

Weighted Model Counting with Conditional Weights for Bayesian Networks

Paulius Dilkas Vaishak Belle

University of Edinburgh, Edinburgh, UK

UAI 2021



THE UNIVERSITY OF EDINBURGH

informatics



EDINBURGH CENTRE FOR

ROBOTICS



Engineering and
Physical Sciences
Research Council

The Problem of Computing Probability

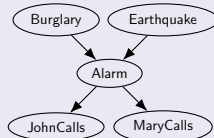
ProbLog

```
0.001 :: burglary.  
0.002 :: earthquake.  
0.95 :: alarm :- burglary, earthquake.  
0.94 :: alarm :- burglary, \+ earthquake.  
0.29 :: alarm :- \+ burglary, earthquake.  
0.001 :: alarm :- \+ burglary, \+ earthquake.  
0.9 :: johnCalls :- alarm.  
0.05 :: johnCalls :- \+ alarm.  
0.7 :: maryCalls :- alarm.  
0.01 :: maryCalls :- \+ alarm.
```

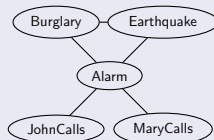
BLOG

```
random Boolean Burglary ~ BooleanDistrib(0.001);  
random Boolean Earthquake ~ BooleanDistrib(0.002);  
random Boolean Alarm ~  
  if Burglary then  
    if Earthquake then BooleanDistrib(0.95)  
    else BooleanDistrib(0.94)  
  else  
    if Earthquake then BooleanDistrib(0.29)  
    else BooleanDistrib(0.001);  
random Boolean JohnCalls ~  
  if Alarm then BooleanDistrib(0.9)  
  else BooleanDistrib(0.05);  
random Boolean MaryCalls ~  
  if Alarm then BooleanDistrib(0.7)  
  else BooleanDistrib(0.01);
```

Bayesian Network



Markov Random Field

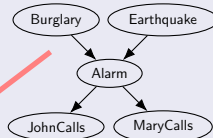


The Problem of Computing Probability

ProbLog

```
0.001 :: burglary.  
0.002 :: earthquake.  
0.95 :: alarm :- burglary, earthquake.  
0.94 :: alarm :- burglary, \+ earthquake.  
0.29 :: alarm :- \+ burglary, earthquake.  
0.001 :: alarm :- \+ burglary, \+ earthquake.  
0.9 :: johnCalls :- alarm.  
0.05 :: johnCalls :- \+ alarm.  
0.7 :: maryCalls :- alarm.  
0.01 :: maryCalls :- \+ alarm.
```

Bayesian Network

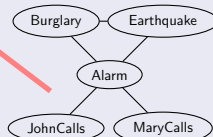


BLOG

```
random Boolean Burglary ~ BooleanDistrib  
random Boolean Earthquake ~ BooleanDistrib  
random Boolean Alarm ~  
  if Burglary then  
    if Earthquake then BooleanDistrib(0.95)  
    else BooleanDistrib(0.94)  
  else  
    if Earthquake then BooleanDistrib(0.29)  
    else BooleanDistrib(0.001);  
random Boolean JohnCalls ~  
  if Alarm then BooleanDistrib(0.9)  
  else BooleanDistrib(0.05);  
random Boolean MaryCalls ~  
  if Alarm then BooleanDistrib(0.7)  
  else BooleanDistrib(0.01);
```

WMC

Markov Random Field



Weighted Model Counting (WMC)

- Generalises propositional model counting ($\#SAT$)
- Applications:
 - graphical models
 - probabilistic programming
 - neural-symbolic artificial intelligence
- Main types of algorithms:
 - using knowledge compilation
 - using a SAT solver
 - manipulating pseudo-Boolean functions

Example

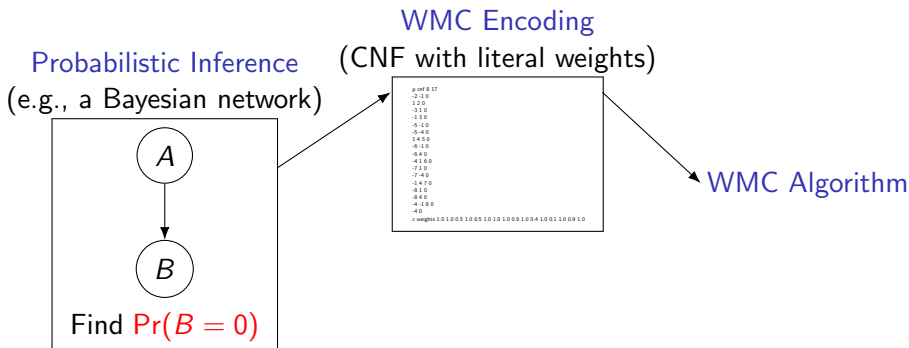
$$w(x) = 0.3, w(\neg x) = 0.7, \\ w(y) = 0.2, w(\neg y) = 0.8$$

$$WMC(x \vee y) = w(x)w(y) + \\ w(x)w(\neg y) + w(\neg x)w(y) = 0.44$$

An Alternative Way to Think About It

Some Theoretical Results

The Big Picture



The Big Picture

