## 2.C DETERMINANTS AND CRAMER'S RULE

Note Title

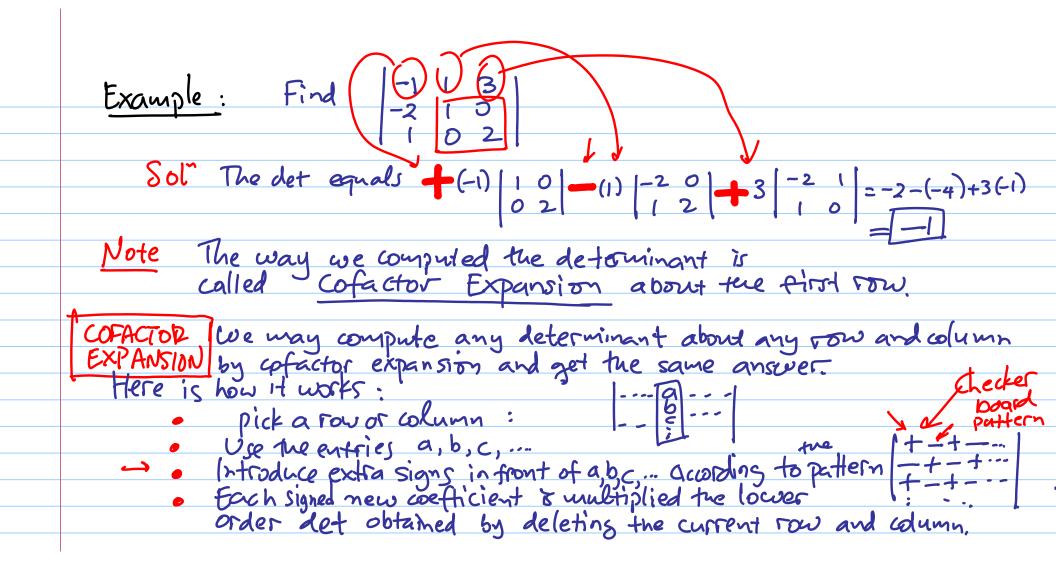
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## Determinants

A 2x2 determinant of a matrix 
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
 is the number  $det(A) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = a \cdot d - cb$ 

cross multiply; take the difference

Similarly, we define the det of any size square matrix



Example: Find  $\begin{vmatrix} -1 & 1 & 3 \\ -2 & 1 & 3 \end{vmatrix}$  by cofactor expansion about the  $2^{10}$  column

Solt  $\begin{vmatrix} -1 & 0 & 3 \\ -2 & 0 & 2 \end{vmatrix} = -1 \begin{vmatrix} -2 & 0 & | & -1 & 1 \\ 1 & 2 & | & -2 & 1 \end{vmatrix} = +4-5+0=-1$ 

Note We can do this for any size and any row or column but it is NOT EFFICIENT.

Triangular Det; a b c = a.d. f = a d e = a.d. f

product of diagonal 0 of

In practice we compute determinants by correct Gauss elimination that brings them to diagonal form.

For this we need 3 basic properties not multiplied! det B = det A det B = c. det A det B= -det A =-(1)(-5)(4)(1)(-11)=-220 -Triangular

Cramer's Rule It applies to square systems Ax=b with det  $A\neq 0$ . X: = det A; where A; is obtained from A by replacing its ith column with b. If x= (x), then Example: Use Cramer's Rule to solve  $x_1 + x_2 - x_3 = 3$   $-x_1 + x_2 + x_3 = 4$   $-x_1 + x_2 + x_3 = 4$  3 - 1 = -10  $x_1 = \frac{\det A}{\det A} = \frac{5}{\det A}$   $x_2 = \frac{\det A}{\det A} = 3$  $\det A_2 = \begin{vmatrix} 1 & 2 & -1 \\ 1 & 3 & 1 \end{vmatrix} = -12 \quad \det A_3 = \begin{vmatrix} 1 & 1 & 2 \\ 1 & -1 & 3 \end{vmatrix} = -14 \quad X_3 = \frac{\det A_3}{\det A} = \frac{7}{2}$