DIPLOMA, CTEVT, Questions and Solutions

2078, 2079

Engineering Mathematics-III

(for Diploma II Yrs. I Part)

Third Semester

Diploma in Engineering

By

Arjun Chaudhary

Published in : www.arjun00.com.np
www.facebook.com/arjun00.com.np

Authors © Arjun Chaudhary

**Download from www.arjun00.com.np

S.No	Exam Year, Month	
1.	2078 Kartik	Regular/Back
2.	2079 Bhadra	Regular/Back

Any Problem Than Contact on
Email:- arjun7.com.np@gmail.com

***Download from www.arjun00.com.np

**Download from www.arjun00.com.np

Engg. Mathematics-III (Engg. All) 3rd Sem

(2078) Question Paper Solution.

Compile by Arjun Chy

Website :- www.arjun00.com.np

Facebook :- www.facebook.com/Arjun00.com.np

Group 'A'

1.a) Using definition find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of $f(x,y) = x^2y - xy^2$

Solution:

Let $u = f(x,y)$ be a function of two variables x and y. Then, the partial derivatives of $u = f(x,y)$ with respect to x is obtained by differentiating $u = f(x,y)$ with respect to x treating y as constant and

denoted by symbol $\frac{\partial y}{\partial x}$ or $\frac{\partial f}{\partial x}, f_x(x,y)$

$$\frac{\partial f}{\partial x} = \lim_{n\to\infty} \frac{f(x+\Delta x,y)-f(x,y)}{\Delta x}$$

$$\text{or, } \frac{\partial f}{\partial y} = \lim_{n\to\infty} \frac{f(x+\Delta x,y)-f(x,y)}{\Delta y}$$

Now,

$$\frac{\partial f}{\partial x} = \lim_{n\to\infty} \frac{f(x+\Delta x,y)-f(x,y)}{\Delta x}$$

$$\frac{\partial f}{\partial x} = \lim_{n\to\infty} \frac{(x+\Delta x)^2y - (x+\Delta x)y^2 - (x^2y - xy^2)}{\Delta x}$$

**Download from www.arjun00.com.np

$$\frac{\partial f}{\partial x} = \lim_{n\to\infty} \frac{[x^2+2x\Delta x+(\Delta x)^2]y-(x+\Delta x)y^2-x^2y+xy^2}{\Delta x}$$

$$\frac{\partial f}{\partial x} = \lim_{n\to\infty} \frac{x^2y+2x\Delta xy+(\Delta x)^2y-xy^2-\Delta xy^2-x^2y+xy^2}{\Delta x}$$

$$\frac{\partial f}{\partial x} = \lim_{n\to\infty} \frac{2x\Delta xy+(\Delta x)^2y-\Delta xy^2}{\Delta x}$$

$$\frac{\partial f}{\partial x} = \lim_{n\to\infty} \frac{\Delta x\left(2xy+\Delta xy-y^2\right)}{\Delta x}$$

$$\therefore \boldsymbol{\frac{\partial f}{\partial x} = 2xy - y^2}$$

Next, $$\boldsymbol{\frac{\partial f}{\partial y} = \lim_{n\to\infty} \frac{f(x,y+\Delta y)-f(x,y)}{\Delta y}}$$

$$\frac{\partial f}{\partial y} = \lim_{n\to\infty} \frac{(y+\Delta y)x^2-(y+\Delta y)^2x-(x^2y-xy^2)}{\Delta y}$$

$$\frac{\partial f}{\partial y} = \lim_{n\to\infty} \frac{(y+\Delta y)x^2-[y^2+2y\Delta y+(\Delta y)^2]x-x^2y+xy^2}{\Delta y}$$

$$\frac{\partial f}{\partial y} = \lim_{n\to\infty} \frac{yx^2+\Delta yx^2-y^2x-2y\Delta yx-(\Delta y)^2x-x^2y+xy^2}{\Delta y}$$

$$\frac{\partial f}{\partial y} = \lim_{n\to\infty} \frac{\Delta yx^2-2y\Delta yx-(\Delta y)^2x}{\Delta y}$$

$$\frac{\partial f}{\partial y} = \lim_{n\to\infty} \frac{\Delta y(x^2-2yx-\Delta yx)}{\Delta y}$$

**Download from www.arjun00.com.np

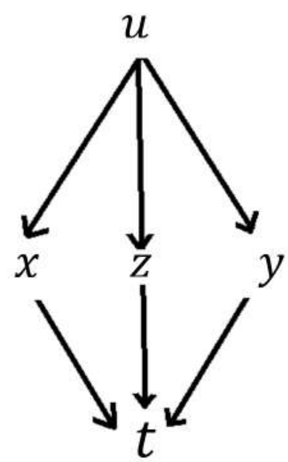
$$\therefore \frac{\partial f}{\partial y} = x^2 - 2yx$$

b) if $u(x,y,z) = x^2 + y^2 + z^2, x = 2t + 1, y = t + 5$ and $z = 7t$ find du/dt.

Solution:

Here u is a function of x, and z while x, y and z are functions of t

$\therefore$ u is a composite function of t



$$\frac{\partial u}{\partial x} = \frac{\partial(x^2 + y^2 + z^2)}{\partial x} = 2x$$

$$\frac{\partial u}{\partial y} = \frac{\partial(x^2 + y^2 + z^2)}{\partial y} = 2y$$

$$\frac{\partial u}{\partial z} = \frac{\partial(x^2 + y^2 + z^2)}{\partial z} = 2z$$

$$\boldsymbol{x} = 2t + 1, dx = 2dt \Rightarrow \frac{dx}{dt} = \mathbf{2}$$

$$\boldsymbol{y} = t + 5, dy = dt \Rightarrow \frac{dy}{dt} = \mathbf{1}$$

$$\boldsymbol{z} = 7t, dz = 7dt \Rightarrow \frac{dz}{dt} = \mathbf{7}$$

$$\boldsymbol{Now,} \frac{\boldsymbol{du}}{\boldsymbol{dt}} = \frac{\boldsymbol{\partial u}}{\boldsymbol{\partial x}} \cdot \frac{\boldsymbol{dx}}{\boldsymbol{dt}} + \frac{\boldsymbol{\partial u}}{\boldsymbol{\partial y}} \cdot \frac{\boldsymbol{dy}}{\boldsymbol{dt}} + \frac{\boldsymbol{\partial u}}{\boldsymbol{\partial z}} \cdot \frac{\boldsymbol{dz}}{\boldsymbol{dt}}$$

$$= 2x \cdot 2 + 2y + 2z \cdot 7$$

$$= \mathbf{2(2\boldsymbol{x} + \boldsymbol{y} + 7\boldsymbol{z})}$$

Copyright © belongs to the Arjun Chy

**Download from www.arjun00.com.np

2. a) State limit comparisons test and use it to test the convergent or divergent of the infinite series

$$\sum\sqrt{n^2+1}-n$$

Let,

$$u_n = \sqrt{n^2+1} - n$$

$$u_n = \sqrt{n^2+1} - n\,\frac{\sqrt{n^2+1}+n}{\sqrt{n^2+1}+n}$$

$$u_n = \frac{n^2+1-n^2}{\sqrt{n^2+1}+n}$$

$$u_n = \frac{1}{\sqrt{n^2+1}+n}$$

Inspecting leading terms of numerator and denominator, we can see effective power of n is

$$0-1=-1$$

$$v_n = n^{-1}$$

$$= \frac{1}{n}$$

Which diverges since $p \leq 1$

Now, taking limit

$$\lim_{n\to\infty}\left(\frac{u_n}{v_n}\right) = \lim_{n\to\infty}\frac{\frac{1}{\sqrt{n^2+1}+n}}{\frac{1}{n}}$$

**Download from www.arjun00.com.np

$$= \lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}+n}$$

$$= \lim_{n\to\infty} \frac{1}{\frac{\sqrt{n^2+1}+n}{n}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{\frac{n^2+1}{n^2}}+\frac{n}{n}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{\frac{n^2}{n^2}+\frac{1}{n^2}}+1}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n^2}}+1}$$

$$= \frac{1}{2}$$

Which is finite and not equal to 0

Hence by comparison test the series diverges since v_n diverges.

b) Find the Fourier series of the function

$$\boldsymbol{f(x) = \begin{cases} 1 & -\pi < x \le 0 \\ -1 & 0 \le x < \pi \end{cases}}$$

The Fourier series of $f(x)$ in the interval $(-\pi, \pi)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

**Download from www.arjun00.com.np

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{-\pi} f(x)\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} dx + \int_{0}^{\pi} -dx\right]$$

$$= \frac{1}{\pi}(x|_{-\pi}^{0} - [x|_{0}^{\pi})$$

$$= \frac{1}{\pi}\{[0-(-\pi)] - [\pi - 0]\}$$

$$= \frac{1}{\pi}[\pi - \pi]$$

$$= \mathbf{0}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{-\pi} f(x)\cos nx dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\cos nx dx + \int_{0}^{\pi} f(x)\cos nx dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} cosnxdx + \int_{0}^{\pi} -\cos nx dx\right]$$

$$= \frac{1}{\pi}\left[\frac{\sin nx}{n}\bigg|_{-\pi}^{0} - \frac{\sin nx}{n}\bigg|_{0}^{\pi}\right]$$

$$= \frac{1}{n\pi}\{(\sin 0 + \sin n\pi) - (\sin n\pi - \sin 0)\}$$

$$= \frac{\mathbf{1}}{\boldsymbol{n\pi}}(\mathbf{0} - \mathbf{0}) = \mathbf{0}$$

**Download from www.arjun00.com.np

$$b_n = \frac{1}{\pi}\int_{-\pi}^{-\pi} f(x)\sin x dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\sin nx dx + \int_{0}^{\pi} f(x)\sin nx dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} sinxdx + \int_{0}^{\pi} - sin\, nxdx\right]$$

$$= \frac{1}{\pi}\left[-\frac{\cos nx}{n}\bigg|_{-\pi}^{0} + \frac{\cos nx}{n}\bigg|_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{\cos nx}{n}\bigg|_{0}^{\pi} - \frac{\cos nx}{n}\bigg|_{-\pi}^{0}\right]$$

$$= \frac{1}{n\pi}[(\cos n\pi - \cos 0) - (\cos 0 - \cos n\pi)$$

$$= \frac{1}{n\pi}[\cos n\pi - 1 - 1 + \cos n\pi]$$

$$= \frac{1}{n\pi}[2\cos n\pi - 2]$$

$$= \frac{1}{n\pi}[2(-1)^n - 2]$$

If n is even $b_n = 0$

If n is odd, $b_n = -\frac{4}{n\pi}$

Substituting the values of a_0, a_n and b_n we get

$$f(x) = \sum_{n=1}^{\infty}\left(-\frac{4}{n\pi}\right)\sin nx$$

$$f(x) = -\frac{4}{\pi}\left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{x} + \cdots\right)$$

**Download from www.arjun00.com.np

3.a) Define a group and prove that the identity element of group is unique. Again prove that inverse of group is unique

- An algebraic structure $(G,*)$ where G is a non-empty set with a operation * defined on it is said to be group, if the operation satisfies following axioms.

  A group is nonempty set G together with a binary operation * that satisfies following axioms.

Closure

If $a \in G, b \in G$ then $aob \in G$ or in other words if $a \in G, b \in G$ then $aob = c$ (closure) where $c \in G$

Associativity

If $a, b, c \in G$ then $ao(boc) = (aob)oc$

Existence of identity

if $a \in G, \exists$ an identity elelment $e \in G$ s.t eoa $= a \forall a \in G$

Existence of inverse

if $a \in G, \exists$ an inverse $a^{-1} \in G$ s.t $a^{-1}oa = e$ where $e \in G,$ being an identity element.

i) The inverse of each element of group is unique

Proof:

Let a be any element of group G and let e be the identity element. Suppose b and c are two inverses of a *i.e.,*

$$ba = e = ab \text{ and } ca = e = ac$$

**Download from www.arjun00.com.np

We have,

$$b(ac) = be[\because ac = e]$$
$$= \boldsymbol{b}[\boldsymbol{e} \textbf{ is identity }]$$

Similarly,

$$(ba)c = ec[\because ba = e]$$
$$= \boldsymbol{c}[\boldsymbol{e} \textbf{ is identity }]$$

But in group the composition is associative.

Therefore, $b(ac) = (ba)c$. Hence $b =$ c. **Hence** inverse of each element of group is unique.

ii) The identity element in a group is unique

Suppose e and e' are two identity elements of group G.We have, $ee' = e$, if e' is identity $ee' = e$, if e is identity,

But ee' is unique element of G

Therefore $ee' = e$ and $ee' = e' \Rightarrow e = e'$

Hence the identity element is unique.

**Download from www.arjun00.com.np

b) Let $s = \{0, 1, 2, 3, 4\}$. Show that S form group under the addition modulo 5 .

Solution: Let $G = \{0,1,2,3,4\}$

x_5	**0**	**1**	**2**	**3**	**4**
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

i) Closure Property

G is closed under addition modulo for 5

ii) Identity Element

$1 +_5 0 = 1 = 0 +_5 1$

$2 +_5 0 = 2 = 0 +_5 2$

0 is the identity element

iii) Existence of Inverse

$1 +_5 4 = 0 = 4 +_5 1$

$2 +_5 3 = 0 = 3 +_5 2$

1 is the inverse of 4.4 is inverse of 1 and so on.

Group G is also associative. Thus, $(G, +_5)$ is a group.

**Download from www.arjun00.com.np

Group'B'

4. Solve by separating the variable $\sqrt{1-x^2}dy+\sqrt{1-y^2}dx=0$

Given that:

$$\sqrt{1-x^2}dy+\sqrt{1-y^2}dx=0$$

On Dividing $\sqrt{1-x^2}\sqrt{1-y^2}$ on both sides, we get

$$\frac{dy}{\sqrt{1-y^2}}+\frac{dx}{\sqrt{1-x^2}}=0$$

On integrating we get,

$$\int\frac{dy}{\sqrt{1-y^2}}+\int\frac{dx}{\sqrt{1-x^2}}=C$$

$\therefore \sin^{-1} y+\sin^{-1} x=C$ **which is the required solution.**

5. Solve the homogeneous differential equation $\frac{dy}{dx}=\frac{x^2+y^2}{2x^2}$

Solution:

$$\frac{dy}{dx}=\frac{x^2+y^2}{2x^2}$$

Let $y=vx$

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

**Download from www.arjun00.com.np

$$\therefore v + x\frac{dv}{dx} = \frac{x^2 + (vx)^2}{2x^2}$$

$$\therefore v + x\frac{dv}{dx} = \frac{x^2(1 + v^2)}{2x^2}$$

$$\therefore v + x\frac{dv}{dx} = \frac{1 + v^2}{2}$$

$$\frac{2v}{1 + v^2}dv = \frac{dx}{2x}$$

On integrating both sides,

$$\int \frac{2v}{1 + v^2}dv = \frac{1}{2}\int \frac{dx}{x}$$

Let $t = 1 + v^2$

$\frac{dt}{dv} = 2v$

$dt = 2vdv$

Now,

$$\int \frac{dt}{t} = \frac{1}{2}\log x$$

$$\log t = \frac{1}{2}\log x$$

$$2\log t = \log x$$

**Download from www.arjun00.com.np

On Substituting $t = 1 + v^2$

$$2\log(1 + v^2) = \log x$$

On substituting $y = vx \Rightarrow v = \frac{y}{x}$

$$2\log\left(1 + \frac{y^2}{x^2}\right) = \log x$$

$$2\log\left(1 + \frac{y^2}{x^2}\right) - \log x = 0$$

$$2\log\left[\frac{1 + \frac{y^2}{x^2}}{x}\right] = 0$$

$$\log\left[\frac{1 + \frac{y^2}{x^2}}{x}\right] = 0$$

$$\frac{1 + \frac{y^2}{x^2}}{x} = e^0$$

$$\mathbf{1} + \frac{\boldsymbol{y^2}}{\boldsymbol{x^2}} = \boldsymbol{x}$$

Which is required solution:

**Download from www.arjun00.com.np

6.Solve the partial differential equation (Any one)

a) $\boldsymbol{z = ax + by + a^2 + b^2}$

Differentiating partially with respect to x and y

$$p = \frac{\partial z}{\partial x} = a + b \cdot 0 + 0 = a$$

$$q = \frac{\partial z}{\partial y} = b$$

Substituting $\boldsymbol{a}$ and $\boldsymbol{b}$ in above equation

$$\boldsymbol{z = px + qy + p^2 + q^2}$$

is required partial differential equation.

b) $\boldsymbol{xp - yq + x^2 - y^2 = 0}$

Solution:

Comparing with equation $\mathbf{Pp + Qq = R}$

$$p = x$$
$$q = y$$
$$R = y^2 - x^2$$

The auxiliary equation are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

**Download from www.arjun00.com.np

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{y^2 - x^2}$$

On choosing multipliers $x, -y$ and, we get,

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{y^2 - x^2} = \frac{xdx - ydy + dz}{0}$$

Hence,

$$xdx - ydy + dz = 0$$

On integration,

$$\int xdx - \int ydy + \int dz = 0$$

$$\frac{x^2}{2} - \frac{y^2}{2} + z = \frac{C_1}{2}$$

$$x^2 - y^2 + 2z = C_1$$

Solving first two fraction, we get,

$$\frac{dx}{x} = -\frac{dy}{y}$$

Integration both sides, we get,

$$\int \frac{dx}{x} = -\int \frac{dy}{y}$$

**Download from www.arjun00.com.np

$$\log x = -\log y - \log C_2$$

$$\log x = -\log(yC_2)\ x = -yC_2$$

$$C_2 = -\frac{x}{y}$$

Since C_2 is arbitrary value, we can write it as, $C_2 = \frac{x}{y}$

Hence a solution is $\frac{x}{y} = f(x^2 - y^2 + 2z)$

7. Solve $(\boldsymbol{mz - ny})\boldsymbol{p} + (\boldsymbol{nx - lz})\boldsymbol{q} = \boldsymbol{ly - mx}$

Solution:-

The given equation is

$$(\boldsymbol{mz - ny})\boldsymbol{p} + (\boldsymbol{nx - lz})\boldsymbol{q} = \boldsymbol{ly - mx}$$

$$\text{Comparing with } \mathrm{Pp + Qq = R}$$

$$\mathrm{P = mz - ny}$$

$$\mathrm{Q = nx - lz}$$

$$\mathrm{R} = l\mathrm{y - mx}$$

The auxillary equation are

$$\frac{dx}{mz - ny} = \frac{dy}{nx - lz} = \frac{dz}{ly - mx}$$

Choosing multipliers $\boldsymbol{x}, \boldsymbol{y}$ and $\boldsymbol{z}$

$$= \frac{xdx + ydy + zdz}{mxz - nxy + nxy - lzy + lyz - mxz}$$

**Download from www.arjun00.com.np

$$= \frac{xdx + ydy + zdz}{0}$$

Hence,

$$xdx + ydy + zdz = 0$$

Integrating we get,

$$\int xdx + \int \; ydy + \int \; zdz = 0$$

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = \frac{C_1}{2}$$

$$\boldsymbol{x^2 + y^2 + z^2 = C_1}$$

On choosing multipliers $\boldsymbol{l}, \boldsymbol{m}$ and $\boldsymbol{n}$

$$= \frac{\mathrm{ld}\, x + mdy + zdz}{lmz - \ln y + mnx - lmz + \ln y - mnx}$$

$$ldx + mdy + zdz = 0$$

On integrating,

$$\int ldx + \int mdy + \int ndz = 0$$

$$lx + my + nz = C_2$$

Hence, the general solution is

$$\boldsymbol{(x^2 + y^2 + z^2) = f(lx + my + nz).}$$

**Download from www.arjun00.com.np

8. Test the convergent of the series and find the sum if convergent

$$\mathbf{3+\frac{3}{-4}+\frac{3}{(-4)^2}+\cdots}$$

The given series,

$$3+\frac{3}{-4}+\frac{3}{(-4)^2}+\cdots+\frac{3}{(-4)^{n-1}}$$

The nth partial sum of this series is

$$\boldsymbol{S_n}=3+\frac{3}{-4}+\frac{3}{(-4)^2}+\cdots\frac{3}{(-4)^2}+\cdots$$

$$=3\left(1+\frac{1}{(-4)}+\frac{1}{(-4)^2}+\cdots.+\frac{1}{(-4)^{n-1}}\right)$$

$$=3\left[\frac{1-\left(-\frac{1}{4}\right)^n}{1-\left(-\frac{1}{4}\right)}\right]$$

$$=3\left[\frac{1-(-1)^n\frac{1}{4^n}}{\frac{5}{4}}\right]$$

$$=\frac{12}{5}\left[1-(-1)^n\frac{1}{4^n}\right]$$

$$\lim_{n\to\infty}S_n=\frac{12}{5}$$ **Which is finite**

Hence, the given series is convergent and it's sum is 12/5

**Download from www.arjun00.com.np

9. Test whether the given series below is absolutely convergent or conditionally convergent

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$

Let,

$$u_n = \frac{(-1)^{n+1}}{\sqrt{n}}$$

Now,

$$|u_n| = \left|\frac{(-1)^{n+1}}{\sqrt{n}}\right| = \frac{1}{n^{1/2}}$$

On comparing with p-series test, $p \leq 1$. Hence it is divergent. Now,

$$|u_n| = \left|\frac{(-1)^{n+1}}{\sqrt{n}}\right|$$

The given series is alternating series

$$|u_n| - |u_{n+1}| = \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}$$

$$= \frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n}\sqrt{n+1}} > 0$$

iii) $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

Hence, by Leibnitz's test, the series is convergent.

The above series is conditionally **convergent series**.

**Download from www.arjun00.com.np

10. Find the interval and radius of convergence of the power series $1+2x+4x^2+8x^3+\cdots$.

Solution:

The given Power series is $\mathbf{1+2x+4x^2+8x^3+\cdots}$

We have,

$$u_n = 2^n x^n$$
$$u_{n+1} = 2^{n+1}x^{n+1}$$

$$\text{so, } \frac{u_{n+1}}{u_n} = \frac{2^{n+1}x^{n+1}}{2^n x^n} = 2x$$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}|2.x| = 2|x|$$

So, by D' Alember's ratio test, the series is convergent For $|x| < \frac{1}{2}$.

i.e., the series is convergent for $-\frac{1}{2} < x < \frac{1}{2}$ and divergent for $|x| > \frac{1}{2}$.

When $x = 1$, the given series becomes;

$1+2+4+8+$………; which is divergent because it is a p-series with $p = -\frac{1}{2} < 1$

**Download from www.arjun00.com.np

When $x = -1$, then the series becomes;

$1 - 2 + 4 - 8 + \ldots \ldots \ldots$; which is divergent.

Thus, the interval of convergence $\left(-\frac{1}{2}, \frac{1}{2}\right)$ radius of convergence= $= \dfrac{\frac{1}{2}+\frac{1}{2}}{2} = \dfrac{1}{2}$.

-The End-

**Download from www.arjun00.com.np

Engg. Mathematics-III (Engg. All) 3rd Sem

(2079) Question Paper Solution.

Compile by Arjun Chy

Website :- www.arjun00.com.np

Facebook :- www.facebook.com/Arjun00.com.np

1.a) Using definition, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ when

$$f(x,y) = x^3 + y^3 + 3axy$$

By definition, $\frac{\partial f}{\partial x} = lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$

$$\frac{\partial f}{\partial x} = lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 + y^3 + 3a(x+\Delta x)y - (x^3 + y^3 + 3axy)}{\Delta x}$$

$$lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 + y^3 + 3axy + 3a\Delta xy - (x^3 + y^3 + 3axy)}{\Delta x}$$

$$= lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 + 3a\Delta xy - x^3}{\Delta x}$$

$$= lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 + 3a\Delta xy - x^3}{\Delta x}$$

$$= lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 + 3a\Delta xy}{\Delta x}$$

$$= lim_{\Delta x \to 0} 3x^2 + 3x.\Delta x + \Delta x^2 + 3ay$$

**Download from www.arjun00.com.np

$$\frac{\partial f}{\partial x} = 3x^2 + 3ay$$

By definition, $\frac{\partial f}{\partial y} = lim_{\Delta y \to 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}$

$$\frac{\partial f}{\partial y} = lim_{\Delta y \to 0} \frac{x^3 + (y + \Delta y)^3 + 3ax(y + \Delta y) - (x^3 + y^3 + 3axy)}{\Delta y}$$

$$lim_{\Delta y \to 0} \frac{x^3 + (y + \Delta y)^3 + 3axy + 3ax\Delta y - (x^3 + y^3 + 3axy)}{\Delta y}$$

$$= lim_{\Delta y \to 0} \frac{(y + \Delta y)^3 + 3ax\Delta y - y^3}{\Delta y}$$

$$= lim_{\Delta y \to 0} \frac{(y^3 + 3y^2\Delta y + 3y\Delta y^2 + \Delta y^3) + 3ax\Delta y - y^3}{\Delta y}$$

$$= lim_{\Delta y \to 0} \frac{3y^2\Delta y + 3y\Delta y^2 + \Delta y^3 + 3ax\Delta y}{\Delta y}$$

$$= lim_{\Delta y \to 0} 3y^2 + 3y.\Delta y + \Delta y^2 + 3ax$$

$$\frac{\partial f}{\partial y} = 3y^2 + 3ax$$

**Download from www.arjun00.com.np

b) Find $\frac{du}{dx}$, of $u = e^{xyz}, x = t^3, y = \frac{1}{t}, z = e^t$.

Solution:-

Here, $u = e^{xyz}, x = t^3, y = \frac{1}{t}, z = e^t$.

$$\frac{\partial u}{\partial x} = e^{xyz}.yz \quad \textbf{Also,} \quad \frac{\mathrm{dx}}{\mathrm{dt}} = 3t^2$$

$$\frac{\partial u}{\partial y} = e^{xyz}.xz \qquad , \quad \frac{\mathrm{dy}}{\mathrm{dt}} = -\frac{1}{t^2}$$

$$\frac{\partial u}{\partial z} = e^{xyz}.xy \qquad , \quad \frac{\mathrm{dz}}{\mathrm{dt}} = e^t$$

Hence, $\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{\mathrm{dx}}{\mathrm{dt}} + \frac{\partial u}{\partial y}\frac{\mathrm{dy}}{\mathrm{dt}} + \frac{\partial u}{\partial z}\frac{\mathrm{dz}}{\mathrm{dt}}$

$$= \mathrm{e^{xyz}yz \times 3t^2 + e^{xyz}zx \times -\frac{1}{t^2} + e^{xyz}xy \times e^t}$$

$$= \mathrm{e^{xyz}(yz \times 3t^2 + zx \times -\frac{1}{t^2} + xy \times e^t)}$$

$$= \mathrm{e^{t^3 \times 1/t \times e^t}(3t^2 \times \frac{1}{t}e^t - e^t t^3 \times \frac{1}{t^2} + t^3\frac{1}{t} \times e^t)}$$

$$= \mathrm{e}^{t^2 \mathrm{e^t}}(\mathrm{3te^t - te^t + t^2e^t})$$

$$= \mathrm{e}^{t^2 \mathrm{e^t}}(\mathrm{2te^t + t^2e^t})$$

**Download from www.arjun00.com.np

$$= e^{t^2 e^t} te^t(2 + t)$$

$$= \mathbf{e^{t^2 e^t}\ te^t(2 + t)}$$

2. a) Define Group. Prove that the identity element of group is unique. Also show that the inverse of group is unique.

Solution:-

Group :- A group is non-empty set 'G' together with an operation $*$ defined on 'G' and satisfying the following axioms.

G.1 Closure axiom

For every a, b $\in$ G, $a * b \in G$

G.2 Associativity axiom

For every a, b, c $\in$ G, $a * (b * c) = (a * b) * c$

G.3 Existence of identity element

For all a $\in$ G, there is an element e $\in$ G, such that

$e * a = a * e = a$

G.4 Existence of inverse element,

For each a $\in$ G, there is an element $a^i \in G$, such that

$a^{-i} * a = a * a^i = e$

**Download from www.arjun00.com.np

For the second part

We need to prove that identity element of a group is unique.

Let e and f be the identity element of group $(G, *)$. Since, e is an identity element,

$$e * f = f$$

Since, 'f' is also an identity element, then,

$$e * f = e$$

so, $f = e$

This proves that the identity element of a group is unique.

For the third part

To prove that the inverse of group is unique, let g' and g'' be the inverse of group $(G, *)$ and $g \in G$ Let, e be the identity element. Then,

$$g * g' = g' * g = e$$

and, $g * g'' = g'' * g = e$

Now,

$$\begin{aligned} g' &= g' * e \\ &= g' * (g' * g'') \\ &=(g' * g) * g'' \\ &= e * g'' \\ &= g'' \end{aligned}$$

Hence, this shows that inverse element of group is unique.

**Download from www.arjun00.com.np

b) If $G = \{\cdots\cdots, -6, -4, -2, 0, 2, 4, 6\cdots\cdots\}$ then prove that (G, +) is a group.

Solution :-

Given that ; $= \{\cdots\cdots, -6, -4, -2, 0, 2, 4, 6\cdots\cdots\} = \mathbb{R}$

Closure:

For any $x, y \in \mathbb{R}, x + y \in \mathbb{R}$

Let, $x = -2$ and, $y = 4$

Then,

$$x + y = -2 + 4 = 2 \in \mathbb{R}$$

Associativity :

For any $x, y, z \in \mathbb{R}$

$$x + (y + z) = (x + y) + z = (x + z) + y$$

Let, $x = -2$, $y = 4$ and $z = 6$

$\therefore\ -2 + (4 + 6) = (-2 + 4) + 6 = (-2 + 6) + 4$

or, $-2 + 10 = 2 + 6 = 4 + 4$

or, $8 = 8 = 8$

Hence, identity element is 0 $\in \mathbb{R}$.

For any $x \in \mathbb{R}$

$$x + 0 = 0 + x = x$$

Let, $x = -2 \in \mathbb{R}$

Then,

$$-2 + 0 = 0 + (-2) = -2$$

Copyright © belongs to the Arjun Chy

**Download from www.arjun00.com.np

$i.e., -2 = -2 = -2$

Inverse element exists:

For any 'x', $-x \in \mathbb{R}$ such that;

$$x + (-x) = 0$$

Let, $x = -2 \in \mathbb{R}$

Then,

$$-2 + [-(-2)] = -2 + 2 = 0$$

Thus, $(\mathbf{G}, +)$ is a group. (Proved)

**Download from www.arjun00.com.np

3. a) Test whether the following series is absolutely or conditionally convergent:

$$\mathbf{1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots\cdots}$$

Solution :-

Here the given series is $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots\cdots$

Here, $\sum |u_n| = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots$

$= \sum \frac{1}{n^{1/2}}$, which is a p-series and n=$\frac{1}{2} < 1.$

Thus the series is divergent.

Now, $\sum u_n$ is an alternating series and $u_n = \frac{1}{\sqrt{n}}$ and

$$u_{n+1} = \frac{1}{\sqrt{n+1}}$$

Clearly $u_n > u_{n+1}$ for all n and $\{u_n\}$ is monotonically decreasing.

Also , $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0.$

Thus by Leibnitz's test, the series is convergent.

Hence, the given series is conditionally convergent.

**Download from www.arjun00.com.np

b) Find the Taylor's series expansion of $f(x) = e^{-x}$ about $x = 2$.

Solution :

We have,

$$f(x) = e^{-x}$$
$$f(2) = e^{-2}$$
$$f^{i}(x) = e^{-x}$$
$$f^{i}(2) = -e^{-2}$$
$$f^{ii}(x) = e^{-x}$$
$$f^{ii}(2) = e^{-2}$$
$$f^{iii}(x) = e^{-x}$$
$$f^{iii}(2) = -e^{-2}$$

The Taylor series at x = 2 is;

$$\mathbf{f(x)} = f(2) + f^{i}\frac{(2)}{1!}(x-2) + f^{ii}\frac{(2)}{2!}(x-2)^2 + f^{iii}\frac{(2)}{3!}(x-2)^3 + \cdots\cdots\cdots$$

$$= e^{-2} + (-e^{-2})(x-2) + \frac{e^{-2}}{2!}(x-2)^2 + \frac{(-e^{-2})}{3!}(x-2)^3 + \cdots\cdots\cdots$$

$$= \mathbf{e^{-2}\left\{1 - (x-2) + \frac{(x-2)^2}{2!} - \frac{(x-2)^3}{3!} + \cdots\cdots\cdots\right\}}$$

Copyright © belongs to the Arjun Chy

**Download from www.arjun00.com.np

4. Solve by separating the variables :

a) $e^{x-y}dx + e^{y-x}dy = 0$

Solution:

We have,

$$e^{x-y}dx + e^{y-x}dy = 0$$

or, $\frac{e^x}{e^y}dx + \frac{e^y}{e^x}dy = 0$

or, $e^{2x}dx + e^{2y}dy = 0$

Integrating ; we get,

$$\int e^{2x}\,dx + \int e^{2y}dy = K$$

or, $\frac{e^{2x}}{2} + \frac{e^{2y}}{2} = K$

or, $e^{2x} + e^{2y} = 2K$; which is the required solution.

Copyright © belongs to the Arjun Chy

**Download from www.arjun00.com.np

b) $\frac{dy}{dx} = -\frac{1+\cos 2y}{1-\cos 2x}$

Solution:

We have,

$$\frac{dy}{dx} = -\frac{1+\cos 2y}{1-\cos 2x}$$

$$\text{or, } \frac{dy}{-(1+\cos 2y)} + \frac{dx}{(1-\cos 2x)} = 0$$

$$\text{or, } \frac{dy}{-2\cos^2 y} + \frac{dx}{2\sin^2 x} = 0$$

$$\text{or, } -\sec^2 y\, dy + \cos^2 x\, dx = C$$

Integrating ; we get,

$$-\int \sec^2 y\, dy + \int \cos^2 x\, dx = C$$

$$\text{or, } -\tan y + \cot x = C$$

or, $\cot x - \tan y = C$; which is the required solution.

**Download from www.arjun00.com.np

5. Solve the homogeneous differential equation : $\frac{dy}{dx} = \frac{x^2y}{x^3+y^3}$.

Solution :

The given differential equation is

$$\text{or, } \frac{dy}{dx} = \frac{x^2y}{x^3+y^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

The equation (i) being homogeneous differential equation, so

Put $y = vx$

$$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$$

After substituting the equation (i) becomes

$$v + x\frac{dv}{dx} = \frac{x^2.vx}{x^3+v^3x^3}$$

$$\text{or, } v + x\frac{dv}{dx} = \frac{v}{1+v^3}$$

$$\text{or, } x\frac{dv}{dx} = \frac{v}{1+v^3} - v$$

$$\text{or, } x\frac{dv}{dx} = \frac{v-v-v^4}{1+v^3}$$

$$\text{or, } x\frac{dv}{dx} = \frac{v^4}{1+v^3}$$

$$\text{or, } \frac{1+v^3}{v^4}dv = -\frac{dx}{x}$$

$$\text{or, } \left(\frac{1}{v^4} + \frac{1}{v}\right) + \frac{dx}{x} = 0$$

**Download from www.arjun00.com.np

Integrating, We get

$$-\frac{1}{3v^3} + \log v + \log x = C$$

Or, $-\frac{1}{3v^3} + \log vx = C$

or, $\log \frac{y}{x}.x = \frac{1}{3\left(\frac{y}{x}\right)^3} + C$

or, $3\log y = \frac{x^3}{y^3} + 3C$

or, $\mathbf{3\log y = \frac{x^3}{y^3} + 3C}$,which is the required equation.

**Download from www.arjun00.com.np

6. Find the Fourier series expansion of $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 \le x < \pi \end{cases}$.

Solution

The Fourier series expansion in the interval $-\pi$ to π is

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx) \qquad \ldots(i)$$

where

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0} f(x)\, dx + \frac{1}{2\pi}\int_{0}^{\pi} f(x)\, dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0} 0\, dx + \frac{1}{2\pi}\int_{0}^{\pi} 1\, dx = 0 + \frac{1}{2\pi}\pi = \frac{1}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} 0 \,.\, \cos nx\, dx + \frac{1}{\pi}\int_{0}^{\pi} 1 \,.\, \cos nx\, dx = 0 + 0 = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx\, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} 0 \,.\, \sin nx\, dx + \frac{1}{\pi}\int_{0}^{\pi} 1 \,.\, \sin nx\, dx$$

$$= 0 + \frac{1}{\pi}\left[-\frac{\cos nx}{n}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{\cos n\pi}{n} + \frac{\cos 0}{n}\right]$$

**Download from www.arjun00.com.np

$$= \frac{1}{\pi}\left[-\frac{(-1)^n}{n} + \frac{1}{n}\right] \qquad [Remember\ \cos n\pi = (-1)^n]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n}{n}\right]$$

Substituting the values in (i), the required Fourier series is

$$f(x) \quad = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1-(-1)^n}{n\pi} \sin nx$$

$$= \frac{1}{2} + \frac{2}{\pi}\sin x + \frac{2}{3\pi}\sin 3x + \frac{2}{5\pi}\sin 5x + \dots$$

$$= \frac{1}{2} + \frac{2}{\pi}\left[\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots\right]$$

7. Define periodic function. Find the smallest positive period of P of sin nx.

- A function f(x) is called **periodic** if it is defined for all real values of 'x' (except for some isolated 'x' e.g. x = $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$, ..., in case of tan x) and if there is some positive number 'p' such that, for all x,

$$f(x + p) = f(x)$$

For Example,

If f(x) = sin x, then f(x +2π) = f(x)

So, Sin x is a period function with a period 2π.

**Download from www.arjun00.com.np

For the Second Part

Solution:

Let, f(x) = Sin nx

If p is a period of f(x); then,

$$f(x + p) = f(x)$$

or, $\sin n(x + p) = \sin nx$

or, $\sin n(x + p) = \sin(nx + 2\pi)$

or, $n(x + p) = nx + 2\pi$

or, $nx + np = nx + 2\pi$

or, $np = 2\pi$

$\therefore$ $\mathbf{p = \frac{2\pi}{n}}$

Thus, the smallest positive period of sin nx is $\frac{2\pi}{n}$.

**Download from www.arjun00.com.np

8. Prepare Cayley table for the set {0, 1, 2, 3, 4, 5} under the operation multiplication module 6. Identify the identity element and the inverse of each element if possible.

Solution:-

Given that;

$S = \{0, 1, 2, 3, 4, 5\}$

Cayley table,

$\times_6$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

Clearly, $0 \times_6 1 = 0$, $1 \times_6 1 = 1, 2 \times_6 1 = 2, 3 \times_6 1 = 3$ **and so on i.e number** $\times_6$ $\boldsymbol{identity\ element = number}$,hence 1 is the identity element

Also, the inverse of 1 = 1, inverse of 5 = 5.

Inverse of 0, 2, 3, 4, doesnot exists .

**Download from www.arjun00.com.np

9. Solve the partial differential equation : (Any One)

i) $\frac{\partial f}{\partial x}\, xz + yz\, \frac{\partial f}{\partial y} = xy.$

Solution :

The given differential equation is

$$\frac{\partial f}{\partial x}\, xz + yz\, \frac{\partial f}{\partial y} = xy.$$

or, $xzp + yzq = xy$

Comparing this equation with $Pp + Qq = R$, We have

$$P = xz, Q = yz \text{ and } R = xy$$

The auxiliary equation are

$$\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{xy}$$

Form the first two relation,

$$\frac{dx}{x} = \frac{dy}{y}$$

or, $\log x = \log y + \log c_1$

or, $\log x - \log y = \log c_1$

or, $\log\left(\frac{x}{y}\right) = \log c_1$

or, $\frac{x}{y} = c_1$ ……………..(i)

**Download from www.arjun00.com.np

Form the last two relation,

$$\frac{dy}{z} = \frac{dz}{x}$$

or, $c_1 y\, dy = z dz$ **[using (i)]**

or, $2c_1 y^2 = 2z dz$

or, $c_1 y^2 = z^2 + c_2$

or, $xy = z^2 + c_2$ **[using (i)]**

or, $\mathbf{xy - z^2 = c_2}$ **................................ (ii)**

From (i) and (ii), the solution is

$$\mathbf{f\left(\frac{x}{y}, xy - z^2\right) = 0 \text{ Ans}}.$$

ii) $\boldsymbol{x\,p - yq + x^2 - y^2 = 0}$

Solution :

Given equation is,

$$x\,p - yq + x^2 - y^2 = 0$$

or, $x\,p - yq + y^2 - x^2$

Comparing this equation with $\mathbf{Pp + Qq = R}$**, We have**

$$P = x, Q = -y \text{ and } R = y^2 - x^2$$

The auxiliary equations are

$$\frac{dx}{x} = \frac{dy}{-y} = \frac{dz}{y^2 - x^2}$$

Copyright © belongs to the Arjun Chy

**Download from www.arjun00.com.np

Form the first two relation,

$$\frac{dx}{x} = \frac{dy}{-y}$$

or, $\log x = -\log y + \log c_1$

or, $\log x + \log y = \log c_1$

or, $\log (xy) = \log c_1$

or, $xy = c_1$ ....(i)

Form the last two relation,

$$\frac{dy}{-y} = \frac{dz}{y^2 - x^2}$$

or, $(y^2 - x^2)dy = -y\,dz$

or, $\left(y^2 - \frac{{c_1}^2}{y^2}\right) dy = -y\,dz$ **[using (i)]**

or, $\left(y - \frac{{c_1}^2}{y^3}\right) dy = -\,dz$

or, $\frac{y^2}{2} + \frac{x^2y^2}{2y^2}) = -z + c_2$ **[using (i)]**

or, $\frac{y^2}{2} + \frac{x^2}{2} + z = c_2$ ....(ii)

Hence, from (i) and (ii), We have

$$\mathbf{f(xy)} = \frac{y^2}{2} + \frac{x^2}{2} + z$$ **, Which is the required solution.**

**Download from www.arjun00.com.np

10. Find the interval and radius of convergence of the series :

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n^2}$$

Solution :

Given series is,

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n^2}$$

We have,

$$u_n = \frac{(-1)^{n-1}x^n}{n^2}$$

$$u_{n+1} = \frac{(-1)^{(n+1)-1}x^{n+1}}{(n+1)^2}$$

$$\frac{u_{n+1}}{u_n} = \frac{(-1)^n x^{n+1}}{(n+1)^2} \times \frac{n^2}{(-1)^{n-1}x^n}$$

$$\frac{u_{n+1}}{u_n} = (-1)^{n-n+1}\left(\frac{n}{n+1}\right)^2 . x$$

$$\left|\frac{u_{n+1}}{u_n}\right| = \left|(-1)^1\left(\frac{n}{n+1}\right)^2 x\right|$$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^2 |x|$$

**Download from www.arjun00.com.np

$$= \lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right)^2 |X|$$

$$= (\frac{1}{1+0})^2.|x|$$

$$= 1.|x|$$

For Convergent,

$$\left|\frac{u_{n+1}}{u_n}\right| < 1$$

$$|x| < 1$$

$$i.e., -1 < x < 1.$$

For Divergent,

$$\left|\frac{u_{n+1}}{u_n}\right| > 1$$

$$|x| > 1$$

For, $x = -1$,

$$u_n = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(-1)^n}{n^2}$$

$$u_n = \sum_{n=1}^{\infty} \frac{(-1)}{n^2}$$

$$v_n = \frac{-1}{n^2}$$

$$\frac{u_n}{v_n} = 1$$

By Comparision test, $\frac{u_n}{v_n} = 1$ (finite value)

**Download from www.arjun00.com.np

$\sum v_n = \frac{1}{n^2}$ for $P = 2$ so, $A + x = -1$

Convgent,,

For $x = 1$.

$$u_n = \sum_{n=1}^{\infty} \frac{(-1)^{x-1}}{x^2}$$

$$u_n = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots\cdots$$

$i.e.$, alternative Series

With $u_n = \frac{1}{n^2}$ so, $u_{n+1} < n$ and

So, $\lim\limits_{n\to\infty} u_n = \frac{1}{n^2} = 0$ so, the series is convergent for $n = 1$

Hence,

Interval of convergent is $-1 \le x < 1$

Note,

$$\left\{\text{radius for } (a < x < b) \Rightarrow R = \frac{b-a}{2}\right\}$$

Radius of convergence $= \frac{1-(-1)}{2} = \frac{2}{2} = 1$.

**Download from www.arjun00.com.np

11. Verify Euler's theorem for homogeneous function

$$\boldsymbol{f(x,y,z) = x^2 + y^2 + z^2.}$$

Solution :

The given function is,

$$u = f(x,y,z) = x^2 + y^2 + z^2.$$

Replacing 'x' by tx and 'y' by ty and 'z' by tz; we get

$$f(tx,ty,tz) = (tx)^2 + (ty)^2 + (tz)^2 = t^2x^2 + t^2y^2 + t^2z^2$$

$$= t^2(x^2 + y^2 + z^2) = t^2 f(x,y,z)$$

Hence, the function homogeneous function of degree 2.

Also, to verify Euler's Theorem;

Now,

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(x^2 + y^2 + z^2\right) = 2x$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(x^2 + y^2 + z^2\right) = 2y$$

and, $\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}\left(x^2 + y^2 + z^2\right) = 2z$

So, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = x \cdot 2x + y \cdot 2y + z \cdot 2z = 2x^2 + 2y^2 + 2z^2$

$$= \mathbf{2(x^2 + y^2 + z^2)} \textbf{ (Verified)}$$

**Download from www.arjun00.com.np

12. Define convergent and divergent series. Determine whether the following series is convergent of divergent by ratio test

$$\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \cdots\cdots$$

➢ A series $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \cdots + a_n + \cdots$, is called **convergent** if the sequence of the partial sums $\{s_n\}$, as defined earlier, converges.

$$i.e. \text{ if } \lim_{n\to\infty} s_n = L \text{ for some finite } L.$$

The limit L is called the **sum** or the value of the Series.

A series that is not convergent is called a **divergent** series.

For the Second Part

Solution :

The given series is $\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \cdots\cdots$

$$a_n = \frac{n}{n+1}x^{n+2}$$

$$a_{n+1} = \frac{(n+1)}{n+2}x^{n+3}$$

Now,

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{(n+1)}{n+2}x^{n+3}}{\frac{n}{n+2}x^{n+2}}$$

$$= \lim_{n\to\infty}\left(\frac{n+1}{n+2}\right)\times\left(\frac{n+1}{n}\right)x$$

**Download from www.arjun00.com.np

$$= \lim_{n\to\infty} \frac{n\left(1+\frac{1}{n}\right)}{n\left(1+\frac{2}{n}\right)} \times \left(1+\frac{1}{n}\right) x$$

$$= x$$

Hence, by ratio test, the given series is convergent for $x < 1$ and Divergent for $x > 1$.

For $x = 1$, the series is,

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \ldots\ldots\ldots + \frac{n}{n+1} + \ldots\ldots\ldots$$

This series diverges by n^{th} term test of divergence because,

$$\lim_{n\to\infty} \frac{n}{n+1} = \lim_{n\to\infty} \frac{n}{n\left(1+\frac{1}{n}\right)} = 1 \neq 0$$

13. Test whether the function is even or odd. Find the corresponding Fourier series

$$f(x) = \begin{cases} \pi, & -1 < x < 0 \\ -\pi, & 0 \le x < 1 \end{cases}$$

Solution :

Here,

$$f(x) = \begin{cases} \pi, & -1 < x < 0 \\ -\pi, & 0 \le x < 1 \end{cases}$$

$$= \begin{cases} \pi, & 0 < x < 1 \\ -\pi, & -1 \le x < 0 \end{cases}$$

$$= -f(x)$$

**Download from www.arjun00.com.np

So, given function $\mathbf{f(x)}$ is odd.

Only sine Series exist.

Then,

$$f(x) = a_0 + \sum_{n=1}^{\infty} b_n \sin nx$$

Where,

$$b_n = \frac{2}{p}\int_p f(x)\sin n\pi x\,dx$$

$$p = 1 - (-1) = 2$$

$$b_n = \frac{2}{2}\int_{-1}^{1} f(x)\sin n\pi x\,dx$$

$$= \frac{2}{2}\int_{-1}^{0} \pi \sin n\pi x\,dx + \int_{-1}^{0} -\pi \sin n\pi x\,dx$$

$$= \pi\left[\left|\frac{-\cos n\pi x}{n\pi}\right|_{-1}^{0} + \left|\frac{-\cos n\pi}{n}\right|_{0}^{1}\right]$$

$$= 2\,\frac{\cos n\pi - 1}{n}$$

$$= 2\,\frac{(-1)^n - 1}{n}$$

So, $\mathbf{f(x) = 2\sum_{n=1}^{\infty}\frac{(-1)^n-1}{n}\sin n\pi x}$ **which is required**.

-The End-