

# Empowering Domain Recursion in Symmetric Weighted First-Order Model Counting

Paulius Dilkas

# Overview

## Hypothesis

Partial injections can be counted in polynomial time using (a modification of) FORCLIFT.

## Key Ideas & Progress

100% Support for cycles in circuits

95% Transforming  $X \neq x$  constraints into domain reductions  $\Delta' := \Delta \setminus \{x\}$

95% A less restrictive version of domain recursion

90% A hybrid search algorithm

0% Complexity identification

## Definition

A **partial solution** (i.e., a search node) is a tuple consisting of:

- a partial circuit, i.e., a FORCLIFT-style DAG,
  - Each node type has a predefined out-degree.
  - Some of them may be 'arcs to nothing'.
- a cache (i.e., a formula  $\rightarrow$  node hashmap), used to identify opportunities for recursion,
- a list of formulas that still need to be compiled, listed in a particular order.

Some rules are applied in a greedy manner:

- identifying the current formula as something that's already been compiled using the cache;
- sink nodes: tautologies, contradictions, unit clauses;
- some formula-simplifying rules;
- unit propagation.

And some rules are used to branch out into several possibilities:

- independent subtheories,
- shattering,
- ground decomposition,
- inclusion-exclusion,
- independent partial grounding,
- counting,
- (my version of) domain recursion.

# WFOMC: State of the Art

I'm excluding techniques that are restricted to two variables and purely theoretical results.

- ① **S. M. Kazemi et al.** "New Lifiable Classes for First-Order Probabilistic Inference". In: *NIPS*. 2016
  - generic domain recursion (implementation unavailable)
- ② **Forclift**: **G. Van den Broeck et al.** "Lifted Probabilistic Inference by First-Order Knowledge Compilation". In: *IJCAI*. 2011
  - somewhat restrictive but well-developed
- ③ **L2C**: **S. M. Kazemi and David Poole.** "Knowledge Compilation for Lifted Probabilistic Inference: Compiling to a Low-Level Language". In: *KR*. 2016
  - very basic
- ④ **Alchemy**: **P. M. Domingos et al.** "Unifying Logical and Statistical AI". In: *AAAI*. 2006
  - old, mostly focused on approximations

# Counting Injections

Theory:

$$\forall x \in M. \forall y, z \in N. P(x, y) \wedge P(x, z) \Rightarrow y = z$$

$$\forall x \in M. \exists y \in N. P(x, y)$$

$$\forall w, x \in M. \forall y \in N. P(w, y) \wedge P(x, y) \Rightarrow w = x$$

Answer:  $n^{\underline{m}} = n \cdot (n - 1) \cdots (n - m + 1)$  if  $m \leq n$  and 0 otherwise (for positive  $m$  and  $n$ ).

## One answer

$$f(m, n) = \sum_{k=0}^m \binom{m}{k} (-1)^{m-k} g(k, n),$$

where

$$g(k, n) = \begin{cases} 1 & \text{if } k = 0 \\ \sum_{l=0}^n [l < 2] g(k-1, n-l) & \text{otherwise.} \end{cases}$$

## Another answer (almost found by Forclift)

(The missing parts are highlighted) Same  $f$  and...

$$g(m, n) = \begin{cases} 1 & \text{if } m = 0 \\ n + 1 & \text{if } m = 1 \\ \sum_{k=0}^{\min\{n,1\}} \sum_{l=0}^{\min\{n-k,1\}} \frac{n!}{(n-m-l)!} g(m-2, n-m-l) & \text{otherwise.} \end{cases}$$

# Counting (Unweighted) Functions: Currently Unliftable (2)

## Surjections

Theory:

$$\forall x \in M. \forall y, z \in N. P(x, y) \wedge P(x, z) \Rightarrow y = z$$

$$\forall x \in M. \exists y \in N. P(x, y)$$

$$\forall y \in N. \exists x \in M. P(x, y)$$

Answer:  $n! \left\{ \begin{matrix} m \\ n \end{matrix} \right\} = \sum_{i=0}^n (-1)^i \binom{n}{i} (n-i)^m.$



# Bijections

## Theory

$$\forall x \in M. \forall y, z \in N. P(x, y) \wedge P(x, z) \Rightarrow y = z$$

$$\forall x \in M. \exists y \in N. P(x, y)$$

$$\forall w, x \in M. \forall y \in N. P(w, y) \wedge P(x, y) \Rightarrow w = x$$

$$\forall y \in N. \exists x \in M. P(x, y)$$

Answer:  $n!$  if  $m = n$ .

Linear (!) answer found by Forclift:

$$\begin{aligned} f(m, n) &= -f(m, n-1) + \sum_{k=0}^m \binom{m}{k} [k < 2] f(m-k, n-1) \\ &= -f(m, n-1) + f(m, n-1) + m \times f(m-1, n-1) \\ &= m \times f(m-1, n-1) \end{aligned}$$

# Counting (Unweighted) Functions: Currently Unliftable (3)

## Partial functions

Theory:  $\forall x \in M. \forall y, z \in N. P(x, y) \wedge P(x, z) \Rightarrow y = z$

Answer:  $(n + 1)^m$ .

## Partial injections

Theory:

$$\forall x \in M. \forall y, z \in N. P(x, y) \wedge P(x, z) \Rightarrow y = z$$

$$\forall w, x \in M. \forall y \in N. P(w, y) \wedge P(x, y) \Rightarrow w = x$$

My answer:  $\sum_{k=0}^{\min\{m,n\}} k! \binom{m}{k} \binom{n}{k}$ .

# Counting (Unweighted) Functions: Currently Unliftable (3)

## Partial functions

Theory:  $\forall x \in M. \forall y, z \in N. P(x, y) \wedge P(x, z) \Rightarrow y = z$

Answer:  $(n + 1)^m$ .

## Partial injections

Theory:

$$\forall x \in M. \forall y, z \in N. P(x, y) \wedge P(x, z) \Rightarrow y = z$$

$$\forall w, x \in M. \forall y \in N. P(w, y) \wedge P(x, y) \Rightarrow w = x$$

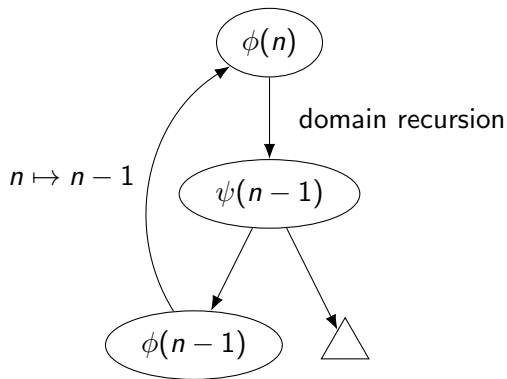
My answer:  $\sum_{k=0}^{\min\{m,n\}} k! \binom{m}{k} \binom{n}{k}$ .

Answer found by Forclift:

$$f(m, n) = \begin{cases} 1 & \text{if } m = 0 \\ \sum_{k=0}^n \binom{n}{k} [k < 2] f(m - 1, k) & \text{otherwise.} \end{cases}$$

(exponential...)

# Conceptual Plan of Action



# Remarks on the Implementation

- WMC computation now loops over the graph.
- We propagate information (e.g., for smoothing) in reverse until convergence.
- Need a good way to recognise equivalent/isomorphic theories.
- Need to be careful about the order of operations:
  - Create a (half-empty) vertex  $v$ .
  - Add it to the cache.
  - Recurse on its direct successors  $S$ .
  - Add the edges from  $v$  to  $S$ .
  - After the graph is constructed, propagate information through the graph that would otherwise cause infinite loops.

# Domain Size Comparisons

$|N| < 2$  (Forclift can already do this!)

$$\phi \equiv \forall x, y \in N \text{ s.t. } x \neq y, P,$$

where  $w(P) = 0$ . The algebraic (multiplicative) contribution of  $\phi$  is  $w(P)^{\text{gr}(\phi)}$ .

- If  $|N| \geq 2$ , then  $\text{gr}(\phi) > 0$ , and so  $0^{\text{gr}(\phi)} = 0$ .
- If  $|N| < 2$ , then we get  $0^0 = 1$ .
- Of course, 2 can be replaced by any other positive integer.

# Domain Size Comparisons

$|N| < 2$  (Forclift can already do this!)

$$\phi \equiv \forall x, y \in N \text{ s.t. } x \neq y, P,$$

where  $w(P) = 0$ . The algebraic (multiplicative) contribution of  $\phi$  is  $w(P)^{\text{gr}(\phi)}$ .

- If  $|N| \geq 2$ , then  $\text{gr}(\phi) > 0$ , and so  $0^{\text{gr}(\phi)} = 0$ .
- If  $|N| < 2$ , then we get  $0^0 = 1$ .
- Of course, 2 can be replaced by any other positive integer.

$m < n$  (possible, but a bit complicated)

$$f(m, n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } m = 0 \text{ and } n > 0 \\ f(m-1, n-1) & \text{otherwise.} \end{cases}$$