REVIEW 1 INTEGRATION BY SUBSTITUTION

Note Title 8/20/2013

Substitution Rule: If $u = g(x)$ is differentiable with range in an interval I and f is continuous on I, then

$$\boxed{\int f(g(x))\, g'(x)\, dx = \int f(u)\, du}$$

Example 1: Find $\int x^3 \cos(5x^4+1)\, dx$

Sol: Let $u = 5x^4+1$. Then $du = 20x^3\, dx \Rightarrow x^3\, dx = \frac{1}{20}\, du$

Hence

$$\int x^3 \cos(5x^4+1)\, dx = \frac{1}{20}\int \cos(u)\, du$$

$$= \frac{1}{20}\sin(u) + C$$

$$= \frac{1}{20}\sin(5x^4+1) + C$$

Example 2: Evaluate $\int \sqrt{3x+2}\, dx$

Sol^n: $u = 3x+2$

$du = 3dx$

$dx = \frac{1}{3} du$

$$\int \sqrt{3x+2}\, dx = \frac{1}{3}\int u^{\frac{1}{2}} du$$

$$= \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{9}(3x+2)^{3/2} + C$$

Example 3: Evaluate $\int \tan x\, dx$ $\left(= \int \frac{\sin x}{\cos x} dx\right)$

Sol^n: $u = \cos x$

$du = -\sin x\, dx$

$\sin x\, dx = -du$

$$\int \tan x\, dx = \int \frac{\sin x}{\cos x} dx$$

$$= -\int \frac{du}{u}$$

$$= -\ln|u| + C$$

$$= -\ln|\cos x| + C$$

$$= \ln|\cos x|^{-1} + C$$

$$= \ln|\sec x| + C$$

Substitution Rule for Definite Integrals

$$\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du \qquad (u = g(x))$$

Example 4: Evaluate $\int_0^4 \sqrt{2x+1}\,dx$

Soln:

$u = 2x+1$ — New limits

$du = 2dx$ — $x=0 \Rightarrow u = 2\cdot 0+1 = 1$

$dx = \frac{1}{2}du$ — $x=4 \Rightarrow u = 2\cdot 4+1 = 9$

$$\int_0^4 \sqrt{2x+1}\,dx = \frac{1}{2}\int_1^9 u^{1/2}\,du$$

$$= \frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big|_1^9$$

$$= \frac{1}{3}\left(9^{3/2} - 1^{3/2}\right)$$

$$= 26/3$$