Weighted Model Counting Without Parameter Variables

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Weighted Model Counting (WMC)

- Generalises propositional model counting (#SAT)
- Applications:
  - graphical models
  - probabilistic programming
  - neural-symbolic artificial intelligence

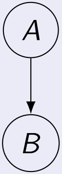
Example

$w(x) = 0.3,\ w(\neg x) = 0.7,$
$w(y) = 0.2,\ w(\neg y) = 0.8$

$\mathrm{WMC}(x \vee y) = w(x)w(y) + w(x)w(\neg y) + w(\neg x)w(y) = 0.44$

The Problem with Assigning Weights to Literals

A Simple Bayesian Network



- from 2 binary variables
- to 8 variables and 17 clauses
- with lots of redundancy

Its WMC Encoding

```
p cnf 8 17
-2 -1 0
1 2 0
-3 1 0
-1 3 0
-5 -1 0
-5 -4 0
1 4 5 0
-6 -1 0
-6 4 0
-4 1 6 0
-7 1 0
-7 -4 0
-1 4 7 0
-8 1 0
-8 4 0
-4 -1 8 0
-4 0
c weights 1.0 1.0 0.5 1.0 \
0.5 1.0 1.0 1.0 0.6 1.0 \
0.4 1.0 0.1 1.0 0.9 1.0
```

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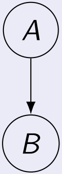

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```

$\neg x_1 \Leftrightarrow x_2$

$x_1 \Leftrightarrow x_3$

$\neg x_1 \wedge \neg x_4 \Leftrightarrow x_5$

$\neg x_1 \wedge x_4 \Leftrightarrow x_6$

$x_1 \wedge \neg x_4 \Leftrightarrow x_7$

$x_1 \wedge x_4 \Leftrightarrow x_8$

$\neg x_4$

A More Expressive Alternative

Definition (Pseudo-Boolean Projection (PBP))

A PBP instance is a tuple (F, X, ω), where X is the set of variables, F is a set of two-valued pseudo-Boolean functions $2^X \to \mathbb{R}$, and $\omega \in \mathbb{R}$ is the scaling factor.

For any propositional formula ϕ over a set of variables X and $p, q \in \mathbb{R}$, let $[\phi]^p_q \colon 2^X \to \mathbb{R}$ be the pseudo-Boolean function defined as

$$[\phi]^p_q(Y) \coloneqq \begin{cases} p & \text{if } Y \models \phi \\ q & \text{otherwise} \end{cases}$$

for any $Y \subseteq X$.

From WMC to PBP

Example

- Indicator variable: x
- Parameter variables: p, q
- Weights: $w(p) = 0.2$, $w(q) = 0.8$, and $w(\neg p) = w(\neg q) = 1$

WMC Clause
$\neg x \Rightarrow p$
$p \Rightarrow \neg x$
$x \Rightarrow q$
$q \Rightarrow x$
$\neg x$

From WMC to PBP

Example

- Indicator variable: x
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WMC Clause	In CNF
$\neg x \Rightarrow p$	$x \vee p$
$p \Rightarrow \neg x$	$\neg x \vee \neg p$
$x \Rightarrow q$	$\neg x \vee q$
$q \Rightarrow x$	$x \vee \neg q$
$\neg x$	$\neg x$

From WMC to PBP

Example

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WMC Clause	In CNF	Pseudo-Boolean Function
$\neg x \Rightarrow p$	$x \vee p$	$[\neg x]_1^{0.2}$
$p \Rightarrow \neg x$	$\neg x \vee \neg p$	
$x \Rightarrow q$	$\neg x \vee q$	$[x]_1^{0.8}$
$q \Rightarrow x$	$x \vee \neg q$	
$\neg x$	$\neg x$	$[\neg x]_0^1$

From WMC to PBP

Example

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WMC Clause	In CNF	Pseudo-Boolean Function	
$\neg x \Rightarrow p$	$x \vee p$	$[\neg x]^{0.2}_{1}$	
$p \Rightarrow \neg x$	$\neg x \vee \neg p$		$[x]^{0.8}_{0.2}$
$x \Rightarrow q$	$\neg x \vee q$	$[x]^{0.8}_{1}$	
$q \Rightarrow x$	$x \vee \neg q$		
$\neg x$	$\neg x$	$[\neg x]^{1}_{0}$	$[\neg x]^{1}_{0}$

Brief Summary

- Fewer variables and clauses/functions
- Improved state of the art for Bayesian network inference
- Some conditions need to be satisfied

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Thank You!