REVIEW 2 INTEGRATION BY PARTS

Note Title 8/21/2013

The Product Rule $(fg)' = f'g + fg'$ implies $\int f(x)g'(x)\,dx + \int g(x)f'(x)\,dx = f(x)g(x)$

Hence,

$$\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int g(x)\,f'(x)\,dx$$

Easier to remember notation: $u = f(x)$, $v = g(x)$. Then $du = f'(x)\,dx$, $dv = g'(x)\,dx$.
So by the Substitution Rule we get

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Integration by Parts

Example 1: $\int x\cos x\,dx$

Sol^{n}

$$\begin{pmatrix} u = x & dv = \cos x\,dx \\ du = dx & v = \sin x \end{pmatrix}$$

$$u\ v - \int \sin x\,dx$$

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$$

$$= x\sin x + \cos x + C$$

Example 2: $\int x^2 \cos x\, dx$

Sol^n $\begin{pmatrix} u = x^2 & dv = \cos x\, dx \\ du = 2x\,dx & v = \sin x \end{pmatrix}$

$uv - \int v\,du$

$$\int x^2 \cos x\, dx = x^2 \sin x - 2\int x \sin x\, dx$$

$\begin{pmatrix} u = x & dv = \sin x\, dx \\ du = dx & v = -\cos x \end{pmatrix}$

$$= x^2 \sin x - 2\left(-x\cos x - \int (-\cos x)\, dx\right)$$

$$= x^2 \sin x + 2x\cos x - 2\sin x + C$$

Example 3: $\int \ln x\, dx$

Sol^n: $\begin{pmatrix} u = \ln x & dv = dx \\ du = \frac{1}{x} dx & v = x \end{pmatrix}$

$uv - \int v\,du$

$$\int \ln x\, dx = x\ln x - \int \frac{1}{x} \cdot x\, dx$$

$$= x\ln x - x + C$$

Example 4: $\int e^x \sin x\,dx$

$(u = e^x,\ dv = \sin x\,dx)$

$$\int e^x \sin x\,dx = -e^x \cos x + \int e^x \cos x\,dx$$

$(u = e^x,\ dv = \cos x\,dx)$

$$= -e^x \cos x + e^x \sin x - \int e^x \sin x\,dx$$

$$\Rightarrow 2\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x$$

$$\Rightarrow \int e^x \sin x\,dx = \frac{1}{2} e^x (\sin x - \cos x) + C$$