

Weighted Model Counting Without Parameter Variables

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The Computational Problem of Probabilistic Inference

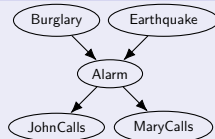
ProbLog

```
0.001 :: burglary.  
0.002 :: earthquake.  
0.95  :: alarm    :- burglary, earthquake.  
0.94  :: alarm    :- burglary, \+ earthquake.  
0.29  :: alarm    :- \+ burglary, earthquake.  
0.001 :: alarm    :- \+ burglary, \+ earthquake.  
0.9   :: johnCalls :- alarm.  
0.05  :: johnCalls :- \+ alarm.  
0.7   :: maryCalls :- alarm.  
0.01  :: maryCalls :- \+ alarm.
```

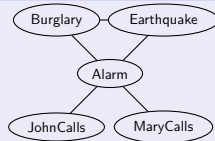
BLOG

```
random Boolean Burglary ~ BooleanDistrib(0.001);  
random Boolean Earthquake ~ BooleanDistrib(0.002);  
random Boolean Alarm ~  
  if Burglary then  
    if Earthquake then BooleanDistrib(0.95)  
    else BooleanDistrib(0.94)  
  else  
    if Earthquake then BooleanDistrib(0.29)  
    else BooleanDistrib(0.001);  
random Boolean JohnCalls ~  
  if Alarm then BooleanDistrib(0.9)  
  else BooleanDistrib(0.05);  
random Boolean MaryCalls ~  
  if Alarm then BooleanDistrib(0.7)  
  else BooleanDistrib(0.01);
```

Bayesian Network



Markov Random Field

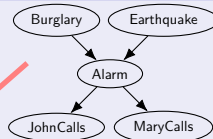


The Computational Problem of Probabilistic Inference

ProbLog

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0.001 :: burglary.  
0.002 :: earthquake.  
0.95  :: alarm    :- burglary, earthquake.  
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0.01  :: maryCalls :- \+ alarm.
```

Bayesian Network

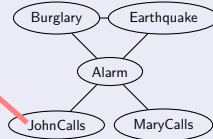


BLOG

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random Boolean Burglary ~ BooleanDistrib(0.001);  
random Boolean Earthquake ~ BooleanDistrib(0.002);  
random Boolean Alarm ~ BooleanDistrib(0.001);  
if Burglary then  
  if Earthquake then BooleanDistrib(0.95)  
  else BooleanDistrib(0.94)  
else  
  if Earthquake then BooleanDistrib(0.29)  
  else BooleanDistrib(0.001);  
random Boolean JohnCalls ~ BooleanDistrib(0.9);  
if Alarm then BooleanDistrib(0.9)  
else BooleanDistrib(0.05);  
random Boolean MaryCalls ~ BooleanDistrib(0.7);  
if Alarm then BooleanDistrib(0.7)  
else BooleanDistrib(0.01);
```

WMC

Markov Random Field



Weighted Model Counting (WMC)

- ▶ Generalises propositional model counting ($\#SAT$)
- ▶ Applications:
 - ▶ graphical models
 - ▶ probabilistic programming
 - ▶ neural-symbolic artificial intelligence
- ▶ Main types of algorithms:
 - ▶ using knowledge compilation
 - ▶ using a SAT solver
 - ▶ manipulating pseudo-Boolean functions

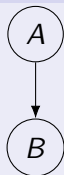
Example

$$w(x) = 0.3, w(\neg x) = 0.7, \\ w(y) = 0.2, w(\neg y) = 0.8$$

$$\text{WMC}(x \vee y) = w(x)w(y) + \\ w(x)w(\neg y) + w(\neg x)w(y) = 0.44$$

The Problem with Assigning Weights to Literals

A Simple Bayesian Network



- ▶ from 2 binary variables
- ▶ to 8 variables and 17 clauses
- ▶ with lots of redundancy

Its WMC Encoding

p cnf 8 17

-2 -1 0

1 2 0

-3 1 0

-1 3 0

-5 -1 0

-5 -4 0

1 4 5 0

-6 -1 0

-6 4 0

-4 1 6 0

-7 1 0

-7 -4 0

-1 4 7 0

-8 1 0

-8 4 0

-4 -1 8 0

-4 0

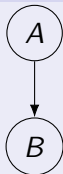
c weights 1.0 1.0 0.5 1.0 \

0.5 1.0 1.0 1.0 0.6 1.0 \

0.4 1.0 0.1 1.0 0.9 1.0

The Problem with Assigning Weights to Literals

A Simple Bayesian Network



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Its WMC Encoding

```
p cnf 8 17
-2 -1 0
1 2 0
-3 1 0
-1 3 0
-5 -1 0
-5 -4 0
1 4 5 0
-6 -1 0
-6 4 0
-4 1 6 0
-7 1 0
-7 -4 0
-1 4 7 0
-8 1 0
-8 4 0
-4 -1 8 0
-4 0
c weights 1.0 1.0 0.5 1.0 \
0.5 1.0 1.0 1.0 0.6 1.0 \
0.4 1.0 0.1 1.0 0.9 1.0
```

$\neg x_1 \Leftrightarrow x_2$

$x_1 \Leftrightarrow x_3$

$\neg x_1 \wedge \neg x_4 \Leftrightarrow x_5$

$\neg x_1 \wedge x_4 \Leftrightarrow x_6$

$x_1 \wedge \neg x_4 \Leftrightarrow x_7$

$x_1 \wedge x_4 \Leftrightarrow x_8$

$\neg x_4$

A More Expressive Alternative

For any propositional formula ϕ over a set of variables X and $p, q \in \mathbb{R}$, let $[\phi]_q^p: 2^X \rightarrow \mathbb{R}$ be the pseudo-Boolean function defined as

$$[\phi]_q^p(Y) := \begin{cases} p & \text{if } Y \models \phi \\ q & \text{otherwise} \end{cases}$$

for any $Y \subseteq X$.

Definition (Pseudo-Boolean Projection (PBP))

A **PBP instance** is a tuple (F, X, ω) , where X is the set of variables, F is a set of two-valued pseudo-Boolean functions $2^X \rightarrow \mathbb{R}$, and $\omega \in \mathbb{R}$ is the scaling factor.

From WMC to PBP

Example

- ▶ Indicator variable: x
- ▶ Parameter variables: p, q
- ▶ Weights: $w(p) = 0.2$, $w(q) = 0.8$, and $w(\neg p) = w(\neg q) = 1$

WMC Clause

$$\neg x \Rightarrow p$$

$$p \Rightarrow \neg x$$

$$x \Rightarrow q$$

$$q \Rightarrow x$$

$$\neg x$$

From WMC to PBP

Example

- ▶ Indicator variable: x
- ▶ Parameter variables: p, q
- ▶ Weights: $w(p) = 0.2$, $w(q) = 0.8$, and $w(\neg p) = w(\neg q) = 1$

WMC Clause	In CNF
$\neg x \Rightarrow p$	$x \vee p$
$p \Rightarrow \neg x$	$\neg x \vee \neg p$
$x \Rightarrow q$	$\neg x \vee q$
$q \Rightarrow x$	$x \vee \neg q$
$\neg x$	$\neg x$

From WMC to PBP

Example

- ▶ Indicator variable: x
- ▶ Parameter variables: p, q
- ▶ Weights: $w(p) = 0.2$, $w(q) = 0.8$, and $w(\neg p) = w(\neg q) = 1$

WMC Clause	In CNF	Pseudo-Boolean Function
$\neg x \Rightarrow p$	$x \vee p$	$[\neg x]_1^{0.2}$
$p \Rightarrow \neg x$	$\neg x \vee \neg p$	
$x \Rightarrow q$	$\neg x \vee q$	$[x]_1^{0.8}$
$q \Rightarrow x$	$x \vee \neg q$	
$\neg x$	$\neg x$	$[\neg x]_0^1$

From WMC to PBP

Example

- ▶ Indicator variable: x
- ▶ Parameter variables: p, q
- ▶ Weights: $w(p) = 0.2$, $w(q) = 0.8$, and $w(\neg p) = w(\neg q) = 1$

WMC Clause	In CNF	Pseudo-Boolean Function
$\neg x \Rightarrow p$	$x \vee p$	$[\neg x]_1^{0.2}$
$p \Rightarrow \neg x$	$\neg x \vee \neg p$	$[x]_{0.2}^{0.8}$
$x \Rightarrow q$	$\neg x \vee q$	$[x]_1^{0.8}$
$q \Rightarrow x$	$x \vee \neg q$	
$\neg x$	$\neg x$	$[\neg x]_0^1$