PROBLEM SET - 6

1a. Given: 59 white balls. {1 → 59}.
In the past 23 has been most likely.

Question ? What is the probability of drawing 23 again ?

→ Probability of drawing 5 balls in which 23 is one of them

$$\rightarrow \frac{1 \times 58 \times 57 \times 56 \times 55 / 4!}{\frac{59 \times 58 \times 57 \times 56 \times 55}{5!}} \quad \begin{matrix}\{\text{order}\} \\ \{\text{all order}\}\end{matrix}$$

$$= \frac{58 \times 57 \times 56 \times 55}{4!} \times \frac{5!}{59 \times 58 \times 57 \times 56 \times 55}$$

$$\rightarrow \frac{5}{59}$$

Hence, the probability of drawing 23 is equal to 5/59.

1b. Given: prediction of win is 0.5. i.e. (81 of 162 games).
: First six games are already won.

winning the first six games does not necessarily mean that they will win the remaining 156 games, and the fact that their winning percentage is only half, the prediction of 162 wins in 162 games is FALSE.

1c. After first six games, the prediction is 81/162.

Answer: Again, winning the first six games does not necessarily mean they will win 81/102 games like it was predicted, they may, or may not win any games after 6th game, or they might even win all the games.

Hence, the last prediction made is FALSE, as a lot of factors do play in determining the predictions.

1d. The Central Limit theorem talks about sample mean or any sample statistic following a normal distribution provided the sample is big enough from the population. So it does not mention anything about population statistics & distribution.

Hence, the statement is FALSE.

1e. Because we are concerned about the sample, ~~mean~~ & the size being sufficiently large, the distribution might approximately be normal, but might be positively or negatively skewed.

Hence, TRUE.

4. $P(H) = 0.5$ {for a fair coin}.

Law of Average: The law states that if you repeat a random experiment (tossing a coin, rolling a die) a very large no. of times, your outcome should on average be equal to the theoritical average.

in this case in the first trial $P(H) = 0.6$ for 100 trial & would it be $P(H) = 0.4$ for the next 100 trials?

that way $(P(H) = 0.5)$ for 200 trials in total.

200 trials is too less of a trial to accept it as a large number. One can observe the law of large numbers kicking in over the trials

more beyond thousands.

for instance,

pbinom(100, 200, 0.5) = 0.52817

pbinom(500, 1000, 0.5) = 0.5126

pbinom(5000, 10,000, 0.5) = 0.5039

pbinom(25,000, 50000, 0.5) = 0.50178

As you can see, As the trials increase, ~~the~~ we approach true probability.

Hence, for a mere 100 trials, the law of averages does not imply one will get 40 heads exactly. One might get less or greater than 40, but it will reach true probability for a large no. of trials.

Q5. GIVEN:

mean = 5 hours.

S.D = 30 minutes = 1/2 hour.

Let's define a new distribution, that is the sum of individual distribution.

Let $X_1, X_2 \dots X_{20}$ be the individual probabilities of the battery, and $X = X_1 + X_2 \dots X_{20}$.

$\therefore E(X) = EX_1 + EX_2 \dots EX_{20}$

$= 20 \times 5 = 100$ hours.

$Var(X) = Var(X_1) + Var(X_2) \dots Var(X_{20})$.

$= (0.5)^2 \times 20$

$= 5$.

$\therefore$ S.D of $X = \sqrt{5} = 2.236$.

OR

$\sigma = \sqrt{20} \times 0.5$

$= 2.236$.

Using 'R', we have.

$P(X > 105) = 1 -$ pnorm$(105, 100, 2.236)$ [CLT Holds].

$= 0.01267143$