Generating Random Logic Programs Using Constraint Programming

Paulius Dilkas

AIAI Seminar

Probabilistic Logic Programs (ProbLog)

"Smokers" (Domingos et al. 2008; Fierens et al. 2015)

```
0.2::stress(P) :- person(P).
0.3::influences(P1, P2) :- friend(P1, P2).
0.1::cancer_spont(P) :- person(P).
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     cancer(P) :- cancer_spont(P).
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     person(michelle).
     person(timothy).
     friend(timothy, michelle).
```

Inference

Let $a \oplus b = a + b - ab$.

$$
\begin{aligned}
\Pr[\texttt{cancer}(\mathit{michelle})] = {} & \Pr[\texttt{cancer\_spont}(\mathit{michelle})] \\
& \oplus \Pr[\texttt{smokes}(\mathit{michelle})] \\
& \times \Pr[\texttt{cancer\_smoke}(\mathit{michelle})]
\end{aligned}
$$

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cancer(P) :- cancer_spont(P).
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Inference

Let $a \oplus b = a + b - ab$.
$\Pr[\texttt{cancer}(\mathit{michelle})] = 0.1 \oplus 0.3 \times \Pr[\texttt{smokes}(\mathit{michelle})]$

$$\boxed{\begin{array}{l} 0.1 :: \texttt{cancer\_spont}(P) \texttt{:-} \texttt{person}(P). \\ 0.3 :: \texttt{cancer\_smoke}(P) \texttt{:-} \texttt{person}(P). \end{array}}$$

Inference

Let $a \oplus b = a + b - ab$.

$$\begin{aligned}
\Pr[\texttt{cancer}(\textit{michelle})] &= 0.1 \oplus 0.3 \times \Pr[\texttt{smokes}(\textit{michelle})] \\
\Pr[\texttt{smokes}(\textit{michelle})] &= \Pr[\texttt{stress}(\textit{michelle})] \\
&\oplus \Pr[\texttt{smokes}(\textit{timothy})] \\
&\times \Pr[\texttt{influences}(\textit{timothy}, \textit{michelle})]
\end{aligned}$$

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smokes(X) :- stress(X).
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Let $a \oplus b = a + b - ab$.

$\Pr[\texttt{cancer}(\mathit{michelle})] = 0.1 \oplus 0.3 \times \Pr[\texttt{smokes}(\mathit{michelle})]$

$\Pr[\texttt{smokes}(\mathit{michelle})] = 0.2 \oplus 0.3 \times \Pr[\texttt{smokes}(\mathit{timothy})]$

$0.2 :: \texttt{stress}(P) \text{ :- } \texttt{person}(P).$

$0.3 :: \texttt{influences}(P_1, P_2) \text{ :- } \texttt{friend}(P_1, P_2).$

Inference

Let $a \oplus b = a + b - ab$.

$\Pr[\mathtt{cancer}(\mathit{michelle})] = 0.1 \oplus 0.3 \times \Pr[\mathtt{smokes}(\mathit{michelle})]$

$\Pr[\mathtt{smokes}(\mathit{michelle})] = 0.2 \oplus 0.3 \times \Pr[\mathtt{smokes}(\mathit{timothy})]$

$\Pr[\mathtt{smokes}(\mathit{timothy})] = \Pr[\mathtt{stress}(\mathit{timothy})] = 0.2$

```
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     smokes(X) :- stress(X).
```
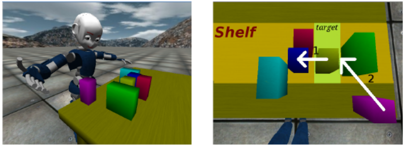

Applications



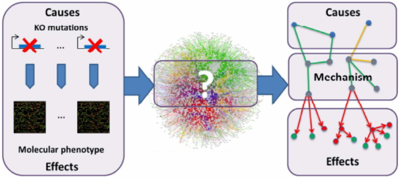
Moldovan et al. 2012

Q1: In a group of 10 people, 60 percent have brown eyes. Two people are to be selected at random from the group. What is the probability that neither person selected will have brown eyes?
Q2: Mike has a bag of marbles with 4 white, 8 blue, and 6 red marbles. He pulls out one marble from the bag and it is red. What is the probability that the second marble he pulls out of the bag is white?

Dries et al. 2017

```
is_malignant(Case):-
        biopsyProcedure(Case,usCore),
        changes_Sizeinc(Case,missing),
        feature_shape(Case).
is_malignant(Case):-
        assoFinding(Case,asymmetry),
        breastDensity(Case,scatteredFDensities),
        vacuumAssisted(Case,yes).
is_malignant(Case):-
        needleGauge(Case,9),
        offset(Case,14),
        vacuumAssisted(Case,yes).
```

Côrte-Real, Dutra, and Rocha 2017



De Maeyer et al. 2013

Inference Algorithms and Knowledge Compilation Maps

NNF	negation normal form
d-DNNF	deterministic decomposable negation normal form
BDD	binary decision diagrams
SDD	sentential decision diagrams
k-Best	only use the k most probable proofs

Example Diagrams for $C \wedge (A \vee \neg B)$

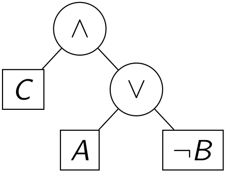


Figure: NNF

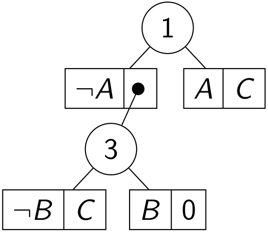


Figure: SDD

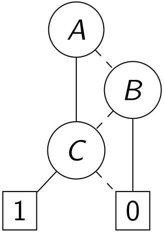


Figure: BDD

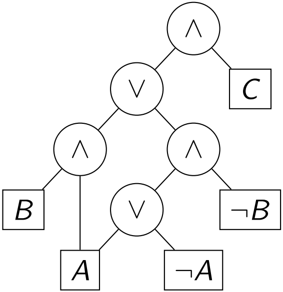


Figure: d-DNNF

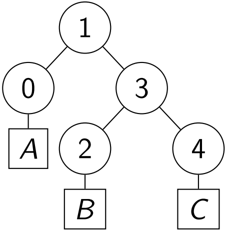


Figure: vtree

How Many Programs Are Used to Test Algorithms?

How Many Programs Are Used to Test Algorithms?

Anytime Inference in Probabilistic Logic Programs with $T_{\mathcal{P}}$-Compilation

4

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Inference and learning in probabilistic logic programs using weighted Boolean formulas

3

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***k*-Optimal: a novel approximate inference algorithm for ProbLog**

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On the Efficient Execution of ProbLog Programs

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On the Implementation of the Probabilistic Logic Programming Language ProbLog

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1

ProbLog Technology for Inference in a Probabilistic First Order Logic

Maurice Bruynooghe and **Theofrastos Mantadelis** and **Angelika Kimmig** and **Bernd Gutmann** and **Joost Vennekens** and **Gerda Janssens** and **Luc De Raedt**[1]

2

What Characterizes a (Probabilistic) Logic Program?

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$\texttt{smokes}(X) \mathrel{:-} \texttt{stress}(X).$

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- complexity

Clauses As Trees

$$\neg \mathtt{p}(X) \vee (\mathtt{q}(X) \wedge \mathtt{r}(X))$$

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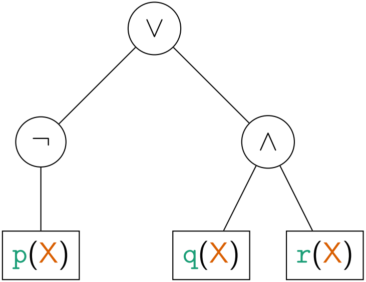

Clauses As Trees

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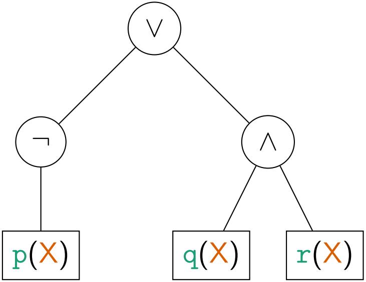
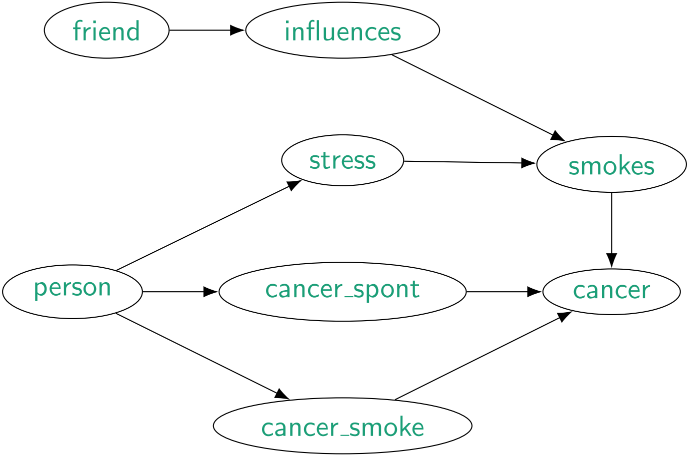
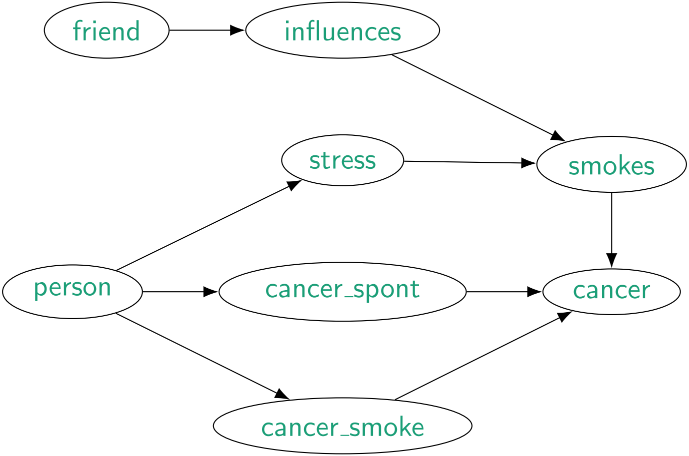


0	0	0	1	2	2	6
$\vee$	$\neg$	$\wedge$	$\mathtt{p}(X)$	$\mathtt{q}(X)$	$\mathtt{r}(X)$	$\top$

Predicate Dependency Graph



Independence: $\mathrm{influences} \perp\!\!\!\perp \mathrm{stress}$



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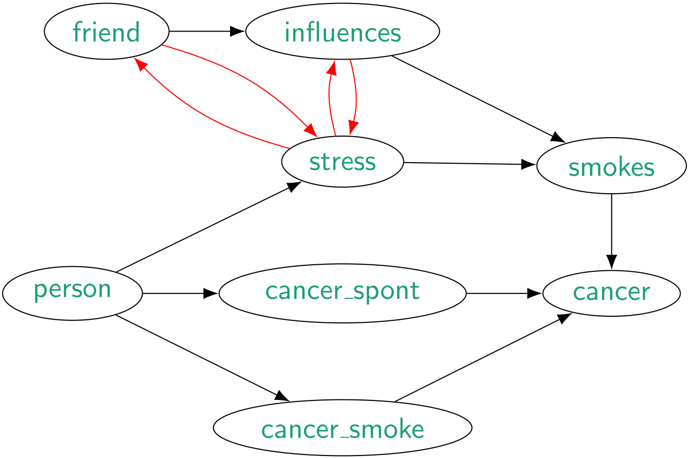
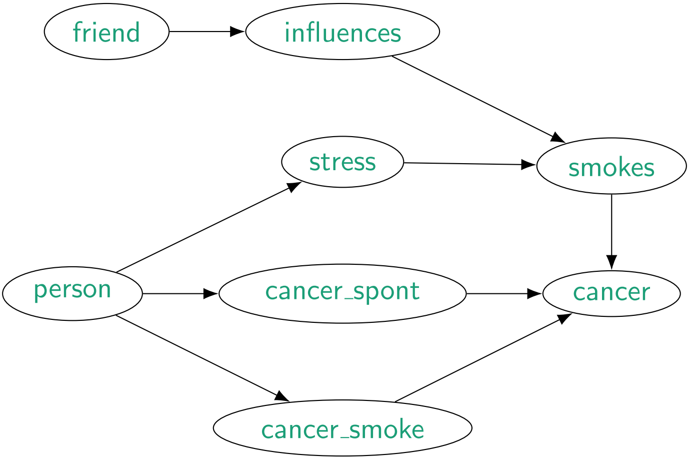


Stratification and Negative Cycles

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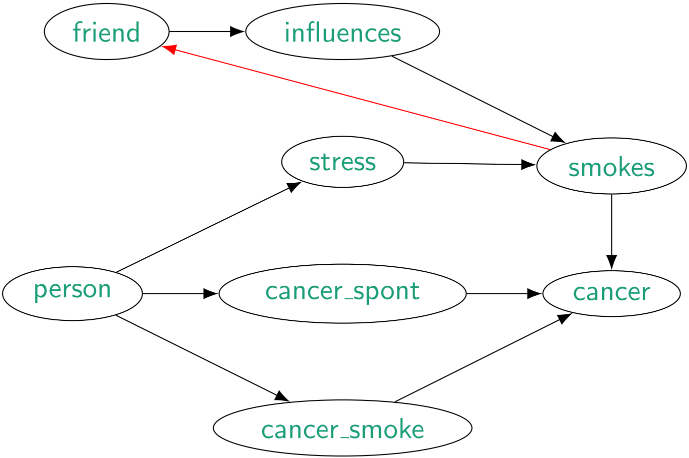
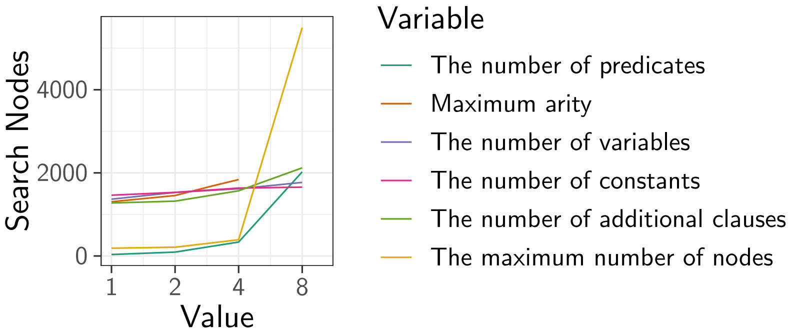
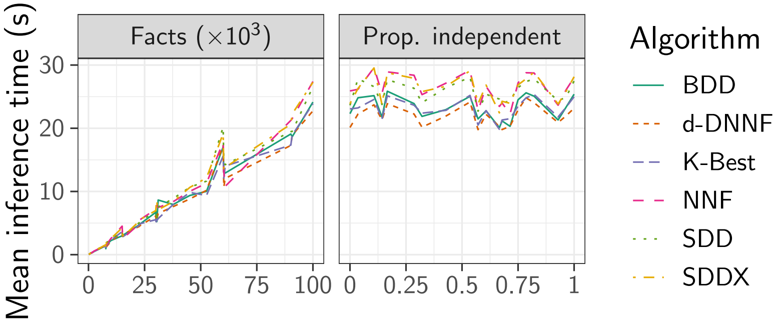


0.1 :: `friend`(X, Y) :- \+ `smokes`(Y).

Scalability



Properties of Programs vs. Inference Algorithms



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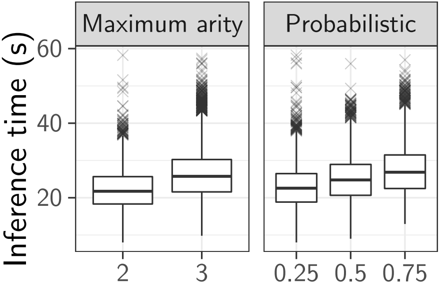

Summary

- foo
- bar
- baz

The implementation of the model is available at

`https://github.com/dilkas/random-logic-programs`

Clause constraints

- $\texttt{treeStructure}$ represents $\texttt{numTrees}$ trees.
- $\texttt{treeStructure}[0] = 0$
- $\texttt{numTrees} + \texttt{numNodes} = \texttt{maxNumNodes} + 1$
- $\texttt{treeStructure}$ is sorted
- For $i = 0, \ldots, \texttt{maxNumNodes} - 1$,
  - If $\texttt{numNodes} \leq i$,
  - then $\texttt{treeStructure}[i] = i$ and $\texttt{treeValues}[i] = \top$,
  - else $\texttt{treeStructure}[i] < \texttt{numNodes}$.
  - has 0 children $\iff$ $\texttt{treeValues}[i]$ is a predicate
  - has 1 child $\iff$ $\texttt{treeValues}[i] = \neg$
  - has > 1 child $\iff$ $\texttt{treeValues}[i] \in \{\wedge, \vee\}$
  - $\texttt{treeStructure}[i] \neq i \implies \texttt{treeValues}[i] \neq \top$
- If the clause should be disabled, $\texttt{numNodes} = 1$ and $\texttt{treeValues}[0] = \top$.

Adjacency matrix representation

$A[i][j] = 0 \iff \nexists k : \texttt{clauseAssignments}[k] = j$ and $i \in \texttt{clauses}[k].\texttt{treeValues}$

New constraints

- No (negative) cycles
  - No clever propagation, just entailment checking.
- Independence. Propagation:
  - Two types of dependencies: determined and one-undetermined-edge-away-from-being-determined.
  - Look up the dependencies of both predicates. For each pair of matching dependencies:
    - If both are determined, fail.
    - If one is determined, the selected edge of the other must not exist.