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1. Give a tight asymptotic upper bound (O notations) on the solution to each of the following recurrences.

- a) $T(n) = 2T\left(\frac{n}{8}\right) + n^{\frac{m}{3}}$
- b) $T(n) = T\left(\frac{9n}{10}\right) + n$
- c) $T(n) = 3T\left(\frac{n}{2}\right) + n \log n$
- d) $T(n) = 4T\left(\frac{n}{2}\right) + n^2\sqrt{n}$
- e) $T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + T\left(\frac{n}{8}\right) + n$

2. An array $A[1 \dots (2n + 1)]$ is **wiggly** if $A[1] \leq A[2] \geq A[3] \leq A[4] \geq \dots \leq A[2n] \geq A[2n + 1]$. Given an unsorted array $B[1 \dots (2n + 1)]$ of real numbers, **design** and **analyse** an efficient algorithm that outputs a permutation $A[1 \dots (2n + 1)]$ of B such that A is a wiggly array.

3. Consider the RSA crypto system. Let $p = 11$ and $q = 13$.

- a) Compute n and $\phi(n)$
- b) Let the public key $e = 7$. Compute the private key d .
- c) Let the message that you want to send is $m = 2$. What is the encryption code c ?
- d) Decrypt the c that you got above. Show all the steps.

4. Consider the **Diffie-Hellman** key exchange as discussed in the class. Let the common modulus be $p = 31$ and let the common base $\alpha = 3$. Let the two parties be **Dilip** and **Aruna**. Let the private key of **Dilip** be $a = 7$ and let the private key of **Aruna** be $b = 11$.

- a) What does **Dilip** send to **Aruna**?
- b) What does **Aruna** send to **Dilip**?
- c) What is the common shared secret?

5. A general had 1200 soldiers at the start of a battle. After the battle, 3 soldiers were left over when they were lined up 5 at a time, 3 soldiers were left over when they were lined up 6 at a time, 1 soldier was left over when they were lined up 7 at a time, and no soldier was left over when they were lined up 11 at a time. How many soldier(s) survived the battle? Show all the steps.

```
while(noSuccess){
    tryAgain();
    if(Dead){
        break;
    }
}
```

Success Algorithm

Answers