# EPISTEMIC MODALS, STRUCTURED PROPOSITIONS, AND LOGICAL OMNISCIENCE

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**Abstract:** This essay argues that the widely-held view that epistemic modals are quantifiers over possibilities of some kind undermines the principal motivation for the widely-held view that propositions are structured. The principal motivation for the structured view is that it avoids the problem of logical omniscience. But if we accept that epistemic modals are quantifiers over possibilities, then we will run into a version of that problem even if we take propositions to be structured. So if advocates of the structured view accept that epistemic modals are quantifiers over possibilities, they cannot avoid the problem of logical omniscience in its full generality. This situation might motivate an advocate of the structured view to deny that epistemic modals are quantifiers overs possibilities. This is indeed a stable position, but reveals an unexpected cost of the structured view: it is in tension with any suitably general semantic account of natural language modality.

## 1

Epistemic modals sit between two types of expression that, until recently, have received more attention in the philosophical literature: metaphysical modals, on the one hand, and attitude verbs, on the other. Like metaphysical modals, epistemic modals are often expressed in English via modal auxiliary verbs such as *might*, *could*, and *must*, and are typically analyzed as quantifiers over possibilities of some kind. Like attitude verbs, epistemic modals seem to create opaque contexts and do not appear to be closed under logical entailment.

In this essay, I argue that the peculiar mixture of features possessed by epistemic modals complicates a familiar debate over the nature of propositions. In particular, I argue that the claim that epistemic modals are quantifiers over possibilities undercuts the principal motivation for the view that propositions are structured. That suggests that advocates of structured propositions ought to reject the claim that epistemic modals are quantifiers over possibilities.

While the costs of *unstructured* views of propositions are well-known, our discussion reveals an unexpected cost of the structured view—it appears to be in tension with any suitably general semantic account of natural language modality.

The essay proceeds as follows. Section 2 introduces the structured and unstructured views of propositions, along with what I take to be the principal motivation for the structured view: the problem of logical omniscience. Section 3 argues that the widely-held view that epistemic modals are to be analyzed in terms of quantification over possibilities leads to a version of the problem of logical omniscience, irrespective of whether propositions are taken to be structured or not. This suggests that advocates of structured propositions ought to reject the quantificational analysis of epistemic modality. Section 4 examines the costs and benefits of that view.

## 2

"Proposition" is a philosopher's term of art. Propositions are thought to play a number of different roles in philosophy. They are said to be the contents of beliefs and desires, to be the primary bearers of truth and necessity, and to be the meanings of sentences relative to contexts. Here I am interested in two related roles for propositions: as the contents of attitudes, and as the semantic values of clauses embedded under attitude verbs and modals. I shall assume one and the same object plays these roles.

A familiar question in the philosophy of language is: are propositions structured? Sentences of natural language have constituents that are hiearchically arranged: they have constituents that themselves have constituents. Are propositions like that? Do they too have constituents structured in a way similar to the way the constituents of sentences are structured? Or are propositions unstructured sets of possibilities of some kind (possible worlds, centered worlds, etc.)? On the latter view, a proposition has *elements* (namely possiblities), but these elements do not correspond to the expressions of any sentence used to express the proposition, and these elements are not hiearchically arranged

The view that propositions are structured has historically been the more popular view in philosophy and almost certainly still is.[1] Fregeans and Russellians disagree on what the constituents of propositions are, but they typically agree that propositions *have* constituents, and that those constituents are structured in a manner similar to sentences. The contrasting view

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[1]See, among many others, Frege (1892), Russell (1903), Kaplan (1989), Soames (1985, 1987, 2002), Salmon (1986), and King (2019).

maintains that propositions are unstructured sets of possibilities. The unstructured view faces well-known problems, as we shall presently discuss. But in spite of these problems, the unstructured view still has advocates, largely because it integrates well with various formal theories in which propositions are employed—theories in formal semantics and formal epistemology readily come to mind. And some philosophers have also argued that, once we get clear on the nature of attitudes like belief and desire, we see that propositions might as well be unstructured sets of possibilities.[2]

The reason the structured view of propositions has won more adherents is largely because of a problem that plagues the unstructured view—the *problem of logical omniscience*.[3] Actually, there are at least two problems that go under this name that are worth distinguishing. We might call one the *problem of necessary equivalence* and the other the *problem of deductive closure*.

The latter problem runs like this. On the typical unstructured view, an agent believes a proposition $p$ just in case $p$ is true at every possibility compatible with what she believes. For example, in natural language semantics, one often encounters semantic clauses like this:

> $[\![\alpha \text{ believes } \phi]\!]^{c,i} = 1$ iff $[\phi]^c$ is true at every possibility $i'$ compatible with what $[\![\alpha]\!]^{c,i}$ believes at $i$.

Here, "$[\![\alpha \text{ believes } \phi]\!]^{c,i} = 1$" says that the sentence $\alpha$ *believes* $\phi$ is true at the context $c$ and possibility $i$, $[\phi]^c$ is the proposition expressed by the sentence $\phi$ at $c$, and $[\![\alpha]\!]^{c,i}$ the referent of the singular term $\alpha$ at $c$ and $i$. Given this, and given the standard assumption that the possibilities in question are closed under logical entailment, it follows that if an agent believes $p_1, ..., p_n$, and if the $p_i$ jointly entail $q$, then the agent believes $q$. For if the agent believes the $p_i$, then the $p_i$ are all true at every possibility compatible with what the agent believes. But if the possibilities in question are closed under logical entailment, it follows that $q$ is true at each possibility compatible with what the agent believes. And so it follows that the agent believes $q$.

It follows, in other words, that an agent's beliefs are closed under entailment. But it seems quite clear that there is some sense in which actual agents are not like this: although Joe believes College Avenue is north of Curtis Street, and that Curtis Street is north of Hillsdale Road, he might nevertheless fail to believe that College Avenue is north of Hillsdale Road. This might

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[2] See Stalnaker (1984, Ch.1); Dennett (1971) might also be read in this way. Unstructured views are also defended (or treated sympathetically) in Hintikka (1962), Lewis (1979), Barwise and Perry (1981), Braddon-Mitchell and Jackson (2007), and almost any article in formal semantics on attitude ascriptions, e.g. Heim (1992) and Schlenker (2003).

[3] For a forceful presentation of the problem, see Soames (1985, 1987). For attempts to respond on behalf of the unstructured view, see, among others, Lewis (1982), Stalnaker (1984), Halpern and Pucella (2011), and Yalcin (2016).

happen if Joe has two 'mental maps' of the city, and hasn't integrated them properly (Lewis 1982).

That's the *problem of deductive closure.* The *problem of necessary equivalence* is slightly different. Suppose propositions are sets of possibilities. In that case, it is natural to suppose that a proposition $p$ is true at a possibility $i$ just in case $i$ is contained in $p$. Then it is quite clear that if propositions $p$ and $q$ are necessarily equivalent—true at all the same possibilities—then $p$ and $q$ are identical. To see this, suppose $p$ and $q$ are true at all the same possibilities, and suppose $i$ is any possibility in $p$. Then $p$ is true at $i$, and so $q$ is true at $i$ (since $p$ and $q$ are true at all the same possibilities). So $i$ is an element of $q$. Since this holds for an arbitrary possibility $i$ in $p$, it holds for them all, which means that $p \subseteq q$. A parallel argument shows that $q \subseteq p$, and so $p = q$. Among other things, this means that there is only one necessary proposition. But this seems false, since it implies that if you believe one necessary truth, you believe them all. But one can know some mathematical truths, for example, without knowing them all.

One difference between these two problems is that the problem of deductive closure arises for anyone who accepts any claim along the following lines:

ATTITUDES ARE QUANTIFIERS

A sentence of the form $x$ *believes* $p$ is true just in case $p$ is true at every possibility compatible with what $x$ believes.

That is, the problem of deductive closure doesn't require the assumption that propositions are sets of possibilities. You can take propositions to be as fine-grained as you like, but if you accept ATTITUDES ARE QUANTIFIERS, then you will run into the problem of deductive closure. That means that if you do accept ATTITUDES ARE QUANTIFIERS, then you *might as well* accept the claim that propositions are sets of possibilities, since any structure you posit in the nature of propositions isn't going to be of any use to you, at least not for the purposes of distinguishing attitudes that are intuitively distinct.[4] This point will be important below.

Another difference between these problems is that you can maintain that propositions are sets of possibilities, and yet deny that one believes all the logical consequences of what one believes. One can do this by denying ATTITUDES ARE QUANTIFIERS. Then even if propositions are sets of possibilities, and $p$ entails $q$, then one can believe $p$ without believing $q$, so long as

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[4]For the sake of simplicity, I am ignoring the possibility that one might motivate the view that propositions are structured in some other way, e.g. by appealing to the idea that certain singular terms are 'directly referential' (Kaplan 1989, King 2019). I take it that the principal motivation for the view that propositions are structured has to do with the problem of logical omniscience, and so that is my focus here.

$q$ does not also entail $p$ (for then $p = q$). At least some accounts of attitude verbs in formal semantics are consistent with this view (e.g. Montague 1973).[5]

Given the way the structured view is usually understood, it appears to avoid both the problem of deductive closure and the problem of necessary equivalence. The former only gets going given ATTITUDES ARE QUANTIFIERS, which advocates of structured propositions typically (implicitly) deny. And structured propositions are assumed to be more finely-grained than sets of possibilities, so that even if structured propositions $p$ and $q$ are true at all the same possibilities, they might nevertheless be distinct. Thus, the proposition that

> everything is self-identical

will be distinct from the proposition that

> for any set $A$, the cardinality of the power set of $A$ is greater than the cardinality of $A$.

For the latter proposition will, for example, contain the *greater than* relation as a constituent, whereas the former proposition will not. Thus, one can believe the former without believing the latter, even though these are both presumably true at the very same set of possibilities, namely the set of all possibilities. The fact that the structured view avoids the problem of logical omniscience is, I take it, the principal motivation for that view. It is not, perhaps, the only motivation for the structured view.

## 3

Now ATTITUDES ARE QUANTIFIERS is, to put it mildly, controversial, at least in philosophical circles. In contrast, when it comes to epistemic modals, the analogous claim is the orthodox view:

EPISTEMICS ARE QUANTIFIERS

> A sentence of the form *must p* is true at a point of evaluation $e$ just in case $p$ is true
> at every possibility compatible with the body of knowledge determined by $e$.

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[5]In Montague Grammar and similar systems, the thesis that one believes or knows all of the logical consequences of what one believes or knows is sometimes added by way of a meaning postulate (Groenendijk and Stokhof 1982). The reason for adding it is that certain explanations of certain linguistic phenomena do not go through without it.

A sentence of the form *might p* is true at a point of evaluationt $e$ just in case $p$ is true at some possibility compatible with the body of knowledge determined by $e$.[6]

(Here we assume the modals are being read epistemically.) But any view of this type will face an analogue of the problem of deductive closure. Here is an example of what I have in mind.

Goldach's conjecture is the claim that every even integer greater than two is the sum of two primes. Since no one knows whether Goldbach's conjecture is true, the following claims both appear to be true (at any relevant point of evaluation):

(1) It might be that every even integer greater than two is the sum of two primes.

(2) It might be that not every even integer greater than two is the sum of two primes.

The difficulty now is this. Suppose first that Goldbach's conjecture is true. Then the proposition expressed by the embedded sentence in (2) is false. Since it is a mathematical falsity, it is presumably false at every possibility. In that case, (2) is itself presumably false (at any point of evaluation), given EPISTEMICS ARE QUANTIFIERS. Suppose now that Goldbach's conjecture is false. Then we get a parallel argument for the conclusion that (1) is false (at any point of evalaution), again assuming EPISTEMICS ARE QUANTIFIERS. Either way, if EPISTEMICS ARE QUANTIFIERS is true, then at least one of (1) and (2) is false (at every point of evaluation).

Note that the problem arises not from assuming that propositions are sets of possibilities—we made no such assumption above. It arises simply from assuming EPISTEMICS ARE QUANTIFIERS. Thus, even if we assume that propositions are structured, we will not succeed in avoiding every version of the problem of logical omniscience if we also accept EPISTEMICS ARE QUANTIFIERS. But since the main motivation for holding that propositions are structured is that doing so allows us to avoid the problem of logical omniscience, that motivation is undermined if we also accept EPISTEMICS ARE QUANTIFIERS. If you accept EPISTEMICS ARE QUANTIFIERS, you *might as well* accept that propositions are unstructured, since since any structure you posit in the nature of propositions isn't going to be of any use to you, at least not for the purposes of assigning the correct interpretation to epistemically modalized sentences.

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[6]There is a large literature on epistemic modals, almost all of which assumes (something like) EPISTEMICS ARE QUANTIFIERS. See, for example, Egan *et al.* (2005), Stephenson (2007), Yalcin (2007), von Fintel and Gillies (2008), Portner (2009), Dowell (2011), Kratzer (2012), MacFarlane (2014), and Mandelkern (2019). A 'point of evaluation' might include a context of utterance and a circumstance of evaluation (cf. Kaplan 1989); and a circumstance of evaluation might, in turn, include an accessibility relation, a 'conversational background', or an 'information parameter'.

Dynamic approaches like Veltman (1996) don't fit precisely into the EPISTEMICS ARE QUANTIFIERS-schema as given above, but they do arguably endorse something similar; see Section 4.

It might be pointed out that this argument only works if it is assumed that the possibilities over which epistemic modals quantify are ones that verify every mathematical truth. If we deny this—if we allow in possibilities at which actual mathematical truths are counted false—then we avoid the trouble.[7]

Such an approach might work—we'd also need to say what it is for such a possibility to be 'compatible' with a body of knowledge. But note that if such a strategy succeeds, then it would presumably *also* succeed in the case of attitude ascriptions. Suppose again that Goldbach's conjecture is true, but that John does not believe it. And consider the following sentence:

(3) John does not believe that every even integer greater than two is the sum of two primes.

If one accepts ATTITUDES ARE QUANTIFIERS, then (3) is false on the assumption that all of the possibilities over which *believes* quantifies are ones which verify every mathematical truth. That is usually take to be an argument against ATTITUDES ARE QUANTIFIERS in favor of the structured propositions view.

But if in order to explain the apparent joint truth of the (1) and (2) we appeal to possibilities at which actual mathematical truths fail, then why not appeal to those same possibilities in order to explain how (3) might be true? If we do this, then the argument from (3) to the structured propositions view is again undercut. For in that case, we may as well treat propositions as sets of these possibilities at which mathematical truths may fail.

I am not necessarily advocating this strategy; I don't know what the best solution to the problem of logical omniscience is, nor whether there really is one. For all I know, that problem might really be fatal for the unstructured view. My point is simply this. If one accepts EPISTEMICS ARE QUANTIFIERS, then one cannot avoid the problem of logical omniscience simply by adopting the view that propositions are structured—this is what our discussion of sentences (1) and (2) revealed. Thus, if, as I am assuming, the principal motivation for accepting that propositions are structured is that it allows one to evade the problem of logical omniscience, that motivation will be undercut if EPISTEMICS ARE QUANTIFIERS is true. Furthermore, if—a big *if*—one can maintain EPISTEMICS ARE QUANTIFIERS in the face of the problem raised by sentences (1) and (2) by appealing to impossible worlds (or some such device), then that solution presumably generalizes to the case of attitude ascriptions as well. And if we have a solution

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[7]The term "possibility" may no longer be appropriate; it may be better to call the sort of thing at which a mathematical or logical truth fails an *impossible* world or situation. The important point is that propositions would still be *unstructured sets* of these things, whatever we call them. For approaches to attitudes that use impossible worlds, see Halpern and Pucella (2011) and Ripley (2012), among others.

to the problem of logical omniscience that makes no appeal to structured propositions, the motivation for that view is again undercut. Either way, if EPISTEMICS AS QUANTIFIERS is true, you *might as well* treat propositions as sets of unstructured possibilities, since any structure you posit in the nature of propositions isn't going to be doing any theoretical work.

Thus, it seems to me that either we accept both ATTITUDES ARE QUANTIFIERS and EPISTEMICS ARE QUANTIFIERS or we reject them both. This is of interest because two philosophical conversations have been going on side-by-side. In one, ATTITUDES ARE QUANTIFIERS is often assumed to be false; in the other, EPISTEMICS ARE QUANTIFIERS is almost always assumed to be true. But once we see how similar they are, it seems that at least one of these conversations is proceeding on the basis of a false assumption.

## 4

I've argued, in effect, for a disjunction: either we treat both epistemic modals and attitude verbs as quantifiers over possibilities, or we treat neither in that manner. We've already seen the principal disadvantages of the former view: it faces the problem of logical omniscience. What are the prospects for the alternative view?

The view that attitude verbs are not quantifiers over possibilities is familiar. According to standard structured proposition views, an attitude verb like *believes* expresses a relation between agents and structured propositions (e.g. Salmon 1986). But the view that epistemic modals are not quantifiers over possibilities has received less discussion in the literature. What would such a view look like?

Some discussions of epistemic modals in the philosophical literature are compatible with this view, for they make epistemically modalized sentences equivalent to knowledge ascriptions or their denial (e.g. Hacking 1967, DeRose 1991). Here is an example of the sort of view that I have in mind:

EPISTEMICS ARE KNOWLEDGE CLAIMS

A sentence of the form *must p* is true at a point of evaluation $e$ just in case $X_e$ knows $p$ at $e$, where $X_e$ is an individual or group determined by $e$.

A sentence of the form *might p* is true at a point of evaluation $e$ just in case it is not the case that $X_e$ knows *not p* at $e$, where $X_e$ is an individual or group determined by $e$.

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If we combine this view with a view on which *knows* expresses a relation between an agent and a structured proposition, we would seem to avoid the epistemic modal version of the problem of logical omniscience. For consider the pair of sentences we discussed earlier:

(1) It might be that every even integer greater than two is the sum of two primes.

(2) It might be that some even integer greater than two is not the sum of two primes.

Now suppose that, we utter and assess these at our current point of evaluation. Relative to that point of evaluation, (1) and (2) are presumably equivalent to (1′) and (2′), respectively:

(1′) We do not know that not every even integer greater than two is the sum of two primes.

(2′) We do not know that every even integer greater than two is the sum of two primes.

These claims are both intuitively true at our present point of evaluation, since we don't know whether Goldbach's conjecture is true. Furthermore, a theory according to which attitude verbs like *knows* expressed relations between agents and structured propositions would be well-placed to accommodate these intuitive truth-value judgments.

Given this result, why do philosophers and linguists interested in these matters tend to assume that EPISTEMICS ARE QUANTIFIERS is true? One answer is that this isn't really a substantive commitment on their part. They simply work with the EPISTEMICS ARE QUANTIFIERS-hypothesis since, for most of the applications with which they are concerned, they need not appeal to semantic values that are more fine-grained than sets of possibilities. This response may be correct as a description of people's actual views. But are there any reasons beyond convenience to adopt EPISTEMICS ARE QUANTIFIERS as a working hypothesis? Are there any reasons to think that hypothesis *true*?

One consideration is that epistemic modals are *modals*, and one would expect all modals to have, at some level of abstraction, a common semantics. To the extent that we think metaphysical, deontic, and other (non-epstemic) kinds of modals are to be analyzed in terms of quantification over possibilities, there is some pressure to analyze epistemic modals in a similar fashion. Is it really plausible that there is a bright line separating epistemic modals from all these other forms of modality? From this point of view, it looks like the default presumption would seem to favor EPISTEMICS ARE QUANTIFIERS.

It is worth adding a more specific consideration. As Kratzer emphasizes, one and the same lexical item can often be used to express both epistemic modality and some other form of

modality (Kratzer 1977, 1981, 1991, 2012). For example:

(4) (a) I <u>might</u> have been an economist. (*metaphysical*)

   (b) Joe <u>might</u> be in Toronto. (*epistemic*)

(5) (a) You <u>must</u> apologize. (*deontic*)

   (b) The game <u>must</u> be over—the fans are leaving. (*epistemic*)

(6) (a) You <u>may</u> have two cookies. (*deontic*)

   (b) Bina <u>may</u> be lying. (*epistemic*)

(7) (a) Mary <u>can't</u> swim. (*ability*)

   (b) That <u>can't</u> be a bluejay—it's too small. (*epistemic*)

This is an instance of something more general: different types of modality can very often be expressed by a single lexical item. The point to appreciate here is that epistemic modals are no exception to this generalization.

Kratzer's response to this observation is extremely natural. She hypothesizes that we have, for example, a single lexical item *might*, and all that is encoded in the context-insensitive meaning of *might* is the fact that it expresses existential quantification over some set of possibilities.[8] Different readings for *might* can then be derived in context by providing appropriate *restrictions* on the (whole) set of possibilities. Thus, the context of utterance plays a central role in determining whether, for example, *may* is read deontically or epistemically on a given occasion of use. And, simplifying the story somewhat, it does this by supplying the set of worlds over which the modal is understood to quantify: metaphysical modals quantify over the set of metaphysically possible worlds, deontics over the set of deontically possible worlds, and epistemics over the (contextually relevant) set of epistemically possible worlds.

There are many qualifications, details, and caveats we might wish to add to the foregoing picture, but even this simplified version offers a very appealing story about modality in natural language—it's no surprise it is the dominant theory. And EPISTEMICS ARE QUANTIFIERS is part of this theory. The alternative story sketched above—EPISTEMICS ARE KNOWLEDGE CLAIMS— would seem to force us to reject this attractive account, at least in part.

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[8]This is a slight exaggeration. A modal might also encode some restrictions as to the type of modality it can express. For example, it seems that *can*, in positive form, may not be able to express epistemic modality. And it may be that *linguistic* context does some work in determining the available readings for a given occurrence of a modal. For example, *John must have left* doesn't seem to have a deontic reading. All this is consistent with Kratzer's basic idea that a single lexical item can express more than one type of modality.

Not everyone accepts Kratzer's story in detail. For example, some dynamic accounts appear to maintain that *might* is ambiguous between epistemic readings and other readings. But even these accounts analyze epistemic modals as quantifiers over possibilities. Here, for example, is a way of stating the standard dynamic semantic account of epistemic *might*:

$$s[\textit{might } \phi] = \begin{cases} s & \text{if there is a possibility } i \text{ in } s[\phi]; \\ \emptyset & \text{otherwise.} \end{cases}$$

Here $s$ is a set of possibilities, and $s[\phi]$ is the result of updating $s$ with sentence $\phi$. So even dynamic semanticists can accept the high-level generalization that modals are quantifiers over possibilities. But again, this generalization would seem to be lost if we accept EPISTEMICS ARE KNOWLEDGE CLAIMS.

Of course, none of this constitutes a decisive objection to the view that propositions are structured. The advocate of that view may still argue that the problem of logical omniscience motivates rejecting both ATTITUDES AS QUANTIFIERS and EPISTEMICS AS QUANTIFIERS. My only hope is to have shown that that position comes with an unexpected cost, insofar as it appears to be in tension with any suitably general semantic account of natural language modality. That may not move the advocated of structured propositions to give up their view, but it may spur the neutral observer to take the unstructured view of propositions more seriously, and to seek a solution to the problem of logical omnscience consistent with it.

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