Design of Experiments

Mann-Whitney U Test

Mann-Whitney U Test

What is it?

Mann-Whitney U Test (aka Wilcoxon Rank-Sum test) is a non-parametric test used to compare two independent groups when the data does not meet the assumptions of normality required for a t-test.

Kinds

- **Two-Sample (Independent)**: Compares two independent groups to test whether their distributions are the same.

- **One-Tailed or Two-Tailed**: Can test for a specific direction (e.g., one group is greater than the other) or a general difference between the two groups.

Mann-Whitney U Test

Key Steps

1. Combine and rank the data

2. Calculate the sum of ranks for the two groups

3. Calculate the U-statistic
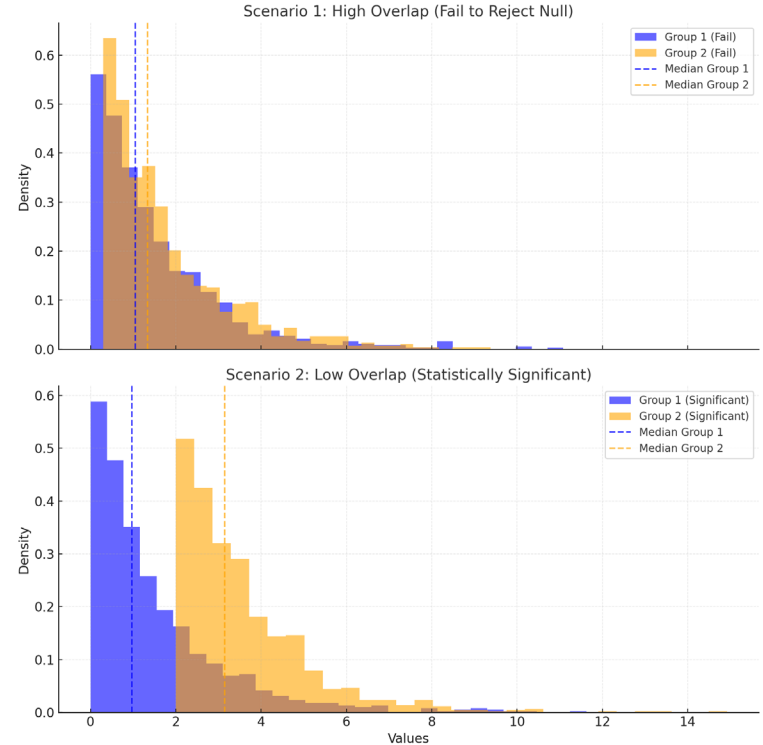
Mann-Whitney U-Test: Mathematical Formulation

- **Combine and Rank the Data**
  - Combine all observations into a single data set, but keep a record of which belongs to which group
  - Assign ranks to each observation, starting with 1; ties all get the same rank

- **U Statistic**

$$U_i = n_1 n_2 + \frac{n_i(n_i + 1)}{2} - R_i$$

$$U = \min(U_1, U_2)$$

Where:
- R_i = Sum of ranks for group i
- n_i: Size of group i

Mann-Whitney U Test

When to use it

- **Non-Normal Data**

- **Ordinal or continuous data**

- **Two independent groups**

When not to use it

- **More than 2 groups**

- **Paired/matched data**

- **Parametric data (e.g. data is normally distributed)**

Hypothesis Testing Framework: Expanded

1. State the Hypotheses:
- **Null Hypothesis (H_0)**: The default assumption (e.g., there is no effect or difference).
- **Alternative Hypothesis (H_1)**: What you're trying to prove (e.g., there is an effect or difference).

2. Choose the Significance Level:
- Typically $\alpha=0.05$. This is the threshold for deciding whether to reject the null hypothesis and acts as a bound on the false positive rate

3. Choose the Power Level:
- Typically $\beta=0.20$. This gives us a statistical power of 80% as a bound on our false negative rate

4. Determine the sample size
- Using the sample size formula

Hypothesis Testing Framework: Expanded

5. Run your experiment and collect the results

6. Calculate the Test Statistic:
- Based on the test you're using (e.g., Z, t, F, χ2).

4. Find the p-value:
- The p-value tells you the probability of observing the data, or something more extreme, assuming the null hypothesis is true.

5. Make a Decision:
- **Reject H_0** if the p-value is less than α, otherwise **fail to reject H_0**.