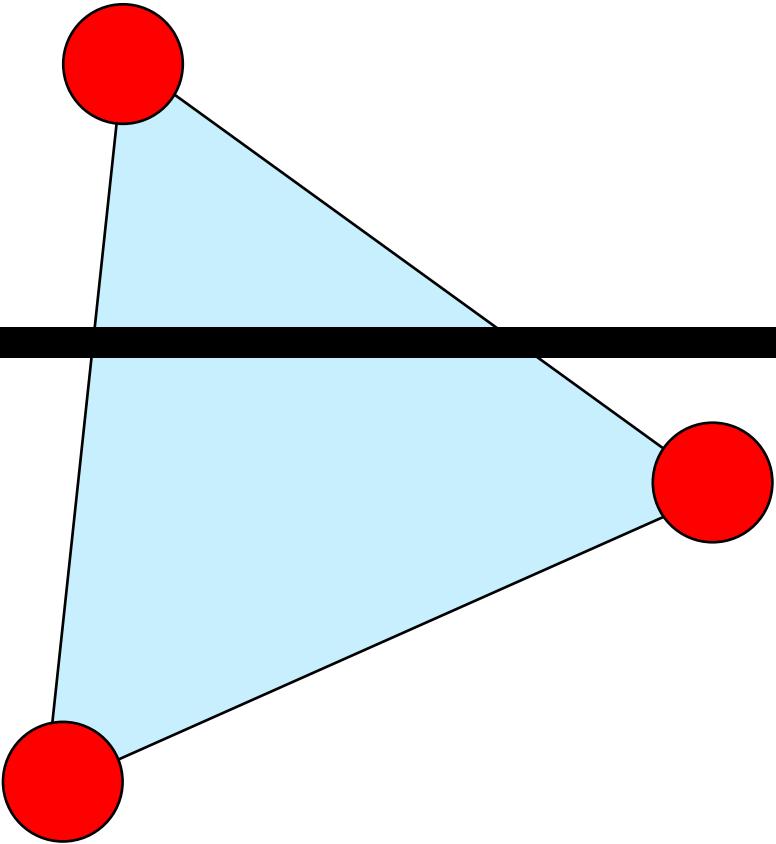
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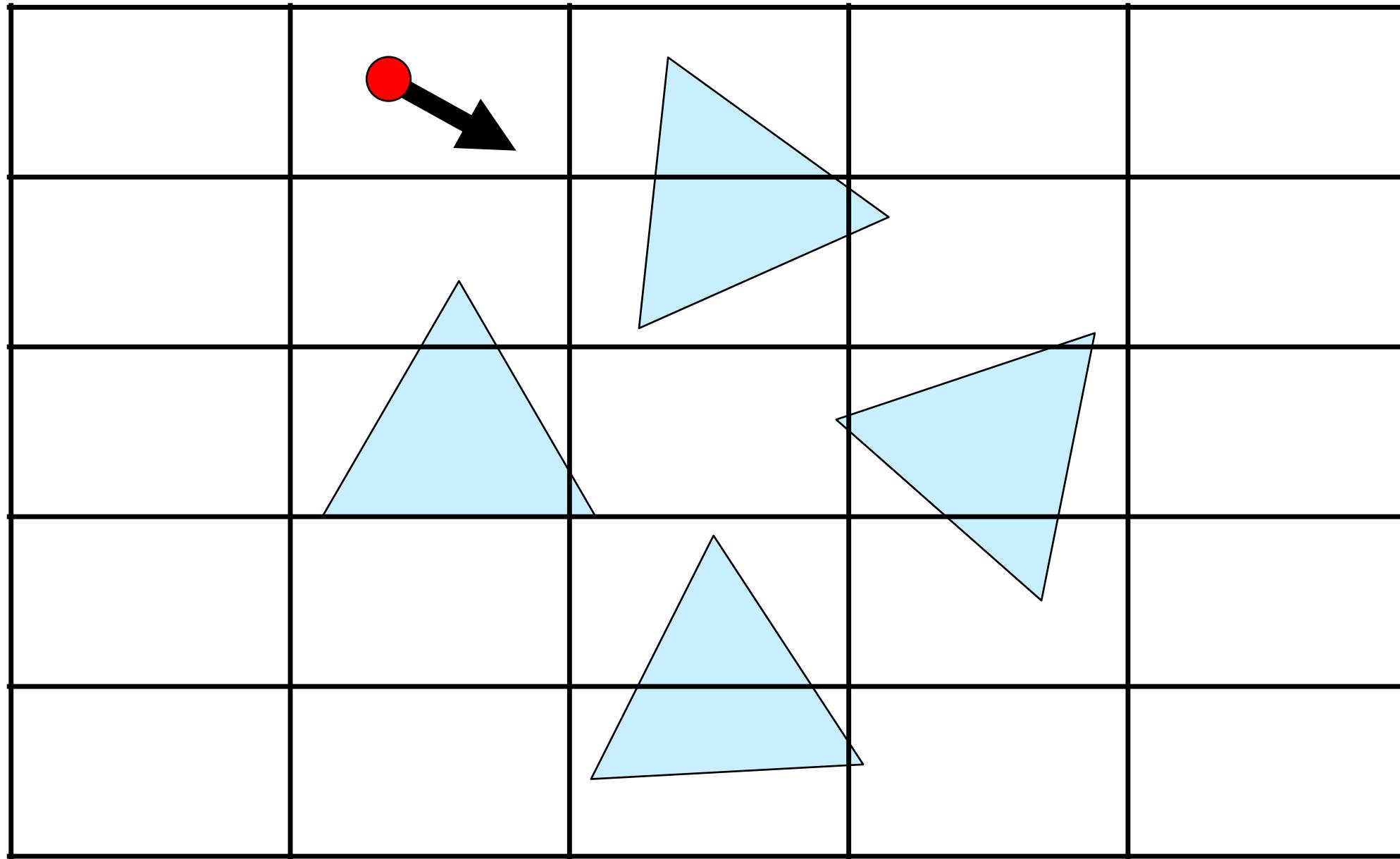
Construction



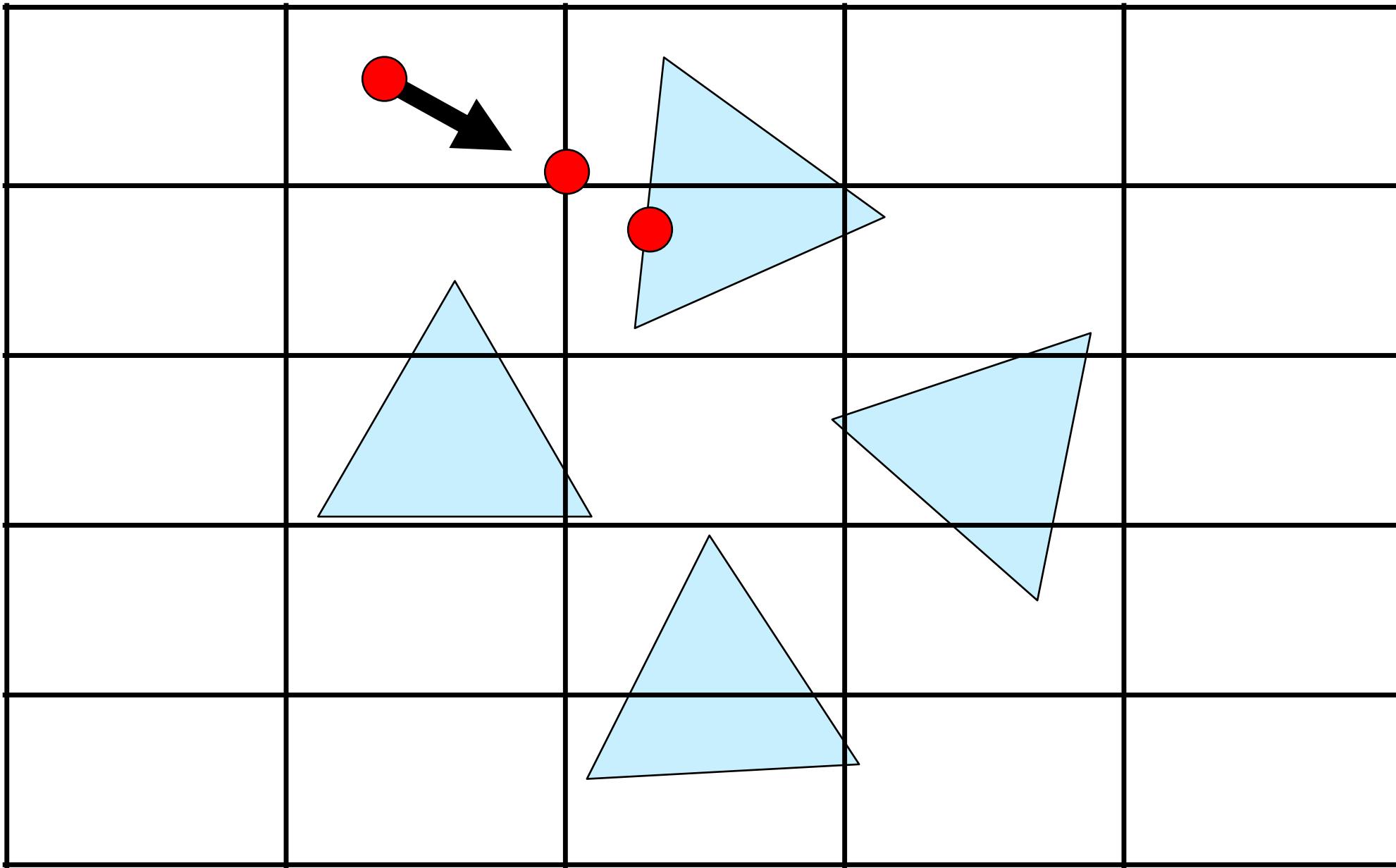
Duplicate Triangle



Intersection Tests

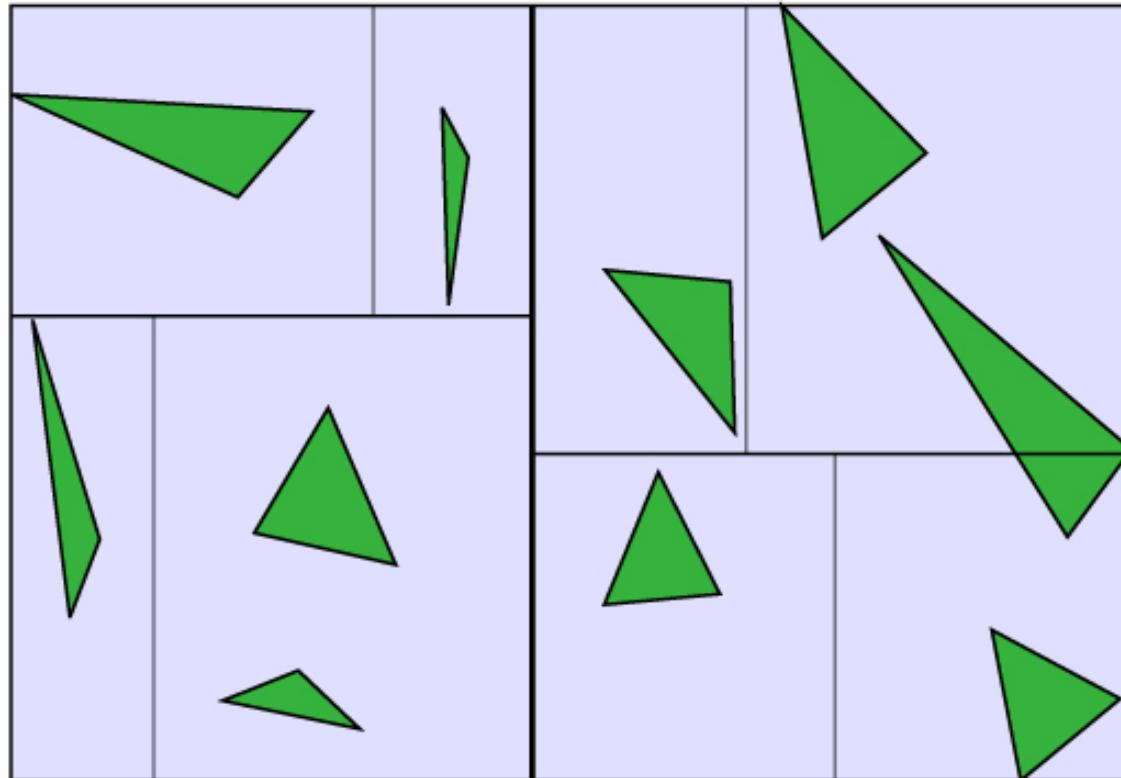


Intersection Tests

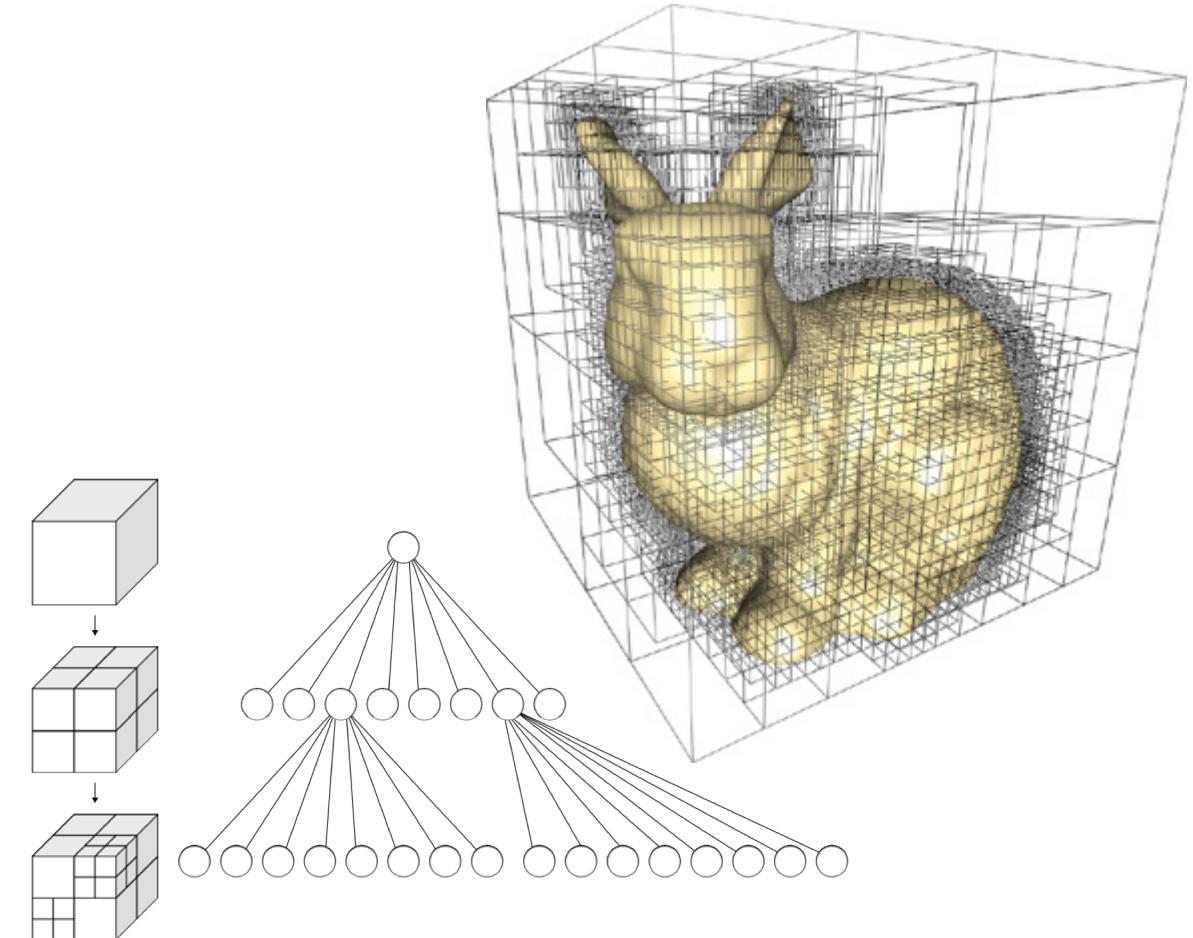


Axis-Aligned Spatial Subdivision (Non-Uniform)

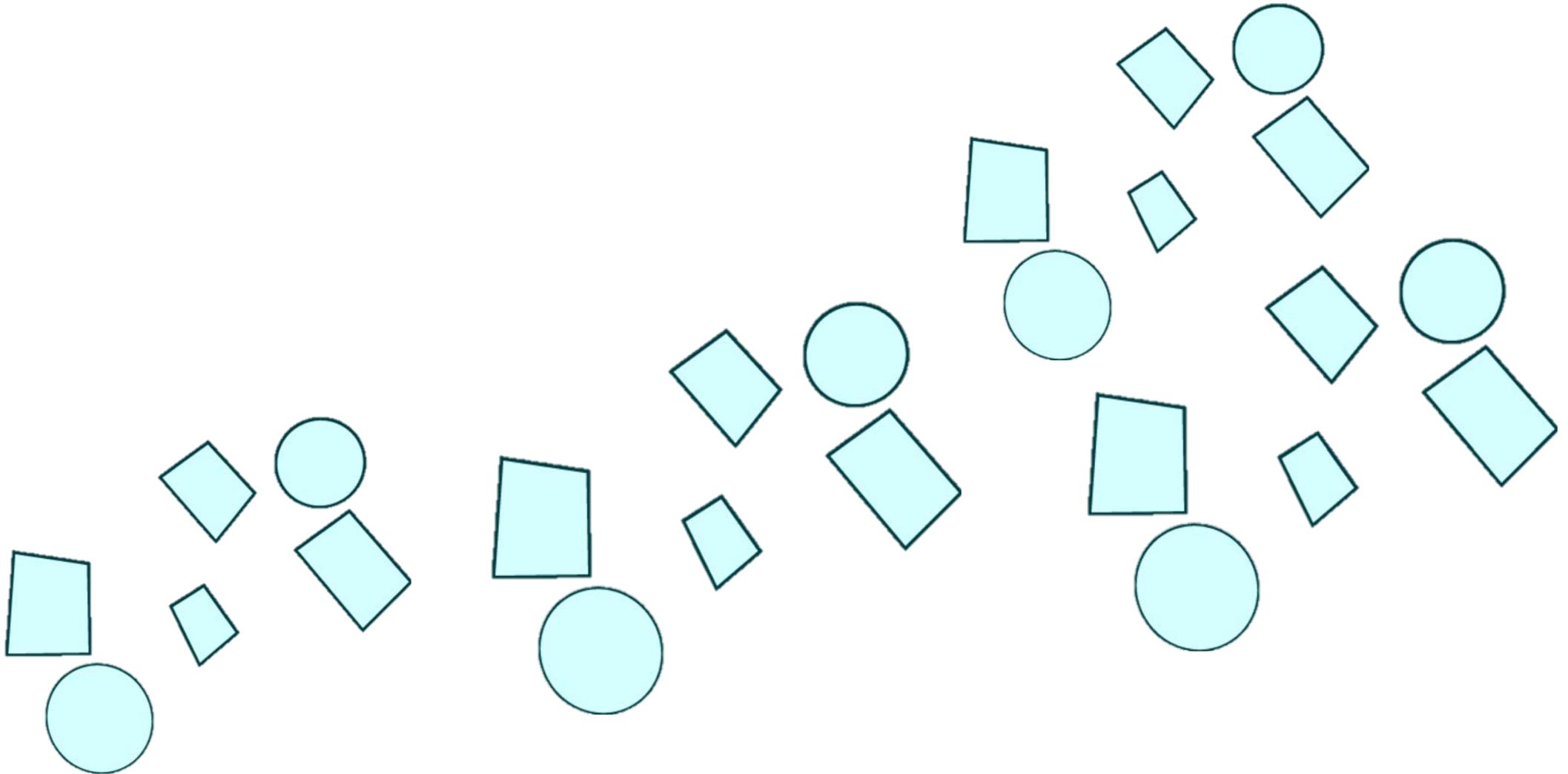
BSP Tree



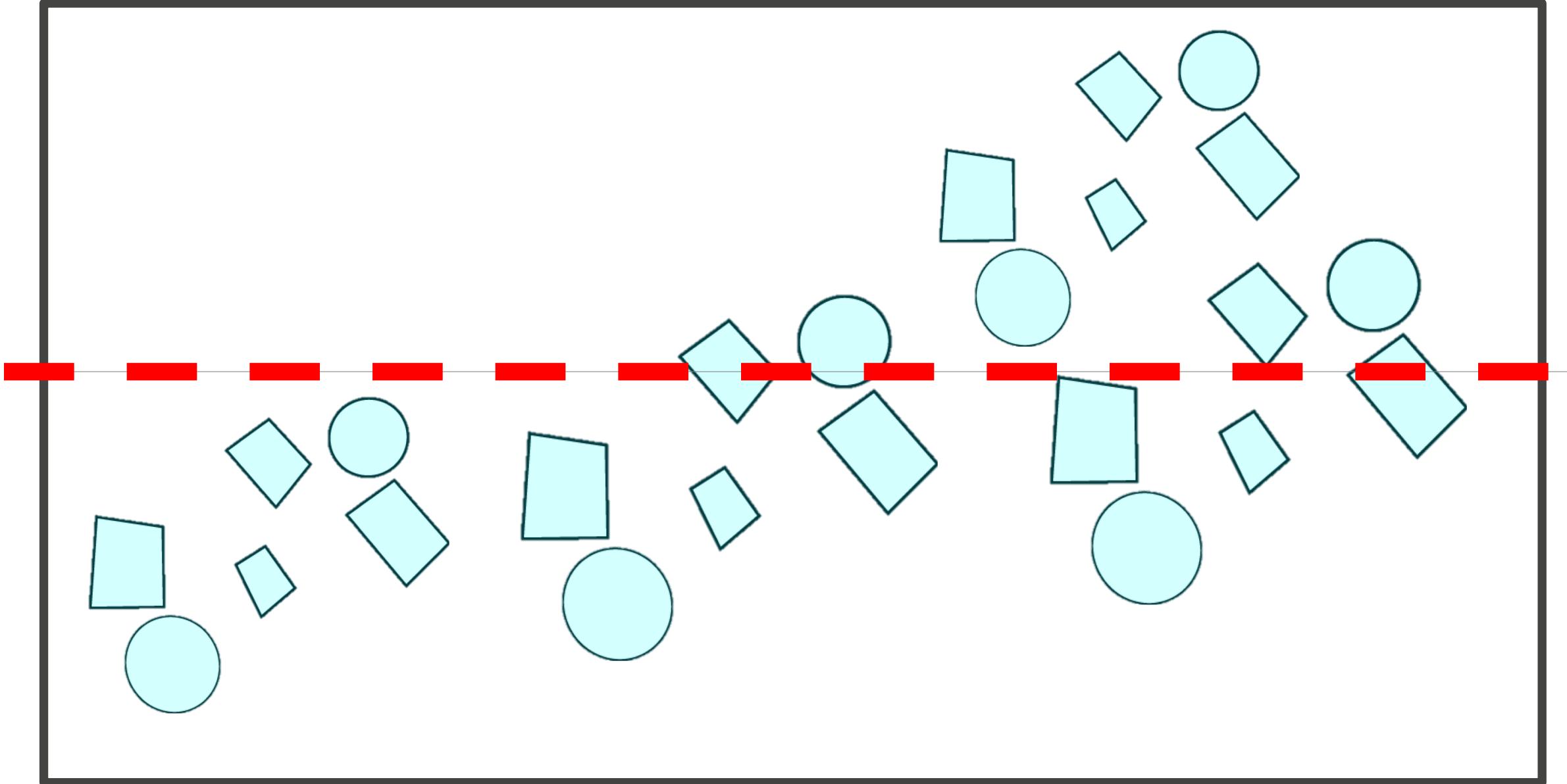
Octree



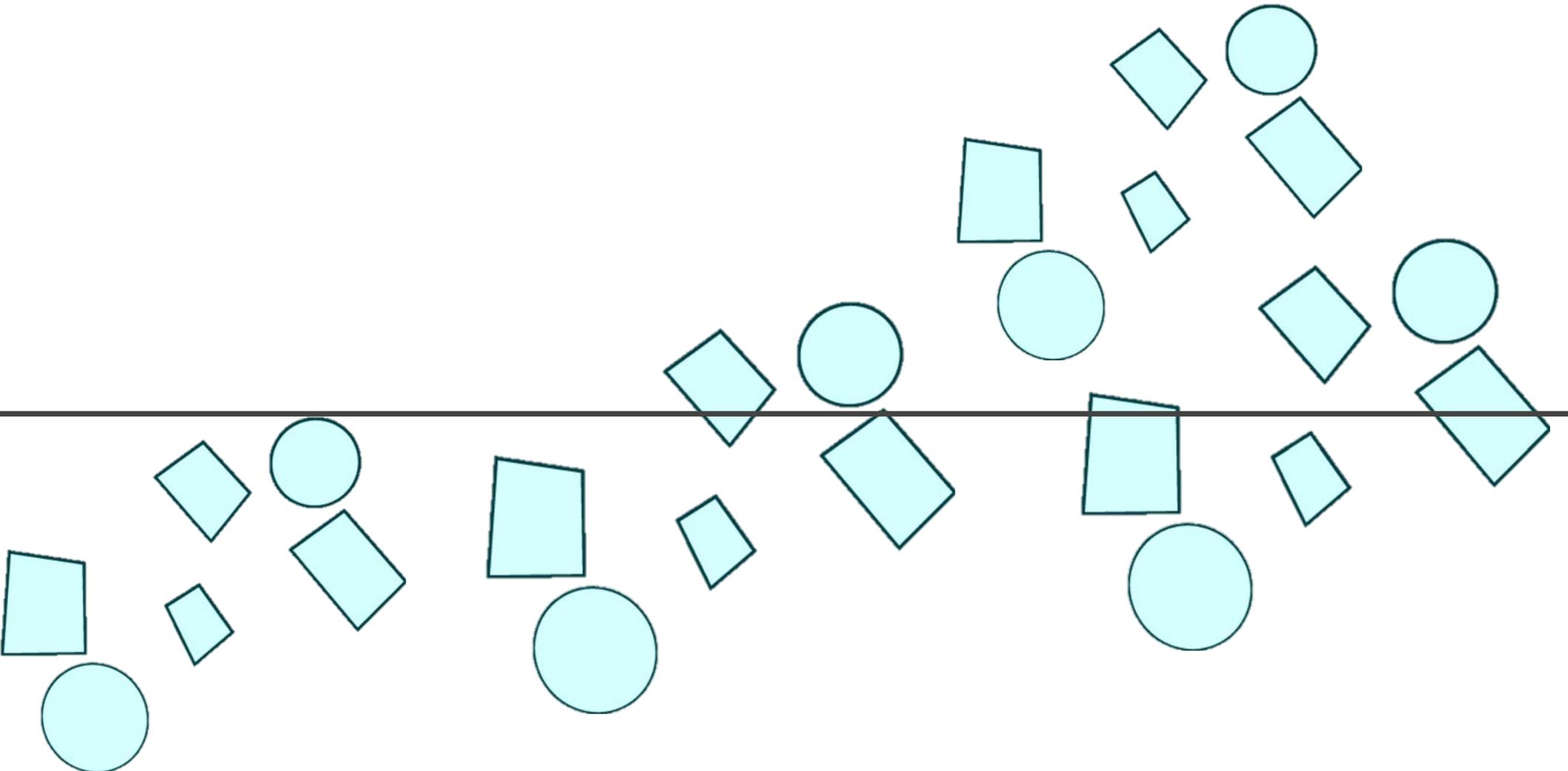
Constructing a k-d Tree



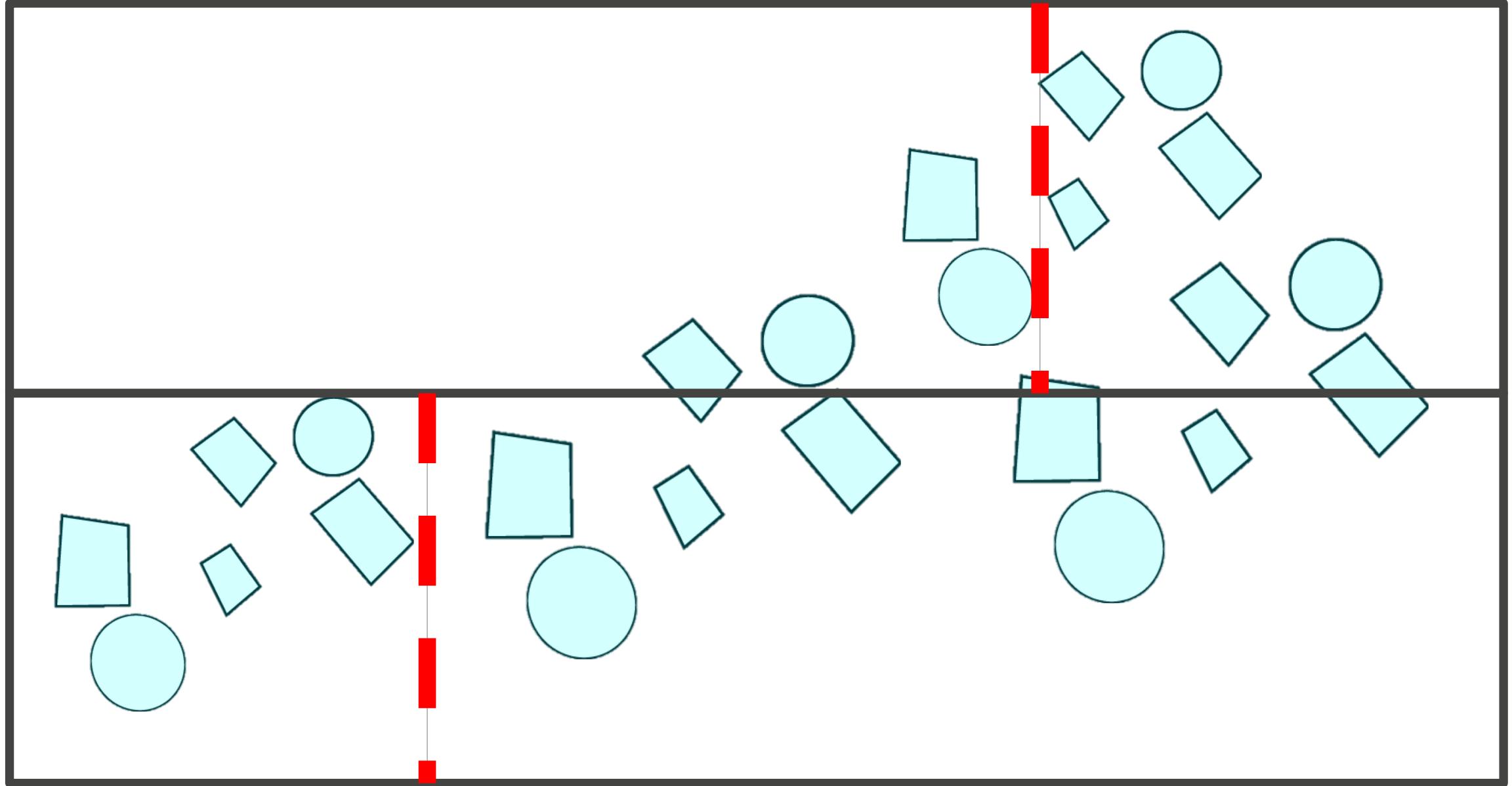
Constructing a k-d Tree



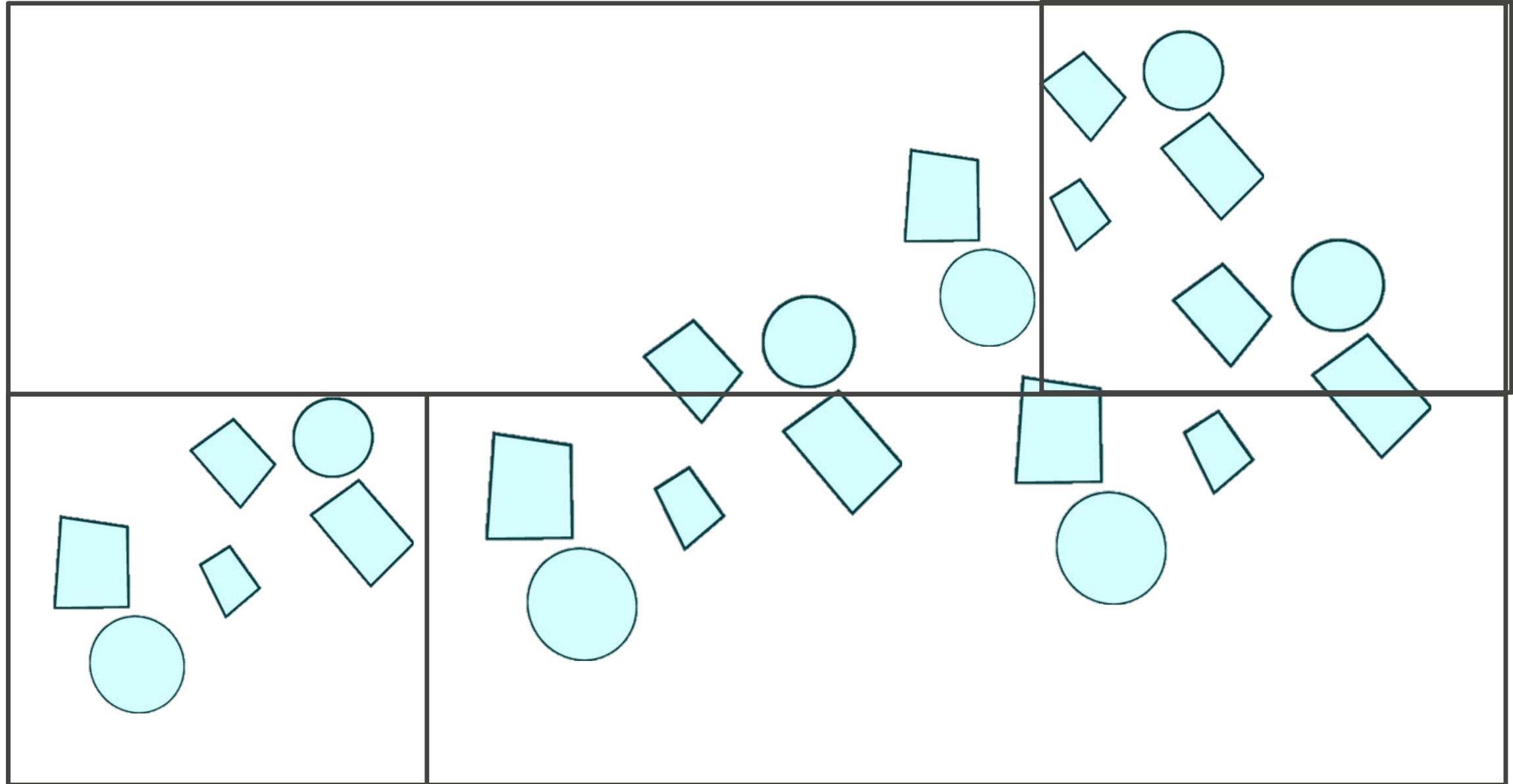
Constructing a k-d Tree



Constructing a k-d Tree



Constructing a k-d Tree



Ray Intersection Tests

Depth First Search Again

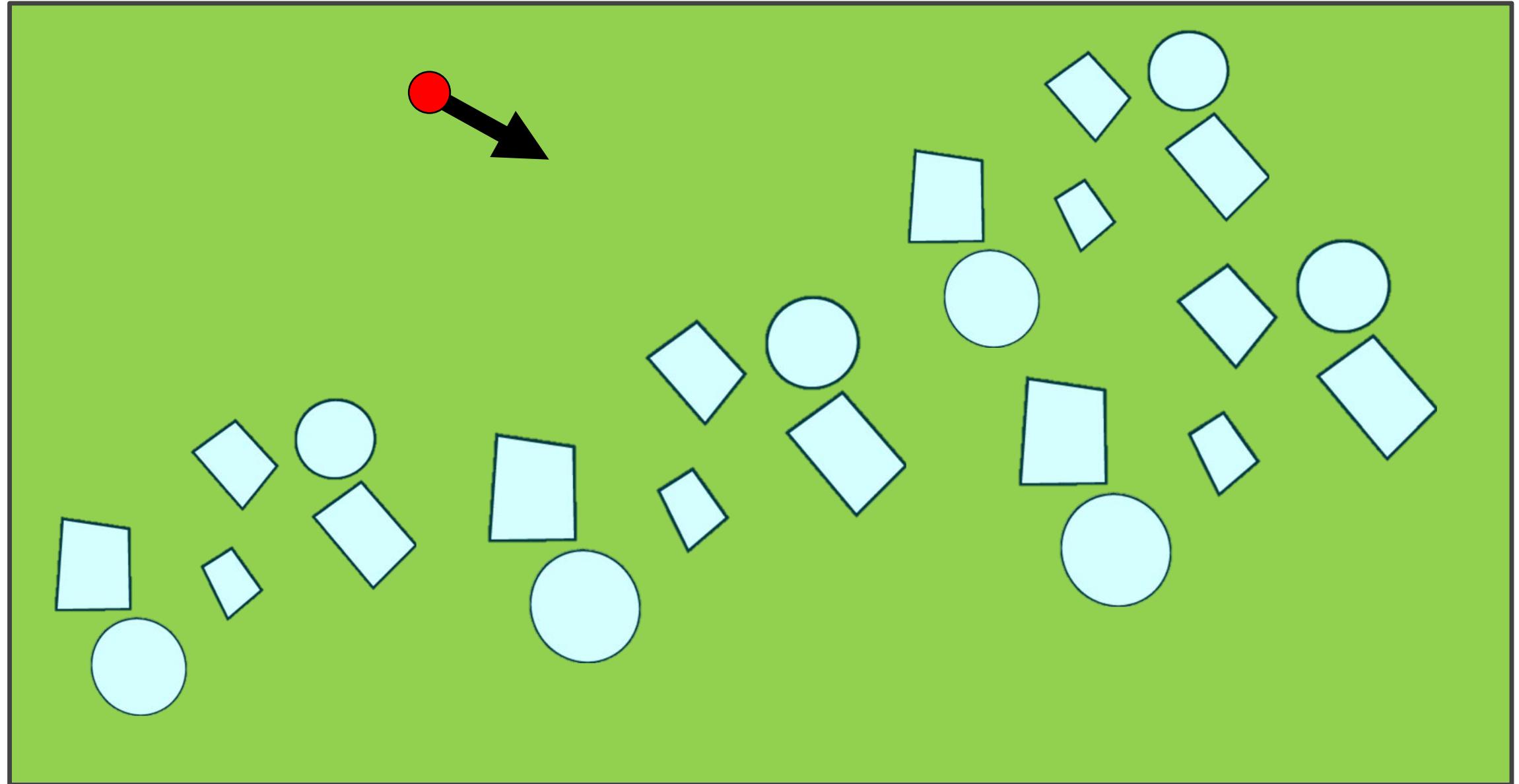
If ray interacts with child node then recurse

Interactions are

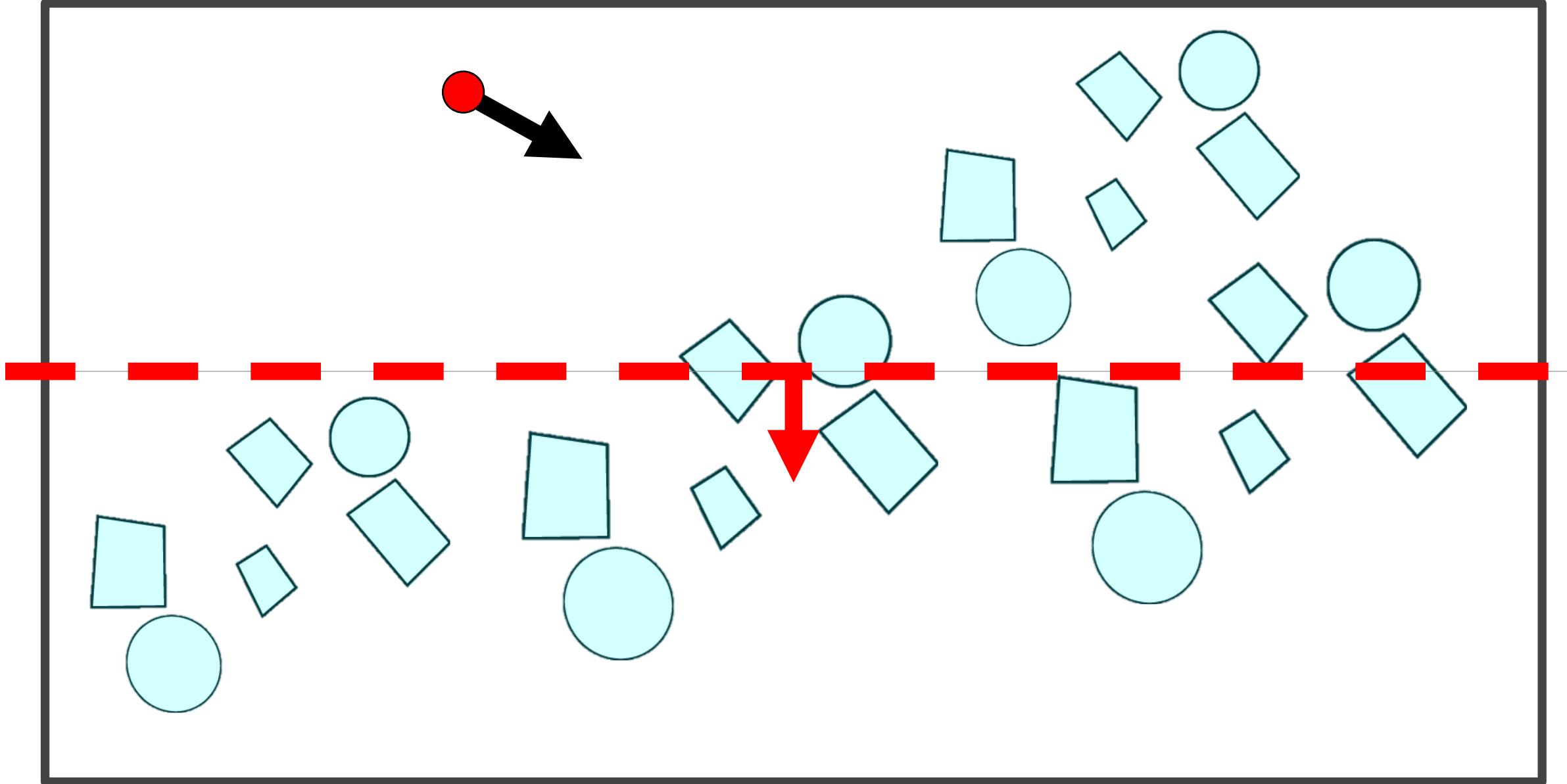
Child contains ray origin point

Ray crossed into child node

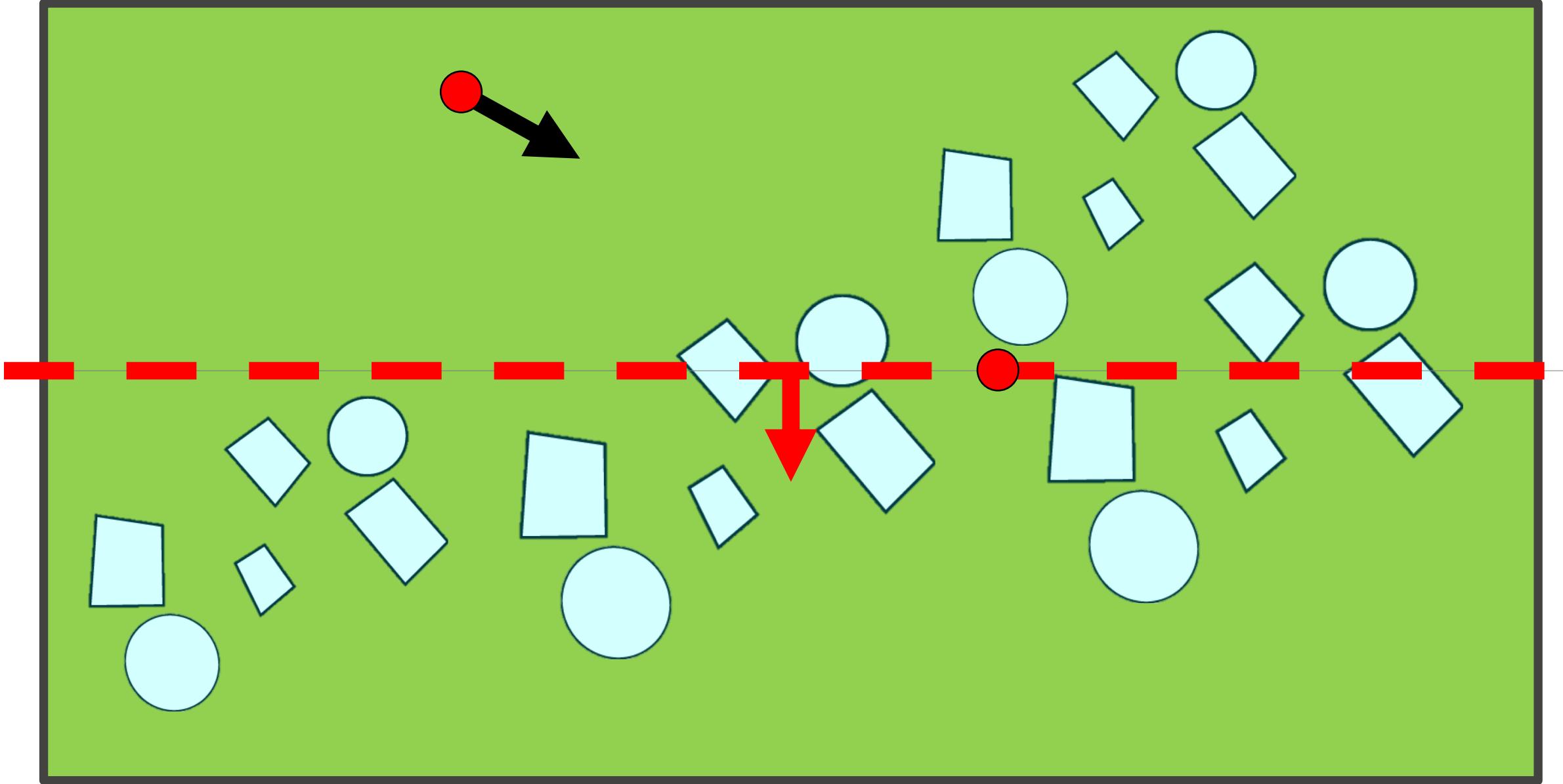
Ray Intersection Tests



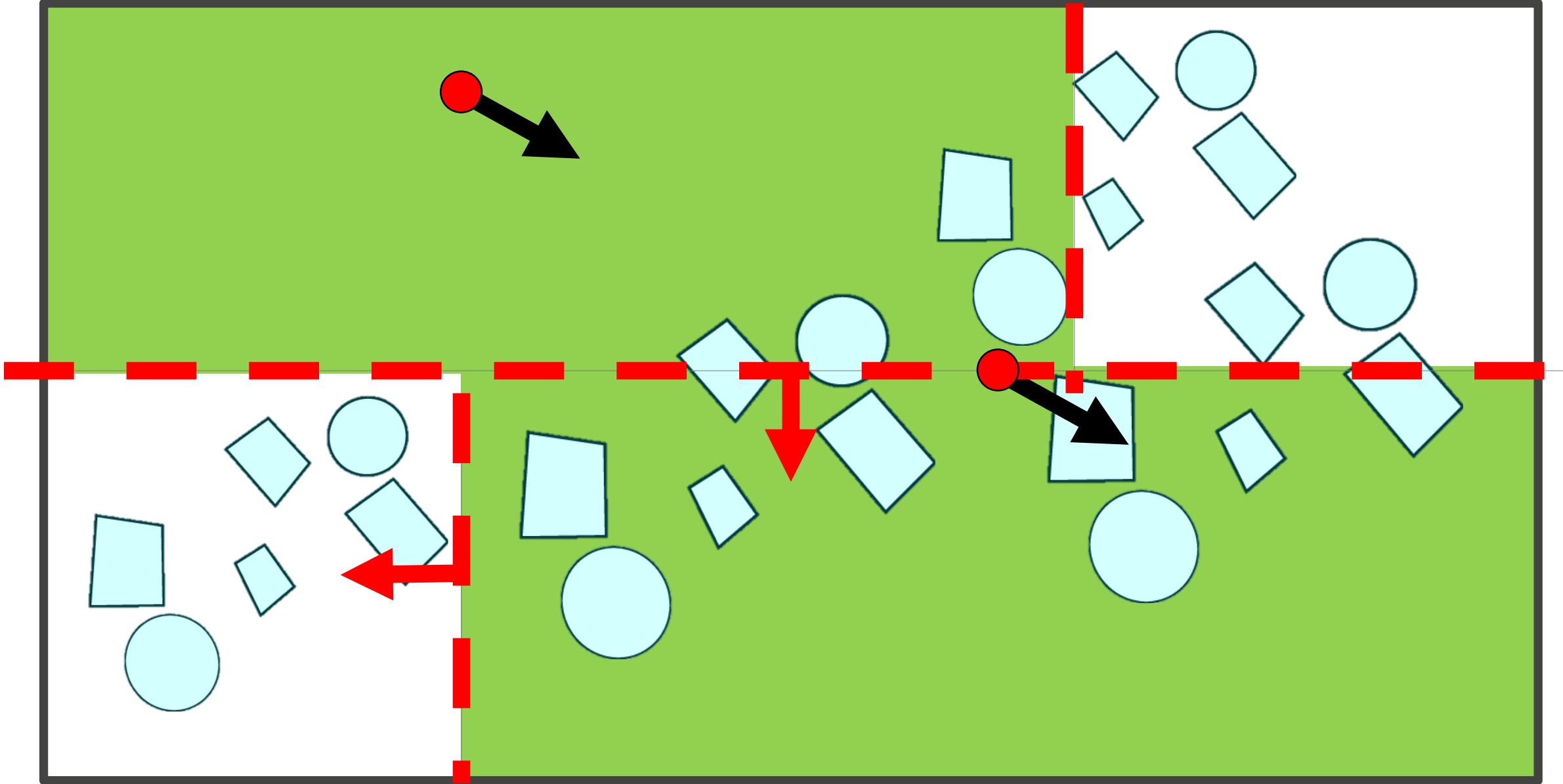
Ray Intersection Tests



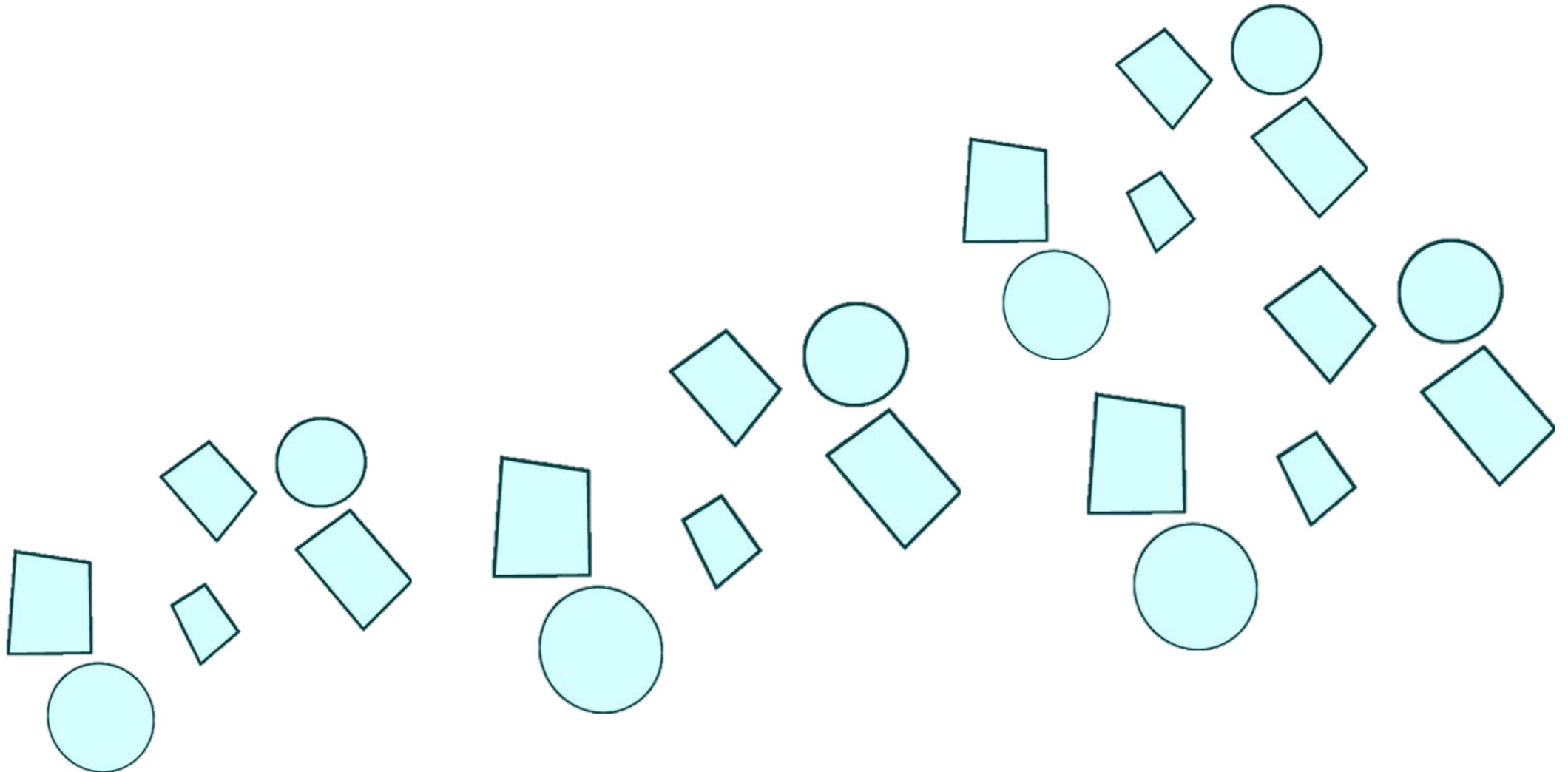
Ray Intersection Tests



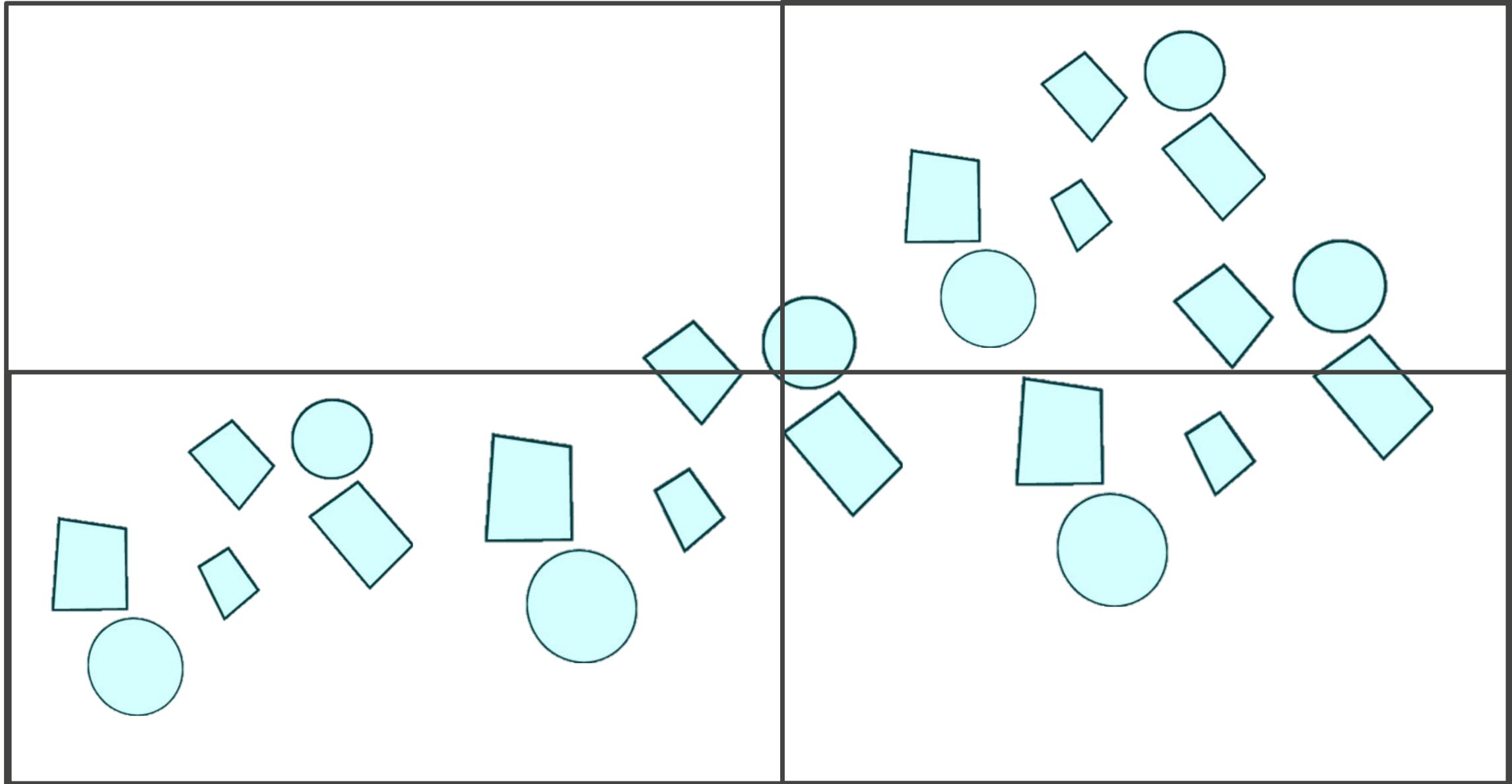
Ray Intersection Tests



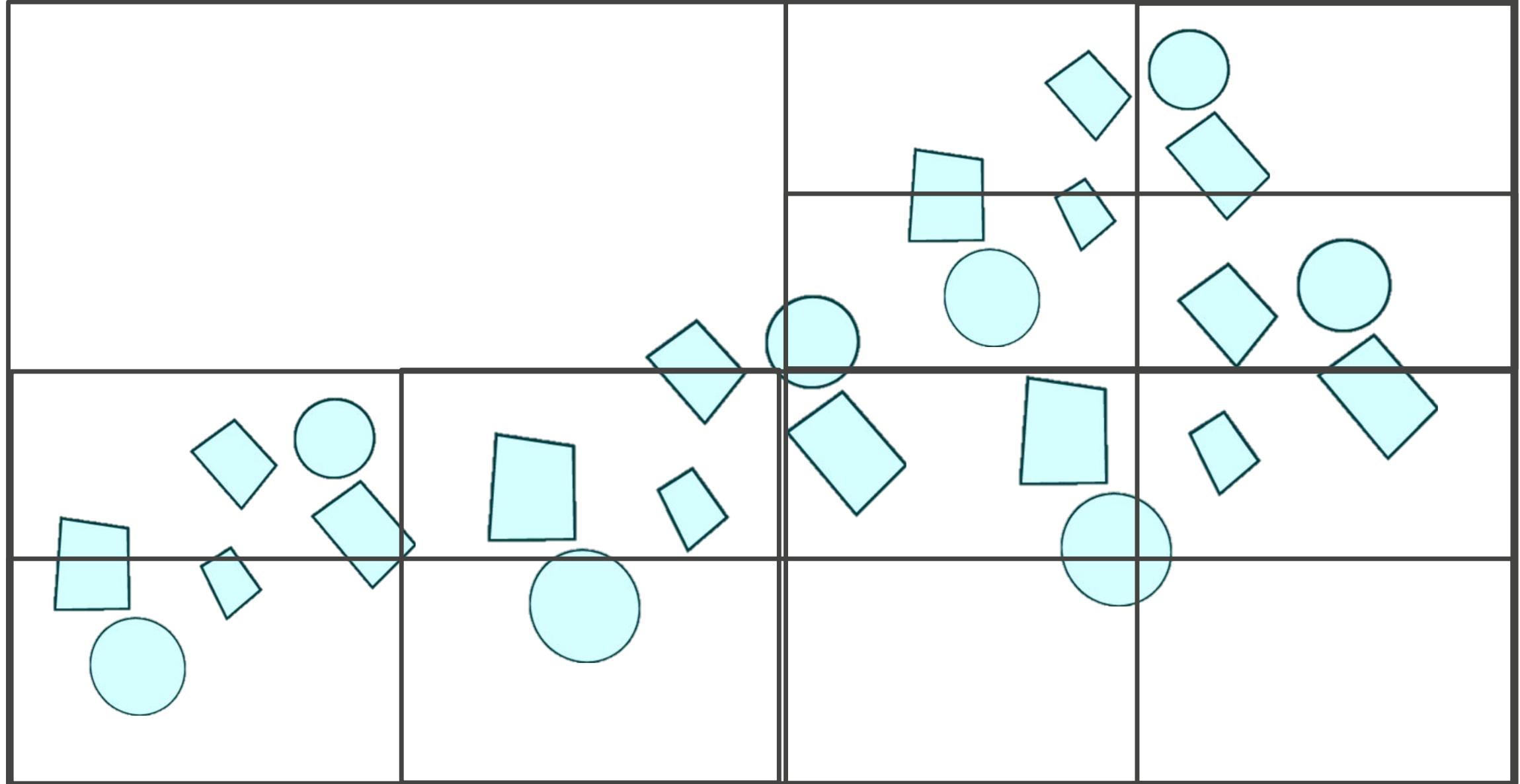
Constructing an Quadtree Tree



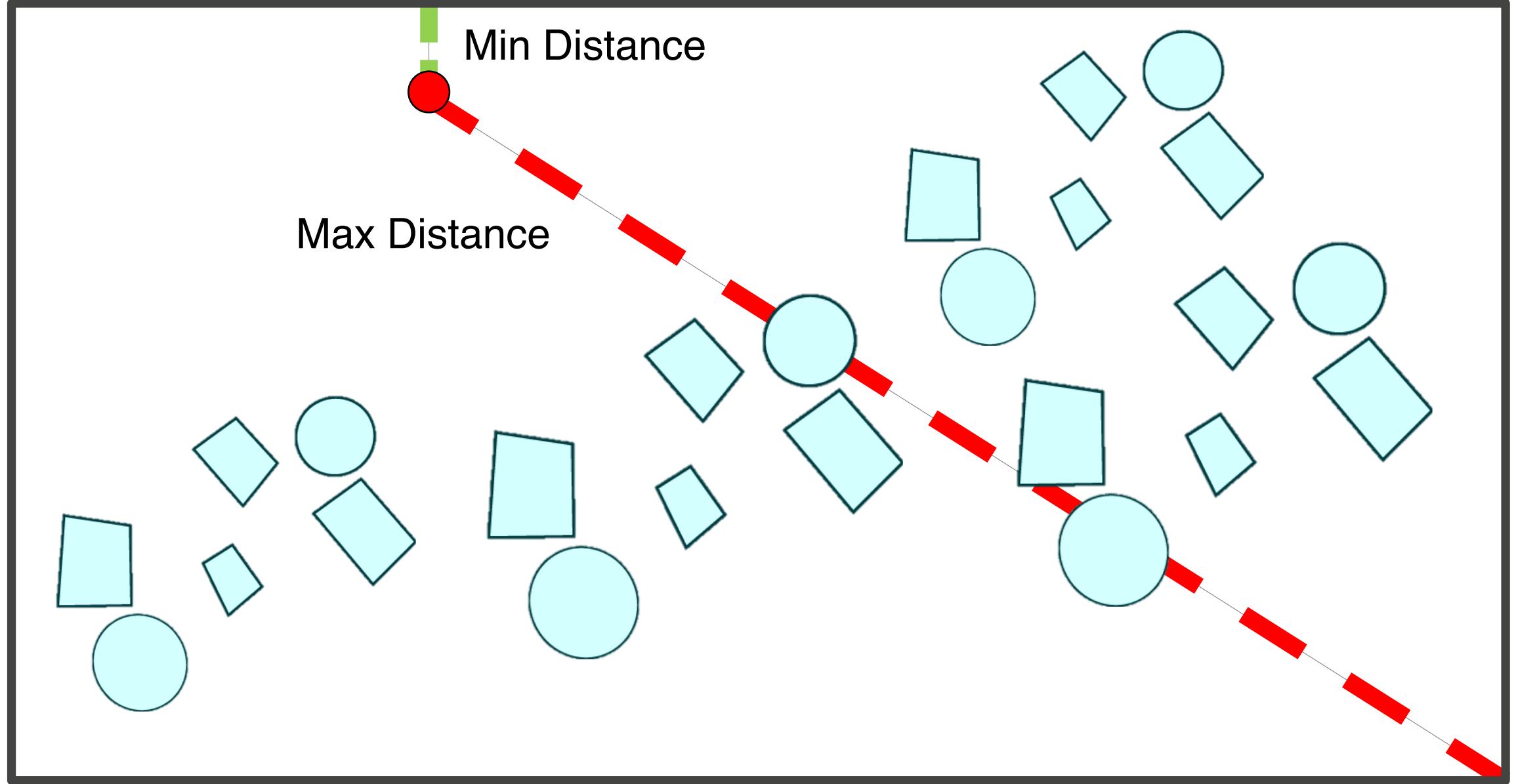
Constructing an Quadtree Tree



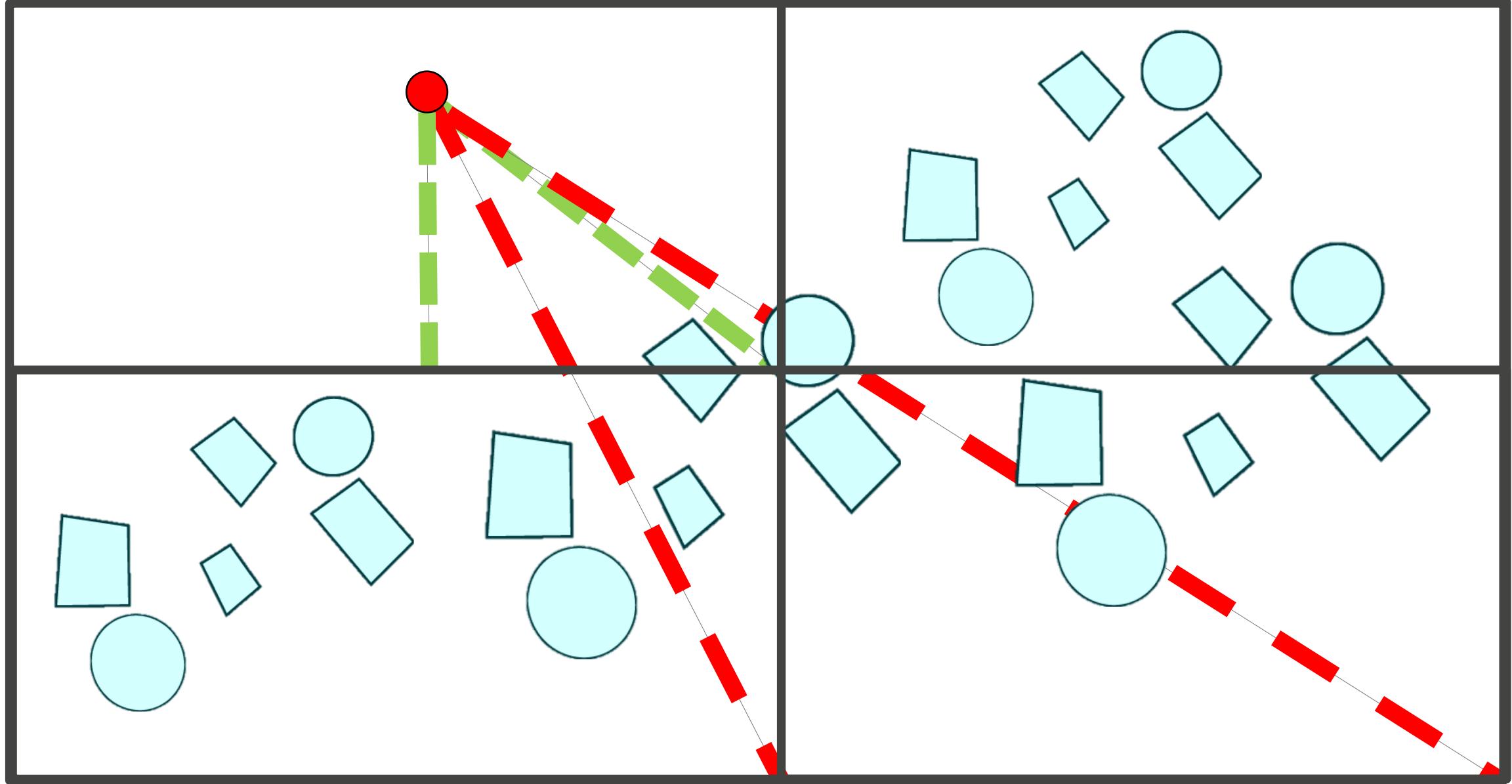
Constructing an Quadtree Tree



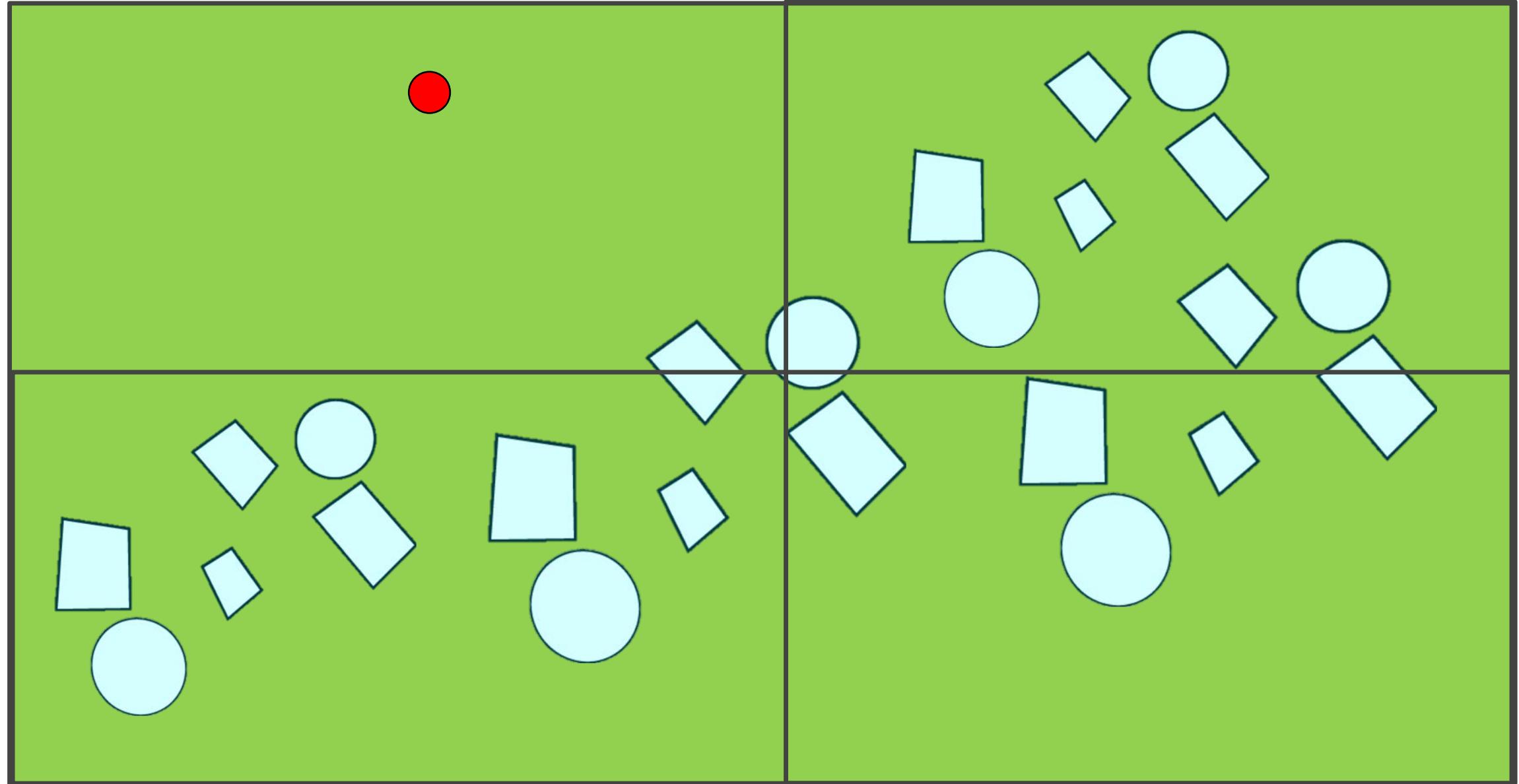
Distance Query



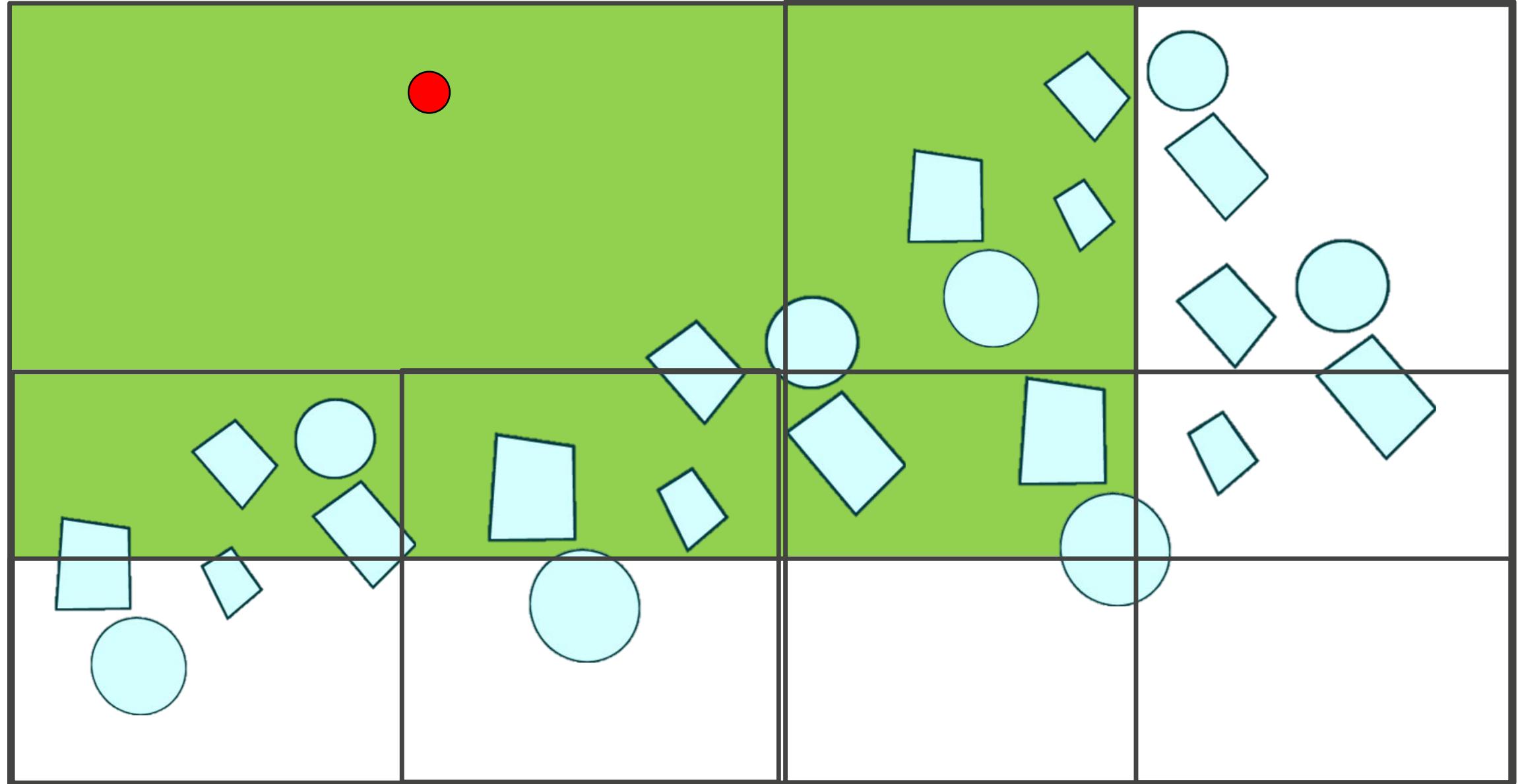
Distance Query



Distance Query



Distance Query



Done

Assignment 3 due on Friday

Assignment 4 out now

Office hours now BA5268