

# GAUSS ELIMINATION 1

Note Title

6/29/2013

Solve  $\begin{cases} x+2y=8 \\ -3x+5y=-13 \end{cases} \Rightarrow \begin{cases} x=8-2y \\ -3(8-2y)+5y=-13 \end{cases} \Rightarrow \begin{cases} x=8-2(1)=6 \\ y=1 \end{cases} \Rightarrow \begin{cases} x=6 \\ y=1 \end{cases} \Rightarrow (6,1)$

$$\begin{array}{r} 3x \\ + \end{array} \begin{array}{r} x+2y=8 \\ -3x+5y=-13 \\ \hline \end{array}$$

$$\begin{aligned} 11y &= 11 \\ \Rightarrow y &= 1 \end{aligned}$$

$$\begin{aligned} x+2(1) &= 8 \\ \Rightarrow x &= 6 \end{aligned}$$

$$\begin{array}{l} x+2y=8 \\ 11y=11 \end{array} \quad \text{echelon form}$$

Elimination:  $E_i + cE_j \rightarrow E_i$

Scaling:  $cE_i \rightarrow E_i$

Interchange:  $E_i \leftrightarrow E_j$

Elementary  
Equation  
Operations

Use them  
to transform  
any Lin. Sys.  
to one  
in echelon  
form

Example : Solve

$$\begin{aligned} x_1 + 2x_2 &= -3 \\ 2x_1 + 3x_2 - 2x_3 &= -10 \\ -x_1 &+ 6x_3 = 9 \end{aligned}$$

Work with the augmented matrix

$$\left[ \begin{array}{ccc|c} \textcircled{1} & 2 & 0 & -3 \\ 2 & 3 & -2 & -10 \\ -1 & 0 & 6 & 9 \end{array} \right] \quad \begin{array}{l} -2R_1 + R_2 \rightarrow R_2 \\ R_1 + R_3 \rightarrow R_3 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 2 & 0 & -3 \\ 0 & -1 & -2 & -4 \\ 0 & 2 & 6 & 6 \end{array} \right] \quad 2R_2 + R_3 \rightarrow R_3$$

c1    c2

$$\left[ \begin{array}{ccc|c} 1 & 2 & 0 & -3 \\ 0 & -1 & -2 & -4 \\ 0 & 0 & \textcircled{2} & -2 \end{array} \right] \quad R_3 + R_2 \rightarrow R_2$$

$$\left[ \begin{array}{ccc|c} 1 & 2 & 0 & -3 \\ 0 & -1 & 0 & -6 \\ 0 & 0 & 2 & -2 \end{array} \right] \quad 2R_2 + R_1 \rightarrow R_1$$

Echelon form

$$\begin{array}{l} (-1)R_2 \rightarrow R_2 \\ (\frac{1}{2})R_3 \rightarrow R_3 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -15 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

$$\begin{aligned} x_1 &= -15 \\ x_2 &= 6 \\ x_3 &= -1 \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -15 \\ 6 \\ -1 \end{bmatrix}$$

Example : Solve

$$\begin{aligned} x + 2y - z &= 4 \\ 2x + 5y + 2z &= 9 \\ x + 4y + 7z &= 6 \end{aligned}$$

By elimination we get

$$\left[ \begin{array}{ccc|c} 1 & 0 & -9 & 2 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$x$  (leading)  $y$  (leading)  $z$  free  
 $x = 2 + 9r$   
 $y = 1 - 4r$   
 $z = r$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9r+2 \\ -4r+1 \\ r \end{bmatrix}$$

$r$  any

Each equation represents a plane in 3-space

$$\begin{bmatrix} 9r+2 \\ -4r+1 \\ r \end{bmatrix} = r \begin{bmatrix} 9 \\ -4 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

line thru  $\begin{bmatrix} 9 \\ -4 \\ 1 \end{bmatrix}$   
 direction  $\begin{bmatrix} 9 \\ -4 \\ 1 \end{bmatrix}$

line thru  $(2, 1, 0)$   
 in the direction of  $\begin{bmatrix} 9 \\ -4 \\ 1 \end{bmatrix}$

So the planes intersect along a straight line

Example : Solve

$$\begin{aligned} q - 2k &= -5 \\ 2p - q + k &= -2 \\ 4p - q &= -4 \end{aligned}$$

By elimination we get

$$\left[ \begin{array}{ccc|c} 2 & -1 & 1 & -2 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & 0 & 5 \end{array} \right]$$

$0p + 0q + 0k = 5$   
 ~~$0 = 5$~~

No solutions