

535.641 Mathematical Methods Assignment 3

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1. Consider a linear transformation $T : P_2 \rightarrow P_3$, where P_2 denotes the set of all 2nd order polynomials and P_3 denotes the set of all 3rd order polynomials. The transform is such that: $T(1) = 1 - 2x - x^2$, $T(x) = 3x + 3x^2 + 2x^3$, and $T(x^2) = -2 + x - x^2 - 2x^3$.
 - (a) Find the standard matrix A of this linear transform.
 - (b) Find the image of $2 - 2x + 3x^2$.
 - (c) Is the polynomial $q = 1 + x + 2x^2 + 2x^3$ in the image of this transform? If it is, then find all P_2 polynomials, p that would satisfy $T(p) = q$.

2. Consider the given vectors that form basis for a subspace of \mathbb{R}^4

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Construct an orthogonal basis for this subspace.

3. For the following matrix

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ c & 0 \end{bmatrix}$$

- (a) Find all real values c for which the matrix \mathbf{A} has real eigenvalues
- (b) Find the eigenvectors which correspond to the situation where all eigenvalues are real ($c \in \mathbb{R}$). *Hint: You should confirm the eigenvectors are reasonable for all c being considered.*

4. Consider the quadratic form,

$$Q(x_1, x_2) = 5x_1^2 + 5x_2^2 - 4x_1x_2$$

- (a) find the maximum value of $Q(x_1, x_2)$ subject to the constraint $x_1^2 + x_2^2 = 1$
- (b) find the location where this maximum is achieved