Empowering Domain Recursion in Symmetric Weighted First-Order Model Counting

Paulius Dilkas

Overview

Hypothesis

Partial injections can be counted in polynomial time using (a modification of) ForcLift.

Key Ideas & Progress

- 100% Support for cycles in circuits
- 95% Transforming $X \neq x$ constraints into domain reductions $\Delta' := \Delta \setminus \{x\}$
- 95% A less restrictive version of domain recursion
- 90% A hybrid search algorithm
- 0% Complexity identification

Search

Definition

A partial solution (i.e., a search node) is a tuple consisting of:

- a partial circuit, i.e., a ForcLift-style DAG,
  - Each node type has a predefined out-degree.
  - Some of them may be 'arcs to nothing'.
- a cache (i.e., a formula $\to$ node hashmap), used to identify opportunities for recursion,
- a list of formulas that still need to be compiled, listed in a particular order.

Search

Some rules are applied in a greedy manner:

- identifying the current formula as something that's already been compiled using the cache;
- sink nodes: tautologies, contradictions, unit clauses;
- some formula-simplifying rules;
- unit propagation.

And some rules are used to branch out into several possibilities:

- independent subtheories,
- shattering,
- ground decomposition,
- inclusion-exclusion,
- independent partial grounding,
- counting,
- (my version of) domain recursion.

WFOMC: State of the Art

I'm excluding techniques that are restricted to two variables and purely theoretical results.

1. S. M. Kazemi et al. "New Liftable Classes for First-Order Probabilistic Inference". In: *NIPS*. 2016
   - generic domain recursion (implementation unavailable)
2. Forclift: G. Van den Broeck et al. "Lifted Probabilistic Inference by First-Order Knowledge Compilation". In: *IJCAI*. 2011
   - somewhat restrictive but well-developed
3. L2C: S. M. Kazemi and David Poole. "Knowledge Compilation for Lifted Probabilistic Inference: Compiling to a Low-Level Language". In: *KR*. 2016
   - very basic
4. Alchemy: P. M. Domingos et al. "Unifying Logical and Statistical AI". In: *AAAI*. 2006
   - old, mostly focused on approximations

Counting Injections

Theory:

$$\forall x \in M. \forall y, z \in N. P(x, y) \land P(x, z) \Rightarrow y = z$$
$$\forall x \in M. \exists y \in N. P(x, y)$$
$$\forall w, x \in M. \forall y \in N. P(w, y) \land P(x, y) \Rightarrow w = x$$

Answer: $n^{\underline{m}} = n \cdot (n-1) \cdots (n-m+1)$ if $m \leq n$ and 0 otherwise (for positive m and n).

One answer

$$f(m,n) = \sum_{k=0}^{m} \binom{m}{k} (-1)^{m-k} g(k,n),$$

where

$$g(k,n) = \begin{cases} 1 & \text{if } k = 0 \\ \sum_{l=0}^{n} [l < 2] g(k-1, n-l) & \text{otherwise.} \end{cases}$$

Another answer (almost found by Forclift)

(The missing parts are highlighted) Same f and...

$$g(m,n) = \begin{cases} 1 & \text{if } m = 0 \\ n+1 & \text{if } m = 1 \\ \sum_{k=0}^{\min\{n,1\}} \sum_{l=0}^{\min\{n-k,1\}} \frac{n!}{(n-m-l)!} g(m-2, n-m-l) & \text{otherwise.} \end{cases}$$

Counting (Unweighted) Functions: Currently Unliftable (2)

Surjections

Theory:

$$\forall x \in M. \forall y, z \in N. P(x, y) \land P(x, z) \Rightarrow y = z$$
$$\forall x \in M. \exists y \in N. P(x, y)$$
$$\forall y \in N. \exists x \in M. P(x, y)$$

Answer: $n! \left\{ {m \atop n} \right\} = \sum_{i=0}^{n} (-1)^i \binom{n}{i} (n-i)^m$.

Counting (Unweighted) Functions: Currently Unliftable (3)

Partial functions

Theory: $\forall x \in M. \forall y, z \in N. P(x, y) \land P(x, z) \Rightarrow y = z$
Answer: $(n + 1)^m$.

Partial injections

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My answer: $\sum_{k=0}^{\min\{m,n\}} k! \binom{m}{k} \binom{n}{k}$.

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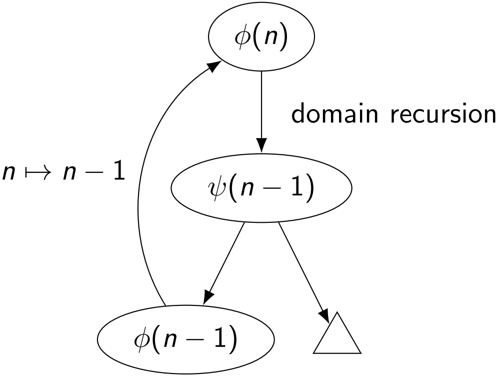
My answer: $\sum_{k=0}^{\min\{m,n\}} k! \binom{m}{k} \binom{n}{k}$.
Answer found by Forclift:

$$f(m, n) = \begin{cases} 1 & \text{if } m = 0 \\ \sum_{k=0}^{n} \binom{n}{k} [k < 2] f(m-1, k) & \text{otherwise.} \end{cases}$$

(exponential...)

Conceptual Plan of Action



Remarks on the Implementation

- WMC computation now loops over the graph.
- We propagate information (e.g., for smoothing) in reverse until convergence.
- Need a good way to recognise equivalent/isomorphic theories.
- Need to be careful about the order of operations:
  - Create a (half-empty) vertex v.
  - Add it to the cache.
  - Recurse on its direct successors S.
  - Add the edges from v to S.
  - After the graph is constructed, propagate information through the graph that would otherwise cause infinite loops.

Domain Size Comparisons

$|N| < 2$ (Forclift can already do this!)

$$\phi \equiv \forall x, y \in N \text{ s.t. } x \neq y,\ P,$$

where $w(P) = 0$. The algebraic (multiplicative) contribution of ϕ is $w(P)^{\mathrm{gr}(\phi)}$.

- If $|N| \geq 2$, then $\mathrm{gr}(\phi) > 0$, and so $0^{\mathrm{gr}(\phi)} = 0$.
- If $|N| < 2$, then we get $0^0 = 1$.
- Of course, 2 can be replaced by any other positive integer.

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$m < n$ (possible, but a bit complicated)

$$f(m, n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } m = 0 \text{ and } n > 0 \\ f(m-1, n-1) & \text{otherwise.} \end{cases}$$

Bijections

$$
\begin{aligned}
&\forall x \in M. \forall y, z \in N. P(x, y) \land P(x, z) \Rightarrow y = z \\
&\quad \forall x \in M. \exists y \in N. P(x, y) \\
&\forall w, x \in M. \forall y \in N. P(w, y) \land P(x, y) \Rightarrow w = x \\
&\quad \forall y \in N. \exists x \in M. P(x, y)
\end{aligned}
$$

Answer: $n!$ if $m = n$ and zero otherwise.

Bijections: a linear answer found by Forclift

$$
\begin{aligned}
f(m,n) &= -f(m,n-1) + \sum_{k=0}^{m} \binom{m}{k} [k < 2] f(m-k, n-1) \\
&= -f(m,n-1) + f(m,n-1) + m \times f(m-1,n-1) \\
&= m \times f(m-1,n-1)
\end{aligned}
$$

This works if supplied with the right base cases:

$$
\begin{aligned}
f(0,0) &= 1, \\
f(m,0) &= 0 \text{ (for all } m > 0\text{)}, \\
f(0,n) &= 0 \text{ (for all } n > 0\text{)}.
\end{aligned}
$$