Engineering Mathematics-III

Question Bank

II Yr. I Part

ENGINEERING MATHEMATICS-III

Examination 2071 Regular/Back

New Course

Time: 3 hrs. Full marks: 80 Pass marks: 32

Candidates are required to give their answers in their own words as far as practicable.
Attempt any Two questions from Group A and Three questions from Group B

Group A

1. a) Prepare Cayley table for the set $\{0, 1, 2, 3\}$ under the operation multiplication modulo 4. Identify the identity element and the inverse of each element if possible.

   b) Define group. Prove that the identity element of group is unique. Also show that the inverse of group is unique.

2. a) Solve: $(x+y+1)\frac{dy}{dx} = 1.$

   b) Show that the given equation is exact and solve;
   $(x+y-1)dx + (x-y-2)dy = 0.$

3. a) Define convergent and divergent series. Test whether series is convergent or divergent;
   $1 + \frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \ldots\ldots\ldots$

   b) Show that the series is conditionally convergent;
   $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots\ldots$

Group B

4. Form a partial differential equations: (Any one)
   i) $z = ke^{ax} \sin ay$
   ii) $lx + my + nz = f(x^2 + y^2 + z^2)$

5. Solve the partial differential equations. [Any one]
   i) $\frac{\partial z}{\partial x} xz + yz \frac{\partial z}{\partial y} = xy$
   ii) $xp - yq + x^2 - y^2 = 0$

6. Find the interval and radius of convergence of power series. [Any one]
   i) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n(n+1)}{2} x^{n-1}$
   ii) $\sum_{n=0}^{\infty} \frac{(x-2)^n}{10^n}$

7. Assuming the convergence of Taylor's series, find the Maclaurin's series expansion of $\sin x$.

8. Using definition, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ from $f(x, y) = x^2 - xy$.

9. Find $\frac{du}{dt}$ (Any one);

i) $u = e^{xyz}, x = t^3, y = \frac{1}{t}, z = e^t$

ii) $x^2 + y^2 + z^2, x = 2t + 1, y = t + 5, z = 7t.$

10. Let $u = \sin^{-1}\frac{x^2 + y^2}{x + y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

11. Find the Fourier series of given function on the given interval;

$$f(x) = \begin{cases} 0, 0 < x < \pi \\ 1, \pi < x < 2\pi \end{cases}$$

13. Test whether the function is even or odd. Also find the corresponding Fourier series;

$$f(x) = \begin{cases} -2x, -\pi < x < 0 \\ 2x, 0 < x < \pi \end{cases}$$

ENGINEERING MATHEMATICS-III

Examination 2072 Back

Full marks: 80
Pass marks: 32
Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.
Attempt any Two questions from Group A and Three questions from Group B

Group A

1. a) Prepare Cayley table for the set {0, 1, 2, 3, 4, 5}. Under the operation multiplication modulo 6. Identify the identity element and the inverse of each element if possible.

b) Define group. Prove that the identity element of group is unique. Also, show that the inverse of group is unique.

2. a) If $u = \sqrt{x^2 + y^2 + z^2}$, then prove that: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{2}{u}$

b) Let $u = \frac{x^4 + y^4}{x + y}$, prove that: $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u$

3. a) Define the p-series. Test the series for convergence by apply ratio of $\frac{1}{3} + \frac{4}{9} + \frac{9}{27} + \dots\dots\dots$

b) Find the interval and radius of convergence of the power series;

$1 + 2x + 3x^2 + 4x^3 + \dots\dots\dots$

Group B

4. Solve: $(xy^2 + x)\,dx + (yx^2 + y)\,dy = 0$

5. Solve: $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$

6. Solve the partial differential equations: (Any one)
   i) $z = ax + by + a^2 + b^2$
   ii) $xp + yq = z$

7. Show that the given equation is exact and solve:
   $$(2\,ax + by)\,y\,dx + (ax + 2\,by)\,x\,dy = 0$$

8. Using definition, find $\frac{\partial f}{\partial y}$ form $f(x, y) = x^2y$

9. Find the Maclaurin's series expansion of $\cos x$.

10. Find the smallest positive period p of $\sin nx$.

11. Find the Fourier series of given function in the given interval,
   $$f(x) = \begin{cases} 0, & -2 < x < 0 \\ 2, & 0 < x < 2 \end{cases}$$

12. Test whether the function is even or odd. Also, find the corresponding Fourier series;
   $$f(x) = \begin{cases} -2x \text{ for } -\pi < x < 0 \\ -2x \text{ for } 0 < x < \pi \end{cases}$$

13. Find $\frac{du}{df}$: $u = x^2 + y^2$, $x = 2t + 1$, $y = t^2 + 2$

ENGINEERING MATHEMATICS-III

Examination 2073 Regular/Back

Time: 3 hrs. | Full marks: 80 | Pass marks: 32

Candidates are required to give their answers in their own words as far as practicable.

Attempt any Two questions from Group A and Three questions from Group B

Group A

1. a) Using definition, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ where $f(x, y) = xy + y^2$.
   b) Solve the following differential questions by separating the variables.
      i) $\tan x\ dy + \tan y\ dx = 0$
      ii) $\frac{dy}{dx} = e^{x-y} + x^2 \times e^{-y}$

2. a) Define p-series with example. Determine whether the following series is convergent or divergent by comparison test $\frac{1}{1+2} + \frac{2}{1+2^2} + \frac{3}{1+2^3} + \dots\dots\dots$

b) Find the Maclaurin's expansion of the function: $f(x) = \log(1+x)$

3. a) Define periodic function. Find the fundamental period of function $f(x) = \cos 3x$.

b) If $G = \{\dots\dots, -6, -4, -2, 0, 2, 4, 6, \dots\dots\}$, then prove that $(G, +)$ is a group.

Group B

4. Verify Euler's theorem for homogenous function if $u = \dfrac{x^2+z^2}{xy+yz}$

5. Find $\dfrac{du}{dt}$ of (any one)

i) $u = x^2 + y^2, x = \cos t + \sin t, y = \cos t - \sin t$ at $t = 0$

ii) $u = e^{xyz}, x = t^3, y = \dfrac{1}{t}, z = e^t$

6. Solve: $\dfrac{dy}{dx} = \dfrac{x+y}{x+y+1}$

7. Prepare clayey table for the set $\{0, 1, 2, 3, 4\}$ under the operation addition modulo 5. Identify the identity element and the inverse of each element.

8. Show that the given equation is exact and solve;

$$(x+y-1)\,dx + (x-y-z)\,dy = 0$$

9. Form P.D.E. by eliminating the form $lx + my + nz = f(x^2+y^2+z^2)$

10. Define Fourier series check whether the function, $f(x) = \begin{cases} -2x \text{ for } -\pi < x < 0 \\ -2x \text{ for } 0 < x < \pi \end{cases}$ is odd or even and hence obtain the corresponding Fourier series.

11. Show that the following series is divergent.

12. Define alternating series with example. Test whether the following series is absolute convergent or conditionally convergent $1 - \dfrac{1}{2} + \dfrac{1}{4} - \dfrac{1}{8} + \dots\dots$

13. Find the interval of convergence and radius of convergence of the given power series $1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dots\dots$

14. Find the Fourier series expansion of $f(x) = x, \; -2 < x < 2$

15. Define group prove that the identity element of group is unique. Also, prove that the inverse of group is unique.

ENGINEERING MATHEMATICS-III

Examination 2074 Regular/Back

Special scholarship

Full marks: 80
Pass marks: 32

Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.

Attempt all questions.

Group A

1. a) If $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 5 \\ 7 & 2 \end{bmatrix}$. Prove that $B^T A^T = (AB)^T$. [5]

   b) Find the acute angle between the lines whose direction cosines are connected by the relations $l + m + n = 0$ and $l^2 + m^2 + n^2 = 0$ [5]

2. a) Solve using row equivalent matrix method or Cramer's rule: [5]

$$x + 2y - 3z = 0$$
$$2x - y + 3z = 4$$
$$3x + 4y + 7z = 14$$

   b) Prove that: $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$ [5]

3. a) Prove that (use vector method): $\cos(A + B) = \cos A \cos B - \sin A \sin B$ [5]

   b) Define collinear vectors. Prove that the three points with the following vectors are collinear: $\vec{i} + 2\vec{j} + 3\vec{k}, -2\vec{i} + 3\vec{j} + 4\vec{k}, 7\vec{i} + \vec{k}$ [5]

Group B

4. Define complex number and find the cube roots of unity. [5]

5. State De-Moivre's theorem and hence use it to find the square roots of $\frac{1}{2} + i\frac{\sqrt{3}}{2}$. [5]

6. Find the local maxima and local minima and point of inflection if exists: [5]

$$f(x) = 2x^3 - 3x^2 - 12x + 4$$

7. Calculate the Karl Pearson's coefficient of correlation: [5]

X	8	4	12	6	10
Y	11	13	8	9	7

8. A stone thrown into a pond produces a circular ripple which expands from the point of impact. If the radius of the ripple increases at the rate of 1.5 ft/sec., how fast is the area growing with radius is 8 ft. [5]

9. Find the area of the region enclosed by $x^2 = 4ay$ and $x = y$. [5]

10. The probability that a student passes a mathematics test is $\frac{3}{5}$ and he passes both mathematics and a chemistry test is $\frac{1}{5}$. The probability that he passes at least one test is $\frac{19}{20}$. What is the probability that he passes the chemistry test? [5]

11. Calculate A.M., G.M. and H.M. from the following data: [5]

Marks	0–10	10–20	20–30	30–40	40–50	50–60
Number of students	5	7	18	10	8	4

12. Find the area enclosed by the circle $x^2 + y^2 = 64$ [5]

13. Determine the maximum value of the objective function $F(x, y) = x + y$ subject to the constraints $2x + y \le 20$, $2x + 3y \le 24$, $x \ge 0$, $y \ge 0$ (Use graph paper)

ENGINEERING MATHEMATICS-III

Examination 2075 Regular/Back

Full marks: 80
Pass marks: 32
Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.
Attempt all questions.

Group A

1. a) Define partial derivative of a function. Using definition, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$: $f(x, y) = xy + y^2$ [5]

   b) Define homogenous function. State and prove Euler's theorem for a two variable cases. [5]

2. a) Define ordinary differential equation. Solve by separation of variables: (any one) [5]

   i) $\frac{dy}{dx} = \frac{x+y}{x+y+1}$

   ii) $\frac{dy}{dx} + 1 = e^{x+y}$

   b) Show that the given function is homogenous and solve: [5]

   $$x \sin\frac{y}{x} dy = \left(y \sin\frac{y}{x} - x\right) dx$$

   or

   Show that the given equation is exact and solve:

   $$(2ax + by)y\, dx + (a + 2by)x\, dy = 0$$

3. a) Discuss the convergence of given geometric series for $r = 1, -1$ and $|r| < 1$, $|r| > 1$ [5]

   $$a + ar + ar^2 + \ldots\ldots\ldots + ar^{n-1} + \ldots\ldots\ldots$$

   b) Test the convergence of series by comparison test or ratio test: (any one) [5]

   i) $\sum \frac{\sqrt{n}}{n^2+1}$

   ii) $\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \ldots\ldots\ldots$

Group B

4. Find $\frac{du}{dt}$: (any one) [5]

   i) $u = z + \sin(xy), x = t, y = \log t, z = e^{t-1}$

   ii) $u = x^3 - y^3, x = \cos t, y = \sin t$

5. Form partial differential equation: $lx + my + nz = f(x^2 + y^2 + z^2)$ [5]

6. Solve the partial differentiate equation: $y^2p - xyq = x(z - 2y)$

7. Find the interval and radius of convergence of the power series: [5]

   $$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots$$

8. Find the Taylor's series expansion of $f(x) = \sqrt{x}$ about $x = 0$ [5]

9. Obtain the Fourier series: $f(x) = \begin{cases} 0, & -2 < x < 0 \\ 2, & 0 < x < 2 \end{cases}$ [5]

10. Find the Fourier series for the function defined as $f(x) = \begin{cases} \pi, & -1 < x < 0 \\ -\pi, & 0 \le x < 1 \end{cases}$ [5]

11. Define binary operation. Show that multiplication (x) is binary on the set $s = \{1, w, w^2\}$; where, w is the cube root of unity. [5]

12. Find the identity element for the binary operation is defined as $x * y = x + y + 1$ for every $x, y \in R$. Also, find inverse of 2 and -3.[5]

13. Let $G = R - \{-1\}$, the set of real numbers without –1. An operation * is defined on G by $x * y = x + y + xy$ for all $x, y \in G$. Show that $(G, *)$ is a group. [5]

ENGINEERING MATHEMATICS-III

Examination 2076 Regular/Back

Time: 3 hrs. **Full marks: 80**
Pass marks: 32

Candidates are required to give their answers in their own words as far as practicable.
Attempt all questions.

Group A

1. a) Define trigonometric and Fourier series. Determine the Fourier coefficient a_0 by Euler's formula. [1+1+3]

   b) Find the Fourier series of the function: [5]

   $$F(x) = \begin{cases} 0 & 0 < x \le \pi \\ 1 & \pi < x \le 2\pi \end{cases}$$

2. a) Define a group. In a group, prove that: $(a \times b)^{-1} = b^{-1} \times a^{-1}$ [5]
   Also, prove that inverse of each element of a group is unique.

b) Given a set $G = \{0, 1, 2, 3, 4\}$ and a binary operation addition modulo $5(+_5)$ is defined on G. Prepare Caley's table for it. Find the identity and inverse element of 3 and 4. [5]

3. a) By using definition of partial derivatives, find F_x and F_y for $F(x, y) = x^2y - xy^3$ [5]

b) If $U = \sqrt{x^2 + y^2 + z^2}$; then prove that; [5]

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{2}{u}$$

Group B

4. Solve: $(1 + \cos y)dy = (1 - \cos x)dx$ [5]
5. Solve: $x\frac{dy}{dx} = y - x\tan\frac{y}{x}$ [5]
6. Form a P.D.E.; $z = \phi(x + iy) + \phi(x - iy)$ [5]
7. By using ratio test, test the convergence or divergence of the series [5]

$$\sum_{n=1}^{\infty} \frac{n!}{(2n-1)!}$$

8. Test the following series for convergence by Cauchy root test. [5]

$$x + \frac{3}{5}x^2 + \frac{8}{10}x^3 + \ldots\ldots\ldots$$

9. Find the radius and interval of convergence of the power series [5]

$$\sum_{n=0}^{\infty} \frac{(x-3)^n}{10^n}$$

10. Define periodic function and its period. Find the least period (fundamental period) of $\tan\frac{3x}{5}$. [5]
11. Find the Taylor's series expansion of $f(x) = \frac{1}{1-x}$ at $x = 0$ [5]
12. Find $\frac{dU}{dx}$ of $U = e^{xyz}, x = t^3, y = \frac{1}{t}, z = e^t$ [5]
13. A binary operation is defined on the set of real numbers as below: $m \times n = 2mn - m + 3n$ for all $m, n, \in R$. Find the identity element and inverse element of 2 and -4. [5]

ENGINEERING MATHEMATICS-III

Examination 2079 Regular/Back

Full marks: 80
Pass marks: 32

Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.

Attempt all questions.

Group A

1. a) Using definition, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ when $f(x, y) = x^3 + y^3 + 3axy$ [5]

   b) Find $\frac{du}{dt}$ of $u = e^{xyz}, x = t^3, y = \frac{1}{t}, z = e^t$ [5]

2. a) Define group. Prove that the identity element of group is unique. Also, show that the inverse of group is unique. [5]

   b) If $G = \{\ldots\ldots, 6, -4, -2, 0, 2, 4, 6, \ldots\ldots\}$ then prove that $(G, +)$ is a group. [5]

3. a) Test whether the following series is absolutely or conditionally convergent: $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \ldots\ldots\ldots$ [5]

   b) Find the Taylor's series expansion of $f(x) = e^{-x}$ about $x = 2$. [5]

Group B

4. Solve by separating the variables: [5]

   a) $e^{x-y}dx + e^{y-x}dy = 0$ b) $\frac{dy}{dx} = -\frac{1 + \cos 2y}{1 - \cos 2x}$

5. Solve the homogeneous differential equation: $\frac{dy}{dx} = \frac{x^2y}{x^3 + y^3}$ [5]

6. Find the Fourier series expansion of $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 \le x < \pi \end{cases}$ [5]

7. Define periodic function. Find the smallest positive period of P of sin nx. [5]

8. Prepare Cayley table for the set {0, 1, 2, 3, 4, 5} under the operation multiplication module 6. Identify the identity element and the inverse of each element if possible. [5]

9. Solve the partial differential equations: (any one) [5]

   a) $\frac{\partial z}{\partial x}xz + yz\frac{\partial z}{\partial y} = xy$ b) $xp - yq + x^2 - y^2 = 0$

10. Find the interval and radius of convergence of the series: $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n^2}$ [5]

11. Verify Euler's theorem for homogeneous function: $f(x, y, z) = x^2 + y^2 + z^2$ [5]

12. Define convergent and divergent series. Determine whether the followings series is convergent of divergent by ratio test. $\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \ldots\ldots\ldots$ [5]

13. Test whether the function is even or odd. Find the corresponding Fourier series $f(x) = \begin{cases} \pi, & -1 < x < 0 \\ -\pi, & 0 \le x < 1 \end{cases}$ [5]

Model Question Set – I

New Course : 2021 (Diploma in Engineering All)
Year: II
Semester: I
Subject: Engineering Mathematics III

F.M. = 80
P.M. = 32
Time: 3 hrs

Group 'A'

$[\mathbf{7} \times (\mathbf{2} + \mathbf{2}) = \mathbf{28}]$

Attempt All Question

1. (a) Find the derivative of $sin^{-1}(3x-4)$.

   (b) Find the slope of the curve $y = 2x^2 - x$ at $(1,0)$.

2. (a) Find the points of stationary points $f(x) = x^3 - 3x^2 + 9$.

   (b) State L' Hospitals's Theorem and use it to evaluate

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$$

3. (a) Find first order partial derivatives of

$$f(x, y) = ax^2 + 2hxy + by^2.$$

   (b) Find $\frac{\partial y}{\partial x}$ if $y = x^2 - 5x + 7$ and $x + 9rs + 2r^2s^2$.

4. (a) Test whether the given function is even, odd or neither where,

$$f(x) = \sqrt{1 + x^2} - \sqrt{1 + x^2}$$

   (b) Find the smallest period of $f(x) = \sin 2x$.

5. (a) Find the area bounded by the curve $y = 4x^2, x - axis$ and the ordinates $x = 0, x = 2$.

   (b) Determine the order and degree of the differential equation:

$$\frac{dy}{dx} = (x + y + 1)^2$$

6. (a) Solve by separation of variable method of

$$(1 + cosx)dy = (1 - cosx)dx \ .$$

(b) Text the exactness of $(x + y^2)\ dx - 2xy\ dy = 0$.

7. (a) State Langrange's Linear Differential Equation with an example.

(b) Define Orthogonality of two function with example.

Group 'B' $[\mathbf{13 \times 4 = 52}]$

Attempt All Questions

8. Find the equation of tangent and normal to the curve

$$f(x) = x^2 + 3x + 1 \text{ at } (0, 1).$$

9. Find the local maxima and local minima of the function $f(x) = 2x^3 - 15x^2 + 36x + 5$. Also, find the point of inflection.

10. Evaluate: $\lim_{x \to 0} \frac{\tan x - x}{x^2 \tan x}$

11. If $u = \log(x^2 + y^2)$ then show that $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 y}{\partial y^2} = 0$.

12. State the Euler's theorem of homogenous function and show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u \text{ for } u = \sin^{-1}\frac{x^2+y^2}{x+y}.$$

13. Integrate the standard integral: $\int \frac{dx}{a\ sin\ x + b\ cos\ x}$.

14. Find the area of the plane region bounded by the x-axis the curve $y = e^x$and the ordinates $x = 0, x = b$ using the limit of sum.

15. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

16. Define linear equation and solve $2\ cos\ x\frac{dy}{dx} + 4y\ sin\ x = sin\ 2x$.

17. Show that the differential equation exact and solve:

$$(x + y - 1)\,dx + (x - y - 2)dy = 0$$

18. If the normal at every point of a curve passes through a fixed point, using first order differential equation so that the curve is circle.

19. Solve: $(mz - ny)p + (nx - lz)q = ly - mx$.

20. Find the Fourier series of $f(x)$ which is defined by

$$f(x) = \begin{cases} 0 \quad for - \pi \le x \le 0 \\ \frac{\pi}{4}x \quad for\ 0 < x \le \pi \end{cases}$$

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Model Question Set-II

New Course - 2021 (Three Years Diploma in Engineering All Programmes)
Year - II
Semester - I
Subject: Engineering Mathematics - III

F.M. : 80
P.M. : 32
Time: 3 hrs

Group 'A'

Attempt All questions. **[(7 × 2) × 2 = 28]**

1. (a) Find the derivative of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$.
   (b) Find the slope of the curve $y = 2x + x^2$ at (0, 1).
2. (a) Define stationery points in a curve and find the points of stationary in the curve $5x^3 - 135x + 22$.
   (b) State L'Hospital's Theorem and use it to evaluate $\lim_{x \to 3} \frac{x^3 - 27}{x - 3}$.
3. (a) Define partial derivative of a function and find fx for $f(x, y) = x^2 y^2 + 2y$
   (b) Find $\frac{\partial y}{\partial x}$ if $y = x^2 - 5x + 7$ and $x = 9rs + 2r^2s^2$.
4. (a) Test whether the given function is even, odd or neither where $f(x) = \sqrt{1+x^2} - \sqrt{1-x^2}$
   (b) Test the periodicity and find the smallest period of $f(x) = \cos 2x$.
5. (a) Find the area bounded by the curve $y = 3x^2$, x-axis and the ordinates $x = 1, x = 2$.
   (b) Determine the order and degree of the differential equation: $\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = xy$.
6. (a) Solve by separation of variable method of : $(1 + x)\, y dx + (1 + y)\, x\, dy = 0$.
   (b) Define exact differentiate equation and test the exactness of $(x + y - 1)\, dx + (x - y - z)\, dy = 0$
7. (a) Define the Langrange's Linear Partial Differential Equation with an example.
   (b) Define Fourier Series and write the formulae to find the Fourier coefficients.

Group 'B'

Attempt All questions. **[13 × 4 = 52]**

8. Find the equation of tangent and normal to the curve $x^2 - y^2 = 7$ at (1, 2).

   OR

   If the area of a circle increases at a uniform rate prove that the rate of increase of the perimeter varies inversely as the radius.
9. Find the local maxima and local minima of the function below and also find the point of inflection $f(x) = 2x^3 - 15x^2 + 36x + 11$.
10. Evaluate : $\lim_{x \to 0} \frac{x \cos x - \log(1 + x)}{x^2}$
11. If $u = \log(x^2 + y^2)$ then show that $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 v}{\partial y^2} = 0$.

12. State the Euler's theorem of homogeneous function and show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$ for $u=\tan^{-1}\frac{x^3+y^3}{x-y}$.

13. Integrate the standard integral : $\int \frac{dx}{3\sin x+4\cos x}$.

14. Find the area of the plane region bounded by the x-axis the curve $y=e^{3x}$ and the ordinates $x=0, x=b$ using the limit of sum.

15. Find the area of the circle $x^2+y^2=16$.

16. Solve the differential equation $x\frac{dy}{dx}=y-x\tan\frac{y}{x}$.

17. Show that the differential equation exact and solve:
$(5x^4+3x^2y^2-2xy^3)\,dx+(2x^3y-3x^2y^2-5y^4)\,dy=0$

18. Define linear differential equation and solve: $x\frac{dy}{dx}+y=x^4$.

19. Solve: $(xz-ny)p+(nx-lz)\,q=ly-mx$.

OR

If the portion of a tangent to a curve included between the co-ordinate axes is bisected by the point of contact then show that the curve is rectangular hyperbola.

20. Find the Fourier series of $f(x)$ which is defined by $f(x)=\begin{cases}1 & \text{for } -\pi<x<0\\ -1 & \text{for } 0\le x<\pi\end{cases}$

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