Symmetric Weighted First-Order Model Counting and Factorial-Like Functions

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WFOMC: State of the Art

I'm excluding techniques that are restricted to two variables and purely theoretical results.

1. S. M. Kazemi et al. "New Liftable Classes for First-Order Probabilistic Inference". In: *NIPS*. 2016
   - generic domain recursion (implementation unavailable)
2. Forclift: G. Van den Broeck et al. "Lifted Probabilistic Inference by First-Order Knowledge Compilation". In: *IJCAI*. 2011
   - somewhat restrictive but well-developed
3. L2C: S. M. Kazemi and David Poole. "Knowledge Compilation for Lifted Probabilistic Inference: Compiling to a Low-Level Language". In: *KR*. 2016
   - very basic
4. Alchemy: P. M. Domingos et al. "Unifying Logical and Statistical AI". In: *AAAI*. 2006
   - old, mostly focused on approximations

Counting (Unweighted) Functions: Currently Unliftable (1)

R. P. Stanley. "Enumerative Combinatorics Volume 1 second edition". In: *Cambridge studies in advanced mathematics* (2011)

Functions $M \to N$ (let $|M| = m$ and $|N| = n$)

Theory:

$$\forall x \in M. \forall y, z \in N. P(x, y) \land P(x, z) \Rightarrow y = z$$
$$\forall x \in M. \exists y \in N. P(x, y)$$

Answer: n^m.

Injections

Theory:

$$\forall x \in M. \forall y, z \in N. P(x, y) \land P(x, z) \Rightarrow y = z$$
$$\forall x \in M. \exists y \in N. P(x, y)$$
$$\forall w, x \in M. \forall y \in N. P(w, y) \land P(x, y) \Rightarrow w = x$$

Answer: $(n)_m = n \cdot (n-1) \cdots (n-m+1)$.

Counting (Unweighted) Functions: Currently Unliftable (2)

Surjections

Theory:

$$\forall x \in M. \forall y, z \in N. P(x, y) \wedge P(x, z) \Rightarrow y = z$$
$$\forall x \in M. \exists y \in N. P(x, y)$$
$$\forall y \in N. \exists x \in M. P(x, y)$$

Answer: $n! \left\{ {m \atop n} \right\} = \sum_{i=0}^{n} (-1)^i \binom{n}{i} (n-i)^m$.

Bijections

Theory:

$$\forall x \in M. \forall y, z \in N. P(x, y) \wedge P(x, z) \Rightarrow y = z$$
$$\forall x \in M. \exists y \in N. P(x, y)$$
$$\forall w, x \in M. \forall y \in N. P(w, y) \wedge P(x, y) \Rightarrow w = x$$
$$\forall y \in N. \exists x \in M. P(x, y)$$

Answer: $n!$ if $m = n$.

Counting (Unweighted) Functions: Currently Unliftable (3)

These ones could contain errors.

Partial functions

Theory: $\forall x \in M. \forall y, z \in N. P(x, y) \wedge P(x, z) \Rightarrow y = z$
Answer: $(n+1)^m$.

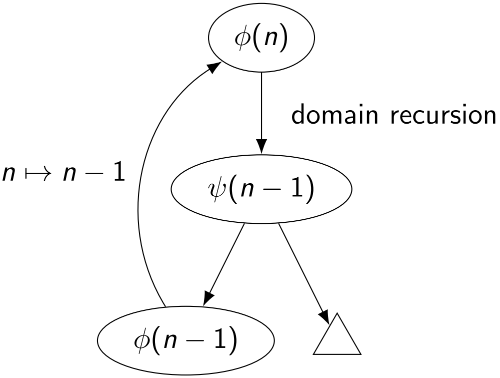
Partial injections

Theory:

$$\forall x \in M. \forall y, z \in N. P(x, y) \wedge P(x, z) \Rightarrow y = z$$
$$\forall w, x \in M. \forall y \in N. P(w, y) \wedge P(x, y) \Rightarrow w = x$$

Answer: $\sum_{k=0}^{\min\{m,n\}} k! \binom{m}{k} \binom{n}{k}$.

Conceptual Plan of Action



Remarks on the Implementation

- WMC computation now loops over the graph.
- We propagate information (e.g., for smoothing) in reverse until convergence.
- Need a good way to recognise equivalent/isomorphic theories.
- Need to be careful about the order of operations:
  - Create a (half-empty) vertex v.
  - Add it to the cache.
  - Recurse on its direct successors S.
  - Add the edges from v to S.
  - After the graph is constructed, propagate information through the graph that would otherwise cause infinite loops.