

535.641 Midterm

Name_____

Instructions:

- Please use this template to submit your answers.
- By completing and submitting this test you also testify this work is entirely your own.
- The test is open book and open notes.
- Please feel free to add pages for more space.
- Please write clearly and show ALL WORK for credit.
- Submit your work as a PDF file, make sure it is easy to read.

1	/16
2	/16
3	/16
4	/16
5	/16
TOTAL	/80

1. (a) Let $S_1 = \{(-3, 0, 3), (6, -4, 7), (2, 1, -3)\}$ and $S_2 = \{(3, 0, 3), (3, -4, -1), (1, 1, 2)\}$. Test each set for linear independence. **Note:** The vectors in the sets are column vectors. They are written in row form in order to save space.
- (b) Use step-by-step **Gauss Elimination** to solve the system.

$$\begin{bmatrix} -1 & 3 & 2 & 0 \\ 2 & 1 & 4 & 0 \\ -3 & 2 & 2 & 0 \\ -2 & 6 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 19 \\ 0 \\ 3 \end{bmatrix}$$

2. (a) Compute the inverse of $\begin{bmatrix} 1 & 0 & -1 \\ -2 & 4 & 0 \\ 2 & 0 & 2 \end{bmatrix}$. Show all steps.

(b) Find matrices P and D that diagonalize $A = \begin{bmatrix} 0 & 5 & -9 \\ 0 & 5 & -9 \\ 0 & 5 & -9 \end{bmatrix}$. Show all steps.

3. (a) Check the set of vectors

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \\ -3 \end{bmatrix} \right\}$$

for orthogonality.

(b) Determine if S is a subspace of \mathbf{R}^4 . If it is, find a basis and the dimension.

$$S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, 2x_2 - 3x_4 = 0 \right\} \subseteq \mathbf{R}^4$$

4. For the linear differential system $\dot{\mathbf{x}} = A\mathbf{x}$, complete the following steps (a)-(c) for each A matrix given below.

- (a) Find the general solution of real functions.
- (b) Sketch **by hand** an approximate phase portrait indicating the direction of the trajectories.
- (c) Discuss why the trajectories are shaped as you drew them. What happens as $t \rightarrow +\infty$ and $t \rightarrow -\infty$?

$$A = \begin{bmatrix} 4 & 4 \\ 4 & -2 \end{bmatrix} \qquad A = \begin{bmatrix} 4 & 4 \\ -4 & 4 \end{bmatrix}$$

5. Please answer each question.

- (a) Use the Laplace transform to solve the initial value problem. **Here you must use unit step functions. Any other method will not earn credit.** Please show all steps.

$$\frac{dy}{dt} + y = f(t), \quad y(0) = -1, \quad \text{where } f(t) = \begin{cases} -2 & \text{if } 0 \leq t < 4 \\ t & \text{if } t \geq 4 \end{cases}$$

- (b) Compute each of the following. Please show all work.

(i) $L(e^{-2t}tu(t-4))$

(ii) $L^{-1}\left(\frac{3s-8}{s^2+2s+5}\right)$