REVIEW 4 IMPROPER INTEGRALS

Note Title 8/21/2013

Improper Integrals of the form $\int_a^{\infty} f(x)\,dx$, $\int_{-\infty}^{b} f(x)\,dx$, $\int_{-\infty}^{\infty} f(x)\,dx$.

$$\int_a^{\infty} f(x)\,dx = \lim_{t \to \infty} \int_a^t f(x)\,dx$$

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{t \to -\infty} \int_t^b f(x)\,dx$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{a} f(x)\,dx + \int_a^{\infty} f(x)\,dx$$

CONVERGES if lim exists

DIVERGES, if Lim DNE

Example 1:

$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{t\to\infty} \int_1^t x^{-2}\,dx =$$

$$\lim_{t\to\infty}\left(-x^{-1}\Big|_1^t\right) =$$

$$\lim_{t\to\infty}\left(-\frac{1}{t}+1\right) = 1$$

$\to 0$

CONVERGES TO 1

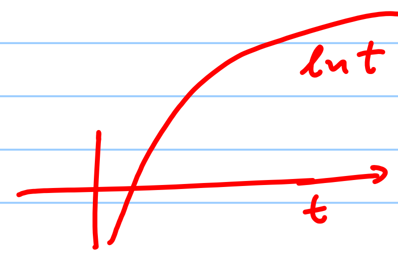
(or the value is 1)

Example 2:

$$\int_1^\infty \frac{1}{x}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{x}\,dx$$

$$= \lim_{t\to\infty} \ln x \Big|_1^t$$

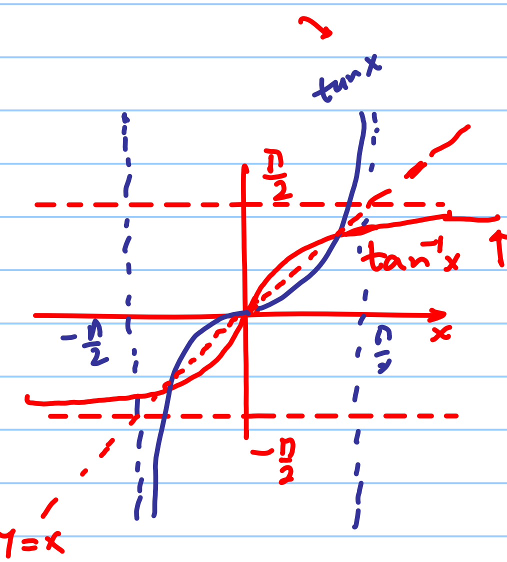
$$= \lim_{t\to\infty}(\ln t - \overset{0}{\overset{\shortparallel}{\ln 1}}) = \infty$$



DIVERGES

Example 3 : $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$

$$= \int_{-\infty}^{0} \frac{1}{1+x^2}\,dx + \int_{0}^{\infty} \frac{1}{1+x^2}\,dx$$



Look at $\displaystyle\int_0^{\infty} \frac{1}{x^2+1}\,dx = \lim_{t\to\infty} \int_0^t \frac{1}{x^2+1}\,dx$

$$= \lim_{t\to\infty} \tan^{-1}(x)\Big|_0^t$$

$$= \lim_{t\to\infty} \left(\tan^{-1}(t) - \underbrace{\tan^{-1}(0)}_{=0}\right)$$

$$= \frac{\pi}{2}$$

Likewise, $\displaystyle\int_{-\infty}^{0} \frac{1}{x^2+1}\,dx = \pi/2$

Total: $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^2+1}\,dx = \frac{\pi}{2} + \frac{\pi}{2} = \pi$