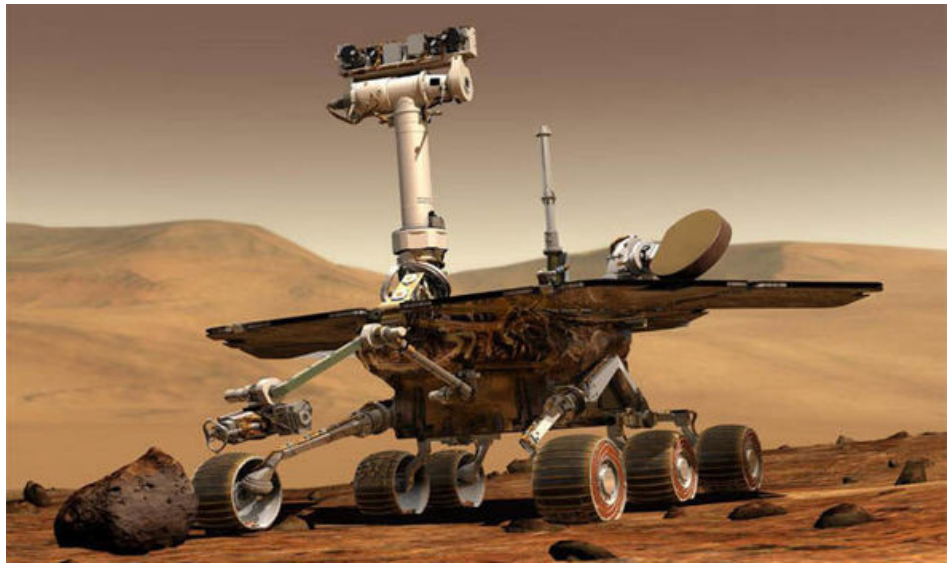
# Nondeterministic Bigraphs and Their Use in Modelling Movement

Paulius Dilkas

Formal Analysis, Theory and Algorithms

16th October 2018

# Motivation

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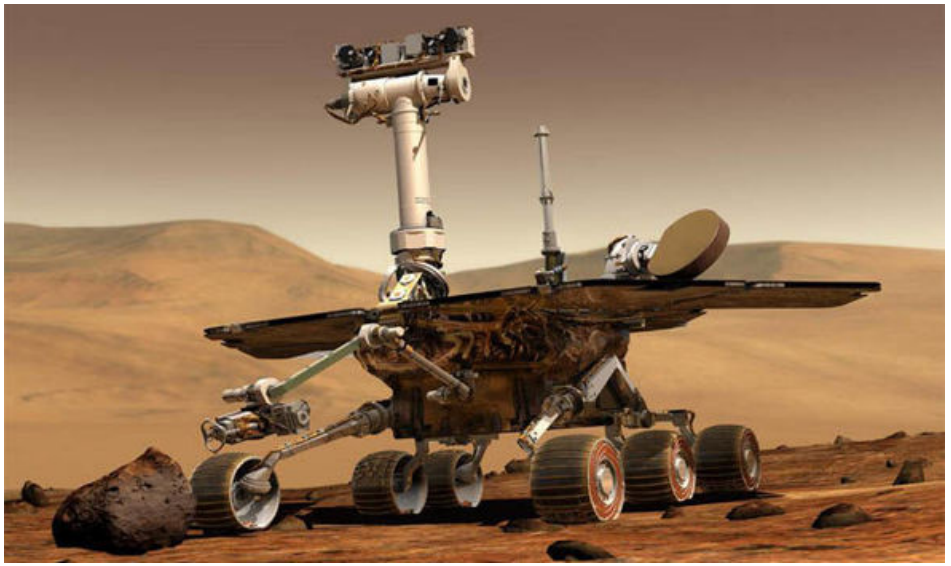

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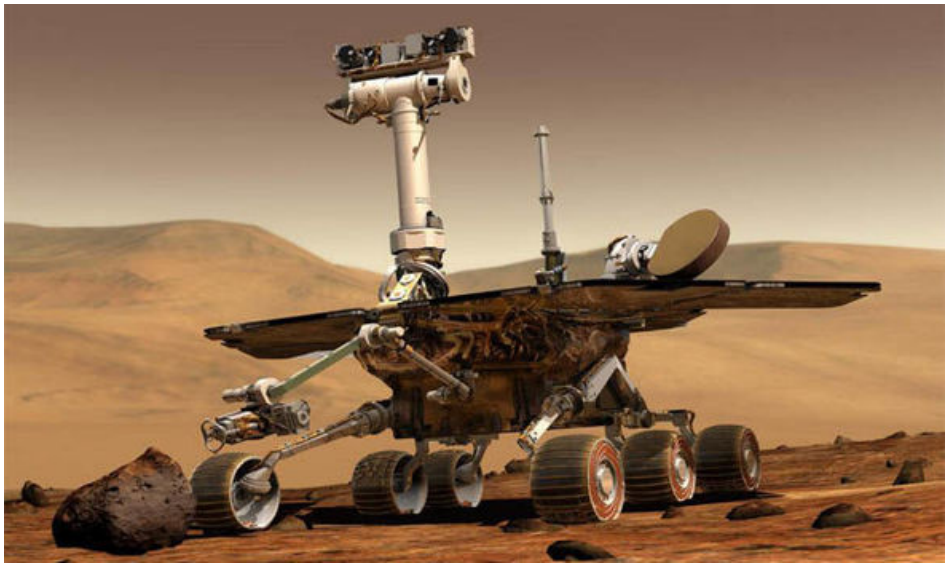

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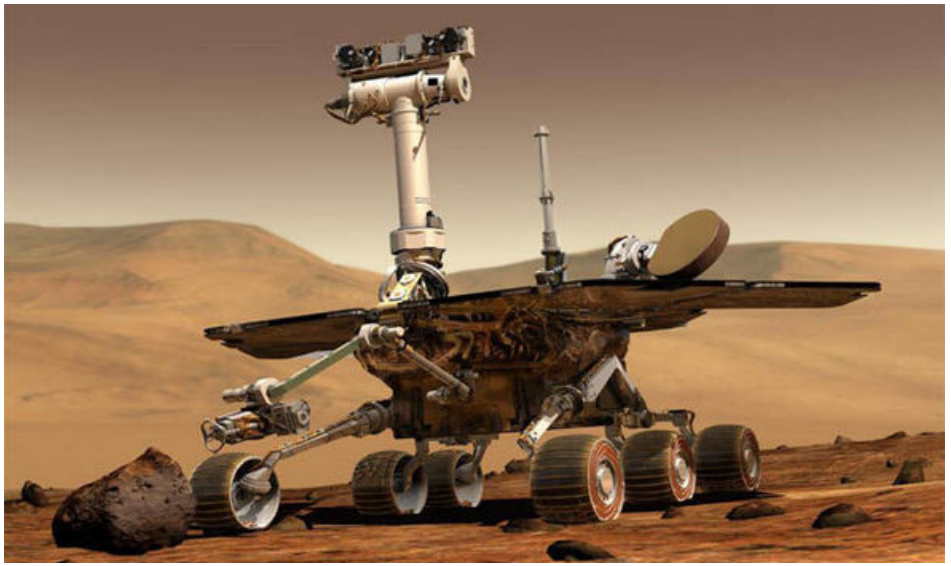
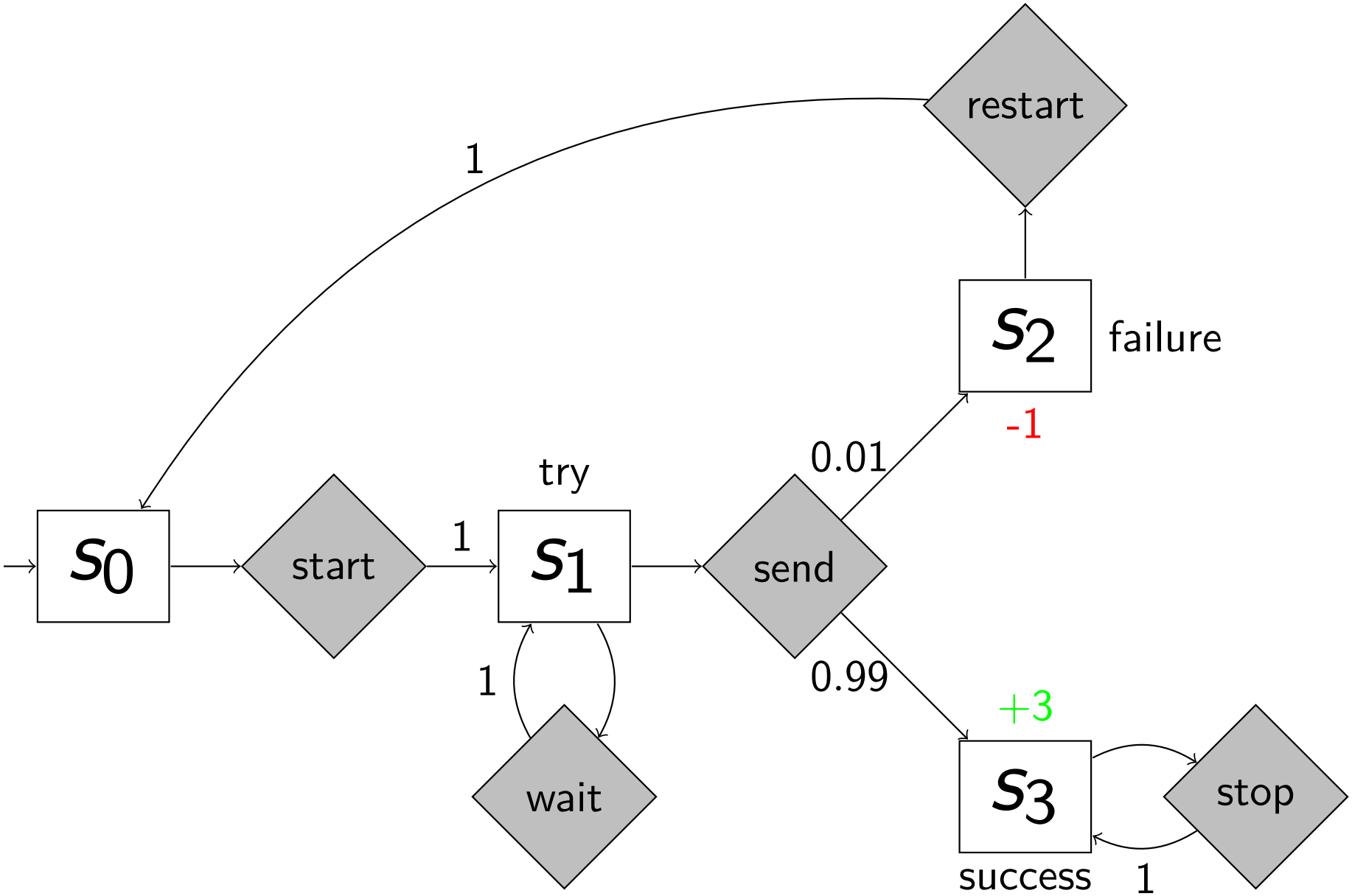


# Markov Decision Process



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- Once a set number of objects is collected, the agent heads home.

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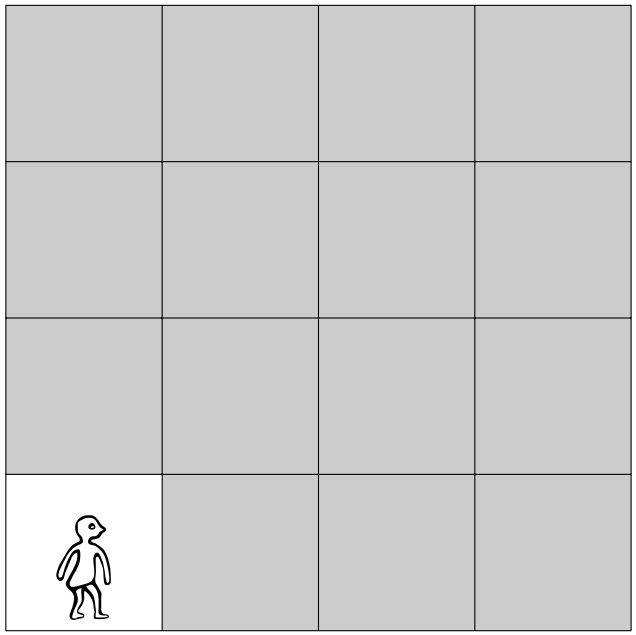

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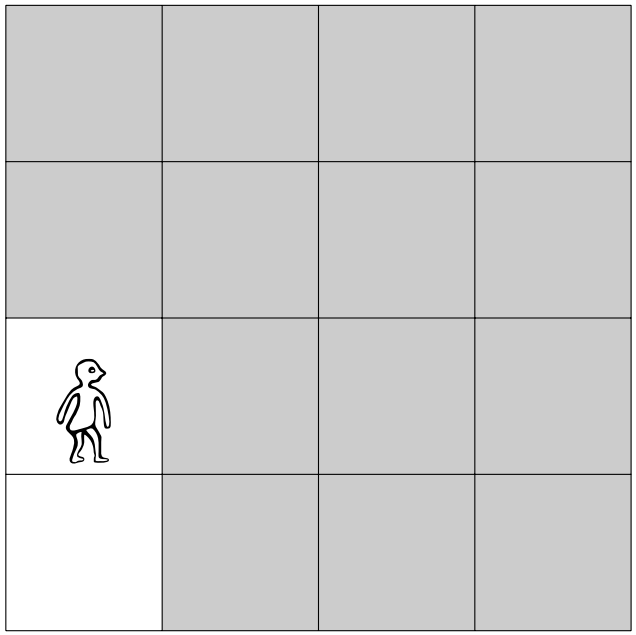

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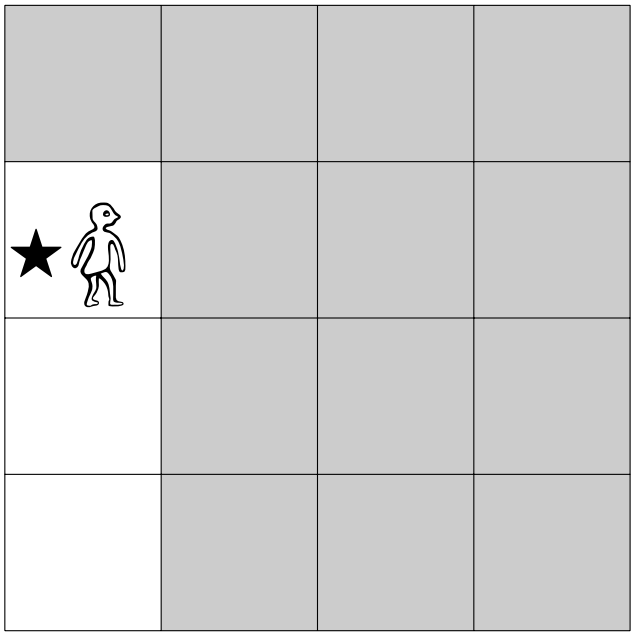

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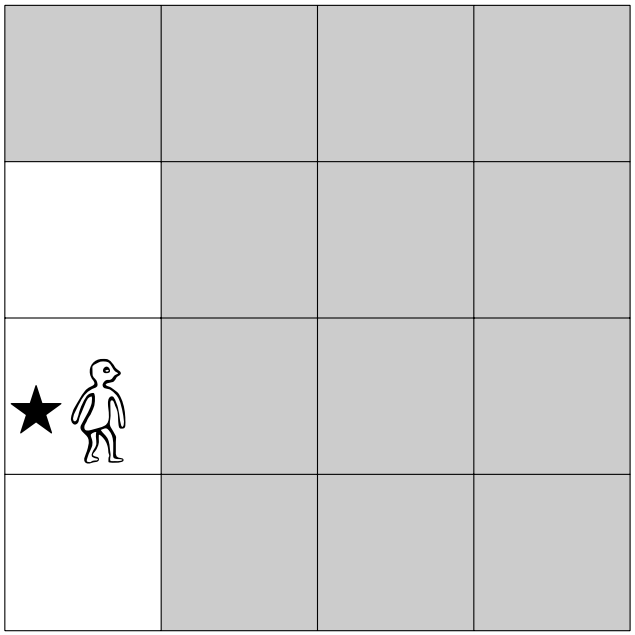

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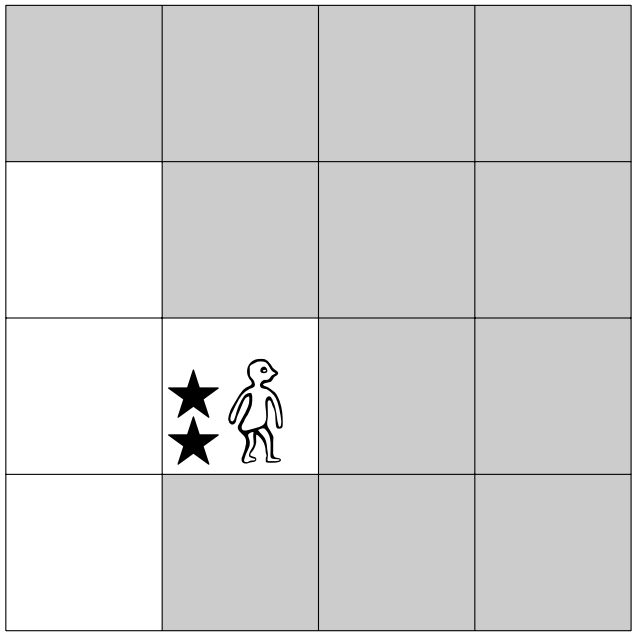

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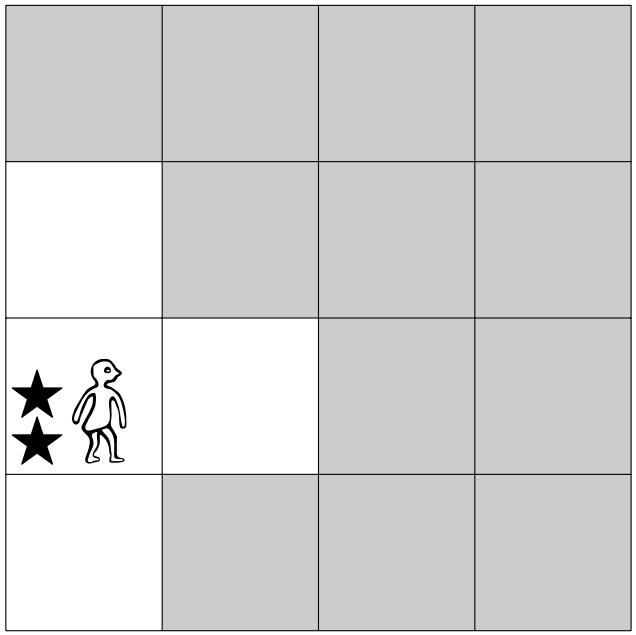

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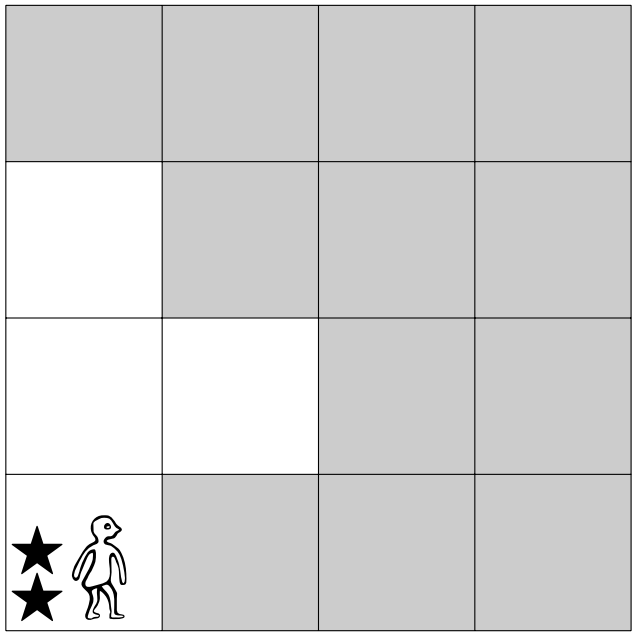

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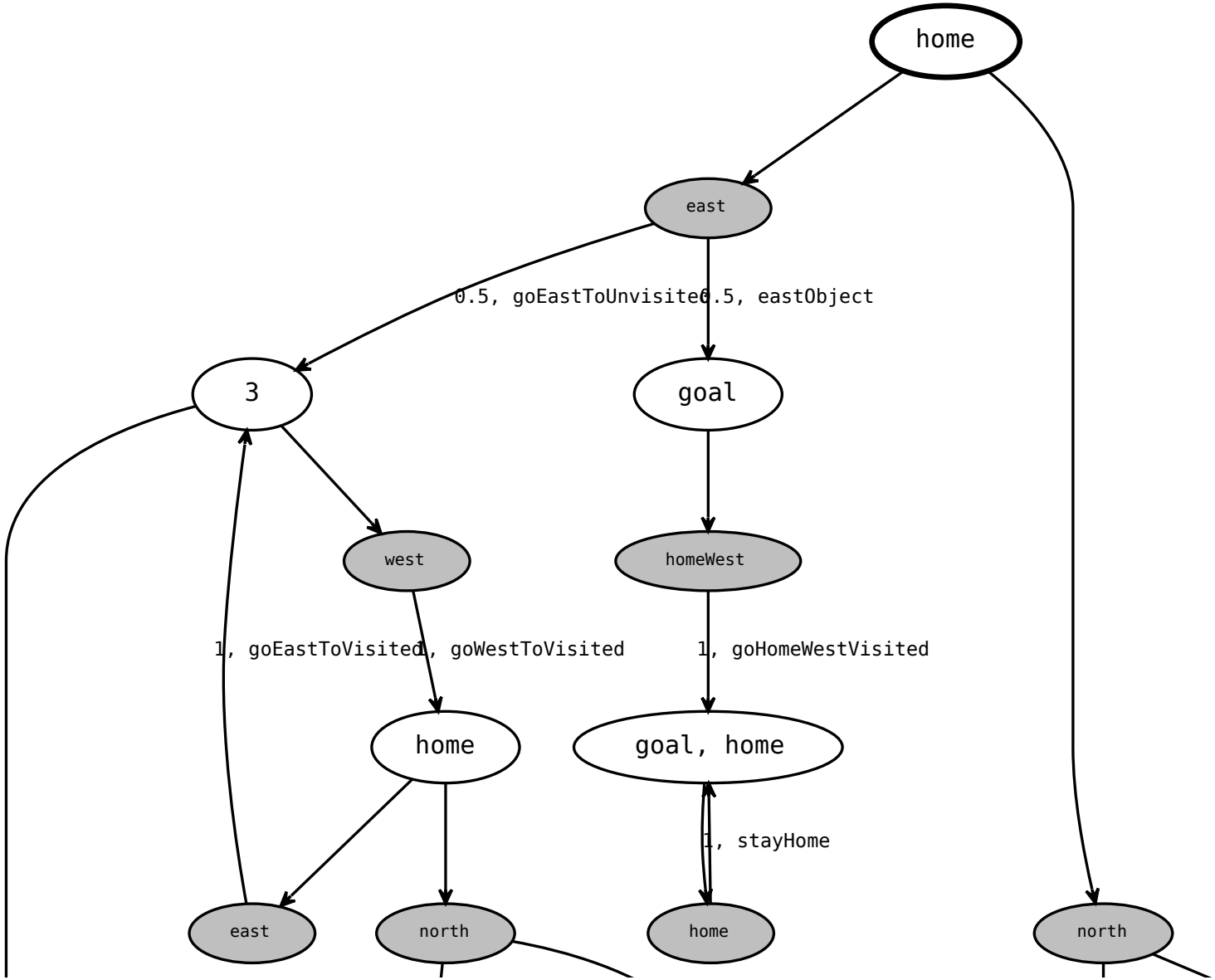
  - Different rules for going to visited and unvisited cells
  - Priority 1: going/staying home (5 rules)
  - Priority 2: 3 rules for each direction
    - ★ visited
    - ★ unvisited
    - ★ unvisited + object

# Transition System



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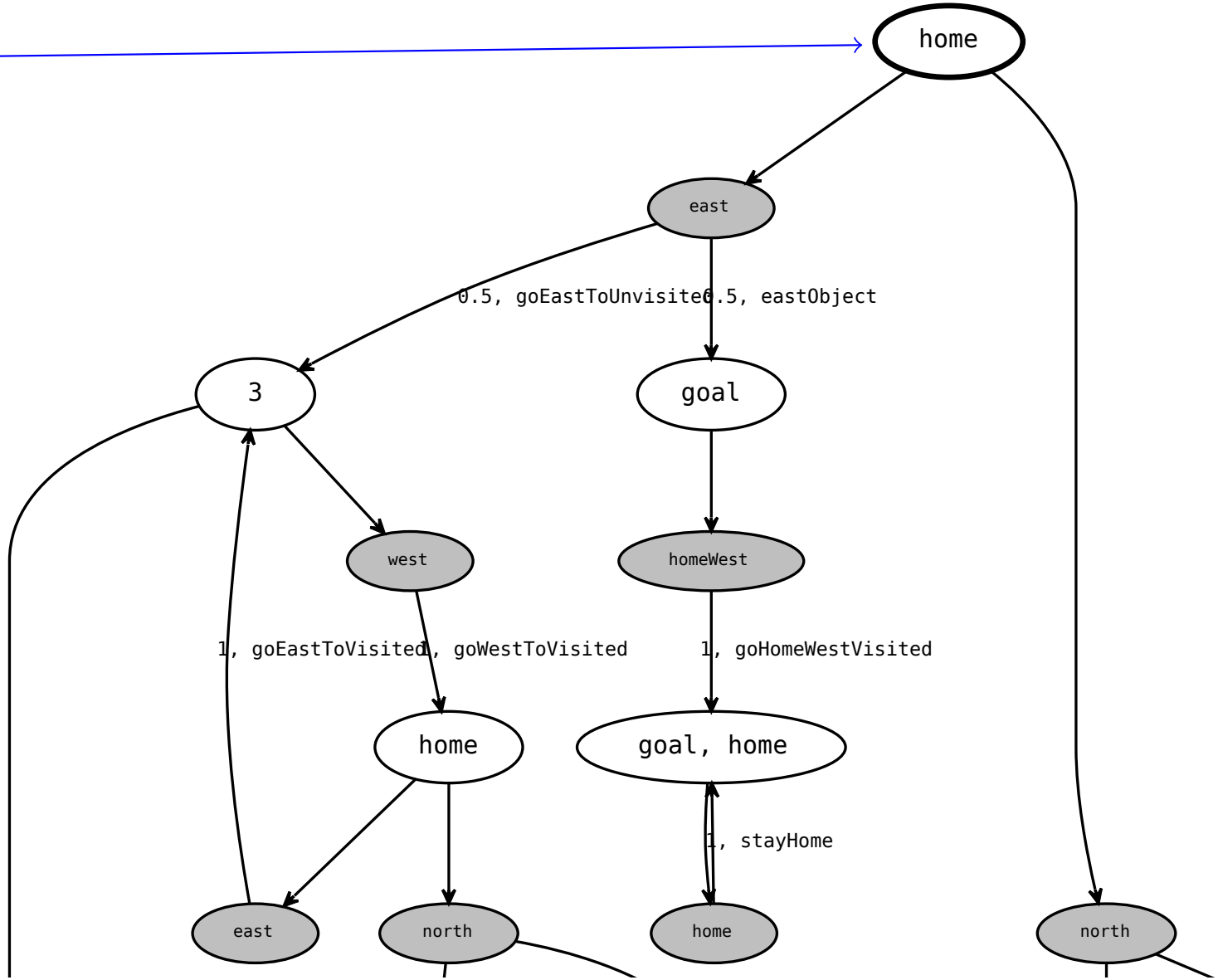

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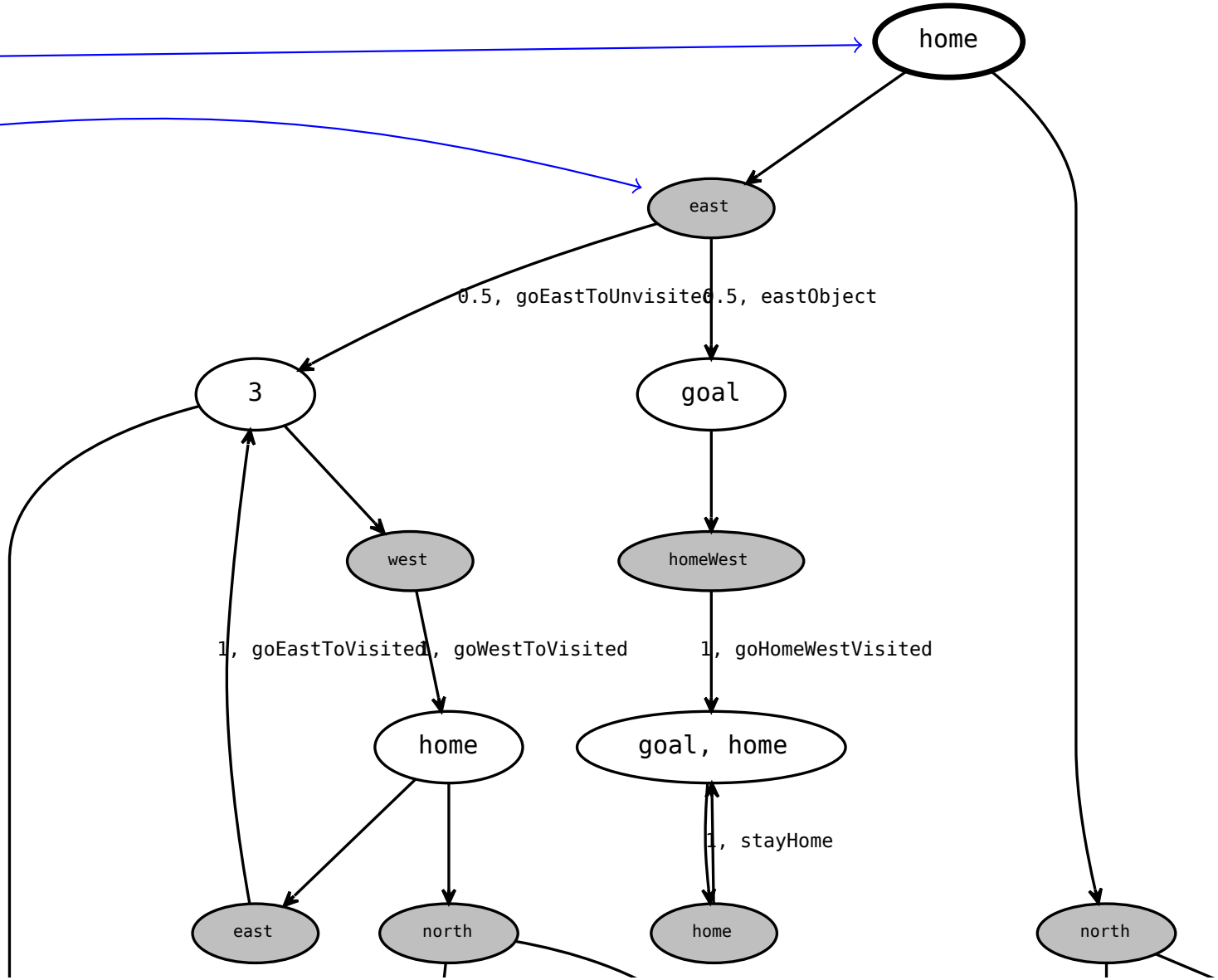

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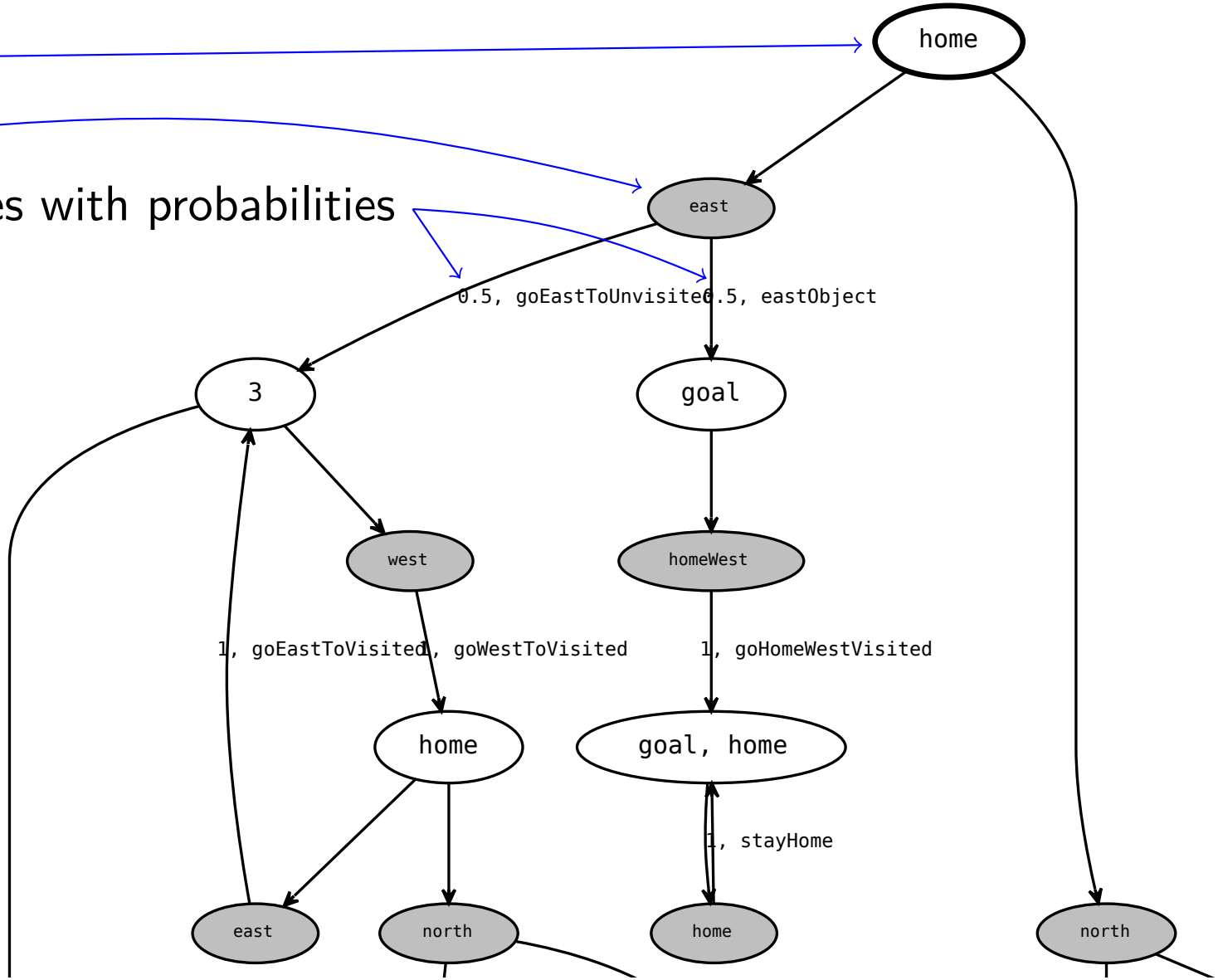

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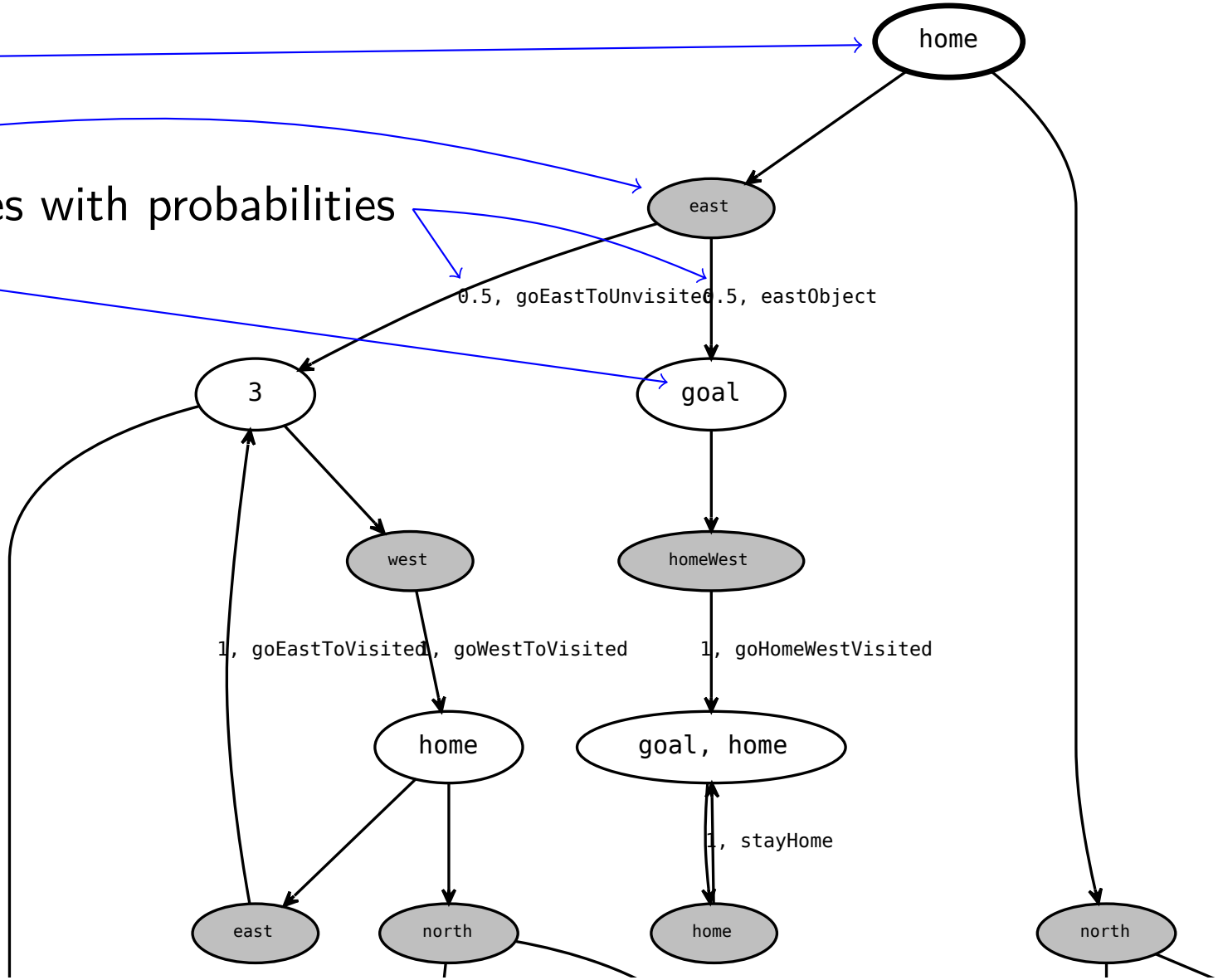

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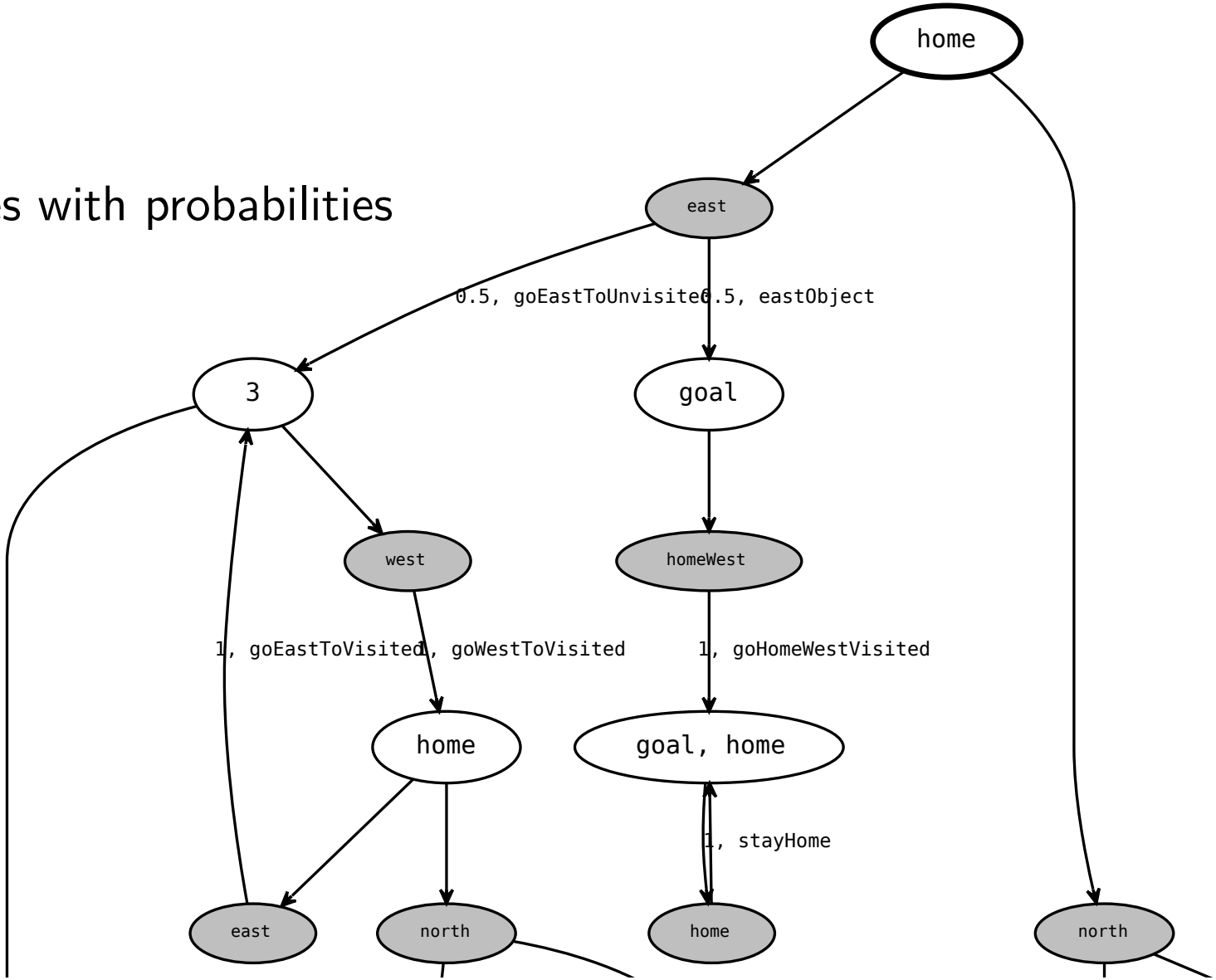

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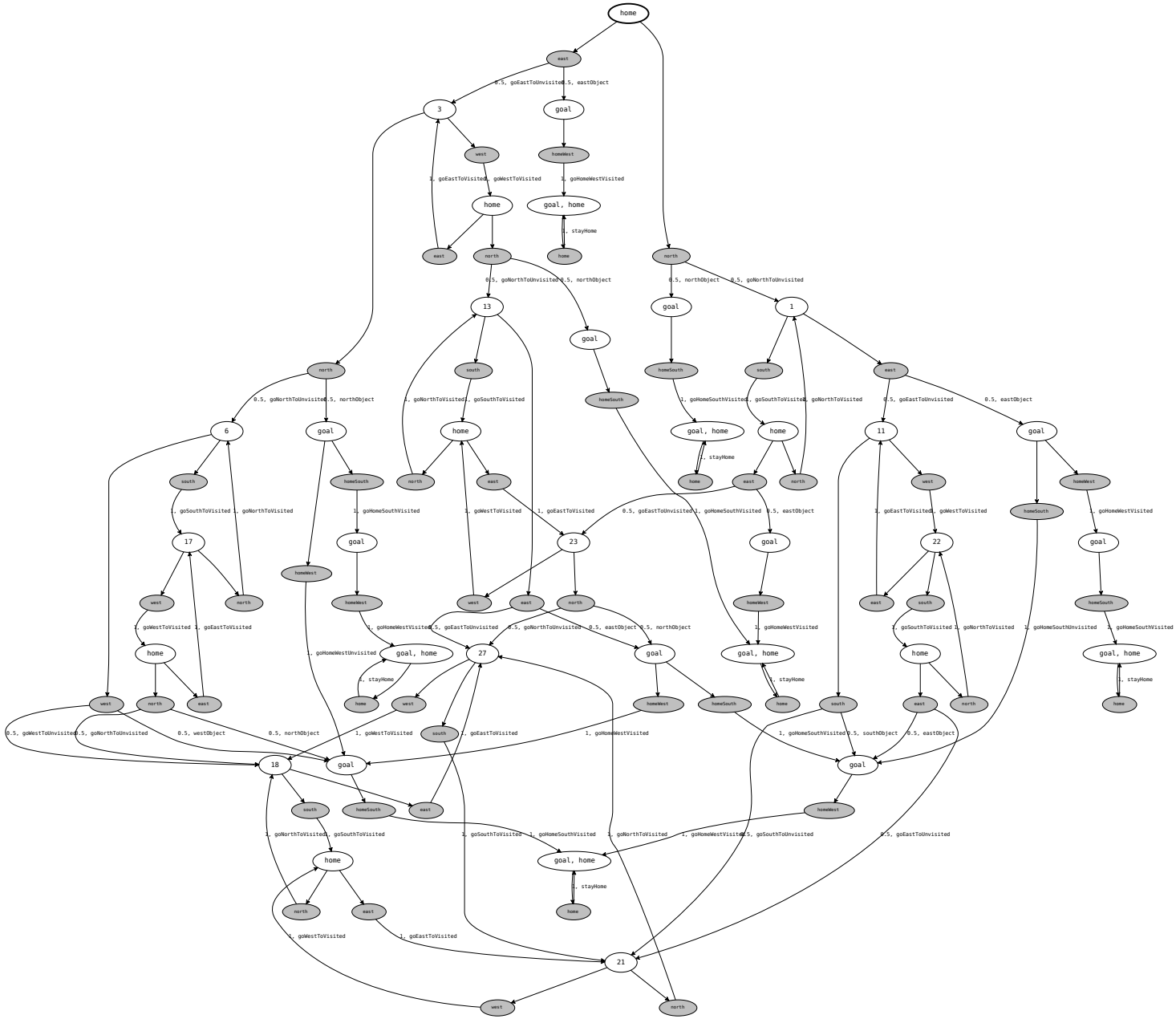

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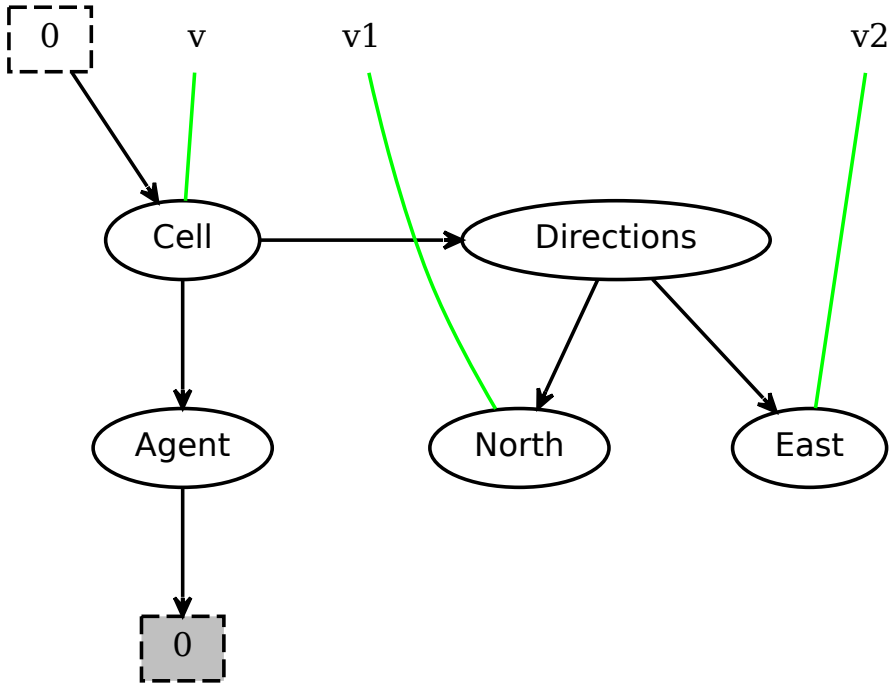
- Normalise probabilities per action
  - Caveat: one rule can sometimes be applied in multiple ways
  - In that case, each outcome is equally likely
- Either:
  - Breadth first search to generate the full transition system
  - Or select the next state randomly for a simulation

# Bigraphs



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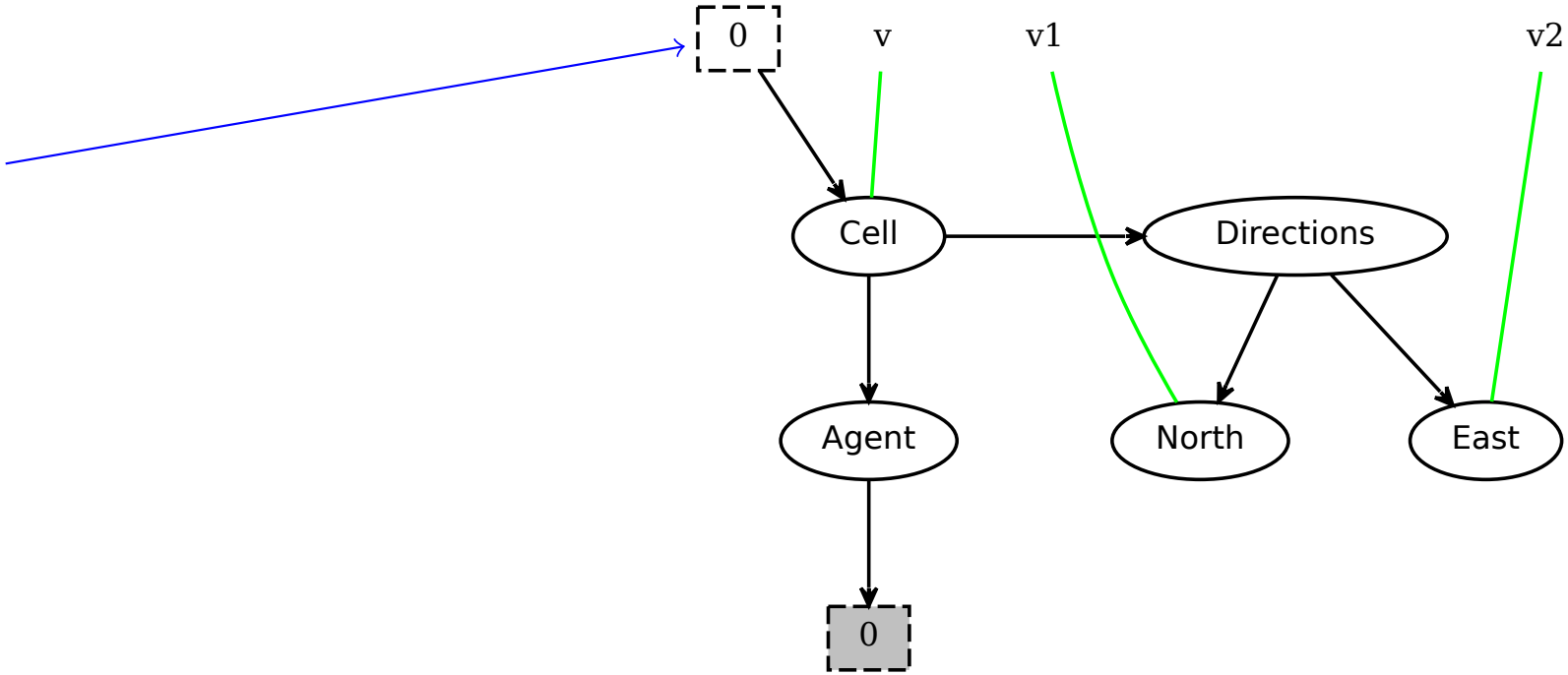
- Region



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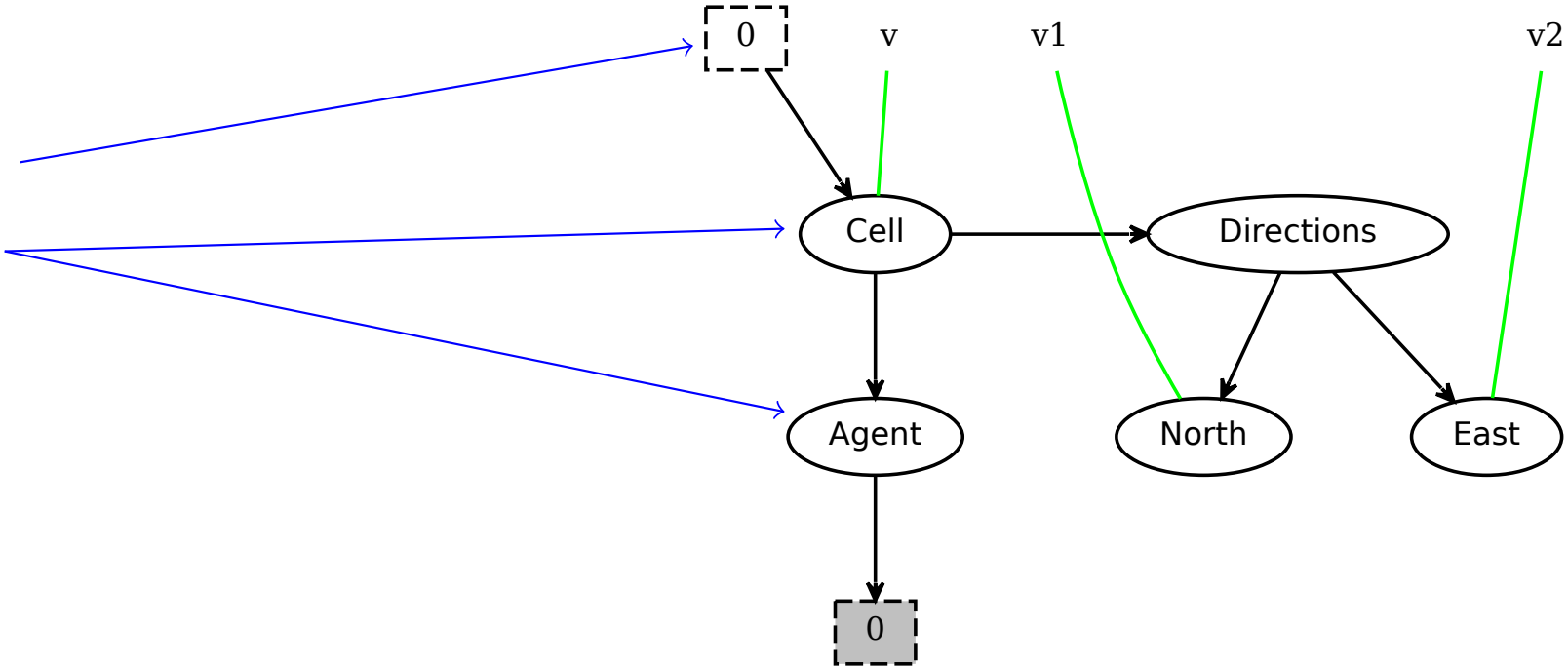
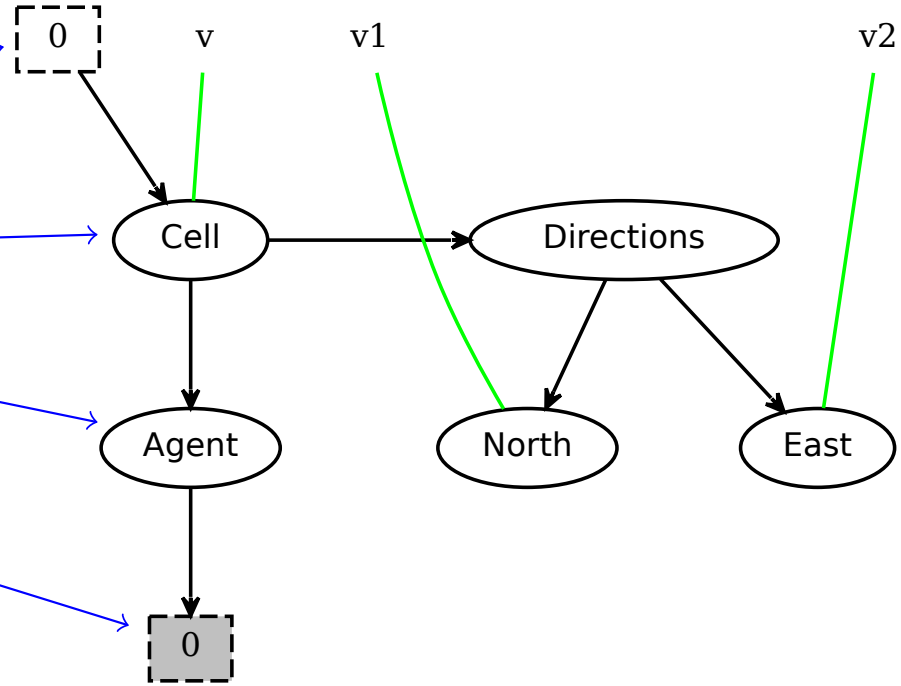
- Region
- Nodes



# Bigraphs

- Region
- Nodes
- Site



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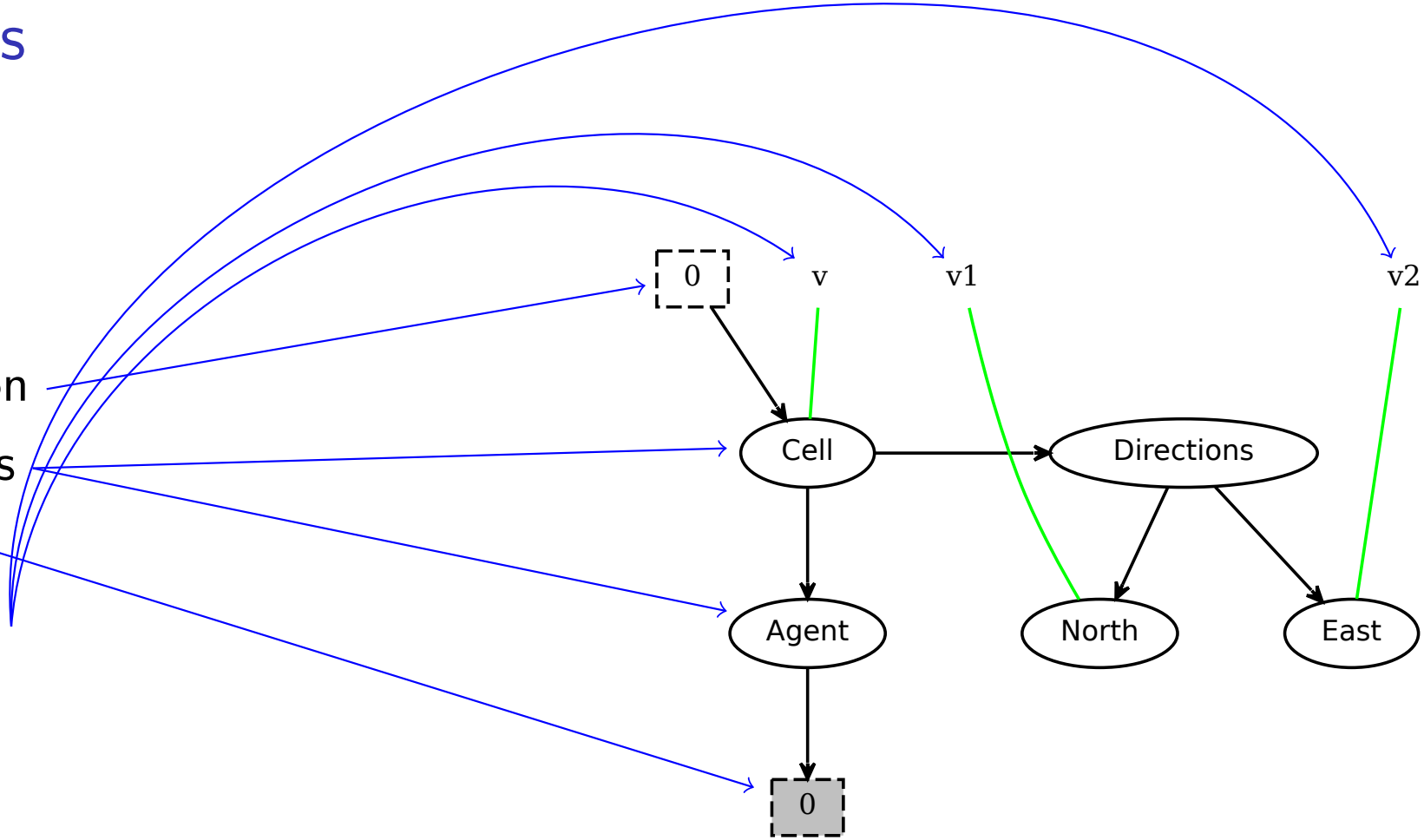

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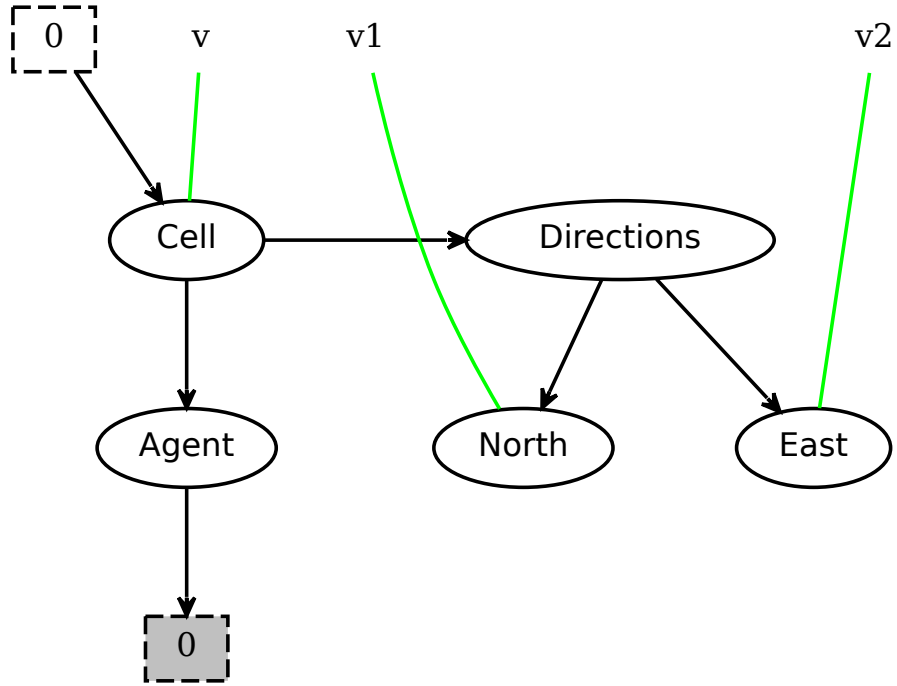
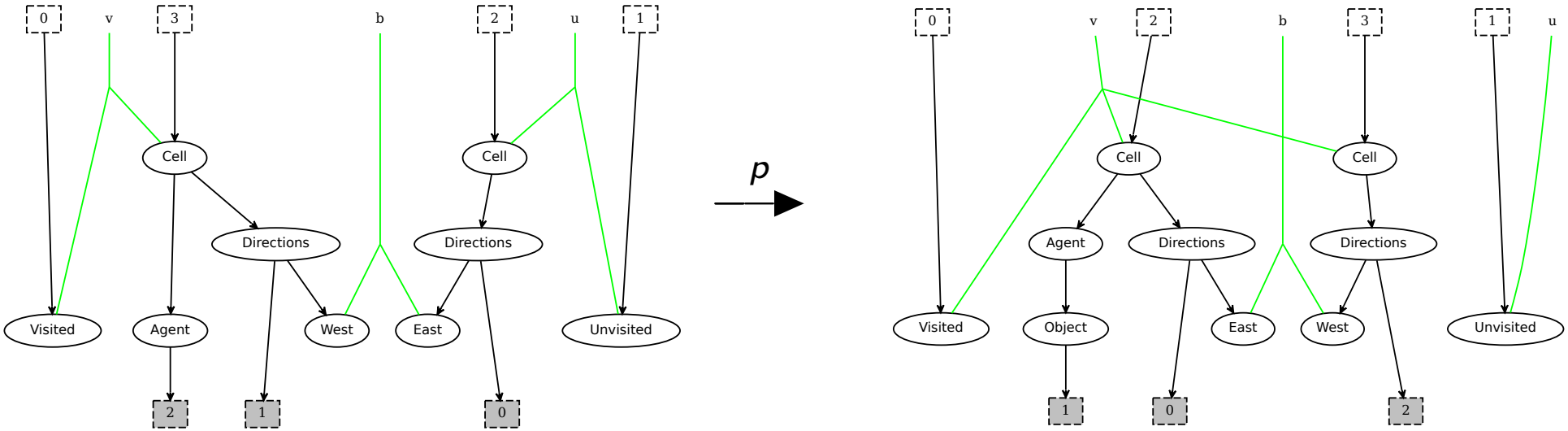



```
big home = Cell{v}.(Directions.(North{v1}
                              | East{v2})
                 | Agent);
```

# Initial State

```
big initial = Visited{v}
            || Unvisited{u}
            # bottom left
            || Cell{v}.(Directions.(North{a}
                                  | East{b})
                        | Agent.1)
            # top left
            || Cell{u}.Directions.(East{c}
                                 | South{a})
            # bottom right
            || Cell{u}.Directions.(North{d}
                                 | West{b})
            # top right
            || Cell{u}.Directions.(West{c}
                                 | South{d});
```

# Reaction Rule: Go West and Collect an Object



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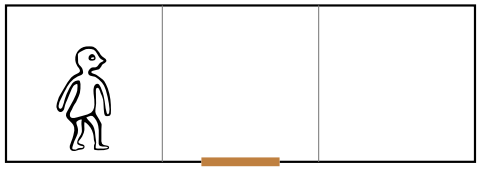

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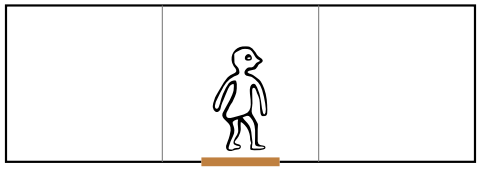

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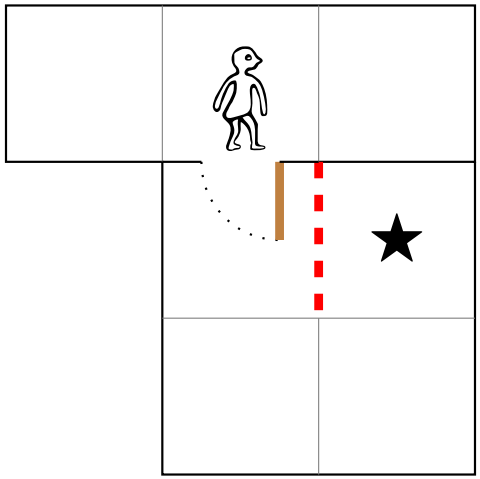

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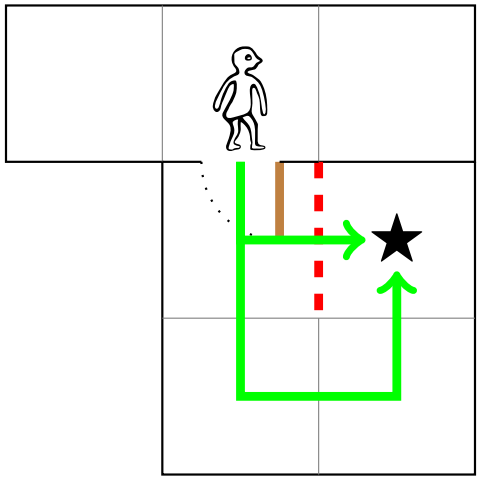

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  - Priority 1: generating the room (2 rules in 1 action)

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    - ★ each rule in a separate action

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- Predicate
  - is `Agent` and `Goal` in the same cell?

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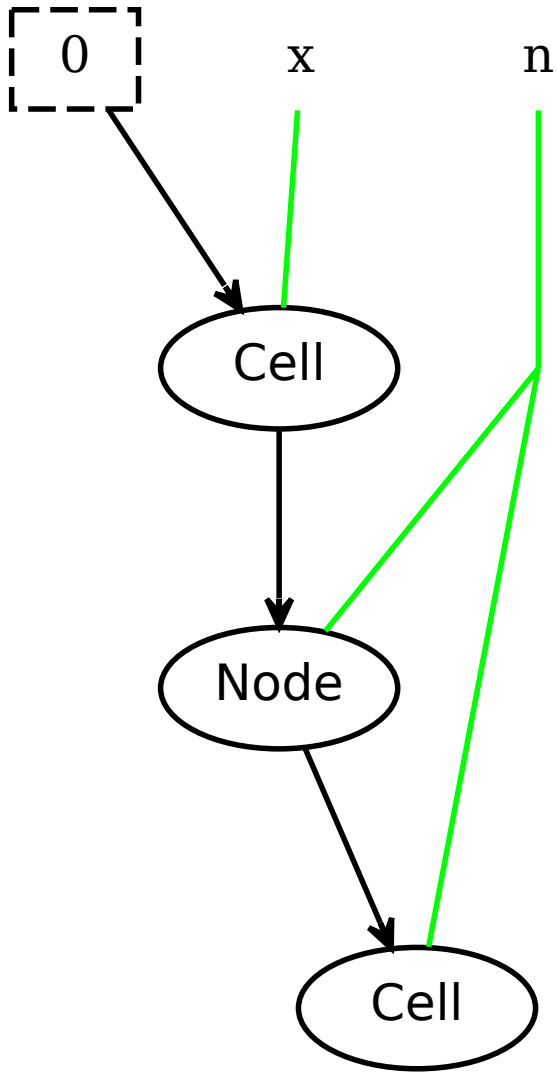

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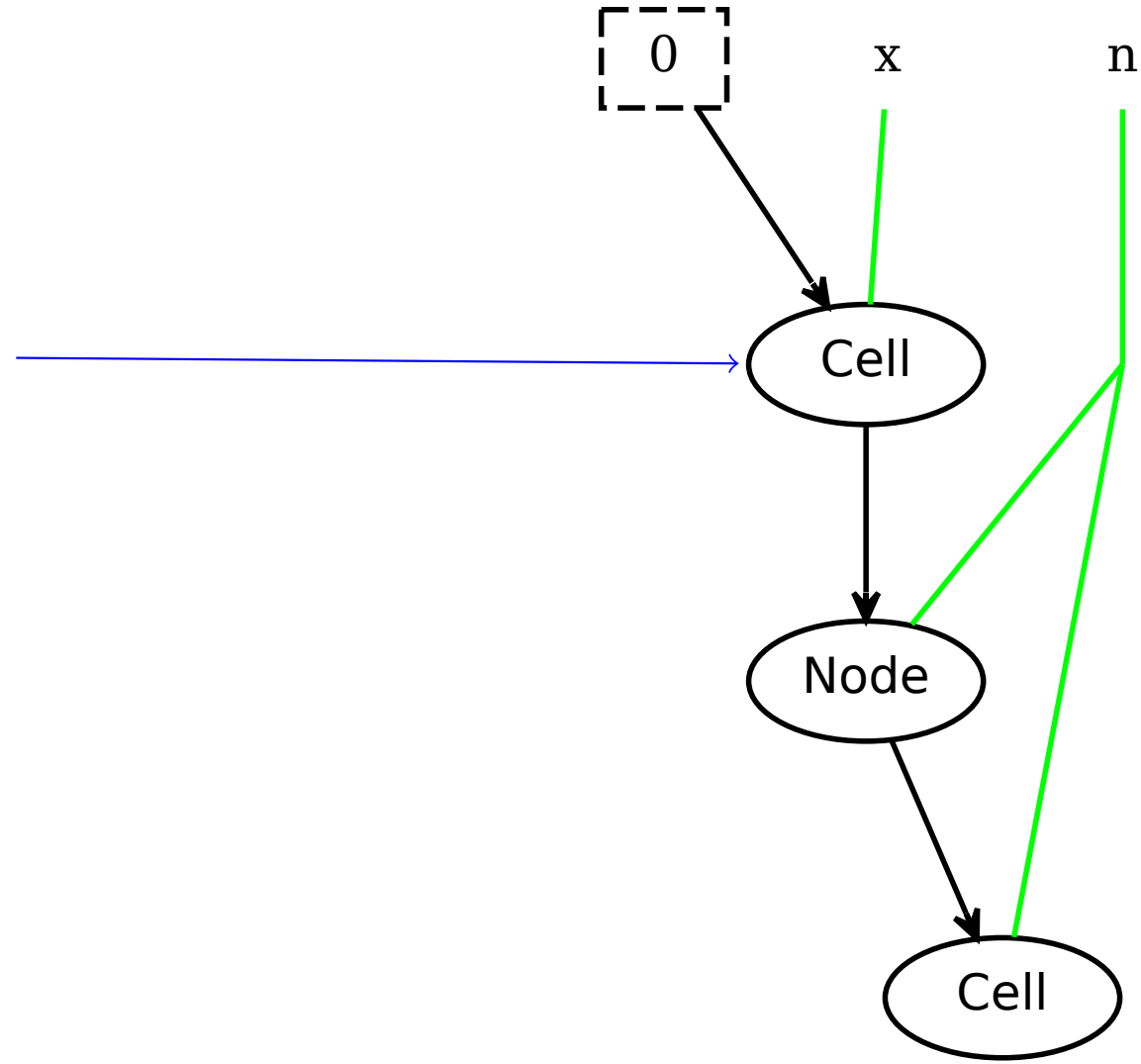
- Outside the door



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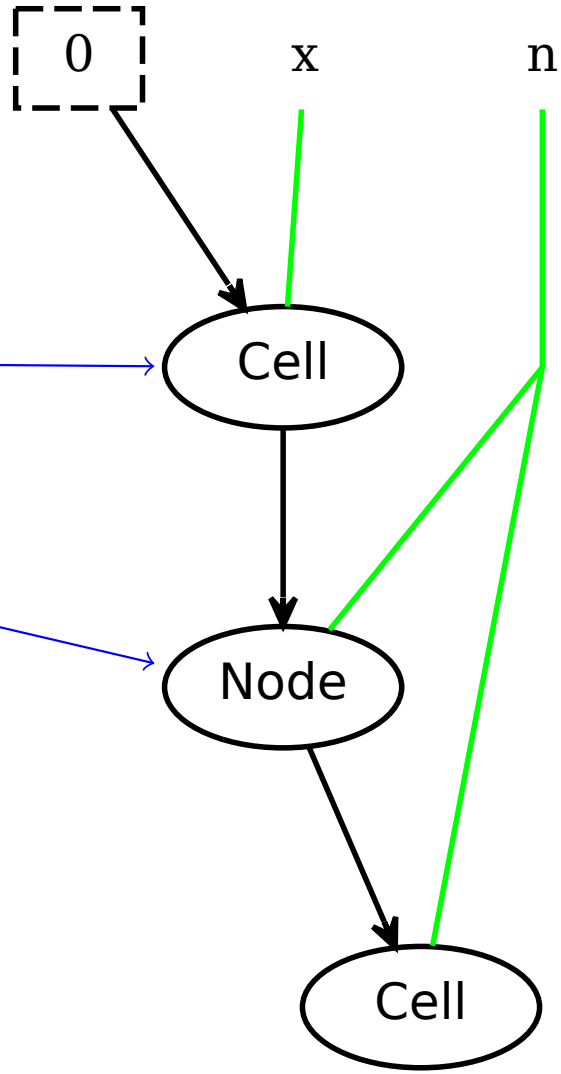

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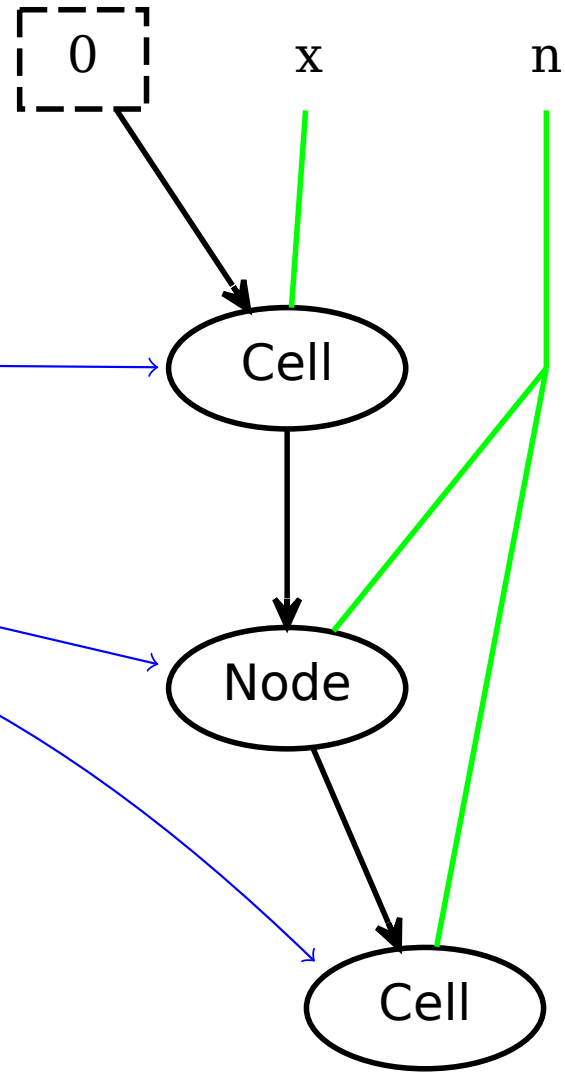
- Inside the door



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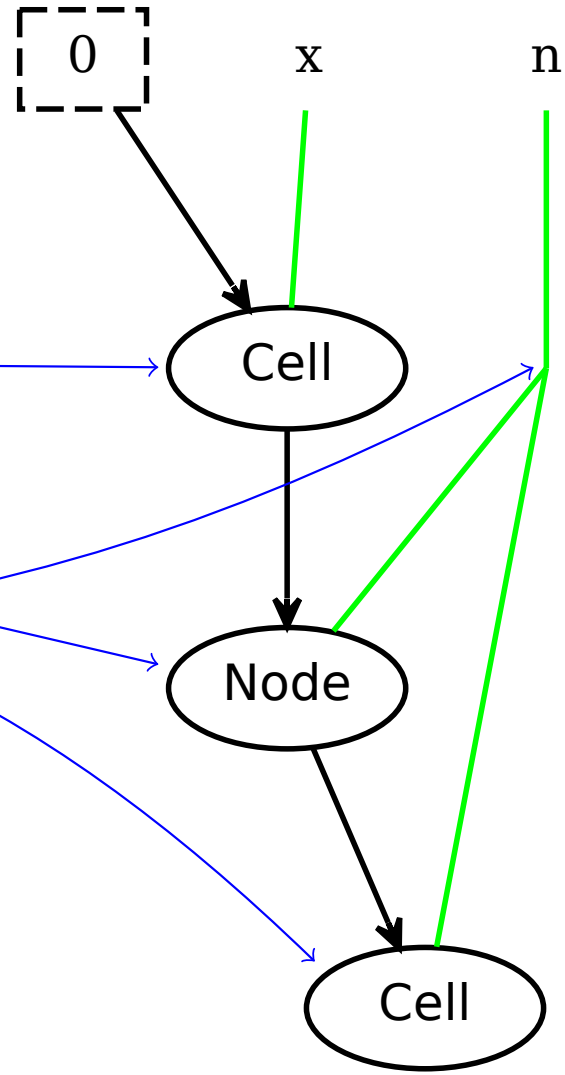
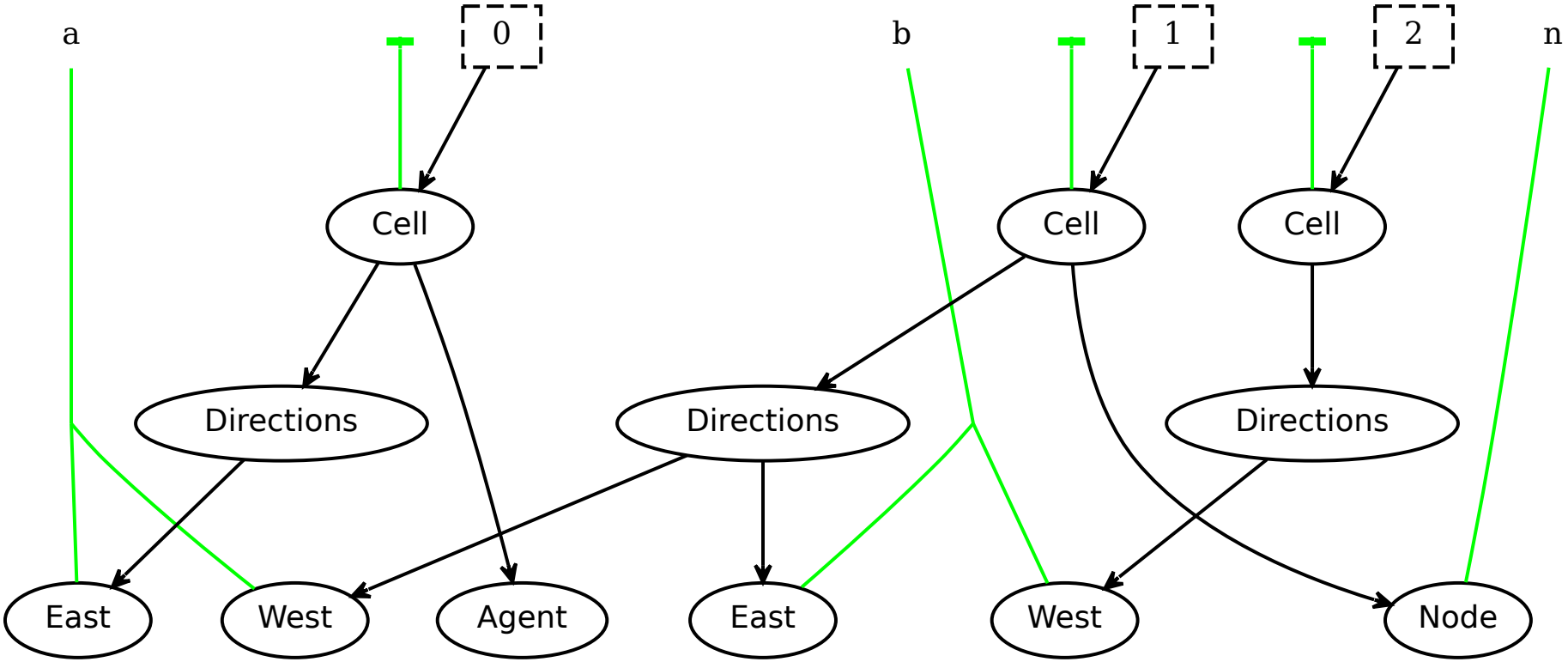
- Outside the door
- The room
- Inside the door
- Which cell is closest to the door?



# Initial State



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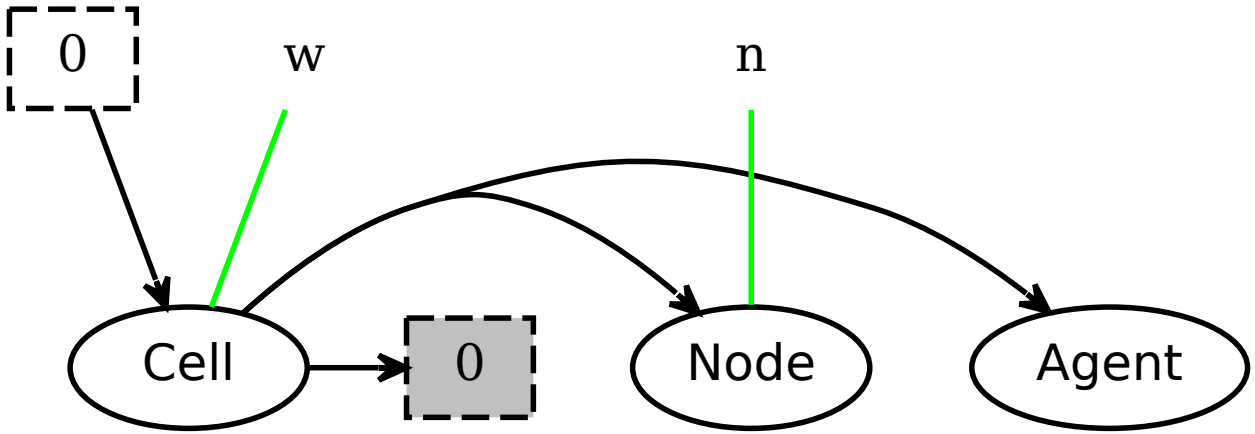

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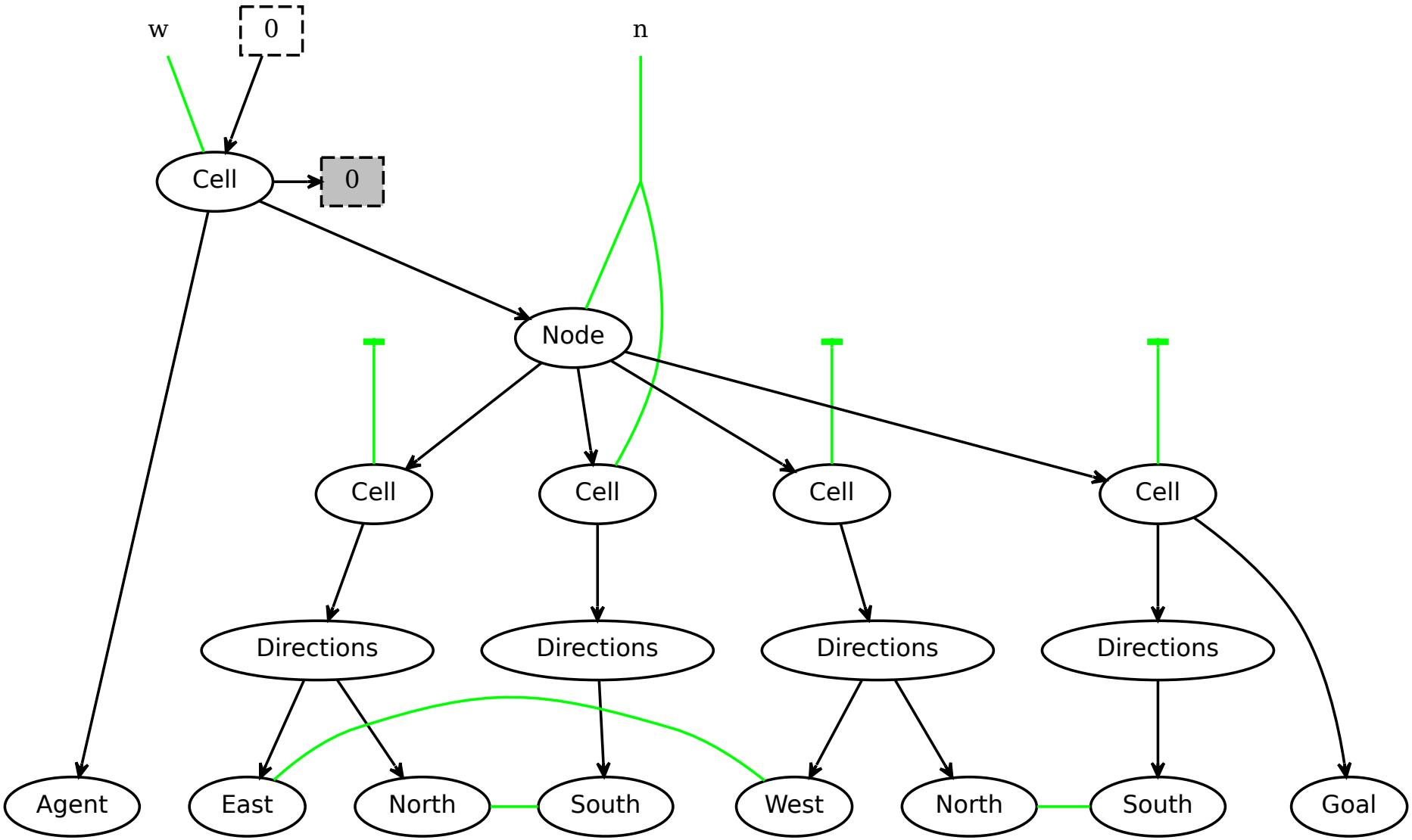
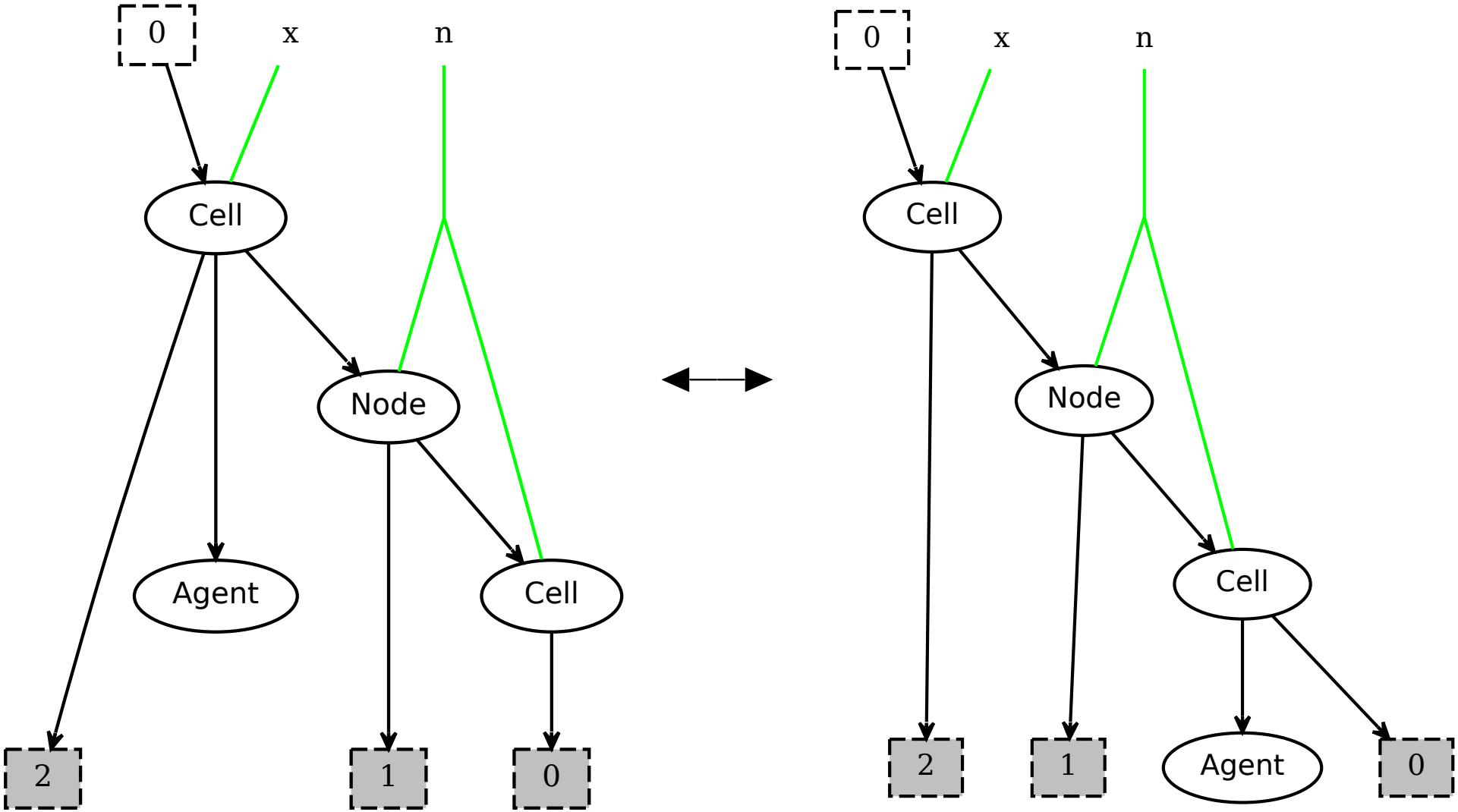


# Entering/Leaving a Room

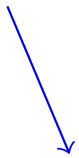


# Entering/Leaving a Room

```
action goIn
  react goIn = Cell{x}.(Agent | Node{n}.(Cell{n}
                                         | id)
                              | id)
               -[1.0]->
               Cell{x}.(Node{n}.(Cell{n}.(Agent
                                          | id)
                                 | id)
                              | id);
end
```

# Entering/Leaving a Room

Action rewards



```
action goIn[1]
   react goIn = Cell{x}.(Agent | Node{n}.(Cell{n}
                                          | id)
                                | id)
                -[1.0]->
                Cell{x}.(Node{n}.(Cell{n}.(Agent
                                           | id)
                                  | id)
                         | id);
end
```

# Tracking Time with State Rewards

```
big agent = Agent;

begin nbrs
  init initialState;
  rules = [ {...}, {...} ];
  preds = { agent[1] };
end
```

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- Blanket predicate

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- List of reaction rules (grouped into priority classes)

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   rules = [ {...}, {...} ];
   preds = { agent[1] };
end
```

- Blanket predicate
- Nondeterministic Bigraphical Reactive System
- Initial state
- List of reaction rules (grouped into priority classes)
- List of predicates

# Tracking Time with State Rewards

```
big agent = Agent;

begin nbrs
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```

- Blanket predicate
- Nondeterministic Bigraphical Reactive System
- Initial state
- List of reaction rules (grouped into priority classes)
- List of predicates
- Predicate rewards (optional)

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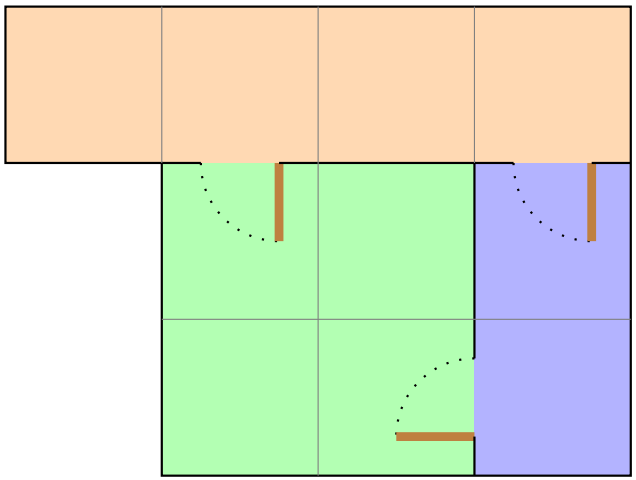

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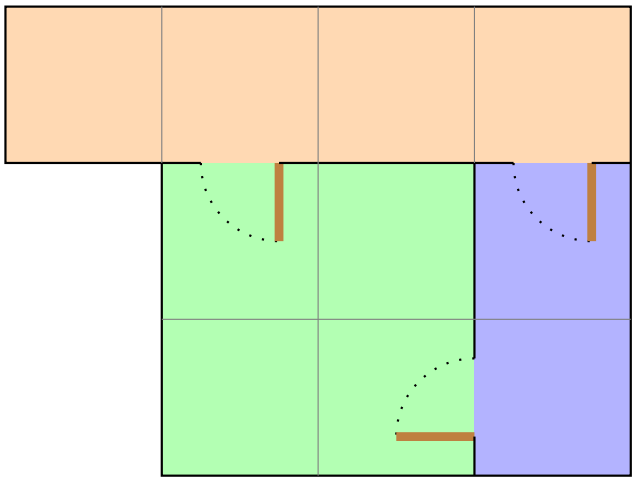

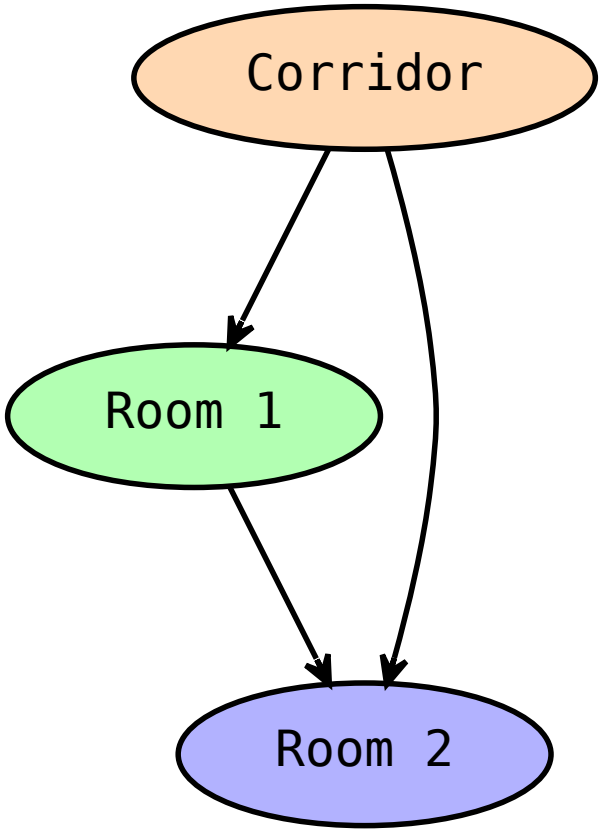

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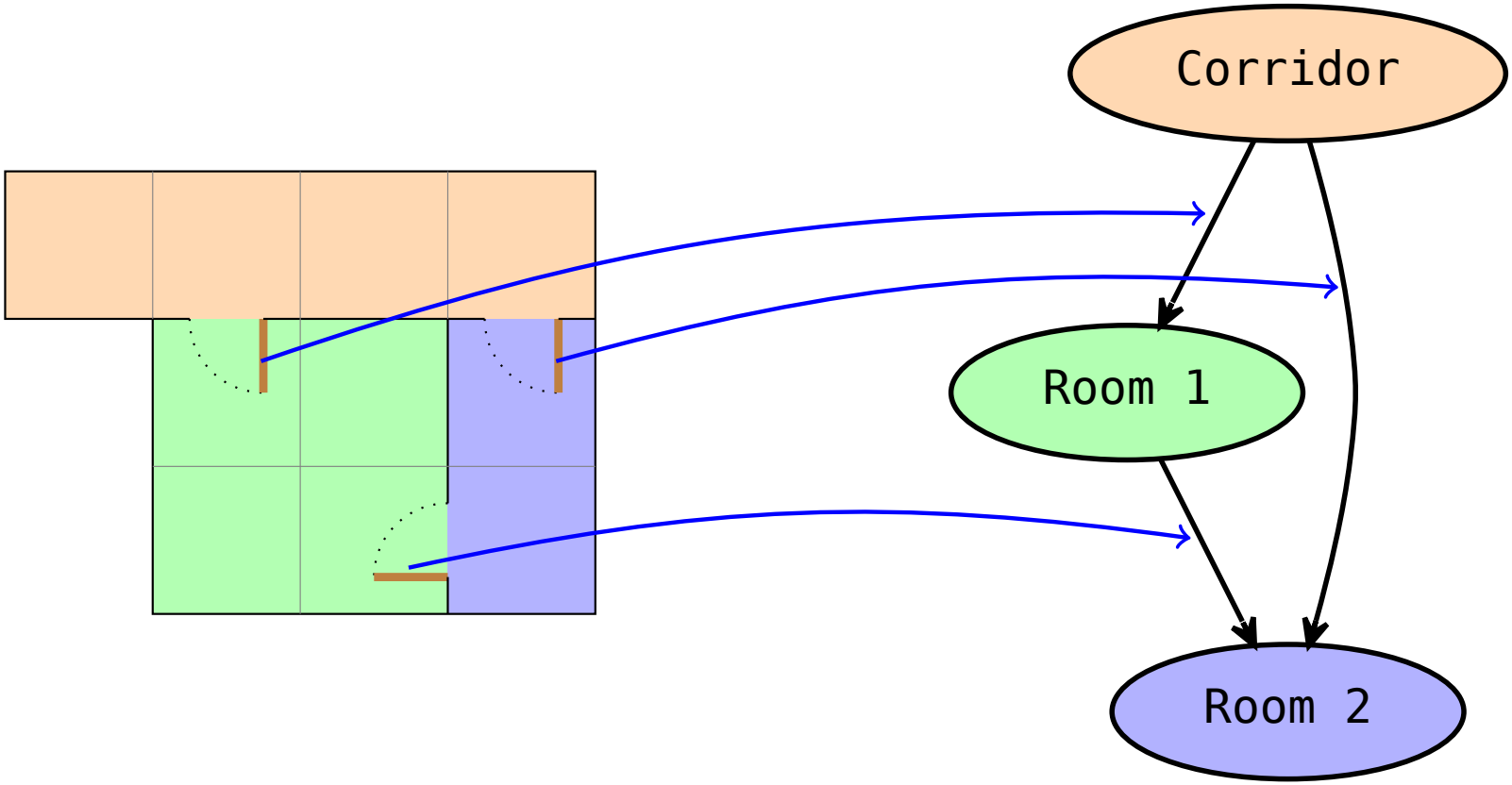

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- Two ideas in one: discovering space & entering an inner space

# A New Interface

jupyter Example Last Checkpoint: a minute ago (autosaved) Logout

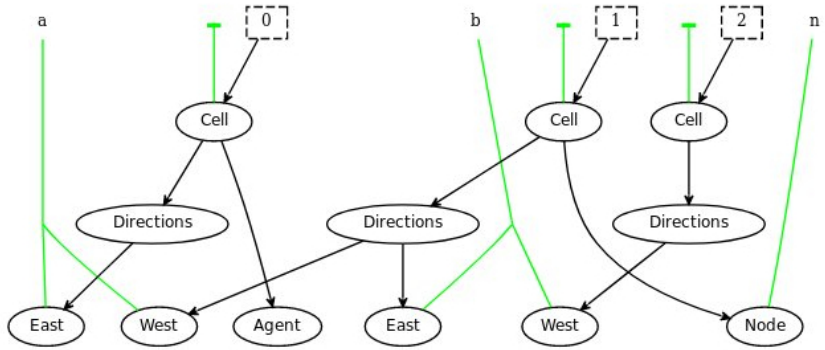
File Edit View Insert Cell Kernel Widgets Help Trusted BigraphER 1.7.0 (OCaml 4.06.0)

Run Code

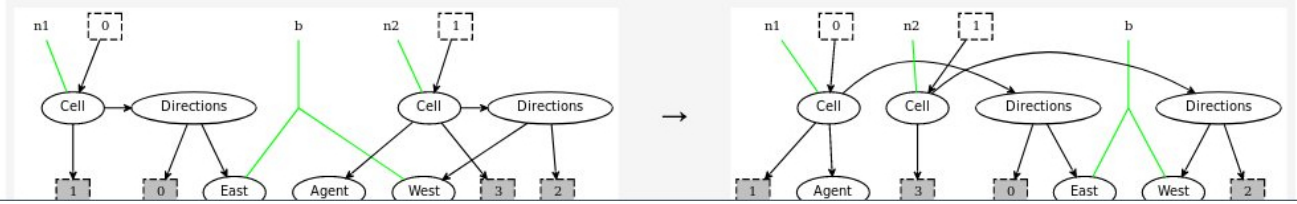
In [1]:

```
1  ctrl Cell = 1;
2  ctrl Directions = 0;
3  ctrl Node = 1;
4  atomic ctrl Agent = 0;
5  atomic ctrl East = 1;
6  atomic ctrl West = 1;
7  big initial = /x /y /z (
8                Cell{x}.(Directions.East{a} | Agent)
9             || Cell{y}.(Directions.(East{b} | West{a})
10                       | Node{n}.1)
11            || Cell{z}.Directions.West{b});
12 react goWest = Cell{n1}.(Directions.(East{b} | id) | id)
13             || Cell{n2}.(Directions.(West{b} | id) | Agent | id)
14             -[1.0]->
15                Cell{n1}.(Directions.(East{b} | id) | Agent | id)
16             || Cell{n2}.(Directions.(West{b} | id) | id);
```

Out[1]: initial




Out[1]: goWest



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- Similar workflow to other Jupyter notebooks

**Available at**

`https://github.com/dilkas/bigrapher-jupyter`

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  - with state bigraph preview on mouseover

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- Visualisation of bigraphs and reaction rules
- Full and partial transition diagrams
  - with state bigraph preview on mouseover
- Backwards compatible to run OCaml code

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*Thank You!*