A Slightly Different Setup

- A theory is a conjunction of clauses
- A clause is a sentence in many-sorted function-free first-order logic with equality
- No restrictions on the number of variables
- Constants are allowed
- Skolemization eliminates existential quantifiers

Example (Injective functions)

$$(\forall x \in \Delta.\ \forall y, z \in \Gamma.\ P(x, y) \land P(x, z) \to y = z) \land$$
$$(\forall x \in \Delta.\ \exists y \in \Gamma.\ P(x, y)) \land$$
$$(\forall w, x \in \Delta.\ \forall y \in \Gamma.\ P(w, y) \land P(x, y) \to w = x)$$

Example (Independence)

Input formula:

$$(\forall x, y \in \Omega.\ x = y) \wedge \tag{1}$$

$$(\forall x \in \Delta.\ \forall y, z \in \Gamma.\ P(x, y) \wedge P(x, z) \rightarrow y = z) \wedge \tag{2}$$

$$(\forall w, x \in \Delta.\ \forall y \in \Gamma.\ P(w, y) \wedge P(x, y) \rightarrow w = x) \tag{3}$$

Compilation Rules

Example (Independence)

Input formula:

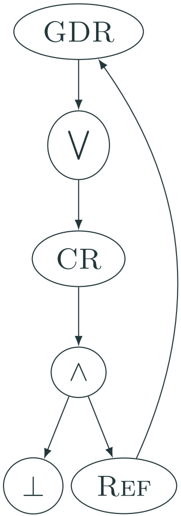
$$(\forall x, y \in \Omega.\ x = y) \land \tag{1}$$

$$(\forall x \in \Delta.\ \forall y, z \in \Gamma.\ P(x, y) \land P(x, z) \rightarrow y = z) \land \tag{2}$$

$$(\forall w, x \in \Delta.\ \forall y \in \Gamma.\ P(w, y) \land P(x, y) \rightarrow w = x) \tag{3}$$

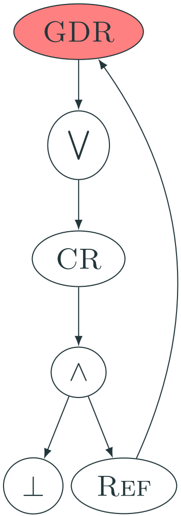
$$G = \begin{tikzcd} & \land \arrow[dl] \arrow[dr] & \\ \star & & \star \end{tikzcd}, \qquad L = \langle (1), (2) \land (3) \rangle$$

The Algebraic Interpretation



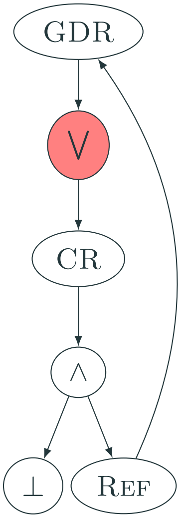
$$f(m, n) =$$

The Algebraic Interpretation



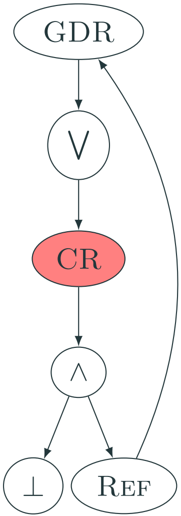
$f(m, n) =$

The Algebraic Interpretation



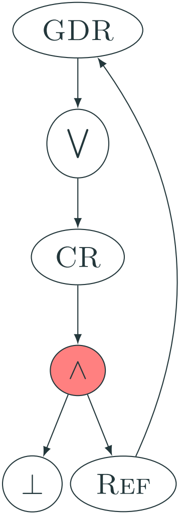
$$f(m,n) = \sum_{l=0}^{m} \binom{m}{l}$$

The Algebraic Interpretation



$$f(m, n) = \sum_{l=0}^{m} \binom{m}{l}$$

The Algebraic Interpretation



$$f(m,n) = \sum_{l=0}^{m} \binom{m}{l} \qquad \times$$

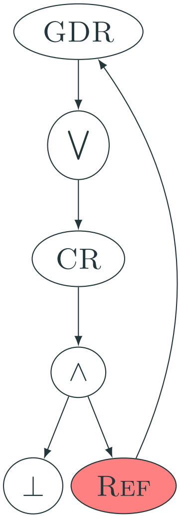
The Algebraic Interpretation

$$\begin{tikzcd}[row sep=large]
 & \mathrm{GDR} \arrow[d] & \\
 & \bigvee \arrow[d] & \\
 & \mathrm{CR} \arrow[d] & \\
 & \wedge \arrow[dl] \arrow[dr] & \\
\bot & & \textsc{Ref} \arrow[uuuul, bend right]
\end{tikzcd}$$

$$f(m,n) = \sum_{l=0}^{m} \binom{m}{l} [l < 2] \times$$

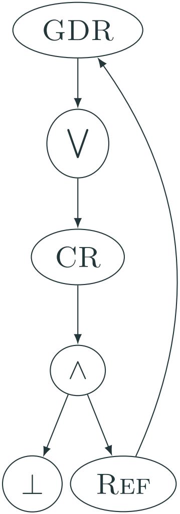
$$[\phi] = \begin{cases} 1 & \text{if } \phi \\ 0 & \text{if } \neg\phi \end{cases}$$

The Algebraic Interpretation



$$f(m,n) = \sum_{l=0}^{m} \binom{m}{l} [l < 2] \times f(m-l, n-1)$$

The Algebraic Interpretation



$$
\begin{aligned}
f(m, n) &= \sum_{l=0}^{m} \binom{m}{l} [l < 2] \times f(m-l, n-1) \\
&= f(m, n-1) + mf(m-1, n-1)
\end{aligned}
$$

Workflow

with ForcLift

1. Compile the formula to a circuit
2. Evaluate to get the answer

with Crane (my work)

1. Compile the formula to a graph
2. Extract the definitions of functions
3. Simplify
4. Supplement with base cases
5. Evaluate to get the answer

Major Directions for Future Work

- An algebraic description of what kind of sequences and functions with domain $\mathbb{N}_0^k$ are computable in this way
  - monotonicity, maximal growth rate, etc.
- A different input format or logic that allows the same approach to capture more computations
  - fixed-point logic with counting
  - 'let domain $\Delta := \{ 1, 2, \dots, \mathrm{MC}(\phi) \}$'
- parameterised Markov logic networks
  - 'What equations do the domain sizes have to satisfy for the probability of event E to be at least 95%?'