LINEAR EQUATIONS

Note Title 6/28/2013

$x_1, x_2, \ldots, x_n$

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

a_i : coefficients
b : constant term

Examples :

linear

$2x = -3$

$3x_1 + 5x_2 = 7$

$x + y - z = 1$

$\vdots$

Non-linear

$xy = 1$

$x^2 - y^3 = 5$

$\ln(x) - \sin(y) = 0$

$x_1, x_2, \ldots, x_5$

$-3x_2 + x_4 - x_5 = 6$

leading

x_1, x_3, x_4, x_5 } free

SOLUTIONS

A solution $(r_1, r_2, \ldots, r_n)$ such that $a_1r_1 + a_2r_2 + \cdots + a_nr_n = b$

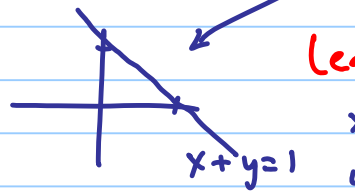
↑ identity

Examples:

$-3x = 5$ $\quad x = -\frac{5}{3}$ $\quad -3\left(\frac{-5}{3}\right) = 5$ ✓

$x + y = 1$ $\quad (1,0)$ $\quad x = 1,\ y = 0$ $\quad 1 + 0 = 1$ ✓ $\quad \left(\frac{1}{2}, \frac{1}{2}\right)$ $\quad x = \frac{1}{2},\ y = \frac{1}{2}$ $\quad \frac{1}{2} + \frac{1}{2} = 1$ ✓

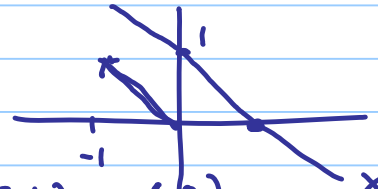
leading, free



$x = 1 - r$, $y = r$, r any ← parameter

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1-r \\ r \end{bmatrix} = r\begin{bmatrix} -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(r any)



$x_2 - x_3 + x_4 = -9$ $\quad (x_1, \ldots, x_4)$

$x_1 = r$
$x_2 = -9 + s - t$
$x_3 = s$
$x_4 = t$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} r \\ -9+s-t \\ s \\ t \end{bmatrix}$ $\quad \underbrace{r, s, t \text{ any}}_{\text{parameters}}$

$= r\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ -9 \\ 0 \\ 0 \end{bmatrix}$