Design of Experiments

DoE in Modeling and Simulation

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Scalability, Reproducibility, & Flexibility

When conducting a new study, the scalability, reproducibility, and adaptability of the framework must be evident for optimal usability.

Scalability

Supports extension from few variables to many without compromising on performance.

Reproducibility

Ensures that findings are not due to random variation or hidden biases.

Flexibility

Experimental setup should allow for modifications without significant rework.

A Design of Experiments (DoE) should maximize the information gained from experimental resources by systematically isolating, measuring, and interpreting the influence of different factors.

Design of Experiments Overview

1. **Problem Definition**
   - ❑ Objective, Hypothesis, Success Criteria.
2. **Factors, Levels, and Ranges**
   - ❑ Key Factors, Control Factors, Noise Factors.
3. **Experimental Design**
   - ❑ Type, Number of Runs, Randomization, Replicates, Blocking.
4. **Execution Plan**
   - ❑ Setup, Resources, Controls, etc.
5. **Data Collection**
   - ❑ Variables, Methods.
6. **Data Analysis**
   - ❑ Software, Statistical Methods, Approach.
7. **Interpretation of Results**
   - ❑ Findings, Insights, Significance.
8. **Validation**
   - ❑ Replication, External Validation.

Problem Definition

Formulate a precise and testable problem with the end in mind.

Research Question

What are we trying to learn?

Response Variables

What outcome(s) are we measuring?

Hypotheses

What do we expect will happen?

Guides factor selection, modeling strategy, and alignment with goals, a well-defined problem creates a focused and interpretable experiment.

Factors, Levels, and Ranges

Systematically and effectively cover the trade space based on requirements.

Factors	Levels	Range
Variables you manipulate (independent). Start simple, add complexity as needed.	Specific values of each factor. Ensure we have normal and edge-case scenarios.	Bounds of possible values for each factor. Should follow directly from your system reqs.

Always start simple and add complexity as needed, additional factors and levels affect the computational complexity of our studies.

Experimental Design

Organize factors combinations to extract meaningful insights from a representative sample.

Full Factorial

All combinations (based on factors, levels, ranges).

Use this when number of runs is within means.

Fractional Factorial

Fewer runs, trades off some interaction terms.

Randomize run order to mitigate time-based bias.

Randomized Block

Controls nuisance variability (groups trials).

Blocking is ideal for environmental drift.

Experimental Design is at the heart of DoE, number of total runs should consider criticality of the study and computational resources.

Execution Plan & Data Collection

Execution Plans ensures that we have a repeatable experiment, considering the data that should be collected during each run.

Setup

- Define run protocol (simulation or physical test).
- Fix control parameters (e.g time step, duration).

Resources

- Schedule compute/lab time.
- Use automation tools (scripting languages, seeds) for repeatability.

Data Logging

- Inputs (factor values)
- Output(s) (response variable(s))
- Metadata (timestamp, run ID, data sources)

A clean pipeline from execution to collection ensures your experiment is reproducible, scalable, and trustworthy.

Data Analysis

Convert experimental data into actionable insights, quantifying the effect of each factor and determining statistical significance.

1. **Descriptive Statistics:** Means, variances, distributions to help spot trends or anomalies early.

2. **Modeling:** To estimate the model error, magnitude, and direction of each factor's influence.

3. **ANOVA (Analysis of Variance):** To evaluate whether the factor-level means are significantly different.

Interpretation and Validation

Which factors actually matter? Are the effects practically significant, not just statistically?

Tools & Visuals

- Main Effect Plots
- Interaction Plots
- Residual Plots
- Prediction Surfaces

Recommendations

- Focus on direction, magnitude, and confidence of effects.
- Watch for anomalous behaviors, unexplained trends may suggest missing variables.
- Use visual aids to communicate findings.

One-on-One Scenario (Ball, 2003)

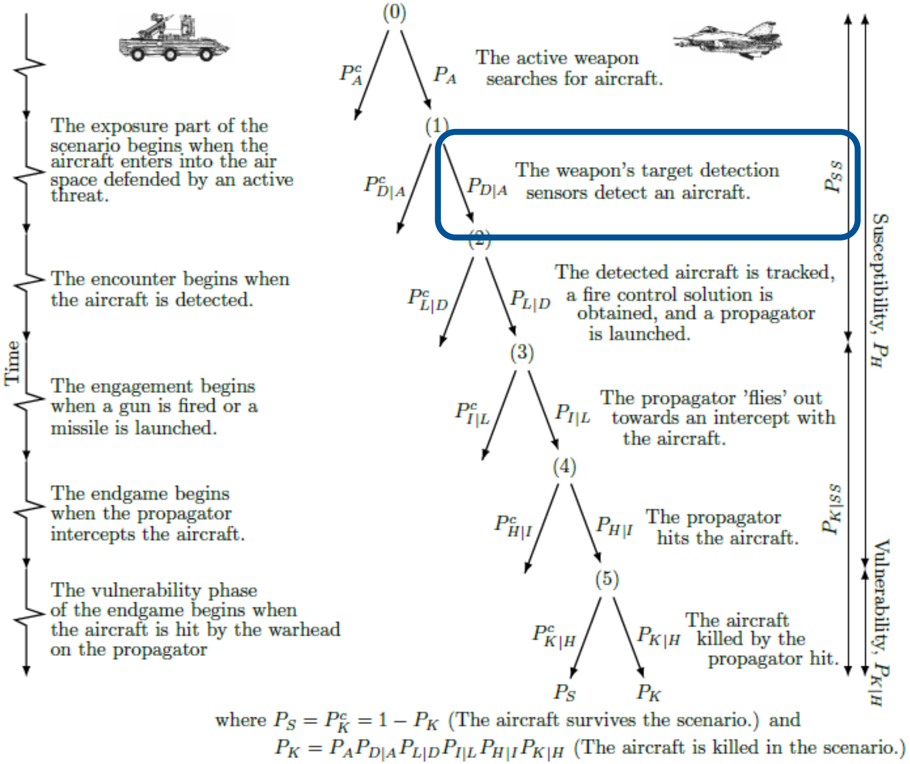


Figure 1. Tree diagram for the one-on-one scenario (single shot) - Original figure from Ball (2003).[1]

Simulation Setup

Hypothesis: Altitude will have a dominant effect on Line-of-Sight determination, showing a smaller ground projection at lower altitudes.

Table 4. Summary of Data Elements Captured in Each Simulation Run

Variable	Description
Ground Latitude	Latitude of the randomly selected ground object location.
Ground Longitude	Longitude of the ground object.
Ground Altitude	Altitude of the ground observer (fixed at 1.5 m).
Aircraft Initial Latitude	Latitude of the aircraft's starting position within the bounding box.
Aircraft Initial Longitude	Longitude of the aircraft's starting position within the bounding box.
Aircraft Altitude	Varied between 45.72 m (150 ft) and 19,812 m (65,000 ft).
Aircraft Speed	Held constant at 250 m/s.
Aircraft Heading	Randomized from 0° to 360°.
Detection Outcome	Binary Los indicator: 0 indicates detection, 1 indicates non-detection.

Simulation Setup

Table 1. Key Factors (Independent Variables)

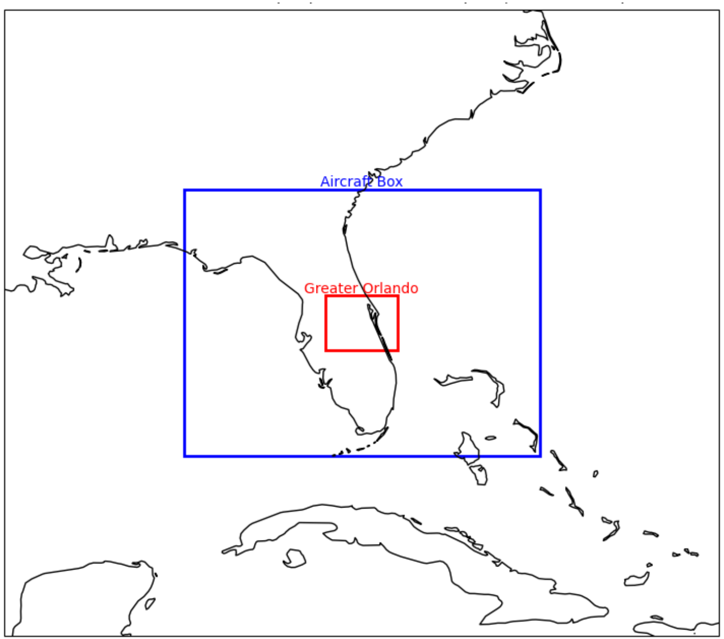
Factor	Description	Range/Values
Aircraft Altitude	Vertical position of the aircraft above ground level.	45.72 m (150 ft) to 19,812 m (65,000 ft)
Horizontal Separation	Relative horizontal distance between the aircraft and the observer.	Variable; determined by simulation geometry
Aircraft Heading	Direction of aircraft motion relative to the observer's location.	Random, 0° to 360°
Ground Observer Location	Geographic coordinates within a defined region representing the observer.	Longitude: [-82, -80], Latitude: [27.5, 29.0]

Table 2. Control Factors

Factor	Description	Fixed Value/Range
Observer Altitude	Height of the ground observer.	1.5 m
Simulation Time Step	Time increment for each simulation iteration.	10 s
Total Simulation Duration	Overall time span for a simulation run.	3600 s (1 hour)
Aircraft Speed	Constant speed of the aircraft during simulation.	250 m/s

Simulation Setup

10,000 Monte Carlo runs of the simulation sampling uniformly from the defined parameter ranges.



Key Assumption: Detection occurs if Line-of-Sight exists.

Simulation Results

Results are captured where the initial values of each parameter represent our input and the target represents output or LOS.

ground_lat	ground_lon	ground_alt	init_plane_lat	init_plane_lon	plane_alt	plane_speed	plane_heading	target
28.159068	-80.913408	1.5	26.830119	-83.453819	18632.338128	250	287.043508	0
28.070547	-81.721285	1.5	31.006451	-82.844182	10766.317053	250	153.786227	0
28.222812	-80.319707	1.5	28.278272	-78.249662	12043.813211	250	7.919106	0
28.171240	-81.355888	1.5	27.189764	-83.471934	2588.453603	250	250.635070	1
28.467847	-81.699527	1.5	26.287418	-84.730112	18255.208534	250	123.598413	0

Modeling

- **Approach:**
  - Trained a random forest classifier on simulated data to predict outcome of detection.
  - **Predictor Variables:**
    - Ground object initial conditions (LLA)
    - Aircraft initial conditions (LLA, speed, heading)
  - **Target Variable:**
    - Detection failure (1)
    - Detection (0)

- **Results:**
  - Achieved 91.7% accuracy on the test set.
  - Minor misclassifications present in borderline horizon distance scenarios.

True label \ Predicted label	Pred=0	Pred=1
True=0	4847	0
True=1	0	3153

True label \ Predicted label	Pred=0	Pred=1
True=0	1150	80
True=1	86	684

RF metamodel effectively captures LOS outcomes, offering a computationally feasible means to conduct experiments at scale.

ANOVA and Modeling Importances

Table 5. ANOVA Results for Key Predictors in the OLS Model

Variable	Sum of Squares	F-value	p-value
Ground Latitude	0.1142	0.6099	0.4349
Ground Longitude	0.1366	0.7295	0.3931
Ground Altitude	0.3321	1.7734	0.1830
Aircraft Initial Latitude	0.0730	0.3900	0.5323
Aircraft Initial Longitude	0.1471	0.7857	0.3754
Aircraft Altitude	412.0531	2200.3590	< 0.001
Aircraft Speed	0.3321	1.7734	0.1830
Aircraft Heading	0.4903	2.6182	0.1057

Table 6. Feature Importances from the Random Forest Classifier

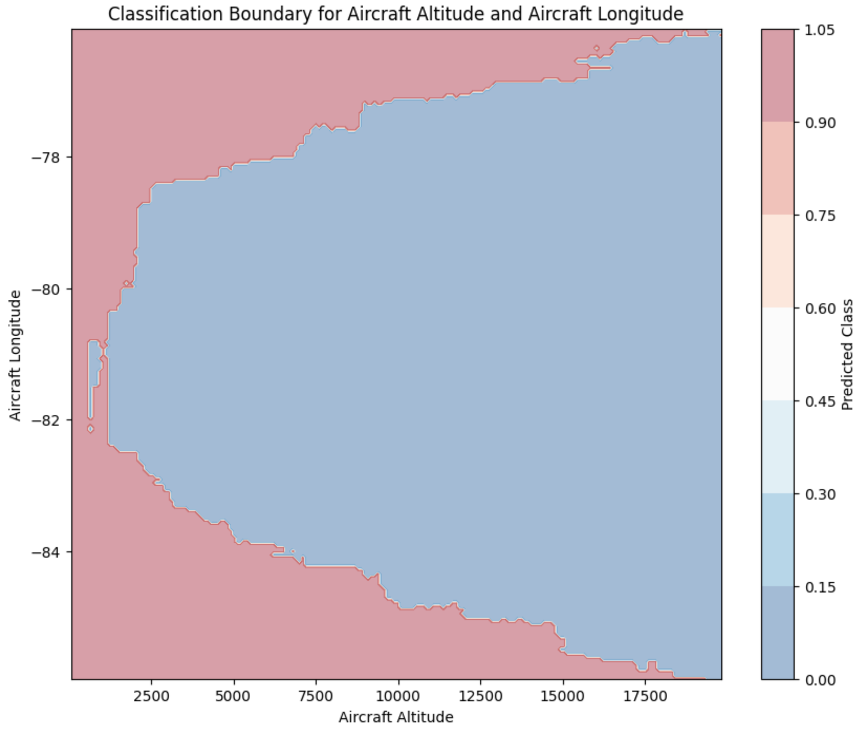
Feature	Importance Score
Aircraft Altitude	0.2972
Aircraft Initial Longitude	0.2396
Aircraft Initial Latitude	0.1578
Aircraft Heading	0.1462
Ground Longitude	0.0820
Ground Latitude	0.0772
Aircraft Speed	0.0000
Ground Altitude	0.0000

Interpretation

- The top two features determined through our analysis highlighted Aircraft Altitude and Aircraft Longitude.

- Based on the response plot, lower altitudes effectively shrinks the longitudinal range for Line-of-Sight detection.

- Hypothesis confirmed, altitude plays the most significant role in detection.



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