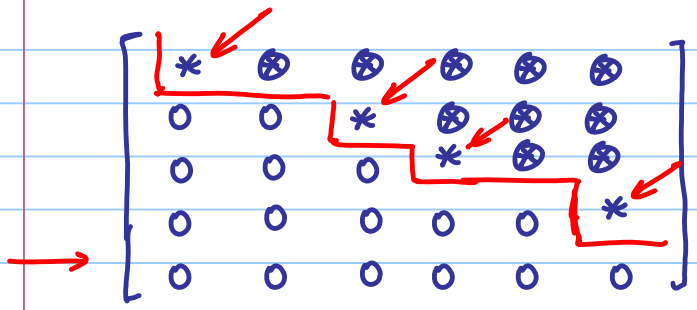
GAUSS ELIMINATION 2

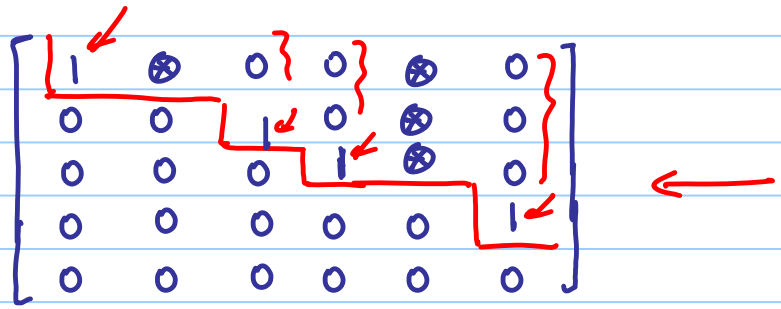
Note Title 6/30/2013

(* any $\neq 0$, ⊛ any)

(Row) Echelon Form (EF)



Reduced Row Echelon Form (RREF)



GAUSS ELIMINATION ALGORITHM :

It transforms any matrix to EF including RREF by using a combination of Elementary Row Operations.

Example: Find the RREF of $A = \begin{bmatrix} 0 & 0 & 1 & -2 & 1 \\ 0 & 2 & 1 & 8 & 1 \\ 0 & 2 & 2 & 6 & 0 \\ 0 & -4 & 1 & -22 & 1 \end{bmatrix}$ by elimination.

Sol^n

$$\begin{bmatrix} 0 & \boxed{0} & 1 & -2 & 1 \\ 0 & 2 & 1 & 8 & 1 \\ 0 & 2 & 2 & 6 & 0 \\ 0 & -4 & 1 & -22 & 1 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 0 & \boxed{2} & 1 & 8 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 2 & 2 & 6 & 0 \\ 0 & -4 & 1 & -22 & 1 \end{bmatrix} \xrightarrow{\substack{-R_1+R_3 \to R_3 \\ 2R_1+R_4 \to R_4}} \begin{bmatrix} 0 & 2 & \boxed{1} & 8 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 & -1 \\ 0 & 0 & 3 & -6 & 3 \end{bmatrix}$$

Top; column $\neq 0$; pivot; pivot

$$\xrightarrow{\substack{-R_2+R_3 \to R_3 \\ -3R_2+R_4 \to R_4}} \begin{bmatrix} 0 & 2 & 1 & 8 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\left(-\frac{1}{2}R_3\right) \to R_3} \begin{bmatrix} 0 & 2 & 1 & 8 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & \boxed{1} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 2 & 1 & 8 & 0 \\ 0 & 0 & \boxed{1} & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & \boxed{2} & 0 & 10 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

pivot; pivot; leading

$$\sim \begin{bmatrix} 0 & 1 & 0 & 5 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \leftarrow \text{RREF}(A)$$

SOLVING ANY LINEAR SYSTEM BY GAUSS ELIMINATION

- Reduce the augmented matrix to RREF. If during any time we get a row of the form $[0\ 0\ \cdots\ 0 \vdots *]$, $*\neq 0$ STOP — NO SOLUTIONS
- From RREF write the free variables (if any) as parameters; solve for the leading.

Example: Solve the system

$$\begin{aligned} 3x_2 - 6x_3 - 4x_4 - 3x_5 &= -5 \\ -x_1 + 3x_2 - 10x_3 - 4x_4 - 4x_5 &= -2 \\ 2x_1 - 6x_2 + 20x_3 + 2x_4 + 8x_5 &= -8 \end{aligned}$$

Sol RREF of aug. matrix is $\begin{bmatrix} 1 & 0 & 4 & 0 & 1 & -3 \\ 0 & 1 & -2 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 2 \end{bmatrix}$

$$\begin{aligned} x_1 + 4x_3 + x_5 &= -3 \\ x_2 - 2x_3 - x_5 &= 1 \\ x_4 &= 2 \end{aligned}$$

leading

$x_1 = -4s - r - 3$
$x_2 = 2s + r + 1$
$x_3 = s$
$x_4 = 2$
$x_5 = r$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -4s-r-3 \\ 2s+r+1 \\ s \\ 2 \\ r \end{bmatrix} = s\begin{bmatrix} -4 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + r\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -3 \\ 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

(r, s any)