BACK-SUBSTITUTION

Note Title 6/29/2013

Echelon Form

$$\begin{aligned} x_1 + 5x_2 + x_3 &= -4 \\ -2x_2 + 4x_3 &= 14 \\ 3x_3 &= 9 \end{aligned}$$

(Triangular Trapezoidal)

leading

Solve by Back-substitution

$x_3 = 3$

Back-substitute $-2x_2 + 4(3) = 14 \Rightarrow x_2 = -1$

Back-substitute $x_1 + 5(-1) + 3 = -4 \Rightarrow x_1 = -2$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix}$$

Solution

Example: Solve

$$\begin{aligned} x_1 - x_2 + x_3 - x_4 + 2x_5 - x_6 &= 1 \\ -x_3 + x_5 &= 1 \\ -x_5 + x_6 &= 3 \end{aligned}$$

leading

free $\begin{cases} x_2 = t \\ x_4 = s \\ x_6 = r \end{cases}$

- Solve the last for x_5 : $x_5 = r - 3$
- Second one : substitute x_5 with $r-3$, solve $x_3 = r - 4$
- First one : substitute all known values, solve for $x_1 = -2r + s + t + 11$

Solution set

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} -2r + s + t + 11 \\ t \\ r - 4 \\ s \\ r - 3 \\ r \end{bmatrix}$$

parameters → r, s, t any