Generating Random WMC Instances

An Empirical Analysis with Varying Primal Treewidth

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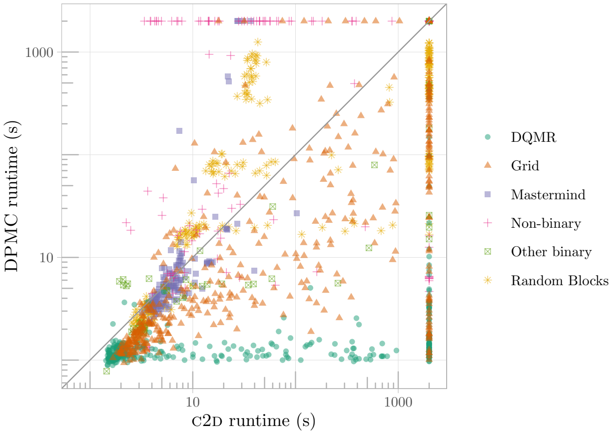
AIAI Seminar

Which Algorithm Is Better? It Depends on the Data



Data from Dilkas and Belle (2021): various Bayesian networks encoded using the method by Darwiche (2002)

The Problem: Weighted Model Counting (WMC)

- A generalisation of propositional model counting (#SAT)
- Applications:
  - graphical models
  - probabilistic programming
  - neural-symbolic artificial intelligence

Example

$w(x) = 0.3,\ w(\neg x) = 0.7,$
$w(y) = 0.2,\ w(\neg y) = 0.8$

$\mathrm{WMC}(x \lor y) = w(x)w(y) + w(x)w(\neg y) + w(\neg x)w(y) = 0.44$

(Some of the) WMC Algorithms

- Cachet (Sang et al. 2004)
  - a SAT solver with clause learning and component caching
- c2d (Darwiche 2004)
  - knowledge compilation to d-DNNF
- d4 (Lagniez and Marquis 2017)
  - knowledge compilation to decision-DNNF
- miniC2D (Oztok and Darwiche 2015)
  - knowledge compilation to decision sentential decision diagrams
- DPMC (Dudek, Phan and Vardi 2020)
  - dynamic programming with algebraic decision diagrams and tree decomposition based planning

Tree Decompositions and Primal Treewidth

Formula in CNF:

$$\phi = (x_4 \vee \neg x_3 \vee x_1) \wedge (\neg x_2 \vee x_4) \wedge (\neg x_1 \vee x_2 \vee x_4)$$

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Its primal graph:

$$\begin{tikzcd} x_1 \arrow[r, dash] \arrow[d, dash] \arrow[dr, dash] & x_2 \arrow[d, dash] \\ x_3 \arrow[r, dash] & x_4 \end{tikzcd}$$

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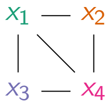
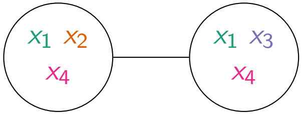


Its minimum-width tree decomposition:



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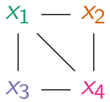

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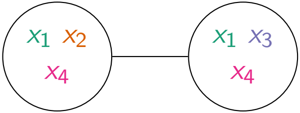


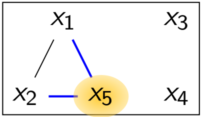
$\therefore$ the primal treewidth of ϕ is 2

From Random SAT to Random WMC

We introduce parameter $\rho \in [0, 1]$ that biases the probability distribution towards adding variables that would introduce fewer new edges to the primal graph.

Example partially-filled formula:
$(\neg x_5 \vee x_2 \vee x_1) \wedge (x_5 \vee ? \vee ?)$

Its primal graph:



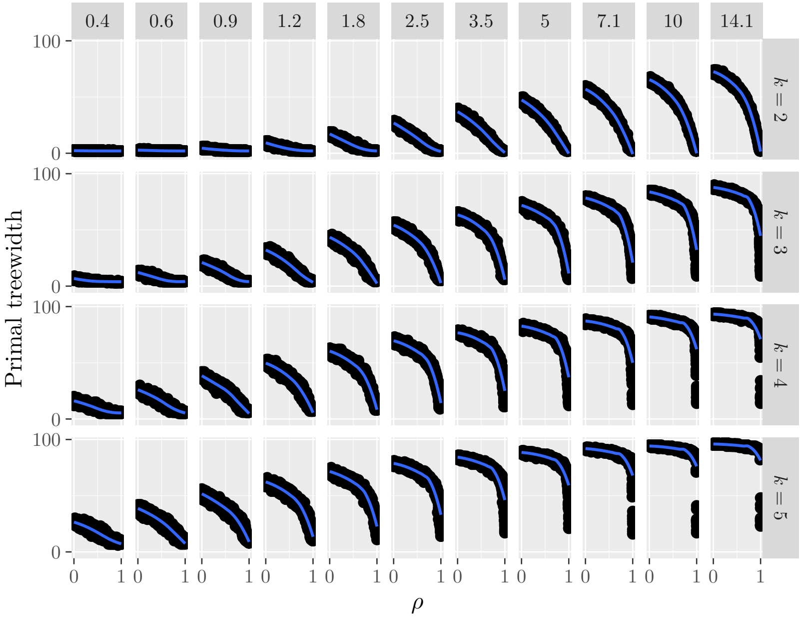
The probability distribution for the next variable

Base probability of each variable being chosen:

$$\frac{1-\rho}{4}.$$

Both x_1 and x_2 get a bonus probability of $\rho/2$ for each being the endpoint of one out of the two neighbourhood edges.

The Relationship Between ρ and Primal Treewidth



Peak Hardness w.r.t. Density

Let μ denote the density, i.e., the number of clauses divided by the number of variables.

- CACHET is known to peak at $\mu = 1.8$ (Sang et al. 2004)
- Bayardo Jr. and Pehoushek (2000) show some #SAT algorithms to peak at $\mu = 1.2$ and $\mu = 1.9$

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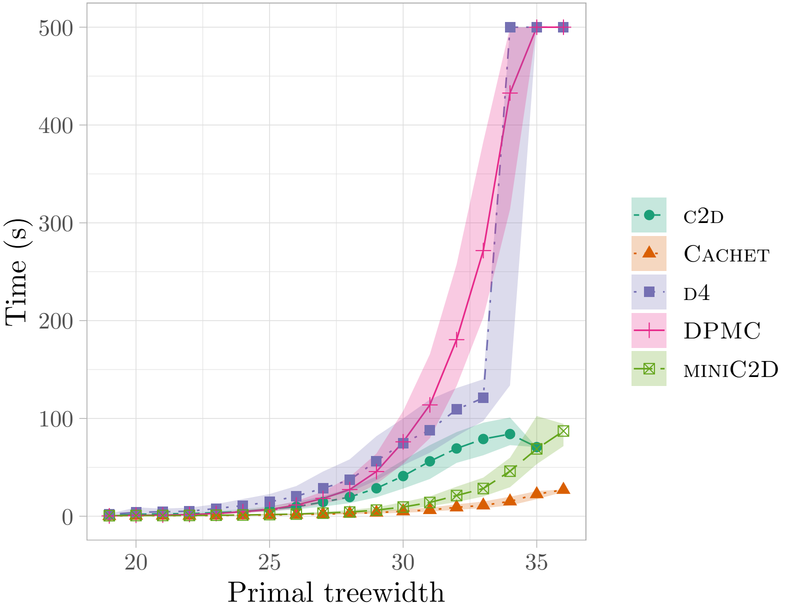
- In our experiments:
  - DPMC peaks at $\mu = 2.2$
  - all other algorithms peak at $\mu = 1.9$

Hardness w.r.t. Primal Treewidth (when $\mu = 1.9$)



Is The Relationship Exponential?

Let us fit the model $\ln t \sim \alpha w + \beta$, i.e., $t \sim e^{\beta}(e^{\alpha})^{w}$, where t is runtime, and w is primal treewidth

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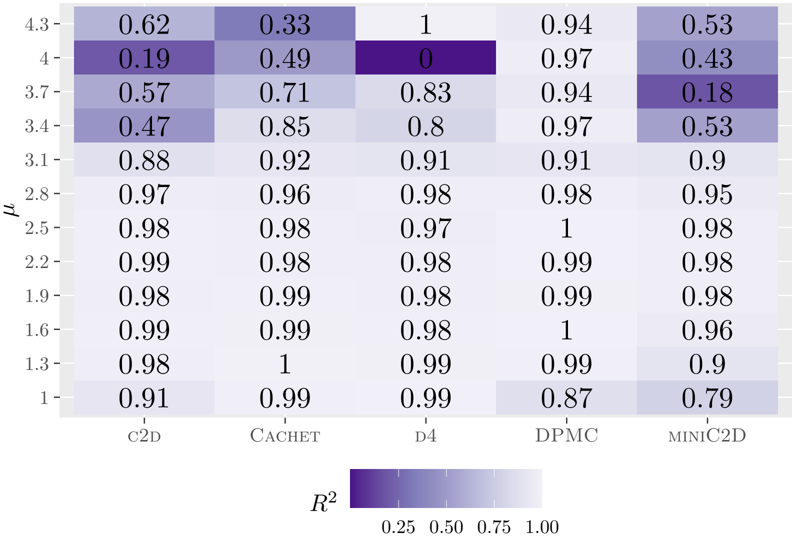

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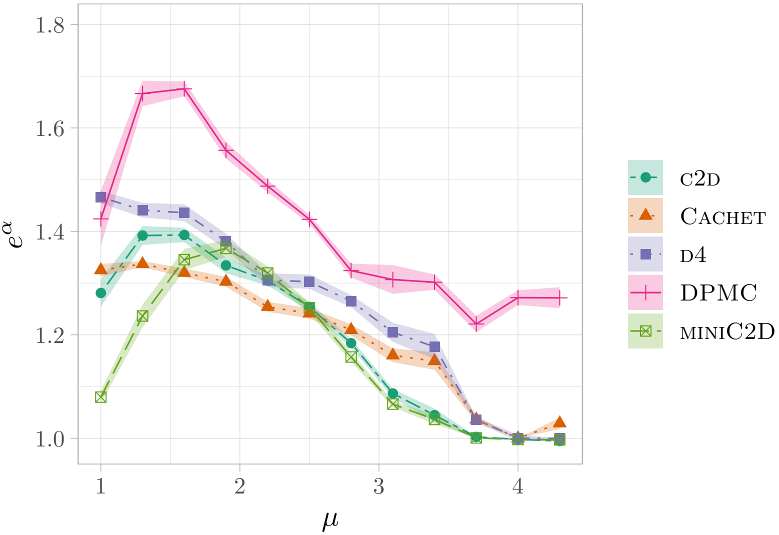

Summary

- Introduced a random model for WMC instances
- Observations:
  - All algorithms scale exponentially w.r.t. primal treewidth
  - The running time of DPMC peaks at a higher density and scales worse w.r.t. primal treewidth
- Future work:
  - A theoretical relationship between ρ and primal treewidth
  - Non-k-CNF instances
  - Algorithm portfolios for WMC