Nondeterministic Bigraphical Reactive Systems for Markov Decision Processes*

Paulius Dilkas[0000−1111−2222−3333] and Michele Sevegnani[1111−2222−3333−4444]

University of Glasgow, Glasgow, UK

Abstract. The abstract should briefly summarize the contents of the paper in 150–250 words.

Keywords: First keyword · Second keyword · Another keyword.

1 Introduction

Definition 1 ([1]). *For any finite set X, let $Dist(X)$ denote the set of discrete probability distributions over X. A* Markov Decision Process *is a tuple $(S, \overline{s}, A, P, L)$, where: S is a finite set of states and $\overline{s} \in S$ is the initial state; A is a finite set of* actions*; $P : S \times A \to Dist(S)$ is a (partial) probabilistic transition function, mapping state-action pairs to probability distributions over S; $L : S \to 2^P$ is a labelling with atomic propositions.*

Definition 2. *A* reward structure *for an MDP $(S, \overline{s}, A, P, L)$ is a pair (ρ, ι), where $\rho : S \to \mathbb{R}$ is the* state reward function*, and $\iota : S \times A \to \mathbb{R}$ is the* transition reward function*.*

From [4].

Definition 3 (Concrete place graph with sharing). *A* concrete place graph with sharing

$$F = (V_F, ctrl_F, prnt_F) : m \to n$$

is a triple having an inner interface m and an outer interface n. These index the sites and regions of the place graph, respectively. F has a finite set $V_F \subset \mathcal{V}$ of nodes, a control map $ctrl_F : V_F \to \mathcal{K}$, and a parent relation

$$prnt_F \subseteq (m \uplus V_F) \times (V_F \uplus n)$$

that is acyclic, i.e., $(v, v) \notin prnt_F^+$ for any $v \in V_F$.

Definition 4 (Composition for place graphs with sharing). *If $F : k \to m$ and $G : m \to n$ are two concrete place graphs with sharing with $V_F \cap V_G = \emptyset$, their composite*

$$G \circ F = (V, ctrl, prnt) : k \to n$$

* Supported by organization x.

has nodes $V = V_F \uplus V_G$ *and control map* $ctrl = ctrl_F \uplus ctrl_G$. *Its parent relation* $prnt \subseteq (k \uplus V) \times (V \uplus n)$ *is given by:*

$$prnt \coloneqq prnt_G^{\lhd} \uplus prnt_{\circ} \uplus prnt_F^{\rhd},$$

where

$$\begin{aligned} prnt_F^{\rhd} &= prnt_F \rhd V_F, \\ prnt_G^{\lhd} &= V_G \lhd prnt_G, \\ prnt_{\circ} &= (m \lhd prnt_G) \circ (prnt_F \rhd m). \end{aligned}$$

Definition 5 (Tensor product for place graphs). *If* $G_0 : m_0 \to n_0$ *and* $G_1 : m_1 \to n_1$ *are two concrete place graphs with sharing with* $V_F \cap V_G = \emptyset$, *their tensor product*

$$G_0 \otimes G_1 = (V, ctrl, prnt) : m_0 + m_1 \to n_0 + n_1$$

has nodes $V = V_{G_0} \uplus V_{G_1}$ *and control map* $ctrl \coloneqq ctrl_{G_0} \uplus ctrl_{G_1}$. *Its parent relation* $prnt \subseteq [(m_0 + m_1) \uplus V] \times [V \uplus (n_0 + n_1)]$ *is defined as*

$$prnt_{G_0} \uplus prnt_{G_1}^{(m_0,n_0)},$$

where

$$\begin{aligned} prnt_{G_1}^{(m_0,n_0)} =& \{(v, w) \mid (v, w) \in prnt_{G_1} \text{ and } v, w \in V_{G_1}\} \\ &\uplus \{(m_0 + i, w) \mid (i, w) \in prnt_{G_1}, w \in V_{G_1} \text{ and } i \in m_1\} \\ &\uplus \{(v, n_0 + i) \mid (v, i) \in prnt_{G_1}, v \in V_{G_1} \text{ and } i \in n_1\} \\ &\uplus \{(m_0 + i, n_0 + j) \mid (i, j) \in prnt_{G_1}, i \in m_1, \text{ and } j \in n_1\}. \end{aligned}$$

Definition 6 (Concrete link graph). *A* concrete link graph

$$F = (V_F, E_F, ctrl_F, link_F) : X \to Y$$

is a quadruple having an inner face X *and an outer face* Y, *both finite subsets of* $\mathcal{X}$, *called respectivelly the inner and outer names of the link graph.* F *has finite sets* $V_F \subset \mathcal{V}$ *of nodes and* $E_F \subset \mathcal{E}$ *of edges, a control map* $ctrl_F : V_F \to \mathcal{K}$ *and a* link map

$$link_F : X \uplus P_F \to E_F \uplus Y,$$

where $P_F \coloneqq \{(v, i) \mid i \in ar(ctrl_F(v))\}$ *is the set of ports of* F. *Thus* (v, i) *is the* i*th port of node* v. *We shall call* $X \uplus P_F$ *the* points *of* F, *and* $E_F \uplus Y$ *its* links.

The sets of points and the set of ports of a link l are defined by

$$points_F(l) \coloneqq \{p \mid link_F(p) = l\}, \quad ports_F(l) \coloneqq points_F(l) \setminus X.$$

An edge is *idle* if it has no points. Identities over name sets are defined by $\mathsf{id}_X = (\emptyset, \emptyset, \emptyset, \mathsf{Id}_X) : X \to X$.

Definition 7 (Concrete bigraph with sharing). *A concrete bigraph*

$$F = (V_F, E_F, ctrl_F, prnt_F, link_F) : \langle k, X\rangle \to \langle m, Y\rangle$$

consists of a concrete place graph with sharing $F^P = (V_F, ctrl_F, prnt_F) : k \to m$ *and a concrete link graph* $F^L = (V_F, E_F, ctrl_F, link_F) : X \to Y$. *We write the concrete bigraph with sharing as* $F = (F^P, F^L)$.

Definitions: support translation, lean-support equivalence, concretion, abstraction.

More definitions: level, normalised levels, occurrence, matching, concrete occurrence, underlying graph

Stochastic bigraphs [2]

From PhD thesis [3]

Definition 8 (reaction rule). *A* reaction rule *is a pair*

$$\mathsf{R} = (R : m \to J, R' : m \to J),$$

sometimes written as $R \longrightarrow R'$, *where* R *is the* redex *and* R' *the* reactum, *and* R *is solid. The rule generates all the* ground reaction rules (r, r'), *where* $r = (R \otimes \mathsf{id}_Y) \circ d$ *and* $r' = (R' \otimes \mathsf{id}_Y) \circ d$ *for some discrete ground* parameter $d : \epsilon \to \langle m, Y\rangle$. *The* reaction relation $\longrightarrow\!\triangleright_{\mathsf{R}}$ *over ground bigraphs is defined by*

$$g \longrightarrow\!\triangleright_{\mathsf{R}} g' \text{ iff } g = Dr \text{ and } g' = Dr'$$

for some bigraph D *and some ground reaction rule* (r, r') *generated from* R.

Definition 9 (bigraphical reactive system (BRS)). *A* bigraphical reactive system *consists of a pair* $(\mathcal{B}, \mathcal{R})$, *where* $\mathcal{B}$ *is a set of agents and* $\mathcal{R}$ *is a set of reaction rules defined over* $\mathcal{B}$. *It has a reaction relation*

$$\longrightarrow\!\triangleright_{\mathcal{R}} \coloneqq \bigcup_{\mathsf{R} \in \mathcal{R}} \longrightarrow\!\triangleright_{\mathsf{R}},$$

which will be written $\longrightarrow\!\triangleright$ *when* $\mathcal{R}$ *is understood. We write* $(\mathcal{B}(\Sigma), \mathcal{R})$ *when the elements of* $\mathcal{B}$ *are* Σ*-sorted.*

1.1 New stuff

Definition 10 (probabilistic reaction rule). *A* probabilistic reaction rule R *is a triple* (R, R', p), *sometimes written* $R \xrightarrow{p} R'$, *where* (R, R') *is a reaction rule and* $p \in [0, 1]$ *is a probability.*

Define NBRS

Lemma: any MDP can be expressed as an NBRS

Changes from PBRS: each reaction rule annotated with an action name (probabilities normalised over each action separately) and an integer reward/-cost, predicates get a reward/cost, too.

2 Jupyter interface & visualisations

We introduce a convenient graphical user interface for working with bigraphs via Jupyter notebooks.

Example 1. Consider an MDP $(S, \overline{s}, A, P, L)$, where... Look at Figure 1.

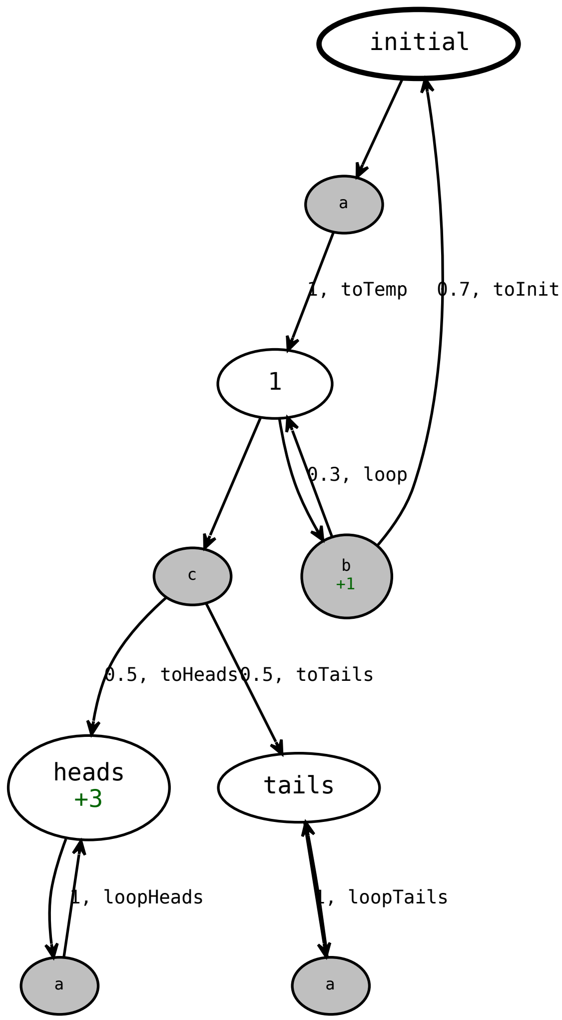

Fig. 1. The full transition system of Example 1.

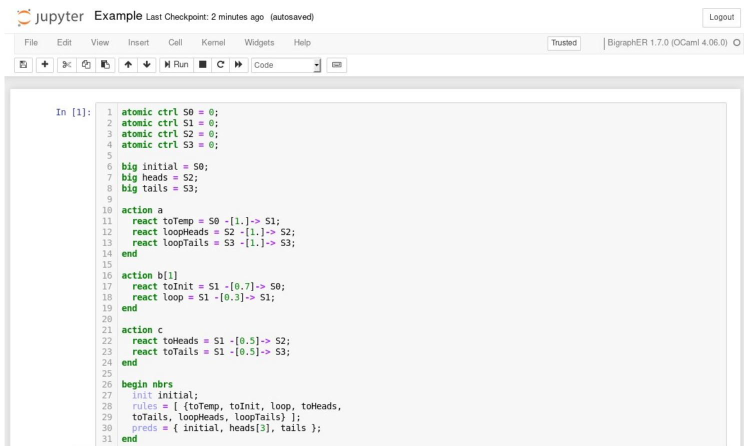

Fig. 2. The BigraphER Jupyter interface with syntax highlighting.

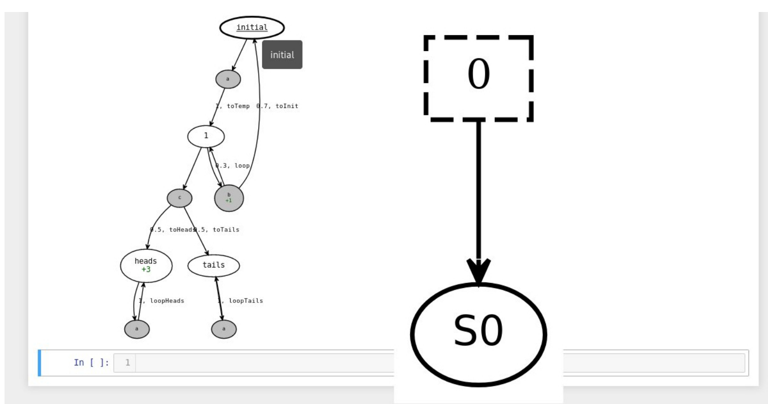

Fig. 3. ...

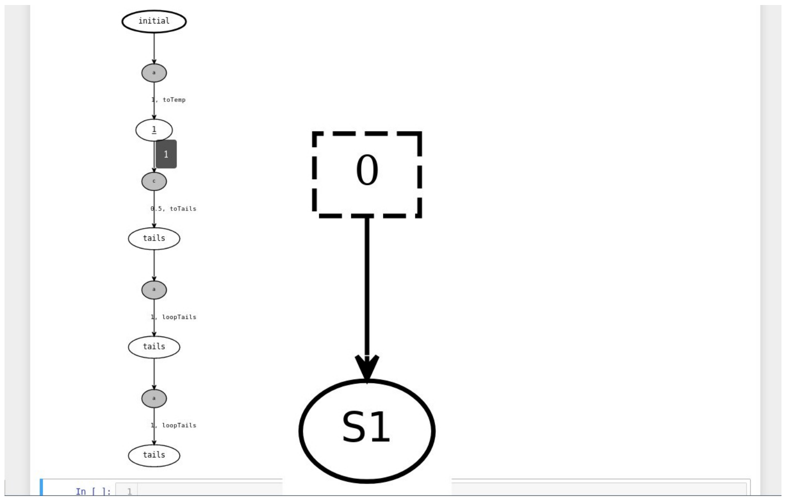

Fig. 4. ...

```
BigraphER: Bigraph Evaluator & Rewriting
========================================
Version:        1.7.0
Date:           Thu Aug 02 11:28:14 2018
Hostname:       tux
OS type:        Unix
Command line:   bigrapher validate -d jupyter-images/4 -f svg [4].big
Parsing model file [4].big ...
Type:           Nondeterministic BRS
Bindings:       14
# of rules:     7
Exporting declarations to jupyter-images/4 ...
Model file parsed correctly
```

Fig. 5. ...

Listing 1.1. ...

```
atomic ctrl S0 = 0;
atomic ctrl S1 = 0;
atomic ctrl S2 = 0;
atomic ctrl S3 = 0;

big initial = S0;
big heads = S2;
big tails = S3;

action a
  react toTemp = S0 -[1.]-> S1;
  react loopHeads = S2 -[1.]-> S2;
  react loopTails = S3 -[1.]-> S3;
end

action b[1]
  react toInit = S1 -[0.7]-> S0;
  react loop = S1 -[0.3]-> S1;
end

action c
  react toHeads = S1 -[0.5]-> S2;
  react toTails = S1 -[0.5]-> S3;
end

begin nbrs
  init initial;
  rules = [ {toTemp, toInit, loop, toHeads,
  toTails, loopHeads, loopTails} ];
  preds = { initial, heads[3], tails };
end
```

3 Exporting to PRISM

Transitions, state rewards, transition rewards

4 Case study in autonomous agents

References

1. Giaquinta, R., Hoffmann, R., Ireland, M., Miller, A., Norman, G.: Strategy synthesis for autonomous agents using PRISM. In: Dutle, A., Muñoz, C.A., Narkawicz, A. (eds.) NASA Formal Methods - 10th International Symposium, NFM 2018, Newport

News, VA, USA, April 17-19, 2018, Proceedings. Lecture Notes in Computer Science, vol. 10811, pp. 220–236. Springer (2018). https://doi.org/10.1007/978-3-319-77935-5_16, https://doi.org/10.1007/978-3-319-77935-5_16
2. Krivine, J., Milner, R., Troina, A.: Stochastic bigraphs. Electr. Notes Theor. Comput. Sci. **218**, 73–96 (2008). https://doi.org/10.1016/j.entcs.2008.10.006, https://doi.org/10.1016/j.entcs.2008.10.006
3. Sevegnani, M.: Bigraphs with sharing and applications in wireless networks. Ph.D. thesis, University of Glasgow, UK (2012), http://theses.gla.ac.uk/3742/
4. Sevegnani, M., Calder, M.: Bigraphs with sharing. Theor. Comput. Sci. **577**, 43–73 (2015). https://doi.org/10.1016/j.tcs.2015.02.011, https://doi.org/10.1016/j.tcs.2015.02.011