A Solutions Found by CRANE

In this appendix, we list the exact function definitions produced by CRANE for all of the problem instances in Section 6 both before and after algebraic simplification (excluding multiplications by one). The correctness of all of them has been checked by identifying suitable base cases and verifying the numerical answers across a range of domain sizes.

1. $\Theta(m)$ solution for counting $\Gamma \to \Gamma$ functions:

$$f(m) = \left(-1 + \sum_{l=0}^{m} \binom{m}{l} [l < 2]\right)^{m} = m^{m}.$$

2. $\Theta(m^3 + n^3)$ solution for counting $\Gamma \to \Delta$ surjections:

$$\begin{aligned} f(m,n) = &\sum_{l=0}^{m} \binom{m}{l} (-1)^{m-l} \sum_{k=0}^{n} \binom{n}{k} (-1)^{n-k} \\ &\left(\sum_{j=0}^{k} \binom{k}{j} [j < 2]\right)^{l} \\ = &\sum_{l=0}^{m} \binom{m}{l} (-1)^{m-l} \sum_{k=0}^{n} \binom{n}{k} (-1)^{n-k} (k+1)^{l}. \end{aligned}$$

3. $\Theta(m^3)$ solution for counting $\Gamma \to \Gamma$ surjections:

$$\begin{aligned} f(m) = &\sum_{l=0}^{m} \binom{m}{l} (-1)^{m-l} \sum_{k=0}^{m} \binom{m}{k} (-1)^{m-k} \\ &\left(\sum_{j=0}^{k} \binom{k}{j} [j < 2]\right)^{l} \\ = &\sum_{l=0}^{m} \binom{m}{l} (-1)^{m-l} \sum_{k=0}^{m} \binom{m}{k} (-1)^{m-k} (k+1)^{l}. \end{aligned}$$

4. $\Theta(mn)$ solution for counting $\Gamma \to \Delta$ injections and partial injections (with different base cases):

$$\begin{aligned} f(m,n) &= \sum_{l=0}^{m} \binom{m}{l} [l < 2] f(m-l, n-1) \\ &= f(m, n-1) + m f(m-1, n-1). \end{aligned}$$

5. $\Theta(m^3)$ solution for counting $\Gamma \to \Gamma$ injections:

$$f(m) = \sum_{l=0}^{m} \binom{m}{l} (-1)^{m-l} g(m,l);$$

$$\begin{aligned} g(m,l) &= \sum_{k=0}^{l} \binom{l}{k} [k < 2] g(m-1, l-k) \\ &= g(m-1, l) + l g(m-1, l-1). \end{aligned}$$

6. $\Theta(m)$ solution for counting $\Gamma \to \Delta$ bijections:

$$f(m,n) = m f(m-1, n-1).$$