Nondeterministic Bigraphs and Their Use in Modelling Movement

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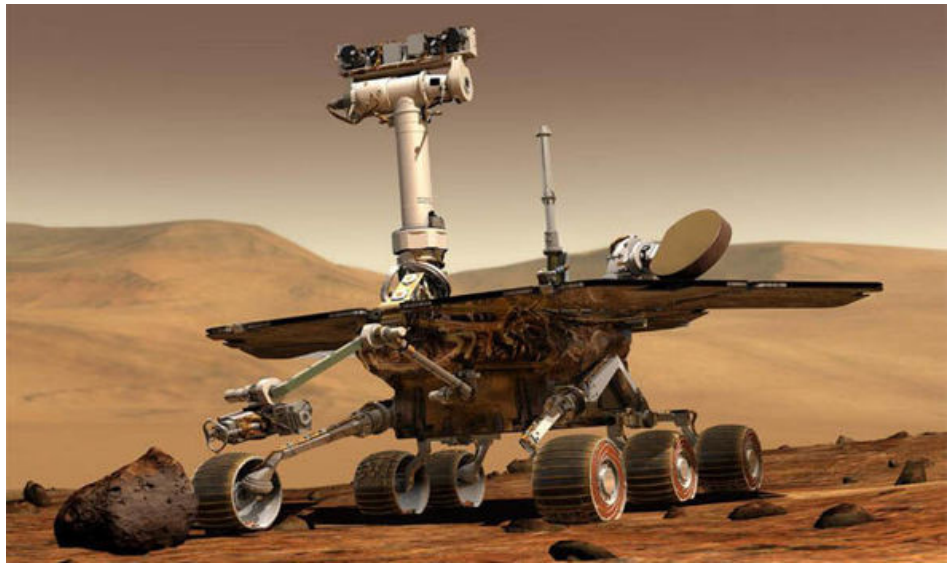
Motivation



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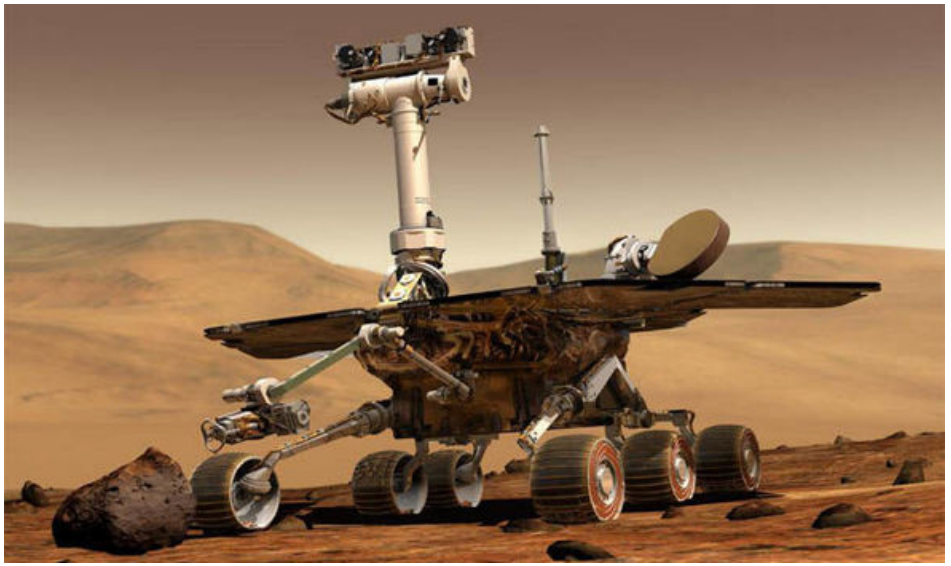

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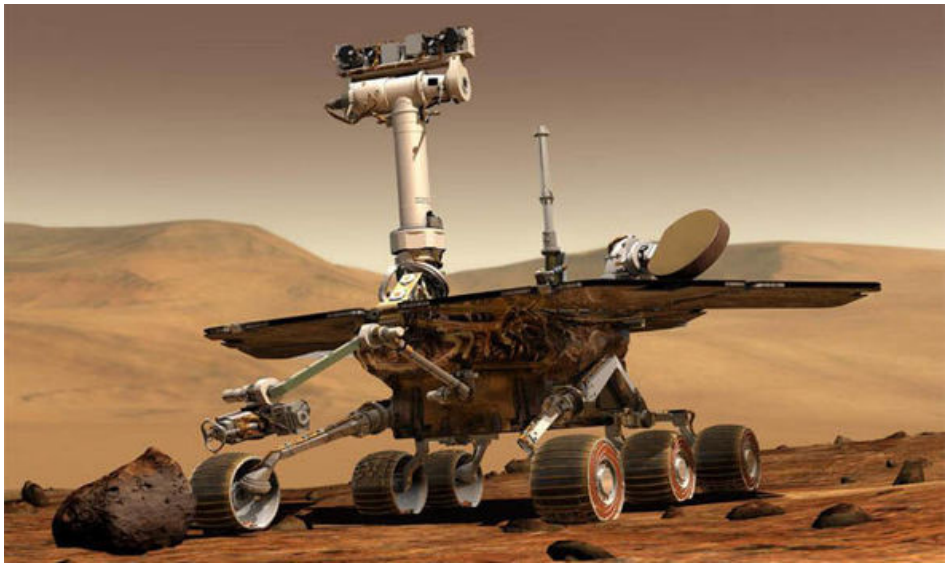

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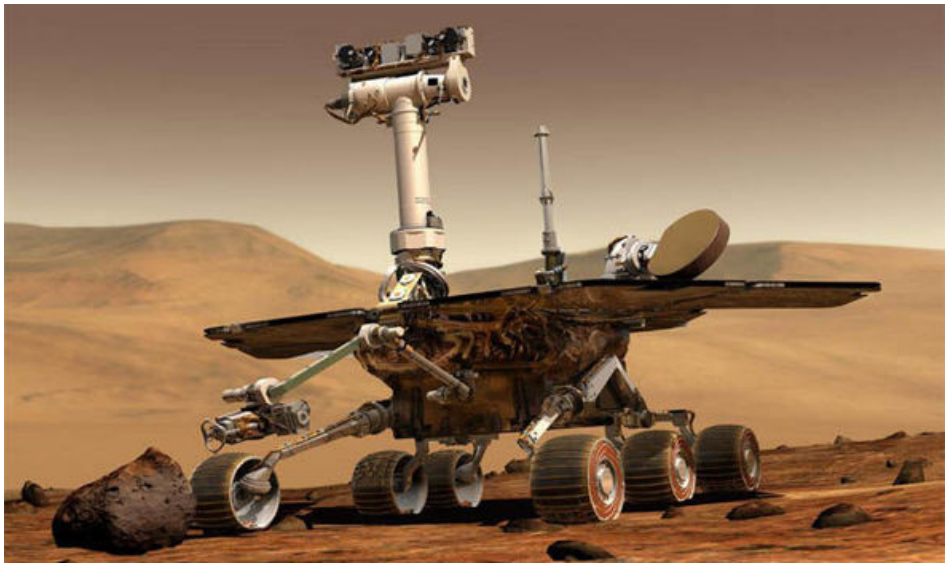
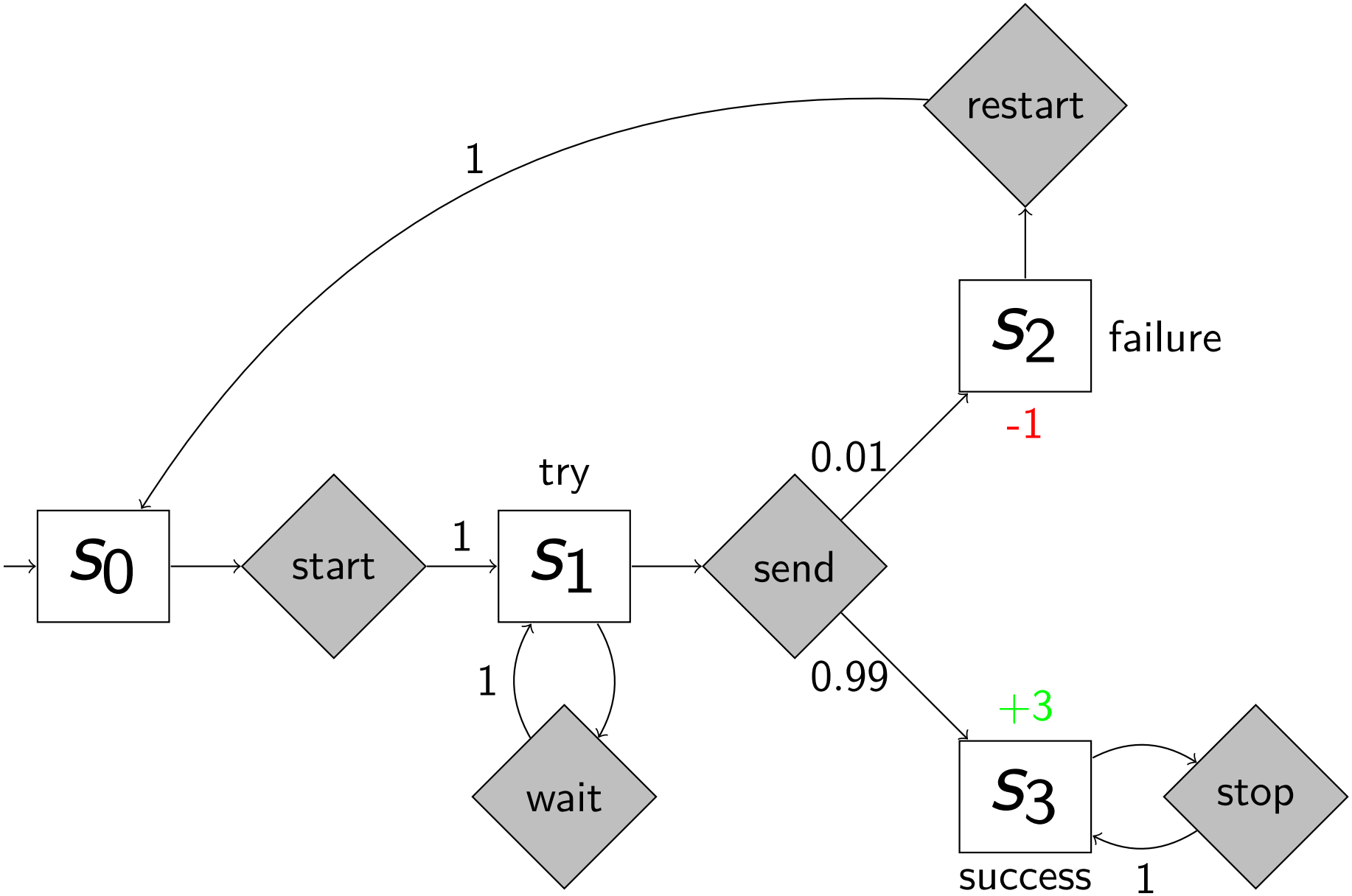


Markov Decision Process



Collecting Objects in a Grid

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- Once a set number of objects is collected, the agent heads home.

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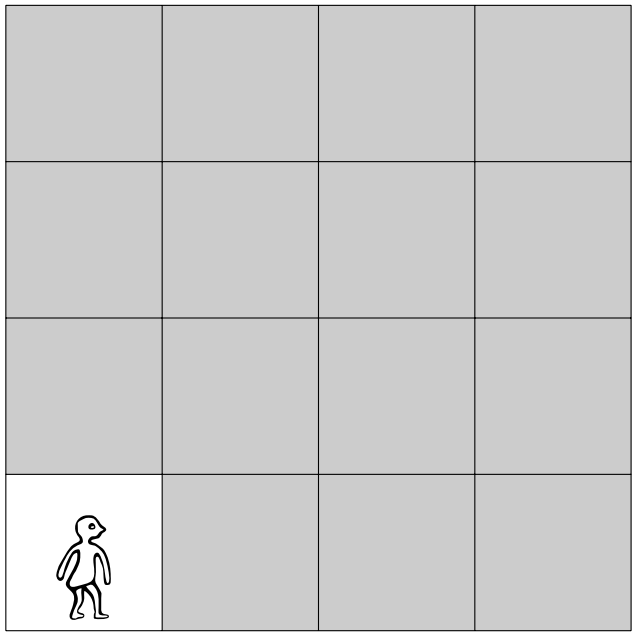

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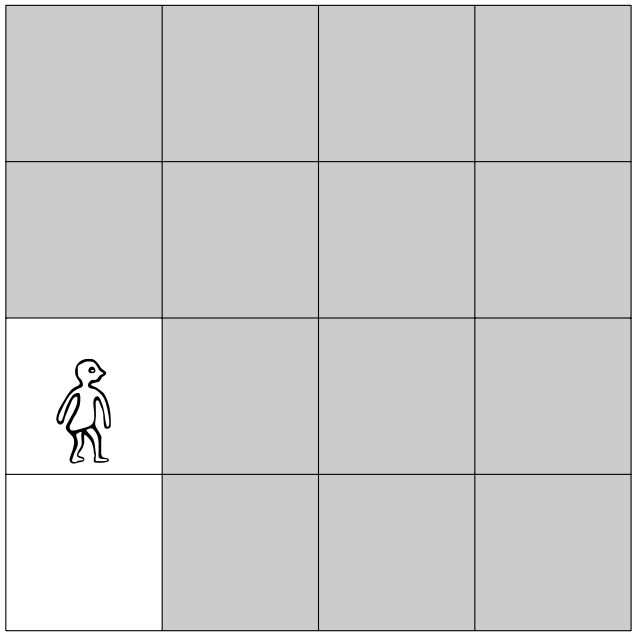

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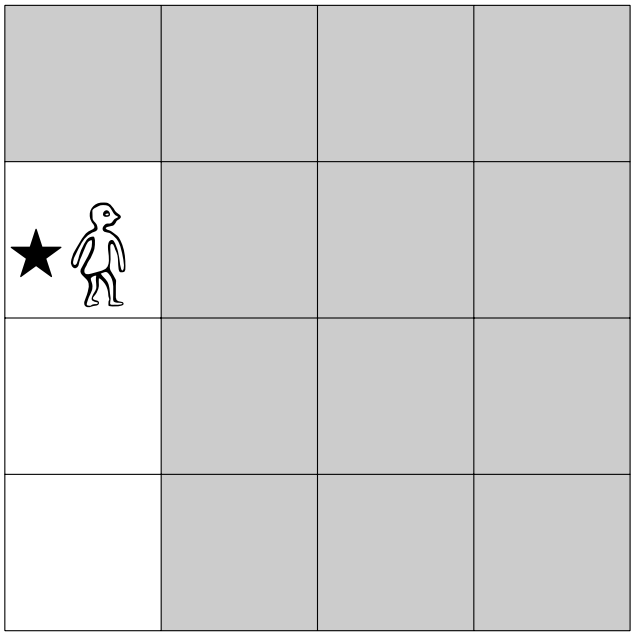

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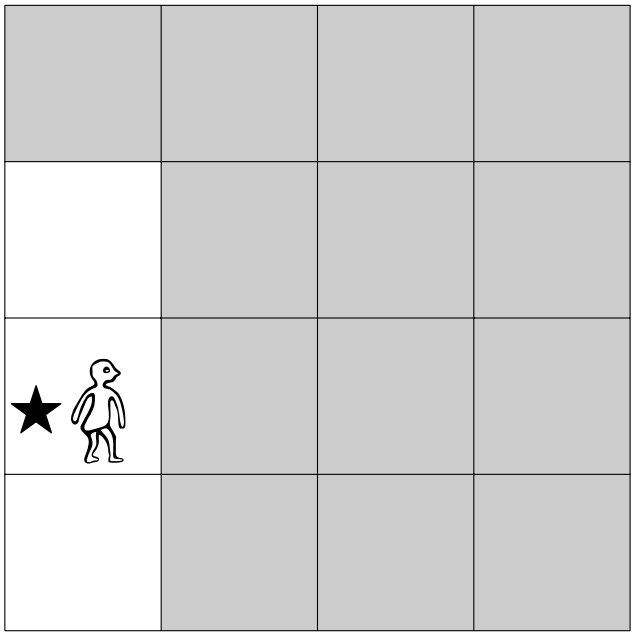

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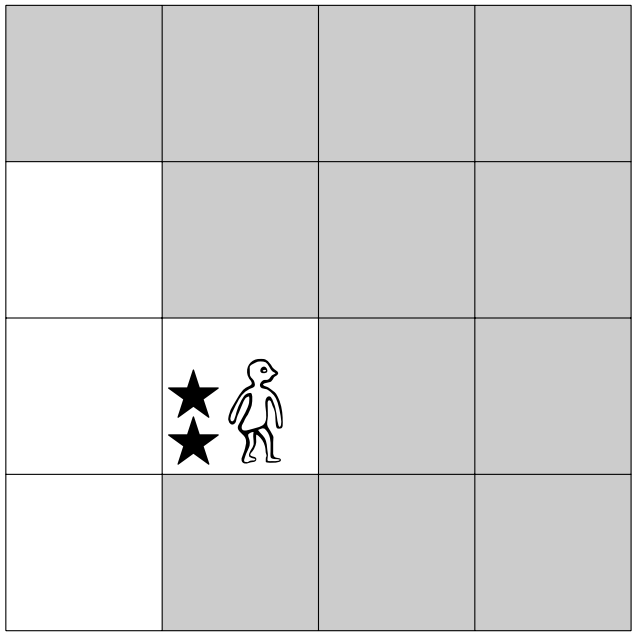

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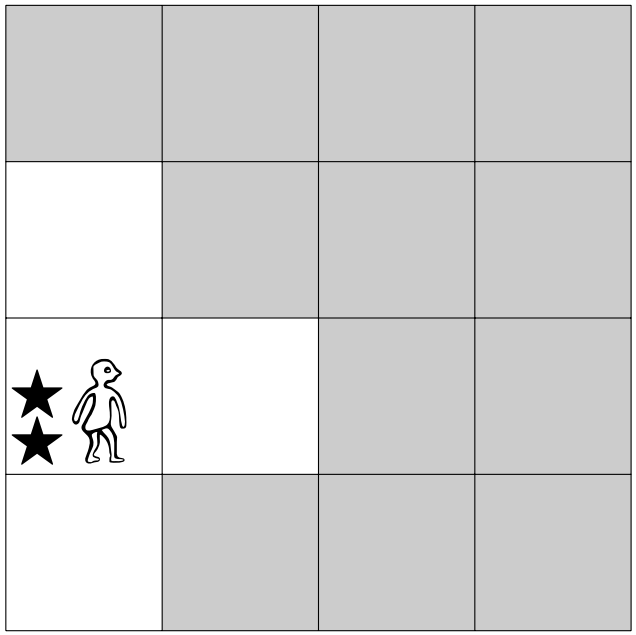

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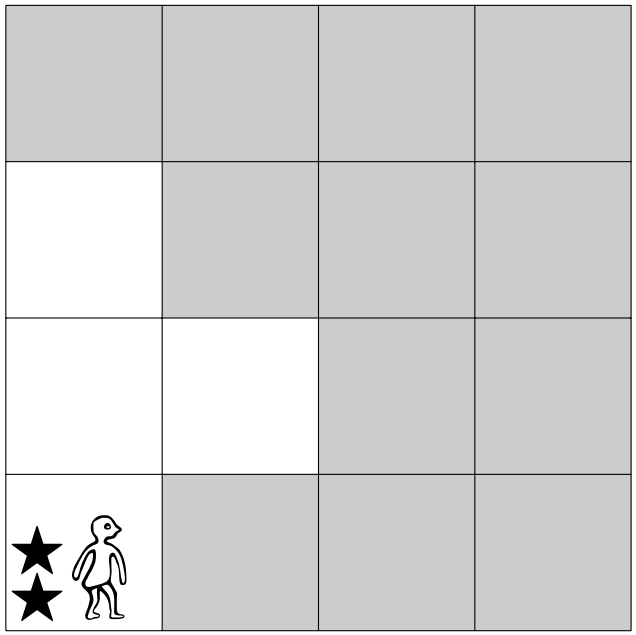

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  - `home`: is in the southwest corner of the grid

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  - Different rules for going to visited and unvisited cells
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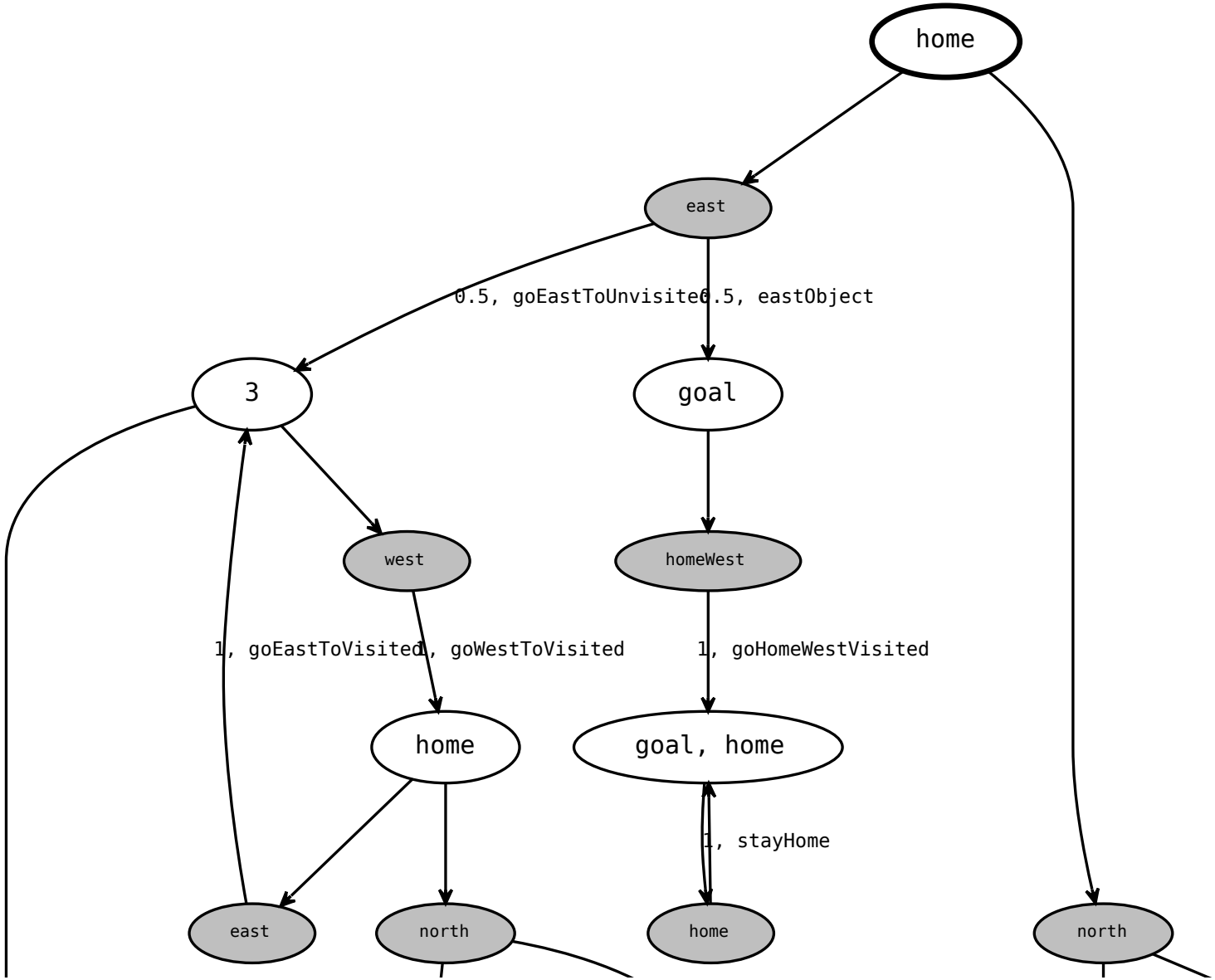
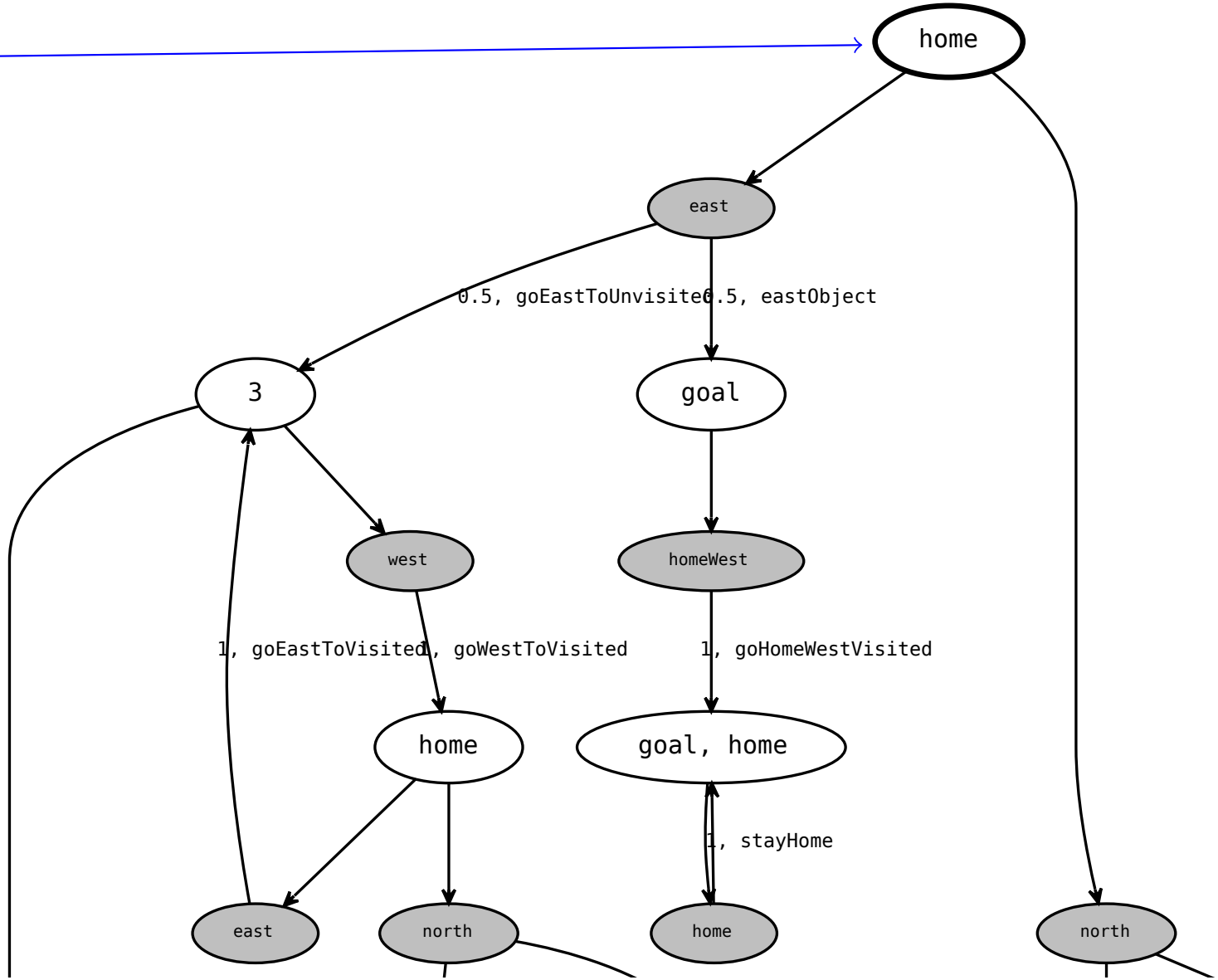
  - Priority 2: 3 rules for each direction
    - ★ visited
    - ★ unvisited
    - ★ unvisited + object

Transition System



Transition System

- States



Transition System

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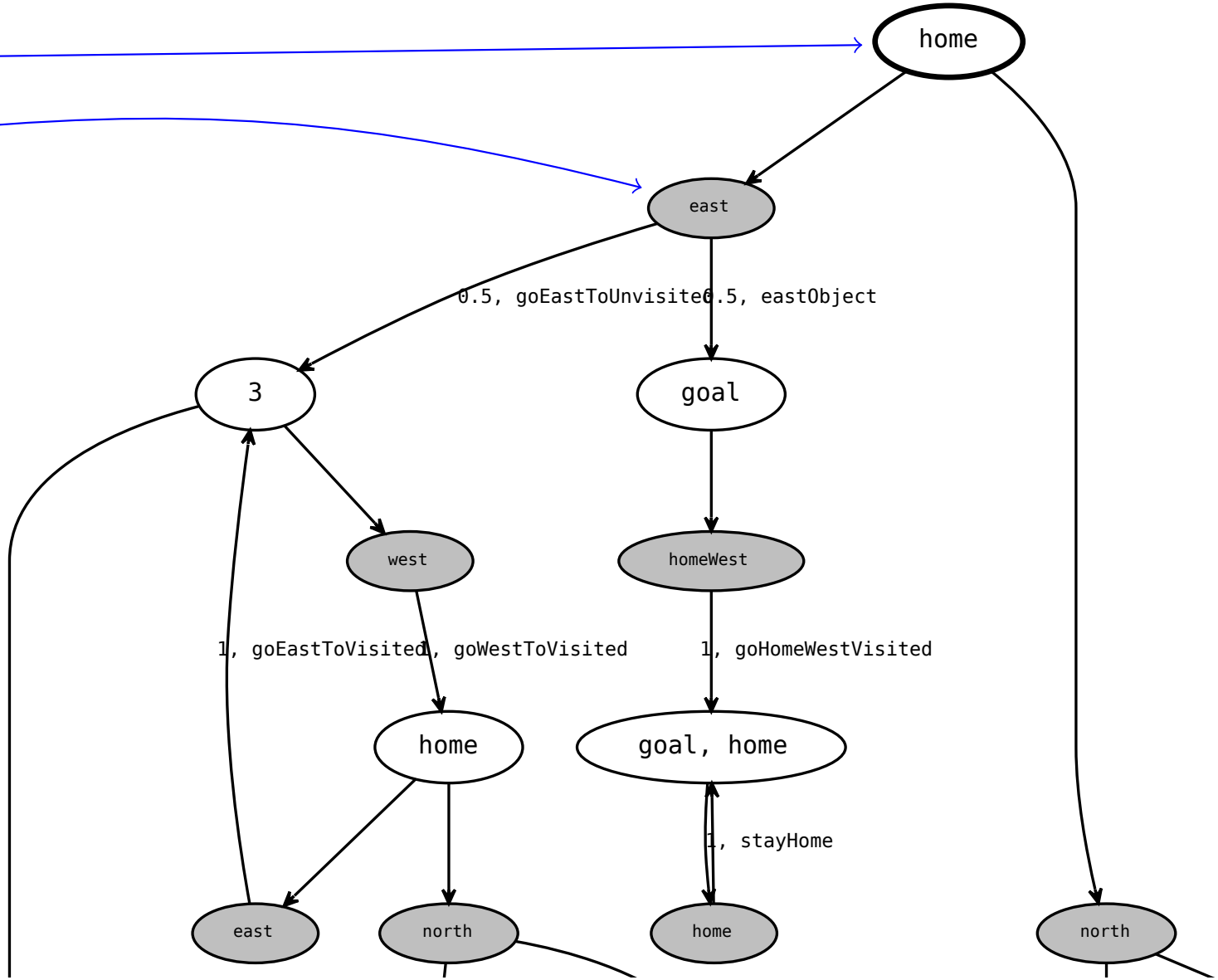
- Actions



Transition System

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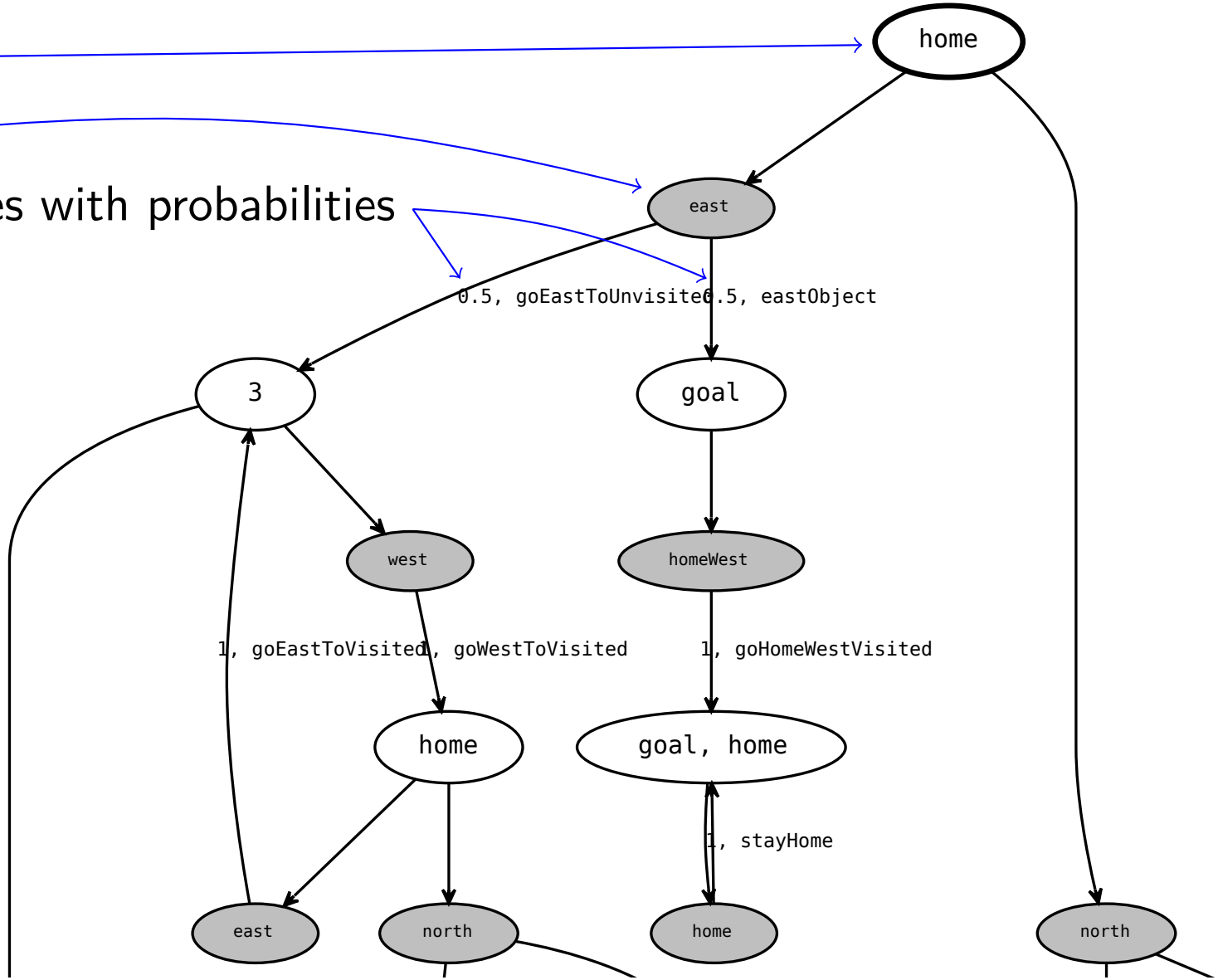
- Reaction rules with probabilities



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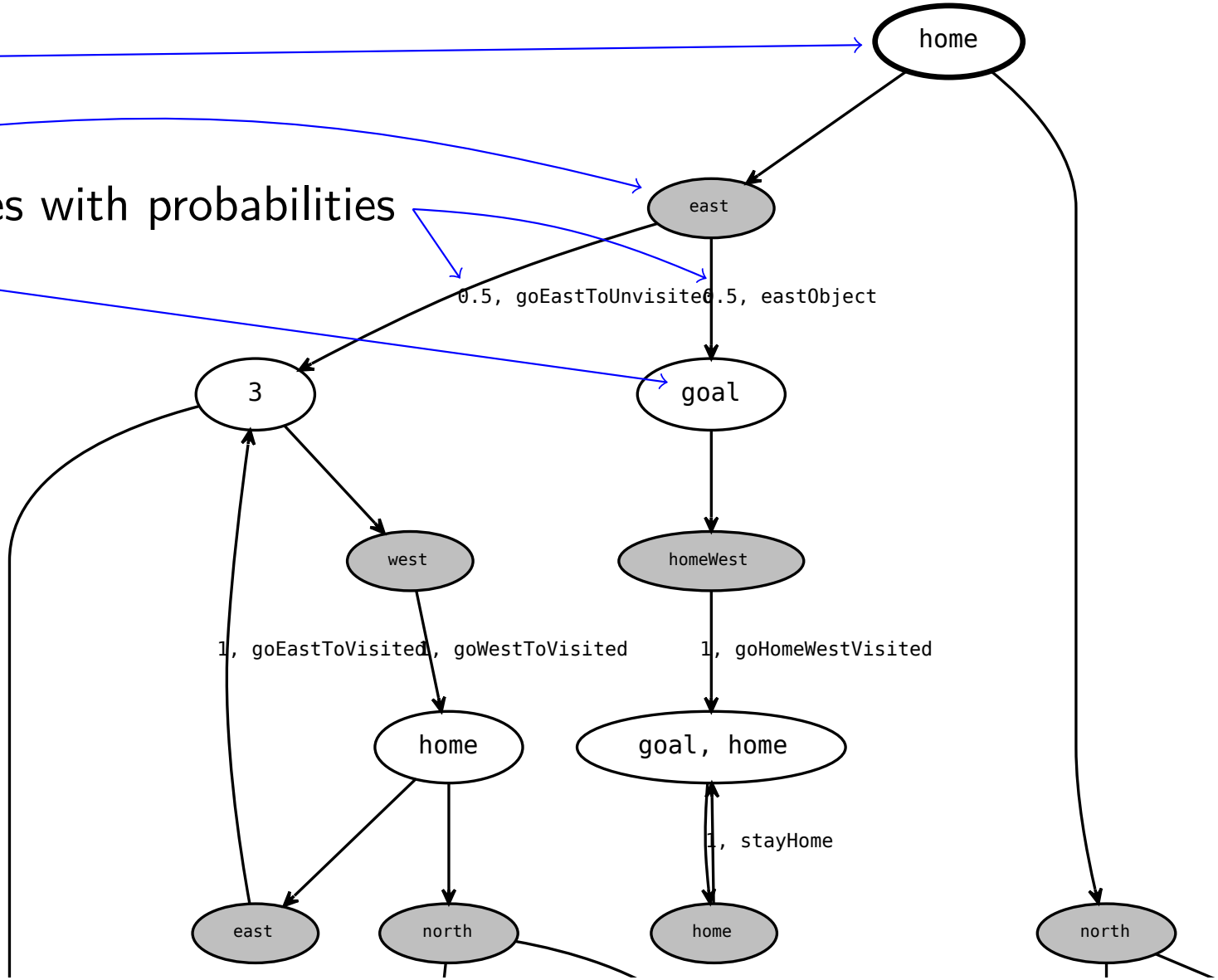

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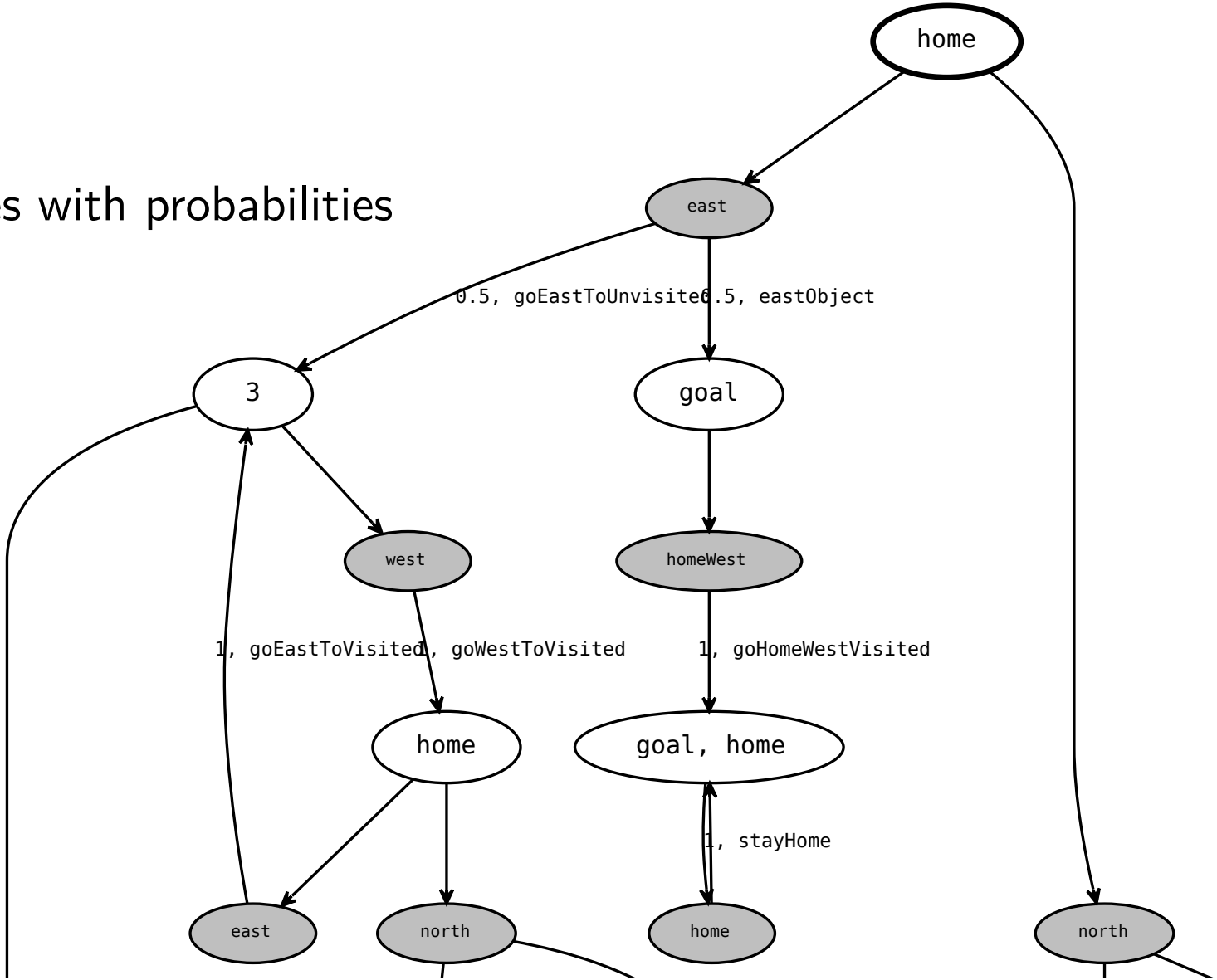

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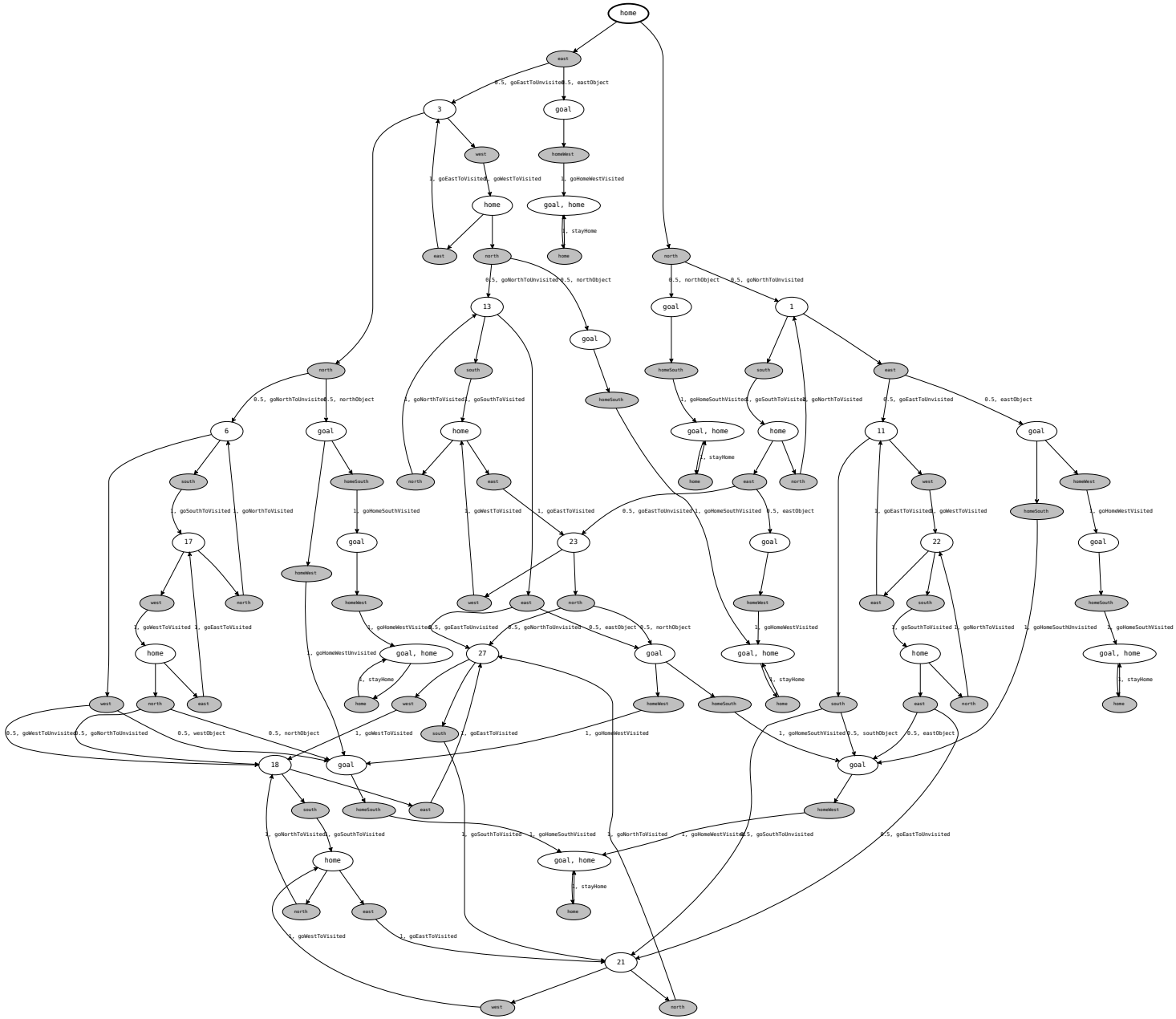

The Workflow

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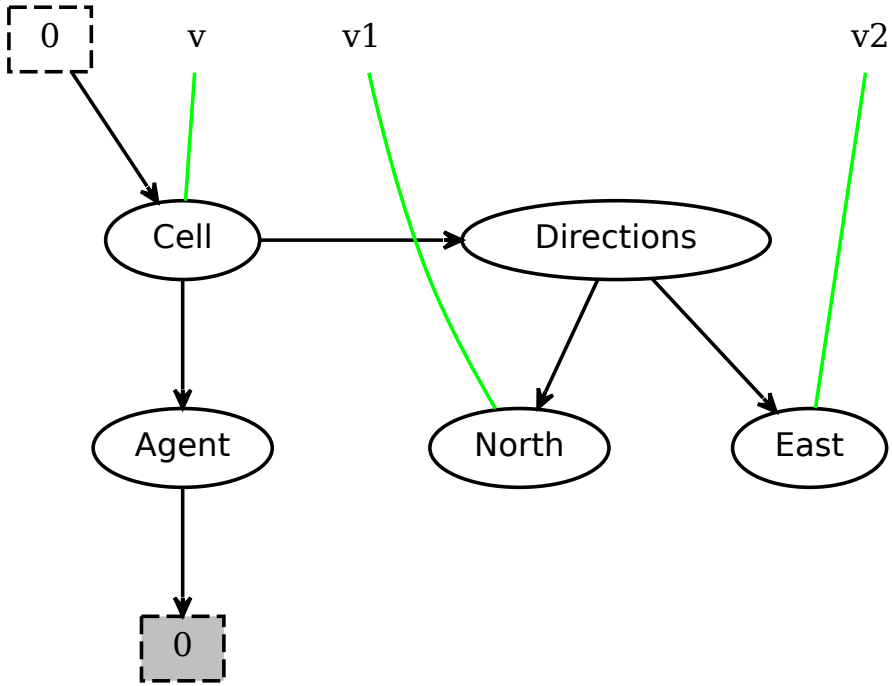
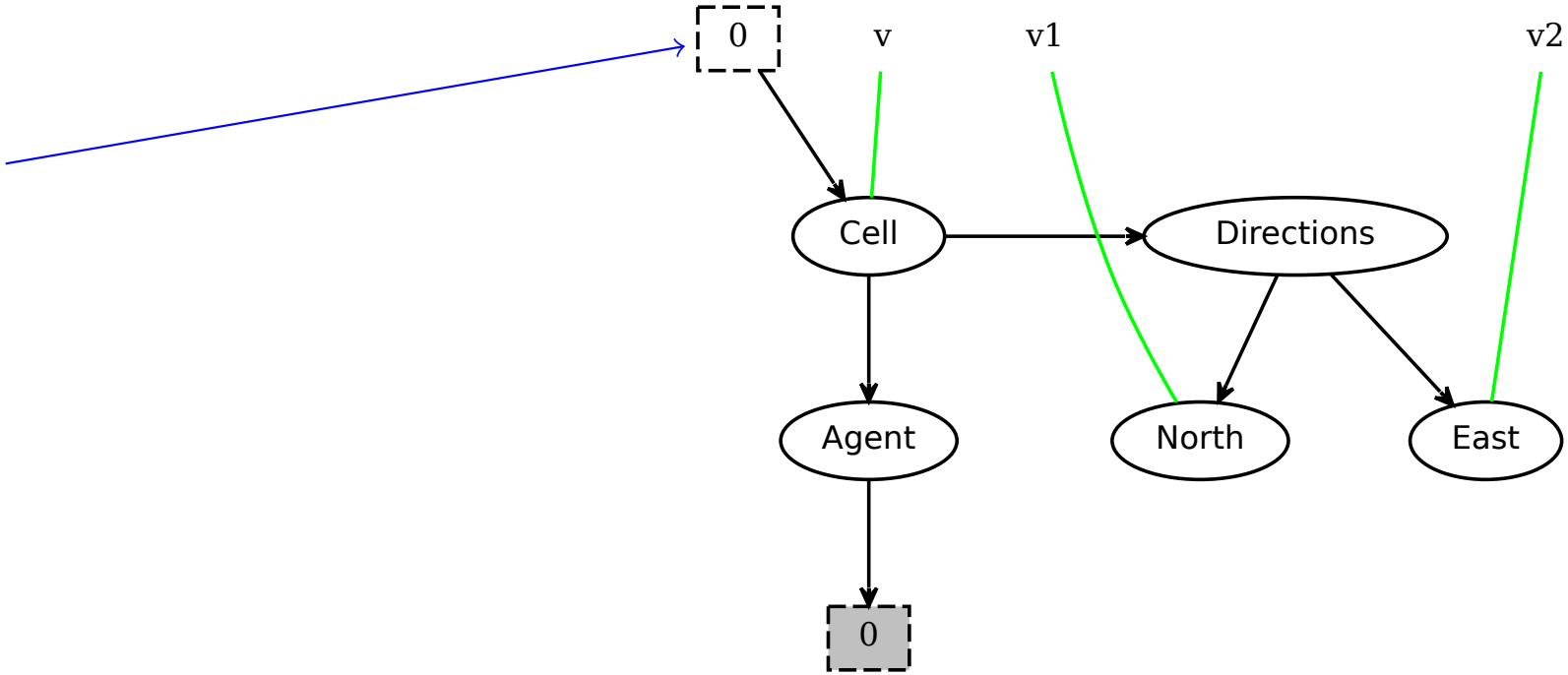
- Either:
  - Breadth first search to generate the full transition system
  - Or select the next state randomly for a simulation

Bigraphs



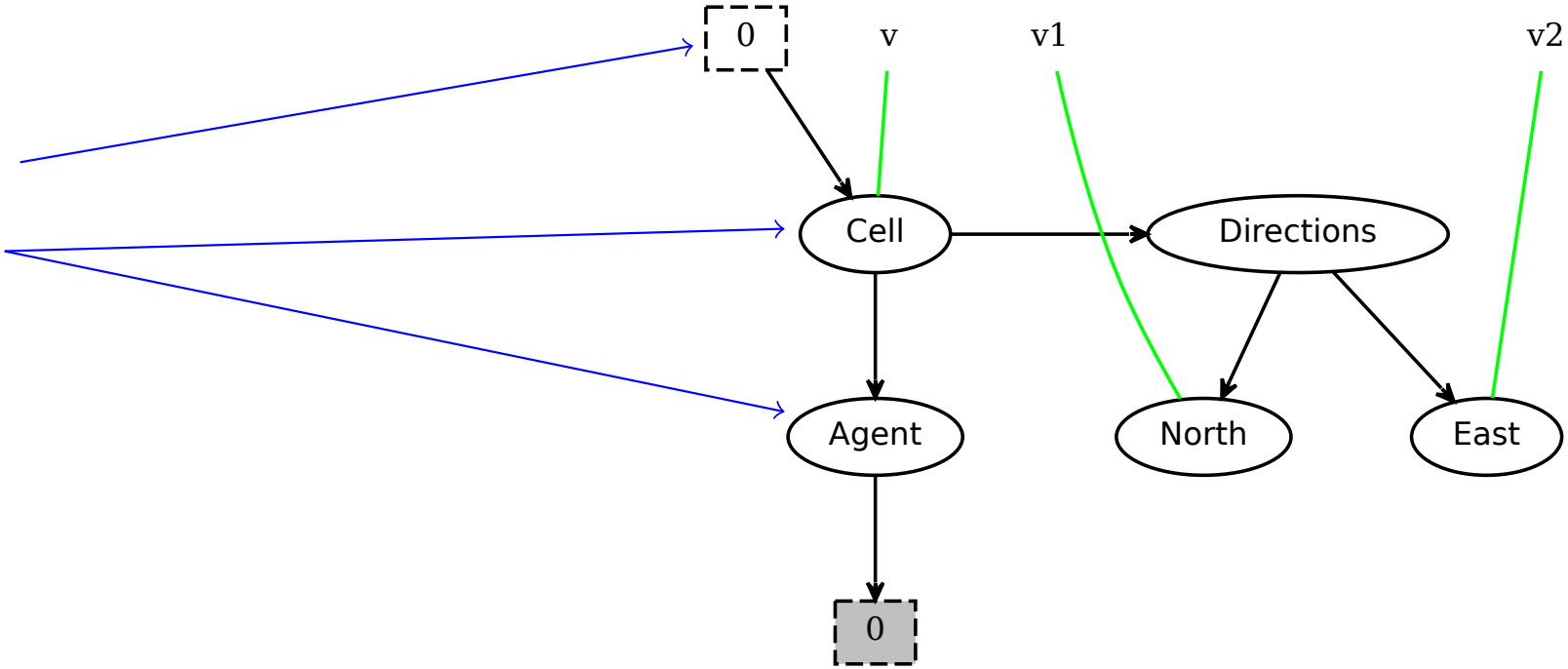
Bigraphs

- Region



Bigraphs

- Region
- Nodes



Bigraphs

- Region
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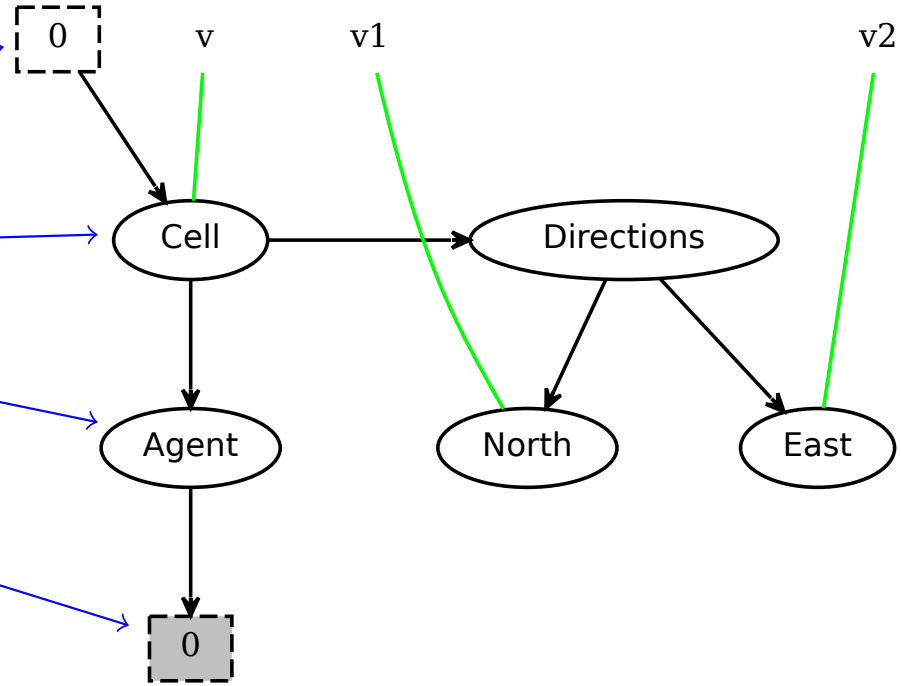
- Site



Bigraphs

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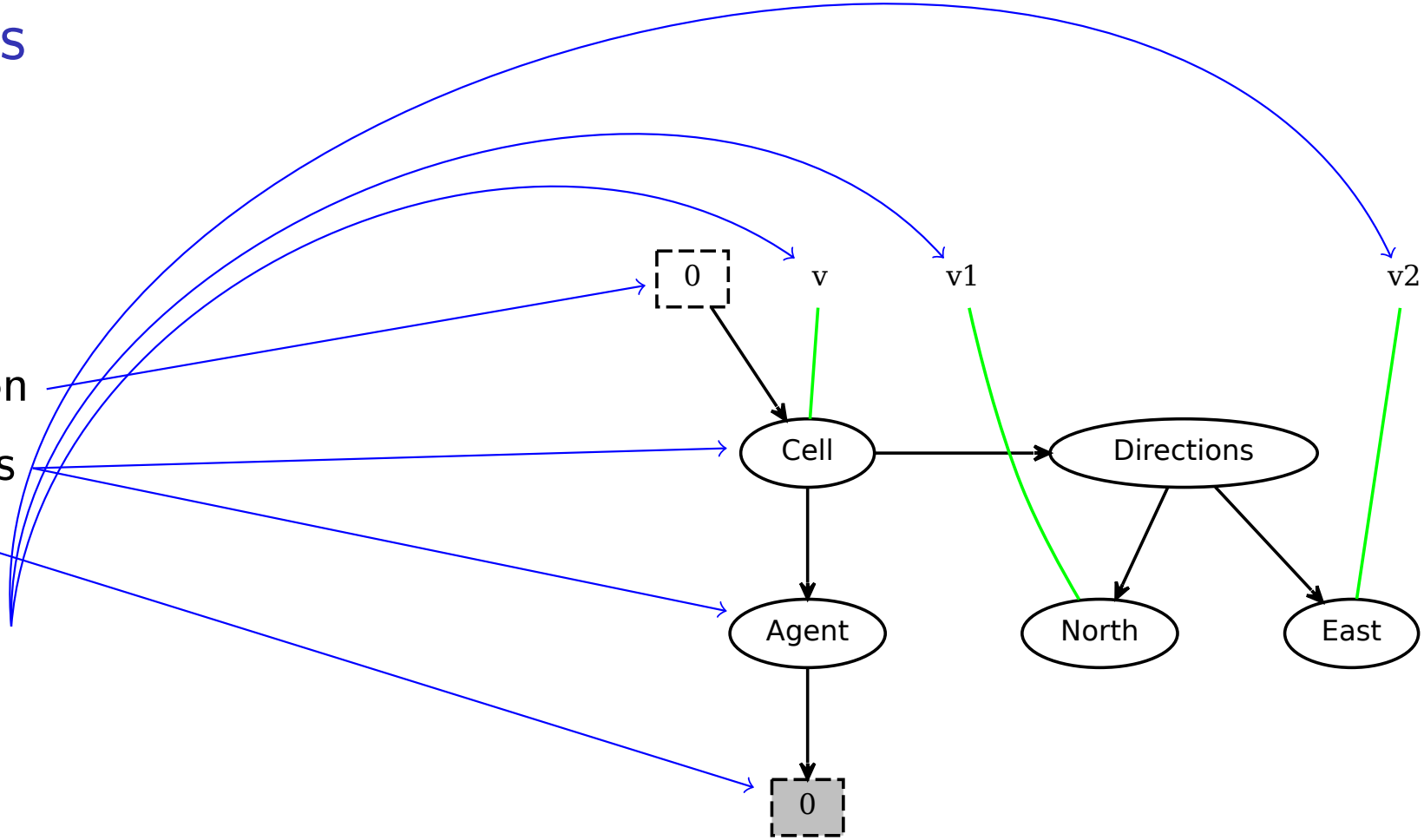

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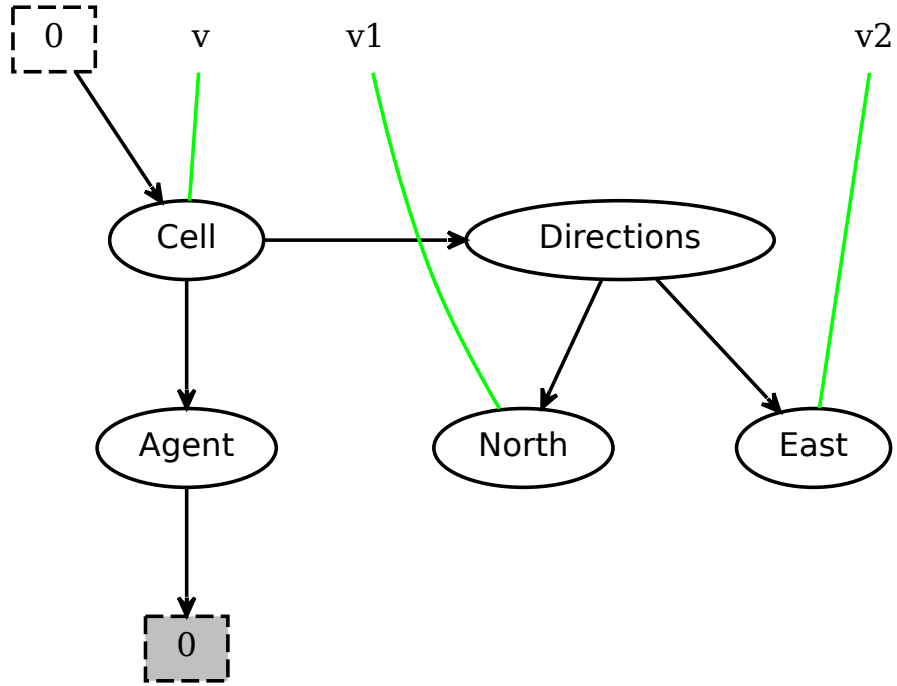
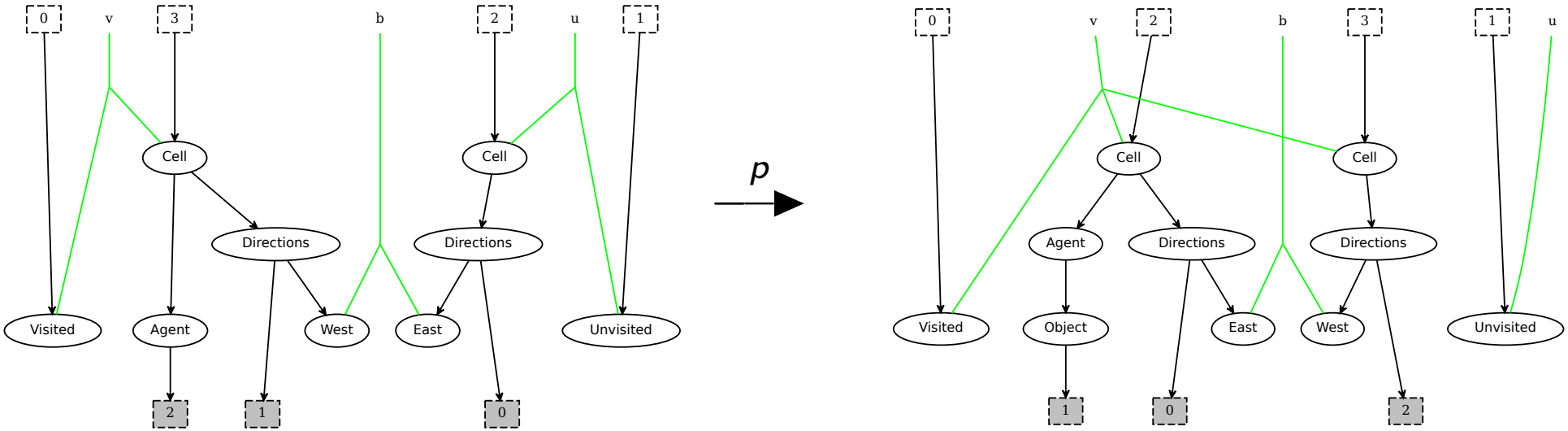



```
big home = Cell{v}.(Directions.(North{v1}
                              | East{v2})
                 | Agent);
```

Initial State

```
big initial = Visited{v}
            || Unvisited{u}
            # bottom left
            || Cell{v}.(Directions.(North{a}
                                  | East{b})
                       | Agent.1)
            # top left
            || Cell{u}.Directions.(East{c}
                                 | South{a})
            # bottom right
            || Cell{u}.Directions.(North{d}
                                 | West{b})
            # top right
            || Cell{u}.Directions.(West{c}
                                 | South{d});
```

Reaction Rule: Go West and Collect an Object



A Tale of Schrödinger's Wall...

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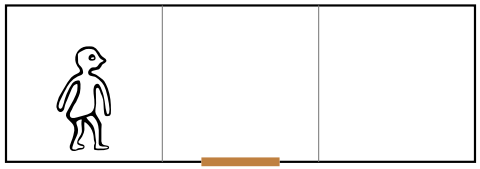

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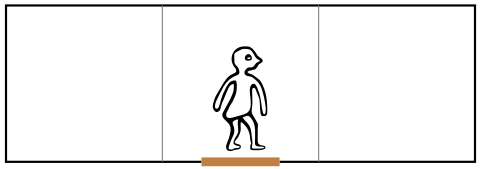

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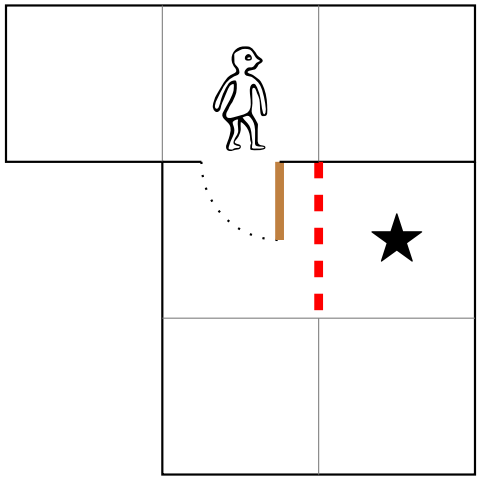

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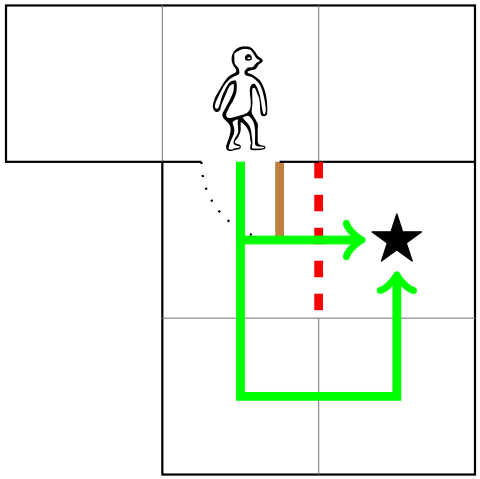

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- Predicate
  - is `Agent` and `Goal` in the same cell?

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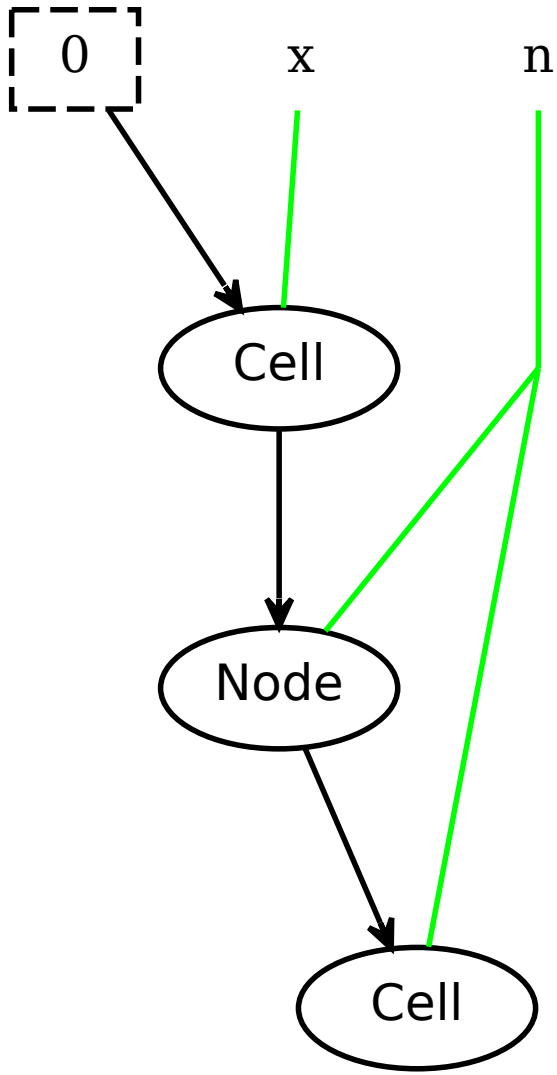

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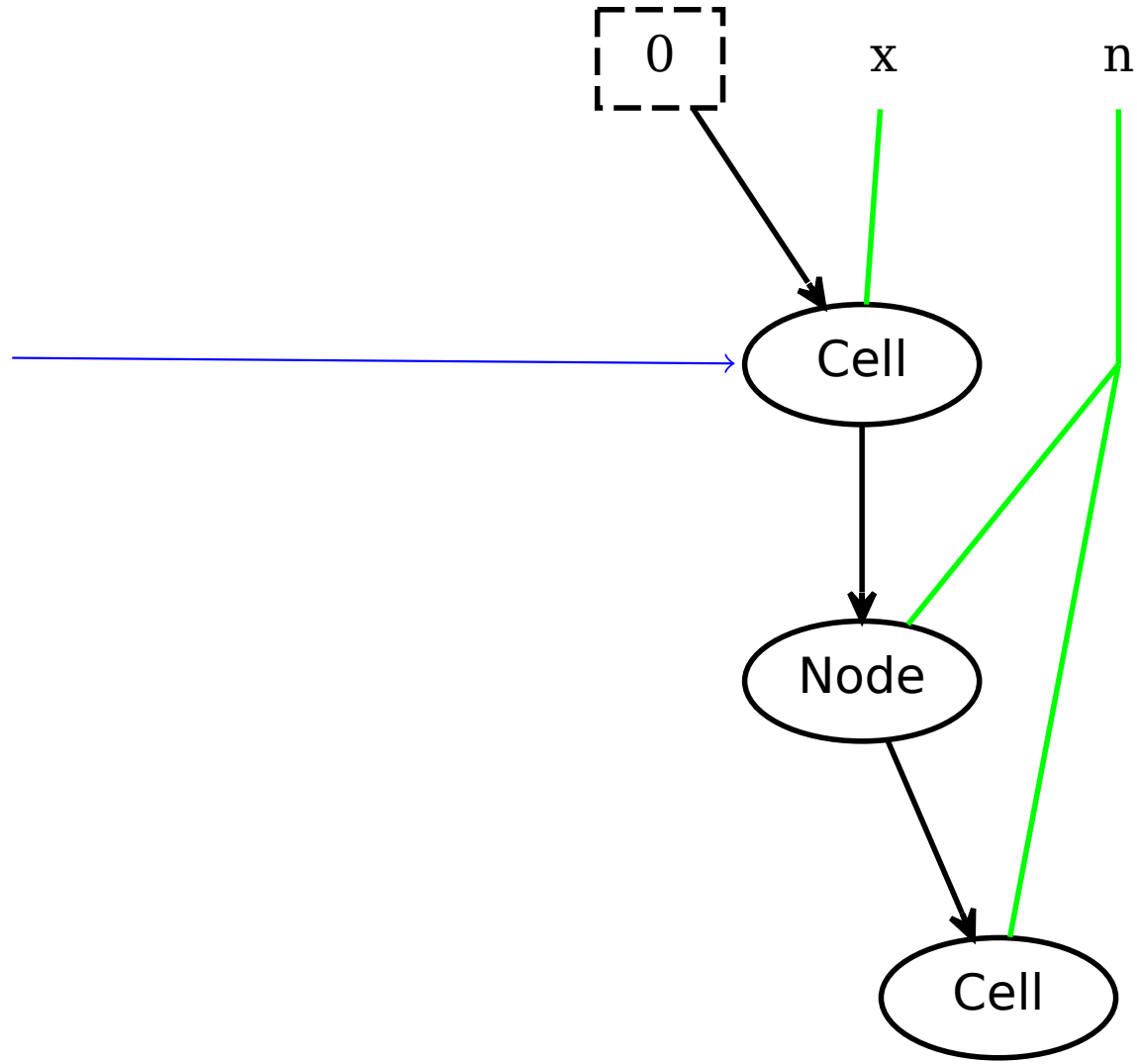

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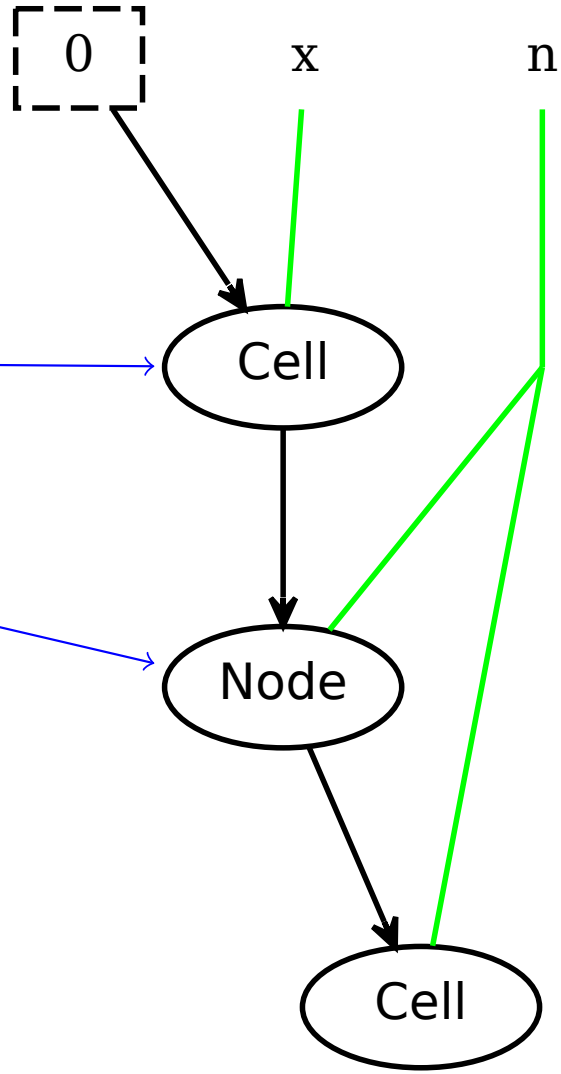

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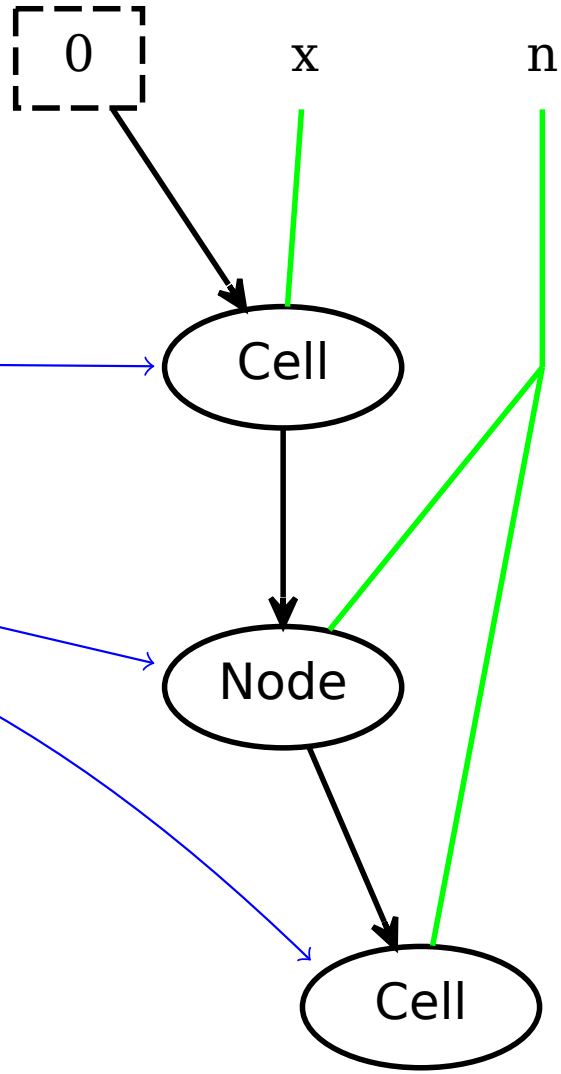
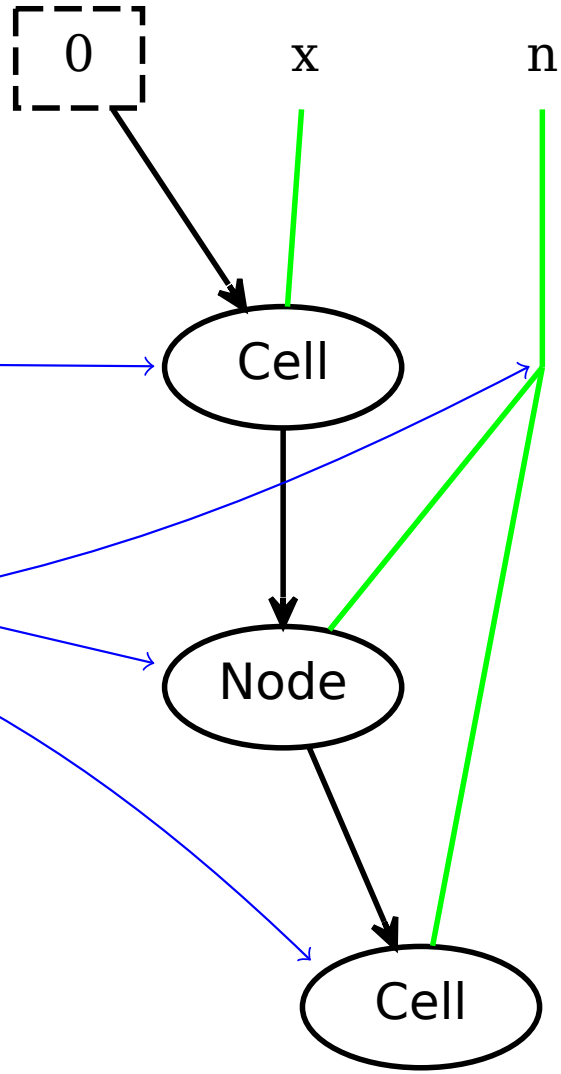
- Inside the door

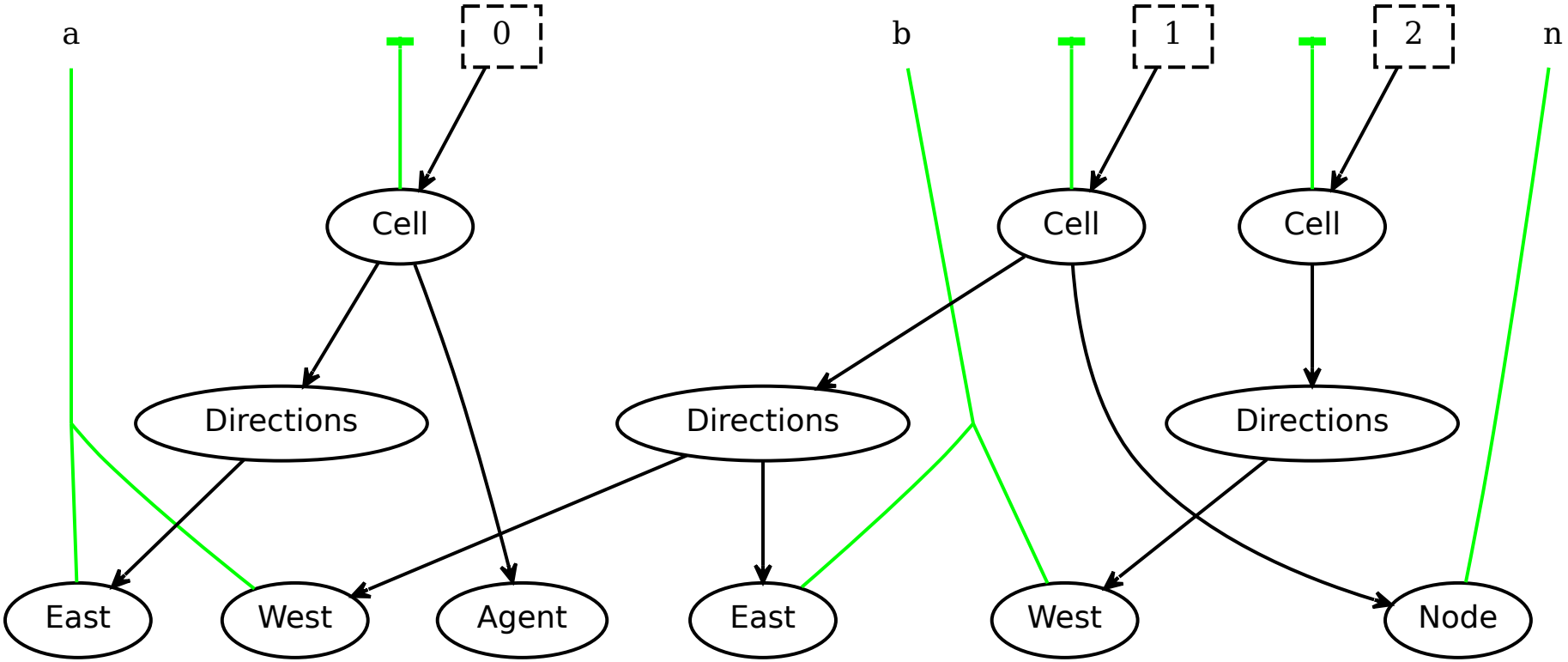


The Main Idea

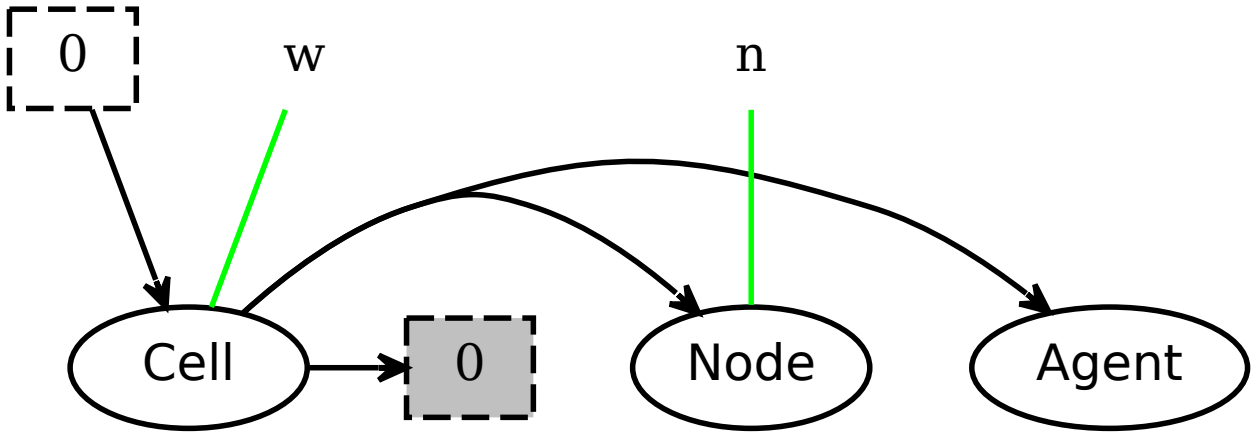
- Outside the door
- The room
- Inside the door
- Which cell is closest to the door?



Initial State



Opening the Door



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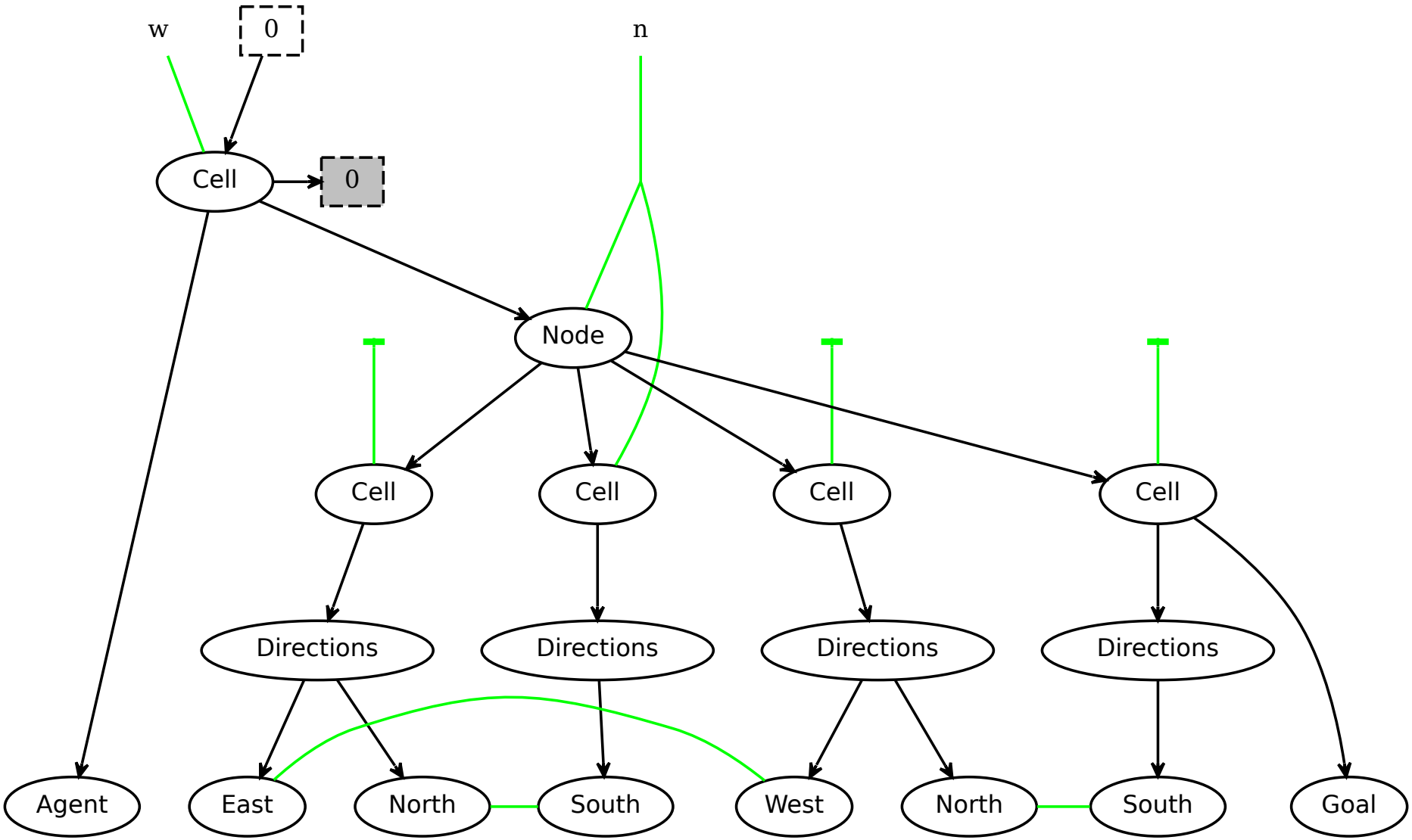
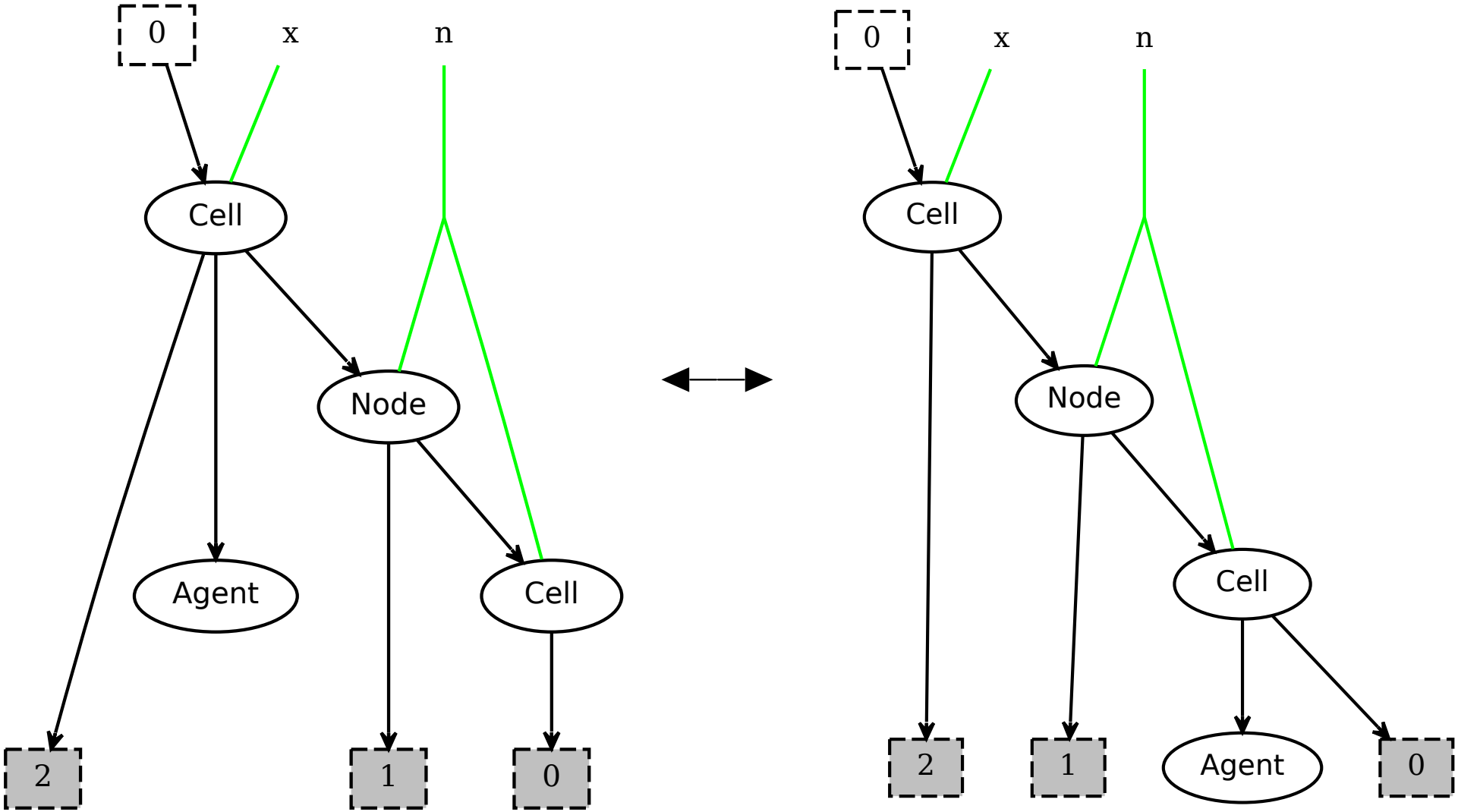


Entering/Leaving a Room

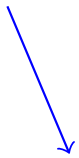


Entering/Leaving a Room

```
action goIn
  react goIn = Cell{x}.(Agent | Node{n}.(Cell{n}
                                         | id)
                              | id)
               -[1.0]->
               Cell{x}.(Node{n}.(Cell{n}.(Agent
                                          | id)
                                 | id)
                              | id);
end
```

Entering/Leaving a Room

Action rewards



```
action goIn[1]
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                                         | id)
                              | id)
               -[1.0]->
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```

Tracking Time with State Rewards

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big agent = Agent;

begin nbrs
  init initialState;
  rules = [ {...}, {...} ];
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```

Tracking Time with State Rewards

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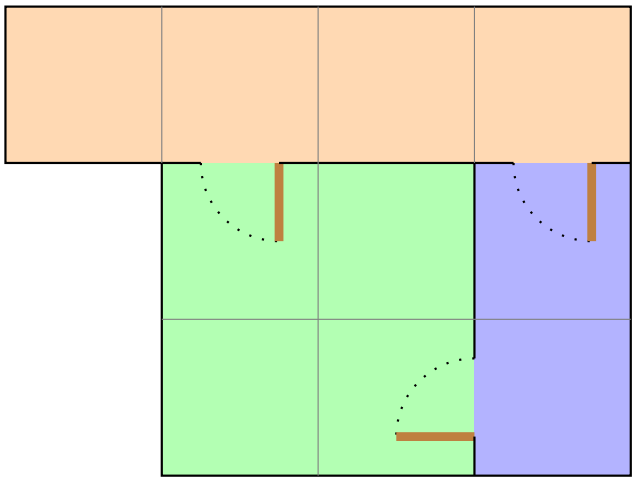

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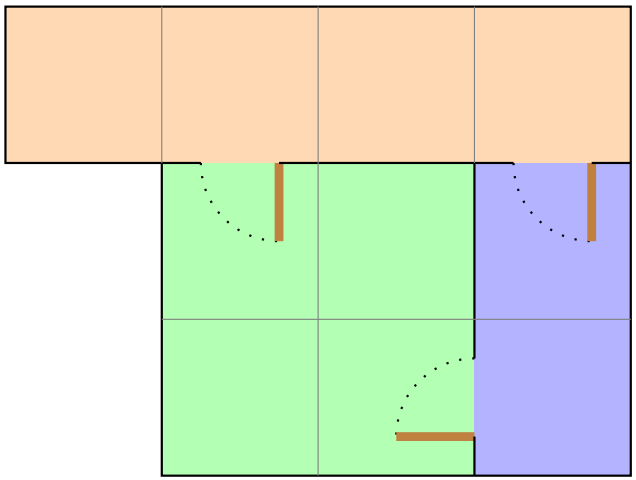

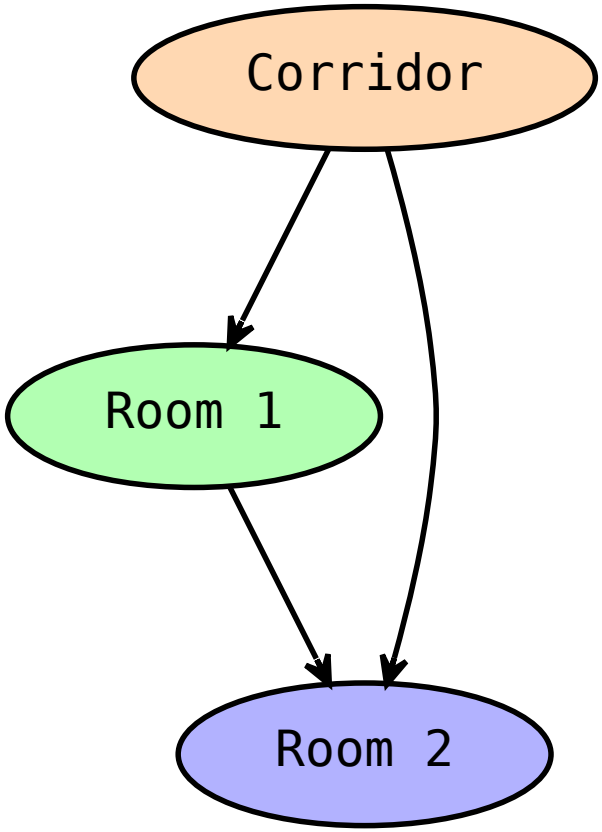

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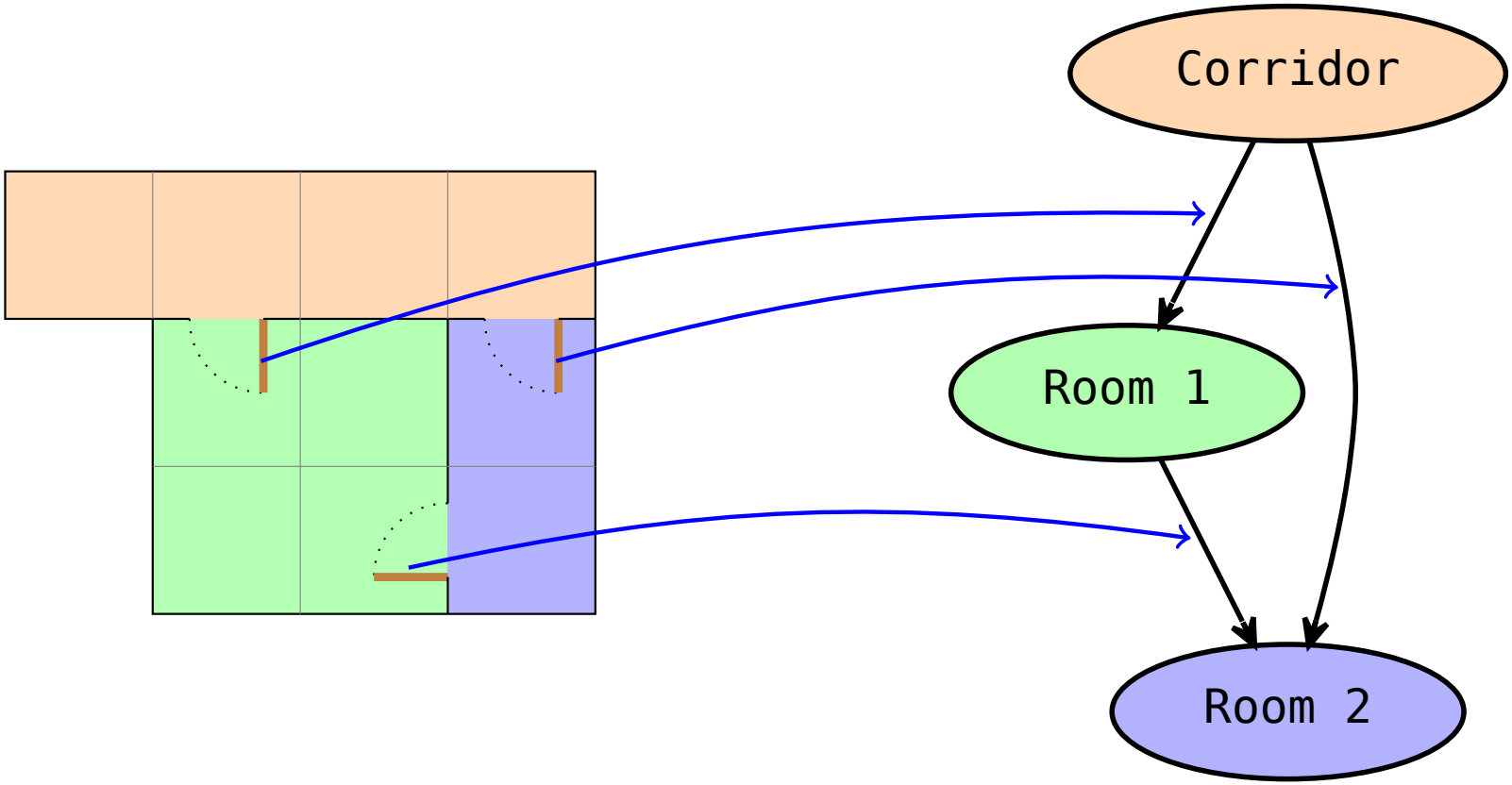

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- Two ideas in one: discovering space & entering an inner space

A New Interface

jupyter Example Last Checkpoint: a minute ago (autosaved) Logout

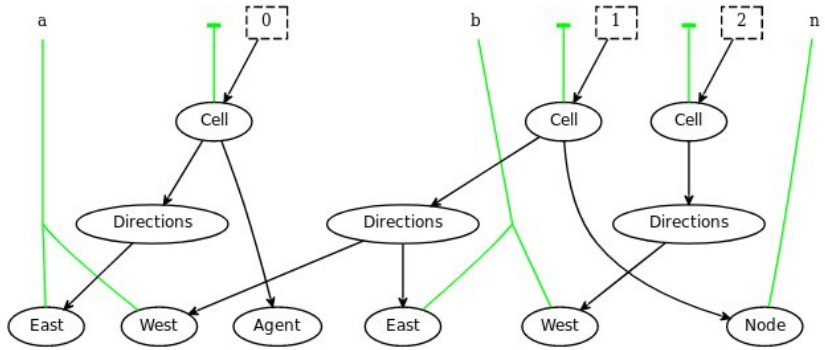
File Edit View Insert Cell Kernel Widgets Help Trusted BigraphER 1.7.0 (OCaml 4.06.0)

Run Code

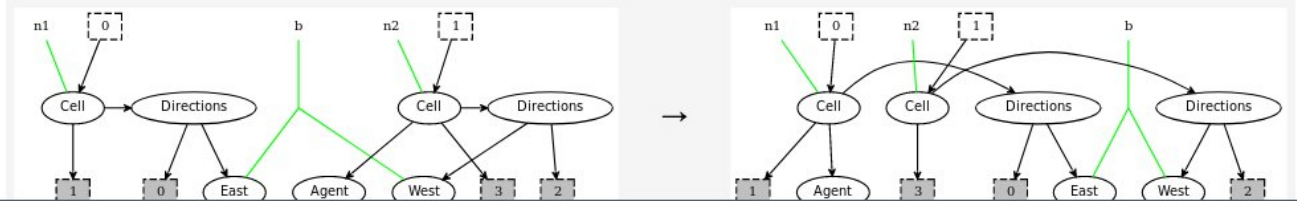
In [1]:

```
1  ctrl Cell = 1;
2  ctrl Directions = 0;
3  ctrl Node = 1;
4  atomic ctrl Agent = 0;
5  atomic ctrl East = 1;
6  atomic ctrl West = 1;
7  big initial = /x /y /z (
8                Cell{x}.(Directions.East{a} | Agent)
9             || Cell{y}.(Directions.(East{b} | West{a})
10                       | Node{n}.1)
11            || Cell{z}.Directions.West{b});
12 react goWest = Cell{n1}.(Directions.(East{b} | id) | id)
13             || Cell{n2}.(Directions.(West{b} | id) | Agent | id)
14             -[1.0]->
15                Cell{n1}.(Directions.(East{b} | id) | Agent | id)
16             || Cell{n2}.(Directions.(West{b} | id) | id);
```

Out[1]: initial



Out[1]: goWest



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- Similar workflow to other Jupyter notebooks

Available at

`https://github.com/dilkas/bigrapher-jupyter`

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- Backwards compatible to run OCaml code

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Thank You!