CATEGORY THEORY 2025

IVAN DI LIBERTI

EXERCISES

savoir faire: Yoneda, adjunctions and limits

Leinster 1 (🪫). 6.2.20

Leinster 2 (🪫). 6.2.21

Leinster 3 (🪫). 6.3.21(a)

Leinster 4 (🪫). 6.3.22

Leinster 5 (🪫). 6.3.26

Leinster 6 (🪫). 6.3.27

monads

Exercise 7 (🪫). Describe the monads (unit and counit) on Set whose algebras are: monoids, groups, semigroups.

Exercise 8 (🪫). Consider the free-forgetful adjunction $D : \mathbf{Set} \leftrightarrows \mathrm{Top} : \mathrm{U}$, where D equips a set with the discrete topology over it. Compute the algebras for the induced monad over **Set**.

Exercise 9 (🔋). Show that the category Suplat whose objects are suplattices and morphisms are suplattices morphisms is monadic over **Set** via forgetful functor $\mathbb{U} : \mathsf{Suplat} \to \mathsf{Set}$. *Hint*: Guess the monad and prove that an algebra is precisely a suplattice.

Exercise 10 (🪫). A monad T on a category C is idempotent if its multiplication if its multiplication is an isomorphism. Show that the fortgetful functor $U_T : \mathrm{Alg}(T) \to C$ of an idempotent monad is fully faithful.

Exercise 11 (🪫). Let C be a category with coproducts and a terminal object. Can you always put a monad structure on the *maybe endofunctor* $c \mapsto c \coprod 1$?

Exercise 12 (🔋). Show that the category of fields is not monadic over Set.

Exercise 13 (🪫). Show that there is a monad on directed graphs whose algebras are small categories.

Exercise 14 (🪫). Show that there is a monad on the category of small categories (and functors) whose algebras are posets.

Date: March 5, 2025.

Jiří's *treat* (⚠, ☕). Let Suplat^{∇} be the category whose objects are suplattices with a unary operation ∇ satisfying $(\forall x)(x \leq \nabla x)$. Morphisms are suplattices morphisms preserving the unary operation. Show that the forgetful functor

$$\mathbb{U} : \mathsf{Suplat}^{\nabla} \to \mathsf{Set}$$

preserves limits but does not have a left adjoint. *Hint*: Show that a free algebra over 1 does not exist.

rules

- Hand your exercises before your **oral interview** via email. In order to make my life easier, make sure to include the word **CT25 in the subject**.
- Pick at least one exercise from each of the yellow groups.
- You must charge at least **1** batteries and a half!
  Example. The vector of exercises [2,7,12,19] would pass this sheet.
- The label **Leinster** refers to the book **Basic Category Theory**, by *Leinster*.
- The label **Riehl** refers to the book **Category Theory in context**, by *Riehl*.