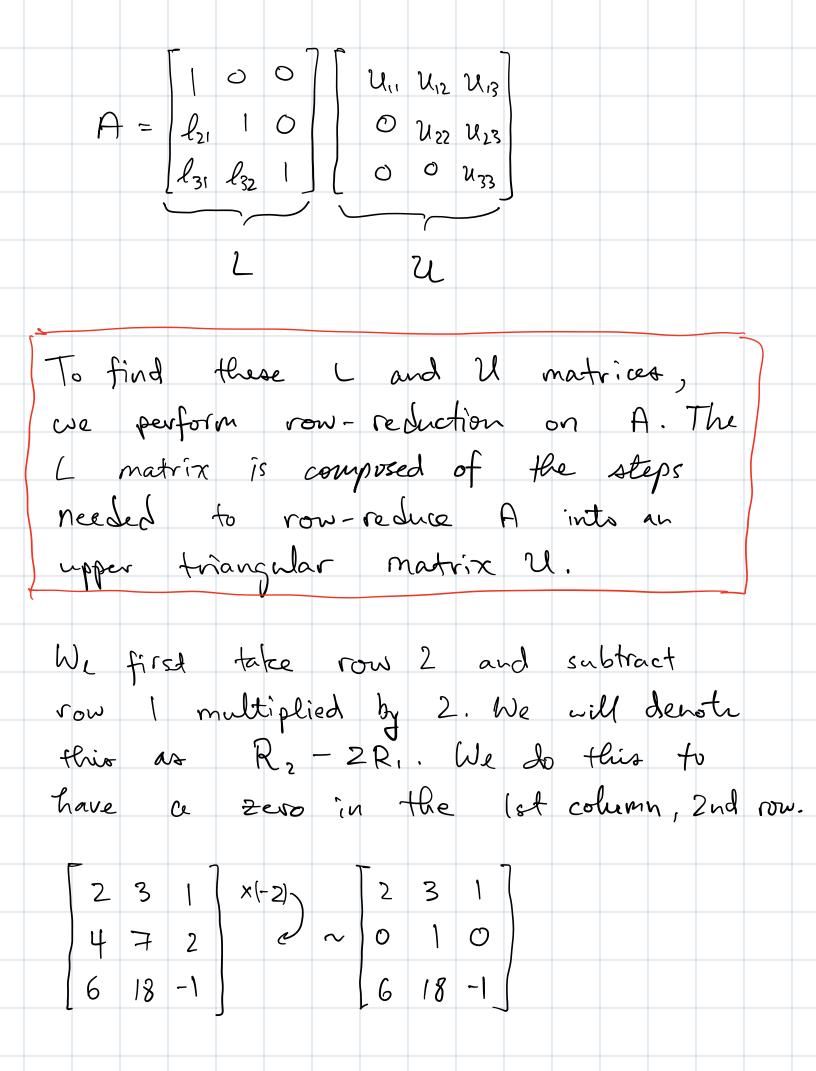
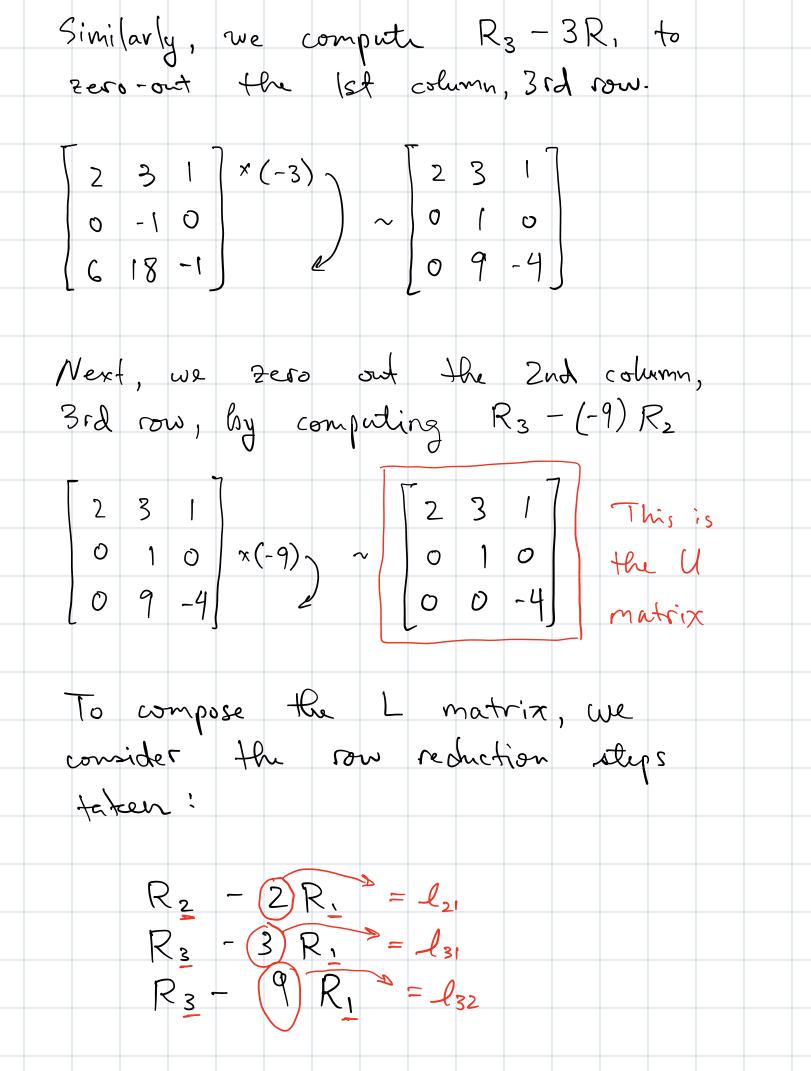
## LU - Decomposition Motivation It is common in engineesing applications to solve a system of equations: A = 6 where $A \in \mathbb{R}^{n \times n}$ , $x, b \in \mathbb{R}^{n \times 1}$ . More so, it is common to solve a sequence of equations in which the right-hand side vector & charges and the coefficient matrix A remains unchanged. For example, if spatially discretizing a problem, the coefficient matrix A depends on the mesh and the vector b depends on the forcing or some

specific configuration. Typically you will want to change the forcing, but use the same mesh. LU decomposition can be useful if solving ce sequence of systems of equations  $Ax_1=B_1$ ,  $Ax_2=B_2$ , ...,  $Ax_k=B_k$ Since A = LU is unchanged for all systems. Example Consider the system of equations Az=6 We want to decompose A such that





The L matrix is then, L = 2 1 0 3 9 1 It is recommended that you confirm by performing matrix multiplication Now we use this LU-decomposition system of equations Ax=b to solve the according to the following This is done steps: Ax = 5 Substitute A=LU  $L(u \times) = 6$ Cet y = Ux

Solve for y Solve for x U = 7 x = 2 7 For the given example, we solve for y. This only involves forward elivination. Solve for y, first, thun yz, then yz.  $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 2 & 1 & 0 & 3 \\ 3 & 9 & 1 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 2y & 4y & 2 \\ 3y & 4y & 3 \\ 3y & 4y & 4y & 5 \\ 3y & 5 & 5$ Now solving for x, noing y. This only involves backwards elimination. Solve for x3 first, then x2, then x.  $\begin{bmatrix} 2 & 3 & 1 & 1 & 2x_1 + 3x_2 + x_3 = 1 & x_1 = -1 \\ 0 & 1 & 0 & 1 & x_2 = 1 & x_2 = 1 \\ 0 & 0 & -4 & -7 & -4x_3 = -7 & x_3 = 7/4 \end{bmatrix}$  $2x_1 + 3x_2 + x_3 = 1 \rightarrow x_1 = -\frac{15}{8}$ [-15/8] Therefore,

