Algorithms for Data Science

Unsupervised Learning: Discrete Cosine Transform (DCT)

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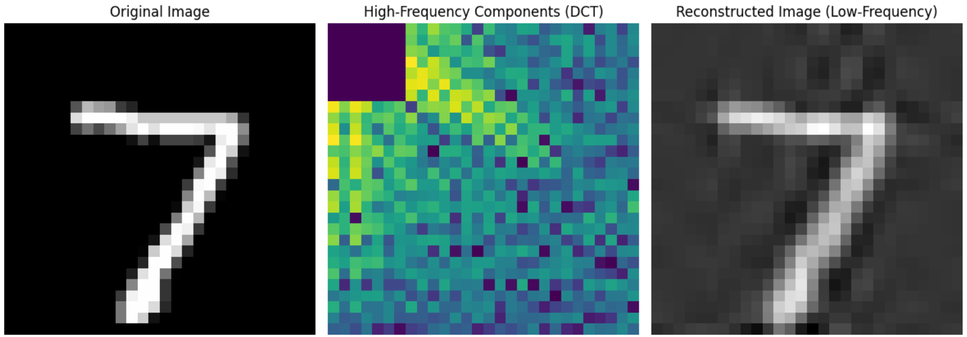
Mathematical Transformations: 2D-DCT

The 2D-DCT transforms spatial-domain data (e.g. images) into the frequency domain.

- **Purpose:** Captures energy distribution in different frequency components (low and high).
- **Applications:** Image compression, feature extraction, noise reduction.
- **Complementary Process:** Inverse 2D-DCT for spatial data reconstruction from frequency coefficients.



2D-DCT: Mathematical Formulation

- 2D-DCT Formula:

$$C_{u,v} = \alpha_u \alpha_v \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} X_{x,y} \cos\left(\frac{\pi(2x+1)u}{2M}\right) \cos\left(\frac{\pi(2y+1)v}{2N}\right)$$

Where:

- $C_{u,v}$: DCT coefficient for frequency indices u, v.
- $X_{x,y}$: Spatial-domain data point at x, y.
- α_u, α_v: Normalization factors

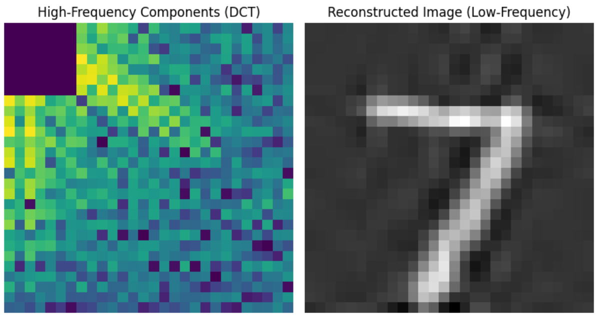
$$\alpha_u = \begin{cases} \frac{1}{\sqrt{M}}, & \text{if } u = 0 \\ \sqrt{\frac{2}{M}}, & \text{if } u \neq 0 \end{cases}$$

Inverse 2D-DCT: Mathematical Formulation

The 2D-DCT is fully reversible.

- Inverse 2D-DCT Formula:

$$X_{x,y} = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} \alpha_u \alpha_v C_{u,v} \cos\left(\frac{\pi(2x+1)u}{2M}\right) \cos\left(\frac{\pi(2y+1)v}{2N}\right)$$



Eigen Decomposition Algorithm Analysis

- **2D-DCT:**
  1. **Compute** frequency coefficients for each pair of indices (u, v).
  2. **Apply** normalization factors α_u, α_v.

Total Complexity: $O(M^2N^2)$

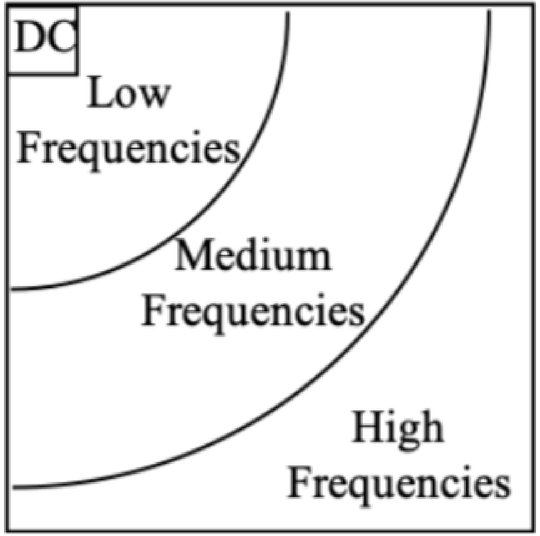
- **Inverse 2D-DCT:**
  1. **Sum** contributions of all frequency components.
  2. **Apply** normalization to reconstruct spatial data.

Optimized Complexity: $O(MNlog(MN))$

Frequency Analysis and Compression

Varying frequency bands unveil distinct layers of an image's structure, from broad patterns to intricate details.

- **Low-Frequency Components:**
  - Represent broad patterns.
  - Retain most of the data's energy.
- **High-Frequency Components:**
  - Represent find details and noise.
  - Can often be discarded for compression.
- **Compression Strategy:**
  - Retain a subset of coefficients (e.g. low-freq.)



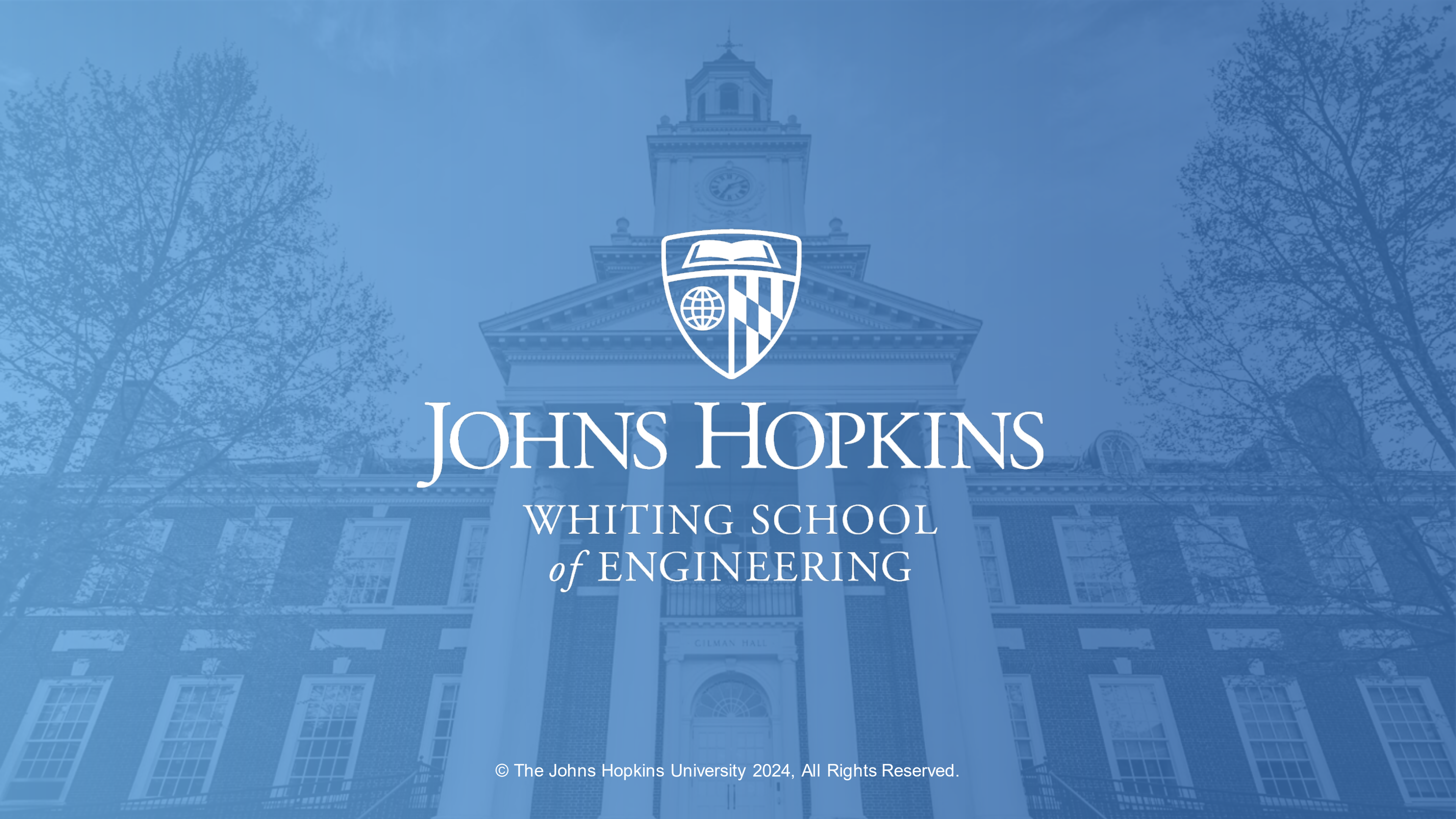
Applications and Limitations

Applications

- **Image Compression:** JPEG format leverages 2D-DCT.
- **Feature Extraction:** Extracting specific frequency components.
- **Noise Reduction:** Remove high-frequency components to denoise.

Limitations

- **Runtime Complexity:** Expensive without optimizations.
- **Assumption:** Assumes stationarity in freq. characteristics.
- **Noise:** Sensitive to noise in high-frequency components.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