Council for Technical Education and Vocational Training

Office of the Controller of Examinations

Sanothimi, Bhaktapur

Regular/Scholarship Exam-2080 Bhadra

Program: Diploma Civil/Hydropower/Information Technology/Computer Engineering — **Full Marks: 80**

Year/Part: II/I (2021, 2022) — **Pass Marks: 32**

Subject: Engineering Mathematics III — **Time: 3 hrs.**

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Group 'A'

Attempt <u>All</u> questions. **[(7×2)×2=28]**

1. a) Find the derivative of $\log(\tan 2x)$.

   b) Find the derivative of $x^{sinh\frac{x}{a}}$.

2. a) Using L – Hospital rule: Evaluate: $\lim_{x\to 0} \frac{logtanx}{logx}$

   b) Find the points on the curve $y = x^3 - 3x^2 + 1$ where the tangent are parallel to $x - axis$.

3. a) If $f(x, y) = \sqrt{x^2 + y^2}$, then find fxx at the point (2, 1).

   b) Find $\frac{du}{dx}$ if $u = x^2 + y^2, x = at^2$ and $y = 2at$.

4. a) Evaluate: $\int \frac{dx}{e^x + e^{-x}}$

   b) Examine whether the function $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ is even or odd.

5. a) Evaluate: $\int \sqrt{2ax - x^2}\, dx$

   b) Solve: $\sqrt{1 - x^2}\, dy + \sqrt{1 - y^2}\, dx = 0$

6. a) Solve: $x\, dy + y\, dx = 0$

   b) Form the partial differential equations: $z = ax + by + a^2 + b^2$

7. a) Solve: $\frac{dy}{dx} = e^{x-y} + x^3 e^{-y}$

   b) Find the fundamental period of $f(x) = sin2\pi x$.

Group 'B'

Attempt <u>ALL</u> questions. **[13×4=52]**

8. Find the maximum and minimum values of the function $f(x) = x^3 - 6x^2 + 9x - 2$.

Cont. .....

OR

A man wishes to fence a rectangular garden with 256-meter fencing material. Find the maximum area he can enclose.

9. A spherical ball of salt dissolving in water decreases its volume at the rate of 0.75 cm^3/min. Find the rate at which the radius of the salt is decreasing when its radius is 6 cm.

10. Find the equation of the tangent and normal to the curve $y = x^3 - 2x^2 + 4$ at (2, 4).

11. Use definition. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of $f(x, y) = x^2y - xy^2$.

12. Verify Euler's theorem for homogeneous function $f(x, y, z) = x^2 + y^2 + z^2$.

13. Sketch the graph of $y = (x - 1)(x - 2)$.

OR

Find the area of circle $x^2 + y^2 = 36$.

14. Evaluate: $\int \frac{dx}{1-2cosx}$

15. Using limit of the sum, find the area bounded by the curve $y = 3x^2$, the x-axis and the ordinates $x = 0$ and $x = 4$.

16. Solve: $\frac{dy}{dx} = \frac{y}{x} + tan\left(\frac{y}{x}\right)$.

17. Solve: $(1 + x^2)\frac{dy}{dx} + 2xy = 3x^2$.

18. The half-life of isotopic radium is 300 years. Find the time required to decay 10% of its initial amount.

19. Solve: $xz\frac{\partial z}{\partial x} + yz\frac{\partial z}{\partial y} = xy$.

OR

Form the partial differential equations if $x + y + z = f(x^2 + y^2 + z^2)$.

20. Find the Fourier series of the function:

$$f(x) = \begin{cases} 1 & -\pi < x < 0 \\ -1 & 0 \le x < \pi \end{cases}$$

Good Luck !

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Full Marks: 80

Year/Part: II/I (2013, 2014, 2016, 2017, 2018)

Pass Marks: 32

Subject: Engineering Mathematics III

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Group'A'

Attempt All questions. **[(5×2)×3=30]**

1. a. Define Fourier series of a function f(x) on the interval $(-\pi, \pi)$.
   Find the Fourier series of :
   $$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 \le x < \pi \end{cases}$$
   b. Test whether the function $f(x)=x^2, -1<x<1$ is even or odd. Also, find the appropriate Fourier series.

2. a. Define group. The identity element in a group is unique. Prove.

   OR

   A set of matrices of the form $A_\theta = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$ where θ is a number, is given:
   i. Show that the operation of matrix multiplication is closed.
   ii. Show that A_0 is the identity element of A_θ.
   iii. Show that $A-\theta$ is the inverse element of A_θ.

   b. Let G = {0, 1, 2}, form a composition table for G under multiplication modulo 3. Find the identify and inverse element of 1 and 2.

3. a. Using definition method, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ where $f(x, y)=x^2y^2$.

   b. Let $u = sin^{-1}\frac{x^2+y^2}{x+y}$, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

Group'B'

Attempt any TEN questions. **[10×5=50]**

4. By separating variables, solve: $(xy + x)dy - (xy + x)dx = 0$

Cont. .....

5. Change the equation $x^2y\ dx - (x^3 + y^3)dy = 0$ into homogeneous differential equation and solve it.

6. Show that the equation is exact and solve it:
$$(x + y - 1)dx + (x - y - 2)dy = 0$$

7. Form a partial differential equation by eliminating 'f' from:
$$lx + my + nz = f(x^2 + y^2 + z^2)$$

8. Solve : $\frac{dy}{dx} = \frac{y}{x} + tan\frac{y}{x}$

9. By using D' Alembert's ratio test, test the convergence or divergence of the series: $\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \cdots$

10. Test whether the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots$ is absolutely convergent of conditionally convergent.

11. Using Maclurin's series, expand the function $f(x) = Sinx$

12. Find the interval of convergence and radius of convergence of the power series:
$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n(n+1)}{2}x^{n-1}$$

13. Define periodic function. Find the fundamental period (P) of $f(x) = Sin2x$.

14. Test the following series for convergence by Cauchy's root test:
$$\frac{2}{3}x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)x^3 + \cdots + \left(\frac{n+1}{n+2}\right)x^n + \cdots \text{ for } x \neq 1.$$

Good Luck !