# 535.641 Mathematical Methods Assignment 4

**Ben Minnick** **Name**________________________________

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|---|---|
| 1 | /20 |
| 2 | /20 |
| 3 | /20 |
| 4 | /20 |
| TOTAL | /80 |

1. Let $f(t)$ and $g(t)$ be piecewise continuous and of exponential order on $[0, \infty)$. Define their convolution as:

$$(f * g)(t) = \int_0^t f(\tau)g(t-\tau)\, d\tau$$

Using the definition of the Laplace Transform:

$$\mathcal{L}\{h(t)\}(s) = \int_0^\infty e^{-st}h(t)\, dt,$$

prove the convolution property of the Laplace Transform:

$$\mathcal{L}\{f * g\}(s) = \mathcal{L}\{f(t)\}(s) \cdot \mathcal{L}\{g(t)\}(s)$$

2. Consider the linear non-homogeneous initial value problem,

$$y'' + y = \sum_{k=1}^{\infty} (-1)^k \delta(t - ak), \quad y(t=0) = 1, \quad y'(t=0) = 0$$

where $a \in \mathbb{R}$ with $a > 0$. Solve for $y(t)$ and sketch the solution for $a = \pi$ and $a = 2\pi$.

3. Consider the integral equation for $y(t)$,

$$y(t) + \int_0^t y(\tau) \cosh(t - \tau)\ d\tau = t + e^t$$

(a) Solve the integral equation for $y(t)$ using Laplace transforms.

(b) Convert the integral equation into an initial value problem by taking two derivatives with respect to $t$, then solve this ODE and verify your solution in part (a).

4. Use the Laplace transform to solve for $y_1(t)$ and $y_2(t)$ that satisfies the coupled differential equation,

$$\frac{d}{dt}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

with initial condition, $y_1(0) = 1$ and $y_2(0) = 1$.