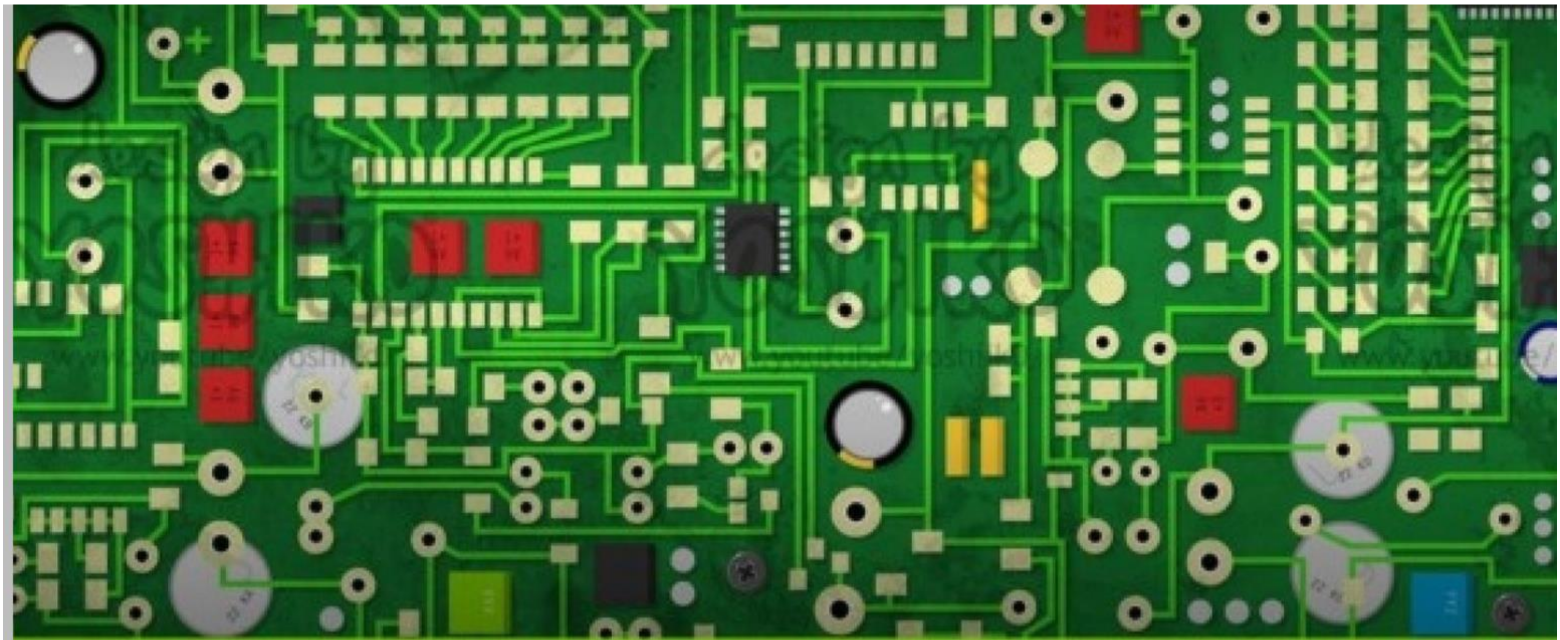

Basic Electrical & Electronics Engineering

CTEVT

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Basic Electrical and Electronics Engineering

Notes Book

Diploma in Engineering

CTEVT

NEW COURSE

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Basic Electrical and Electronics Engineering
EG2106CT

Year: II
Part: I

Total: 7 hours /week
Lecture: 3 hours/week
Tutorial: 1 hour/week
Practical: … hours/week
Lab: 3 hours/week

Course description:
This course is designed to understand fundamental concept of electric and electronic circuits.

Course objectives:
After completion of this course students will be able to:
1. Differentiate between active and passive elements and circuits.
2. Identify and explain the working principle of electric circuits.
3. Identify and explain the working principle of electronic circuits.

Course Contents:

Theory

Unit 1. Basic Electric System
1. Constituent parts of an electric system (Source, Load, Communication and Control)
2. Current flow in a circuit
3. Electromotive Force and Potential Difference
4. Electrical Units
5. Passive Components: Resistance, Inductance & Capacitance, Series and Parallel Combinations
6. Voltage and Current Sources: Independent, Dependent, VCVS, VCCS, CCCS, CCVS
7. Ohm's Law
8. Temperature rise and Temperature Coefficient of Resistance

Unit 2. DC Circuits and Network Theorems
1. Power and Energy
2. Kirchhoff's Law and Its Application: Nodal Analysis and Mesh Analysis
3. Star – Delta and Delta – Star Transformation
4. Superposition Theorem
5. Thevenin's Theorem
6. Norton's Theorem
7. Maximum Power Transfer Theorem
8. Reciprocity Theorem

Unit 3. Alternating Quantities

1. AC system
2. Waveform, Terms and Definitions
3. Average and rms values of Current and Voltage
4. Phasor Representation

Unit 4. Single – Phase AC Circuits

1. AC in Resistive Circuits
2. Current and Voltage in an Inductive circuit
3. Current and Voltage in an Capacitive circuit
4. Concept of Complex Impedance and Admittance
5. AC Series and Parallel Circuits
6. RL, RC and RLC Circuit Analysis and Phasor Representation

Unit 5. Power in AC Circuits

5.1. Power in Resistive Circuits
5.2. Power in Inductive and Capacitive Circuits
5.3. Power in Circuits with Resistance and Reactance
5.4. Active and Reactive Power: Power Factor, Importance and Measurement of Power Factor

Unit 6. Diode

6.1. Conductor, Insulator and Semiconductor
6.2. Types of Semiconductors: Intrinsic and Extrinsic, P type and N type
6.3. Semiconductor Diode Characteristics
6.4. Diode Circuits: Clipper and Clamper Circuits
6.5. Zener Diode, LED, Photodiode, Varacter Diode, Tunnel Diode
6.6. DC Power Supply: Rectifier (Half – Wave and Full - Wave), Zener Regulated Power Supply

Unit 7. Transistor

7.1. BJT: Types, Configurations, Modes of Operations, Working Principle
7.2. Biasing of BJT
7.3. BJT as an Amplifier and a Switch
7.4. Small and Large Signal Models
7.5. BJT as Logic Gates
7.6. Concept of Differential Amplifier using BJT

Unit 8. MOSFET

8.1. Types and Construction of MOSFET
8.2. Working Principle of MOSFET
8.3. Biasing of MOSFET
8.4. Construction and working of CMOS
8.5. MOSFET and CMOS as Logic Gates

Unit 9. The Operational Amplifier (Op - Amp)

9.1. Basic Model, Ideal and Real Characteristics, Virtual Ground Concept
9.2. Inverting and Non – Inverting Mode Amplifier
9.3. Some Applications: Summing Amplifier, Differentiator, Integrator, Comparator

Practical:

1. Measurement of Voltage, Current and Power in DC Circuits
   a) Verification of Ohm's Law
   b) Temperature Effect in Resistance
2. Kirchhoff's Current and Voltage Law
   a) Evaluate Power from V and I
   b) Note Loading Effects in Meters
3. Measurement of Amplitude, Frequency and Time in Oscilloscope
   a) Calculate and Verify Average and rms Values
   b) Examine Phase Relation in RL and RC Circuits
4. Measurement of Alternating Quantities
   a) R, RL, RC Circuits with AV Excitation
   b) AC Power, Power Factor, Phasor Diagram
5. Diode Characteristics, Rectifiers and Zener Diode
6. BJT Characteristics
7. MOSFET Characteristics
8. Voltage Amplifier using OP – Amp, Comparators

Final written exam evaluation scheme			
Unit	**Title**	**Hours**	**Marks Distribution***
1	Basic Electric System	6	10
2	DC Circuits and Network Theorems	6	10
3	Alternating Quantities	4	8
4	Single – Phase AC Circuits	4	8
5	Power in AC Circuits	5	8
6	Diode	6	10
7	Transistor	6	10
8	MOSFET	4	8
9	The Operational Amplifier (Op - Amp)	4	8
	Total	**45**	**80**

* There may be minor deviation in marks distribution.

CHAPTER 1

GENERAL ELECTRIC SYSTEM

1.1 CONSTITUENT PARTS OF AN ELECTRICAL CIRCUIT

A circuit is a closed path through which an electric current either flows or tends to flow. Circuit consists of active and passive elements in it.

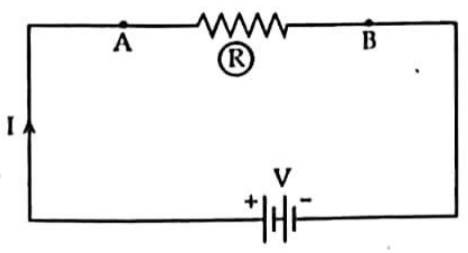


Figure: An Electric circuit

Following are the various consistent parts of an electrical circuit.

(i) Source: Which supply energy *i.e.*, battery, generator, etc.

(ii) Conductor: Used to carry current through circuit *i.e.*, wire.

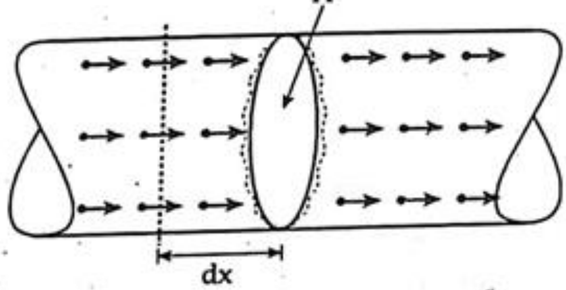
(iii) Load: That utilizes electrical energy and convert into different usable form. *i.e.*, bulb, TV, fan, etc.

(v) Safety devices: Fuse, etc.

(vi) Controlling devices: Switch

1.2 CURRENT FLOW IN A CIRCUIT

The rate of flow of electric charge in an electric circuit is known as electric current. It is denoted by 'I' and its unit is Ampere (A).



Consider, in a given conductor,

n = number of free electron per m^3 volume of the conductor

v = axial drift velocity

dx = distance travelled in small time dt

$\therefore \quad \boxed{dx = v \times dt}$

A = cross-sectional area of conductor

V = volume of conductor

Then, Volume of conductor = A dx = Av dt

If e is the charge of each electron then total charge which crosses the section in time dt is;

$[\because V' = A\,dt\,v]$

$dq = neV$

or, $dq = ne\,Av\,dt$

Since, rate of flow of charge is current,

$\therefore \quad I = \frac{dq}{dt} = \frac{ne\,Av\,dt}{dt}$

or, $\boxed{I = ven\,A}$

Also, the current per unit cross-sectional area of the conductor is called current density. It is a vector quantity. It is denoted by $\vec{j}$. Mathematically,

$\vec{j} = \frac{I}{A} \Rightarrow \boxed{I = \vec{j}\cdot\vec{A}}$ unit of $\vec{j}$ is ampere per m^2

Also, $\vec{j} = \frac{ven\,A}{A} = ven.$

For an irregular area,

$\boxed{I = \int \vec{j}\cdot d\vec{A}}$

1.3 ELECTROMOTIVE FORCE AND POTENTIAL DIFFERENCE

1.3.1 Concept of Electric Potential

When a body is charged, work is done in charging it. This work done is stored in the body in the form of potential energy. The charged body has the capacity to do work by moving other charges either by attraction or repulsion. The ability of the charged body to do work is called electric potential. The greater the capacity of a charged body to do work, the greater is its electric potential

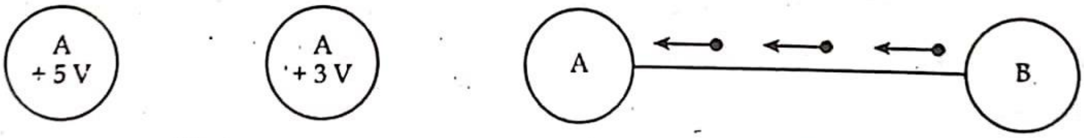
i.e., Electric potential, $V = \frac{\text{Work done}}{\text{Charge}} = \frac{W}{Q}$

The work done is measured in joules and charge in coulombs. Therefore, the unit of electric potential will be joules/coulomb or volt. If W = 1 joule, Q = 1 coulomb, then V = 1/1 = 1 volt.

Hence a body is said to have an electric potential of 1 volt if 1 joule of work is done to give it a charge of 1 coulomb. Thus, when we say that a body has an electric potential of 5 volts, it means that 5 joules of work has been done to charge the body to 1 coulomb. In other words, every coulomb of charge possesses an energy of 5 joules. The greater the joules/coulomb on a charged body, the greater is its electric potential.

1.3.2 Potential Difference

The difference in the potentials of two charged bodies is called potential difference. If two bodies have different electric potentials, a potential difference exists between the bodies. Consider two bodies A and B having potentials of 5 volts and 3 volts respectively as shown in figure below. Each coulomb of charge on body A has an energy of 5 joules while each coulomb of charge on body B has an energy of 3 joules. Clearly, body A is at higher potential than the body B.

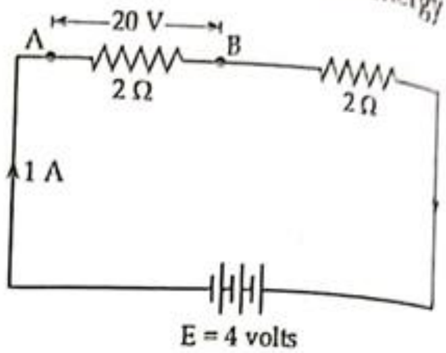


(i) (ii)

If the two bodies are joined through a conductor [See figure (ii)], then electrons will *flow from body B to body A. When the two bodies attain the same potential, the flow of current stops. Therefore, we arrive at a very important conclusion that current will flow in a circuit if potential difference exists. No potential difference, no current flow. It may be noted that potential difference is sometimes called voltage and unit of potential difference is volt.

1.3.3 Concept of E.M.F. and Potential Difference

There is a distinct difference between e.m.f. and potential difference. The e.m.f. of a device, say a battery, is a measure of the energy the battery gives to each coulomb of charge. Thus if a battery supplies 4 joules of energy per coulomb, we say that it has an e.m.f. of 4 volts. The energy given to each coulomb in a battery is due to the chemical action. The potential difference between two points, say A and B, is a measure of the energy used by one coulomb in moving from A to B. Thus if potential difference between points A and B is 2 volts, it means that each coulomb will give up an energy of 2 joules in moving from A to B.



i) The name e.m.f. is not a force but energy supplied to charge by some active device such as a battery.

ii) Electromotive force (e.m.f.) maintains potential difference while p.d. causes current to flow.

1.4 ELECTRICAL UNITS

Quantity	Unit	Symbol
Electric current	Ampere or Columb/s	A
Potential difference	Volt	V
Power	Watt	W or J/s
Resistance	Ohm	Ω, V/A
Capacitance	Farad	F
Inductance	Henry	H

Example 1.1

Find the velocity of charge, when 1 A current flows in a conductor of cross-sectional area 1 cm² and 1 km length. Number of free electron per m³ is 8.5×10^{28}. Also calculate the time required to travel an electric charge to 1 km length?

Solution:

Given that,

$$I = 1\ A$$

$$n = 8.5 \times 10^{28} \text{ per m}^3$$

$$A = 1\ cm^2 = 1 \times 10^{-4}\ m^2$$

Now, $I = venA$

or, $v = \dfrac{1}{neA} = \dfrac{1}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 1 \times 10^{-4}}$

or, $v = 7.35 \times 10^{-7}$ m per sec

Again, time taken by the charge to travel conductor of length 1 km is,

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{1 \times 10^3}{7.35 \times 10^{-7}} = 1.36 \times 10^9 \text{ second}$$

1.5 OHM'S LAW

Statement: Under the given condition of temperature, the ratio of potential difference (V) between any two points on a conductor to the current 'I' flowing between them is constant. Mathematically,

$$V \propto I$$

or, $V = IR$

or, $\boxed{R = \frac{V}{I}}$

where, $R = \frac{V}{I}$ = constant of proportionality, which is also called resistance of a conductor between two points.

Consider an electrical circuit, as shown in figure below:

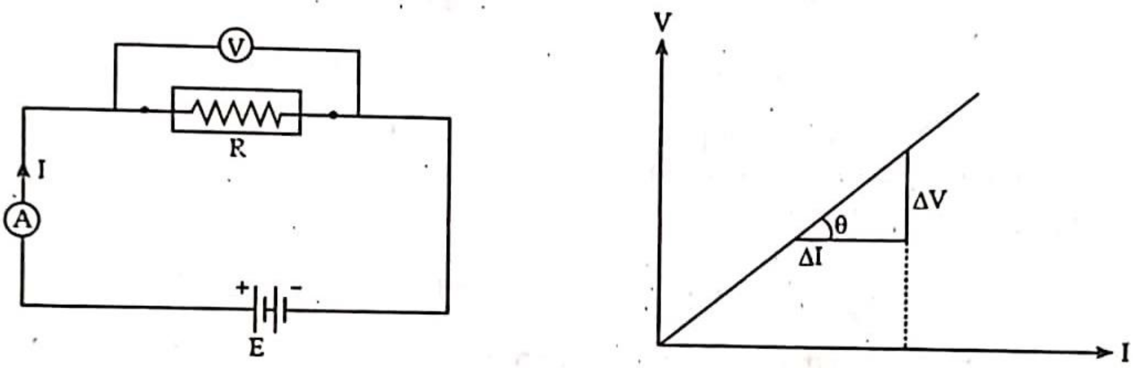


Figure: Arrangement for verification of OHM's Law

From experiment;

$$\frac{\Delta V}{\Delta I} = \text{constant} = R$$

The conductor for which voltage current graph is linear is called ohmic conductor. *For example* all metallic conductor.

The conductor for which voltage. current graph is non linear is called non ohmic conductor. *For example* diode semiconductor, etc.

1.6 RESISTANCE, RESISTIVITY AND CONDUCTANCE

Resistance of a conductor may be defined as the property of conductor which opposes the flow of current through it. It is denoted by letter 'R' and its unit is ohm (Ω). Resistance is measured by ohm meter.

Law of resistance

The resistance of a conductor in a circuit depends upon the;

(i) nature of conductor materials

(ii) directly proportional to the length of the conductor material

i.e., $R \propto l$

(iii) inversely proportional to the cross-sectional area of conductor

i.e., $R \propto \frac{l}{a}$

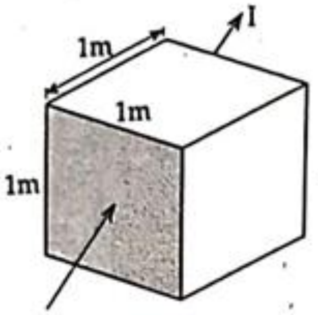
(iv) temperature

Mathematically, combining (ii) and (iii) we get,

$$R \propto \frac{l}{a}$$

or $$R = \frac{\rho l}{a};$$

where, ρ is the constant of proportionality.



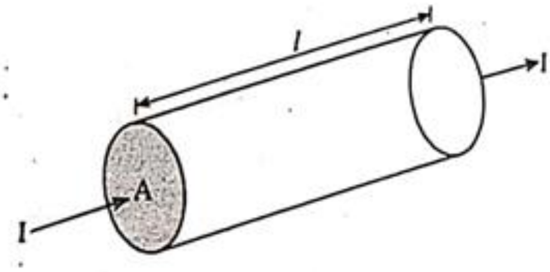


Figure: Cubical conductor

Circular conductor

where, ρ is a constant depending upon the nature of the conductor material and is known as specific resistance or resistivity of conductor.

We have,

$$R = \frac{\rho l}{a} \qquad \text{when, } a = 1\text{ m}^2 \text{ and } l = 1\text{ m, then } R = \rho.$$

Hence, specific resistance of material may be defined as the resistance offered by unit length and having unit cross-sectional area of conductor.

NOTE

Conductance is the reciprocal of resistance. Conductance measure the ease of flow of current through the conductor.

$$\text{Conductance} \Rightarrow G = \frac{1}{R} = \frac{a}{\rho l}.$$

Unit = mho (℧)

Example 1.2

Cross-Sectional area of an aluminium wire is 0.009 sq. cm, and specific resistance is 2.69×10^{-8} ohm-meter. Potential difference between two end points of the conductor is 20V. If 2A current is flowing through this, what is the length of the conductor?

Solution:

Given that;

Area of cross section (a) = 0.009 cm² = 0.009×10^{-4} m²

Specific resistance (ρ) = 2.69×10^{-8} ohm – m (Ω – m)

Potential difference (V) = 20 V

Current (I) = 2 A

Now,

$$\text{Resistance } R = \frac{V}{I} = \frac{20}{2} = 10\ \Omega$$

Again,

$$R = \frac{\rho l}{a}$$

or, $l = \frac{R \times a}{\rho} = \frac{10 \times 0.009 \times 10^{-4}}{2.69 \times 10^{-8}}$

or, $l = 334.5$ m

Example 1.3

1 km of wire having a diameter of 11.7 mm and of resistance 0.031 Ω is drawn so that its diameter becomes 5 mm. What does its resistance become?

Solution:

Given that;

Length of wire (l) = 1 km

Initial diameter of wire (d_1) = 11.7 mm = 11.7×10^{-3} m

$\therefore \quad r_1 = \frac{1}{2} \times 11.7 = 5.85 \times 10^{-3}$ m

Final diameter (d_2) = 5 mm = 5×10^{-3} m

$\therefore \quad r_2 = \frac{5}{2} = 2.5 \times 10^{-3}$ m

and, Initial resistance (R_1) = 0.031 Ω

We know that;

$$R = \frac{\rho l}{a} = \rho \frac{\frac{v}{a}}{a} = \rho \frac{v}{a^2}$$

where, volume (v) = al

Since, volume of material remains constant, as we know that, the resistance is inversely proportional to the cross section area of the conductors

$$R \propto \frac{1}{a^2}$$

$\therefore \quad \frac{R_1}{R_2} = \left(\frac{a_2}{a_1}\right)^2$

or, $R_2 = R_1 \times \left(\frac{a_1}{a_2}\right)^2$

or, $R_2 = R_1 \times \left(\frac{\pi r_1^2}{\pi r_2^2}\right)^2$ [where $a = \pi r^2$]

or, $R_2 = R_1 \times \left(\frac{r_1}{r_2}\right)^4$

Now, putting values of R_1, r_1, r_2; we get,

$$R_2 = 0.031 \times \left(\frac{5.85 \times 10^{-3}}{2.5 \times 10^{-3}}\right)^4 = 0.929\ \Omega$$

Example 1.4

An aluminium wire 7.5 m long is connected in parallel with a copper wire 6 m long. When a current of 5 A is passed through the combination, it is found that the current in the aluminium wire is 3 A. The diameter of the aluminium wire is 1 mm. Determine the diameter of the copper wire. Resistivity of aluminium is 0.028 $\mu\Omega$ – m and that of copper is 0.017 $\mu\Omega$ – m.

Solution:

Let the subscript 1 represent aluminium and subscript 2 represent copper.

Given that;

Aluminium	Copper
$l_1 = 7.5$ m	$R_2 = 6$ m
$I_1 = 3$A	$d_2 = ?$
$d_1 = 1$ mm	
$\rho_1 = 0.028\ \mu\Omega$-m	$\rho_2 = 0.017\ \mu\Omega$-m

Then,

$$R_1 = \left(\frac{\rho l}{a}\right)_1 \qquad (1)$$

$$R_2 = \left(\frac{\rho l}{a}\right)_2 \qquad (2)$$

We know that;

$$\frac{R_1}{R_2} = \frac{\rho_1 l_1}{\rho_2 l_2} \times \frac{a_2}{a_1}$$

or, $$a_2 = a_1 \times \frac{R_1}{R_2} \times \frac{\rho_2}{\rho_1} \times \frac{l_2}{l_1} \qquad (3)$$

Since, $I_1 = 3$ A

$\therefore \quad I_2 = 5 - 3 = 2$ A

If V is the common voltage across aluminium and copper wires; then,

$$V = I_1R_1 = I_2R_2$$

or, $$\frac{R_1}{R_2} = \frac{I_2}{I_1} = \frac{2}{3}$$

Now, substituting all the values of R, ρ and L; we get,

$$a_2 = a_1 \times \frac{2}{3} \times \frac{0.017}{0.028} \times \frac{6}{7.5}$$

or, $$\frac{\pi d_2^2}{4} = \pi \frac{d_1^2}{4} \times 0.3238$$

or, $$d_2^2 = (1)^2 \times 0.3238$$

or, $$d_2 = 0.569 \text{ mm}$$

$\therefore$ Diameter of the copper wire = 0.569 mm.

Example 1.5

A lead wire and an iron wire are connected in parallel. Their respective specific resistance are in the ratio of 49 : 24. The former caries 80% more current than the latter one, and the latter is 47% longer than former one. Determine the ratio of their cross sectional area.

Solution:

For lead (suffix = 1)
Current = I_1
Specific resistance = ρ_1
Length = l_1
c/s area = a_1

For iron (suffix = 2)
Current = I_2
Specific resistance = ρ_2
Length = l_2
c/s area = a_2

$$\therefore \quad \frac{\rho_1}{\rho_2} = \frac{49}{24}$$

and, if $I_2 = I$; then,

$$I_1 = 1 + 0.8 = 1.8\ I$$

Also, if $l_1 = 1$ then $l_2 = 1.47\ l$.

$$\text{Now,} \quad R = \frac{\rho l}{a}$$

$$\therefore \quad R_1 = \frac{\rho_1 l_1}{a_1}$$

$$\text{and,} \quad R_2 = \frac{\rho_2 l_2}{a_2}$$

Since, both wires are in parallel, V remains constant.

i.e., $V = I_1R_1$ and $V = I_2R_2$

$$\therefore \quad V_1 = V_2$$

$$\text{or,} \quad I_1R_1 = I_2R_2$$

$$\text{or,} \quad \frac{I_2}{I_1} = \frac{R_1}{R_2}$$

$$\text{or,} \quad \frac{I_2}{I_1} = \left(\frac{\rho_1 l_1}{a_1}\right) \times \left(\frac{a_2}{\rho_2 l_2}\right)$$

$$\text{or,} \quad \left(\frac{a_2}{a_1}\right) = \left(\frac{I_2}{I_1}\right) \times \left(\frac{\rho_2 l_2}{\rho_1 l_1}\right)$$

$$\text{or,} \quad \left(\frac{a_2}{a_1}\right) = \left(\frac{1}{1.8}\right) \times \frac{24}{49} \times \frac{1.47}{1}$$

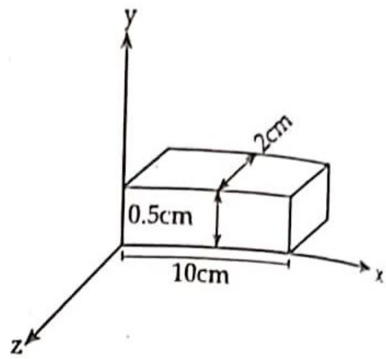
$$\therefore \quad \left(\frac{a_2}{a_1}\right) = 0.4$$

Example 1.6

A rectangular metal strip has the dimension x = 10 cm, y = 0.5 cm, z = 0.2 cm. Determine the ratio of the resistance R_x, R_y, R_z between the respective pair of opposite faces.

Solution:

Given that, the various values of length and cross section area along x/y/z direction are



$l_x = 10$ cm, $l_y = 0.5$ cm, $l_z = 0.2$ cm

$a_x = 0.5 \times 0.2 = 0.1\ cm^2$

$a_y = 10 \times 0.2 = 2\ cm^2$

$a_z = 10 \times 0.5 = 5\ cm^2$

Now, using the relation for R, l and a, for x, y and z direction, we get;

$$R_x = \rho \frac{l_x}{a_x}$$

$$R_y = \rho \frac{l_y}{a_y}$$

$$R_z = \rho \frac{l_z}{a_z}$$

Combining all three

$$R_x : R_y : R_z = \rho \frac{l_x}{a_x} : \rho \frac{l_y}{a_y} : \rho \frac{l_z}{a_z}$$

$$= \rho \frac{10}{0.1} : \rho \frac{0.5}{2} : \rho \frac{0.2}{5}$$

$$= 100\,\rho : 0.25\,\rho : 0.04\,\rho$$

$$= 10000 : 25 : 4$$

1.7 TEMPERATURE RISE AND TEMPERATURE COEFFICIENT OF RESISTANCE

Effect of temperature on resistance is

The effect of temperature on resistance is;

(i) The resistance increases when temperature increases in metal like copper, iron, etc. From this, we can understand that pure metals have positive temperature coefficient of resistance.

(ii) In alloys like magnesium and Eureka, resistance increases is relatively small with increase in temperature.

(iii) In electrolyte, mica, glass, insulator and rubber, resistance decreases with increase in temperature. Hence they have negative temperature coefficient of resistance.

Temperature coefficient of resistance

The difference in resistance while increasing temperature from 0 °C to 1 °C is called temperature coefficient of resistance.

Let, R_o is the resistance of conductor at 0 °C and R_t is the resistance of conductor at t°C then change in resistance is found to be:

(i) directly proportional to its initial resistance (*i.e.*, R_0)
(ii) directly proportional to rise in temperature (*i.e.*, Δt)
(iii) depends upon the nature of the conductor material

Combining all three, we get,

$$R_t - R_o \propto R_o \Delta t$$

or, $$\boxed{R_t - R_o = \alpha R_o \Delta t} \quad (1)$$

where, α = is constant and known as temperature coefficient of resistance.

Now, from above equation **(1)**;

$$\alpha = \frac{R_t - R_o}{R_o \Delta t} \quad (2)$$

If $R_o = 1\ \Omega$ and $\Delta t = 1\ °C$; then,

$$\boxed{\alpha = R_t - R_o}$$

Hence, the temperature coefficient of resistance is also defined as the increase in resistance per °C rise in temperature.

Also, from equation **(2)**; we have,

$$R_t = R_o + \alpha R_o \Delta t$$

or, $$R_t = R_o (1 + \alpha \Delta t) \quad (3)$$

NOTE

1. If the resistance of a conductor is R_2 at t_2 °C and R_1 at t_1 °C such that $t_2 > t$; then,
$$R_2 = R_1 [1 + \alpha_1 (t_2 - t_1)]$$
2. If α_1 is the temperature coefficient of resistance of conductor at t_1 °C then temperature coefficient of resistance at t_2 °C is given as
$$\alpha_2 = \frac{\alpha_1}{1 + \alpha_1 (t_2 - t_1)}$$

Example 1.7

A platinum coil has resistance of 3.146 Ω at 40 °C and 3.767 Ω at 100 °C. Find the resistance at 0 °C and the temperature coefficient of resistance at 40 °C.

Solution:

At 40 °C, $R_{40} = 3.146\ \Omega$

At 100 °C $R_{100} = 3.767\ \Omega$

Then, we know,

$$R_t = R_o (1 + \alpha t)$$

$$\therefore \quad \frac{R_{100}}{R_{40}} = \frac{R_o (1 + \alpha_o \times t_{100})}{R_o (1 + \alpha_o t_{40})}$$

or, $$\frac{3.767}{3.146} = \frac{1 + 100\ \alpha_o}{1 + 40\ \alpha_o}$$

or, $$\alpha_o = 0.00379 \text{ per °C}$$

Again, at 100 °C

$$3.767 = R_o (1 + 0.00379 \times 100)$$

or, $R_o = 2.732\ \Omega$

Now, Temperature coefficient of resistance at 40 is

$$\alpha_{40} = \frac{\alpha_o}{1 + 40\,\alpha_o} = \frac{0.00379}{1 + 40 \times 0.00379} = 0.00329 \text{ per °C.}$$

Example 1.8

A coil has a resistance of 40 Ω when its mean temperature is 30 °C and of 45 Ω when its mean temperature is 60 °C, find its mean temperature rise when its resistance is 47 Ω and surrounding temperature is 25 °C.

Solution:

Let R_o is the resistance and α_o is the temperature coefficient of coil at zero °C then,

$$R = R_o (1 + \alpha_o t)$$

$$\therefore \quad 40 = R_o (1 + 30\,\alpha_o) \qquad (1)$$

$$\text{and,} \quad 45 = R_o (1 + 60\,\alpha_o) \qquad (2)$$

Solving equation **(1)** and **(2)**,

$$\frac{40}{45} = \frac{1 + 30\,\alpha_o}{1 + 60\,\alpha_o}$$

$$\text{or,} \quad \alpha_o = \frac{1}{210} \text{ per °C}$$

If t °C is the temperature of coil when its resistance is 47 Ω then,

$$47 = R_o\left(1 + \frac{t}{210}\right) \qquad (3)$$

From **(2)** and **(3)**,

$$\frac{47}{45} = \frac{1 + \frac{t}{210}}{1 + \frac{60}{210}}$$

$$\text{or,} \quad 1.0444 = \frac{1 + \frac{t}{210}}{1.286}$$

$$\text{or,} \quad 1 + \frac{t}{210} = 1.343$$

$$\text{or,} \quad t = 72 \text{ °C}$$

$\therefore$ Rise in temperature = 72 – 25 = 47 °C.

Example 1.9

The filament of a 240 V metal filament lamp is to be constructed from a wire having a diameter of 0.02 mm and a resistivity at 20 °C of 4.3 micro ohm cm. If α = 0.005 per °C. What length of filament is necessary if the lamp is to dissipate 60 watts at a temperature of 2420 °C.

Solution:

We know that,

$$\text{Power (P)} = I^2R = \frac{V^2}{R} \text{ watts.}$$

Now, $\frac{V^2}{R} = 60$

or, $\frac{(240)^2}{R} = 60$

or, $R = 960\ \Omega$

This is resistance at 2420 °C because it dissipate 60 W power at that temperature.

Now, $R_{2420} = R_{20}\,[1 + (2420 - 20) \times 0.005]$

or, $960 = R_{20}\,(1 + 12)$

or, $R_{20} = 73.85\ \Omega$

Again; we have,

$$R = \frac{\rho l}{a}$$

or, $L = \frac{R \times a}{\rho} = \frac{73.85 \times \pi(0.002)^2}{4 \times 4.3 \times 10^{-6}} = 54$ cm.

Example 1.10

A 60 W, 240 V filament lamp is switched on at 20 °C. The operating temperature of the filament is 2000 °C. Determine the current taken by the lamp at the instant of switching on. The temperature coefficient of resistance of filament = 0.0045 per °C.

Solution:

The lamp is in operation at temperature 2000 °C. Hence, to get resistance at 2000 °C,

$$P = \frac{V^2}{R}$$

or, $60 = \frac{240^2}{R}$

$\therefore$ $R_{2000} = \frac{240^2}{60} = 960\ \Omega$

To get resistance of filament at 20 °C

$$R_{2000} = R_{20}\,[1 + \alpha_{20}\,(2000 - 20)]$$

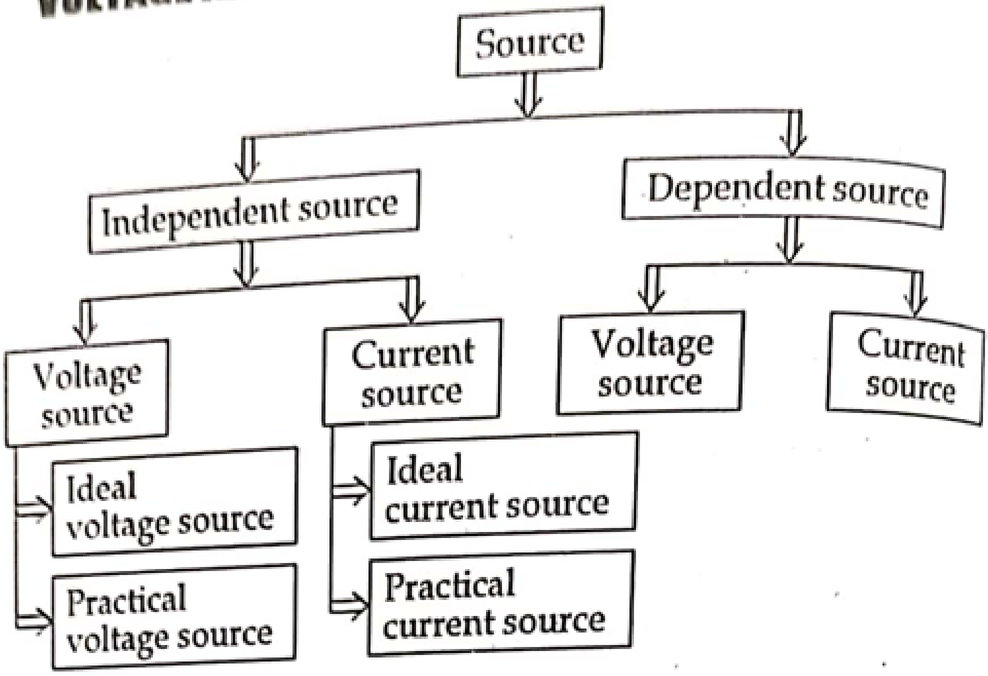
$$960 = R_{20}\,(1 + 0.0045 \times 1980)$$

or, $R_{20} = 96.9\ \Omega$

Therefore, the current taken by the lamp at the instant of switching on (*i.e.*, at the temperature of 20 °C) is

$$I = \frac{V}{R} = \frac{240}{96.9} = 2.48 \text{ A.}$$

1.8 VOLTAGE AND CURRENT SOURCES



1.8.1 Independent Sources

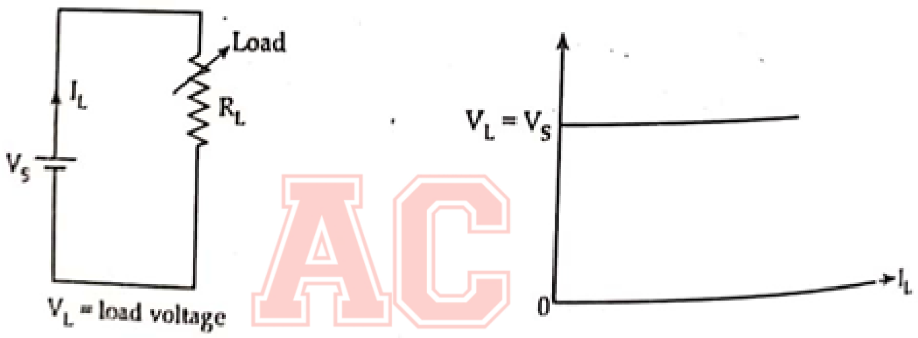
The sources (voltage and current sources) which do not depends on any other quantity in the circuit, then it is called independent source, Following are two types of independent source.

A. Voltage Sources

(i) Ideal voltage source

An ideal voltage source is that which produces a constant voltage across its terminal (V = E), no matter what current is drawn from it. Terminal voltage is dependent of load resistance connected across the terminals.

The circuit diagram and terminal I – V characteristics of ideal voltage source is shown in figure below:



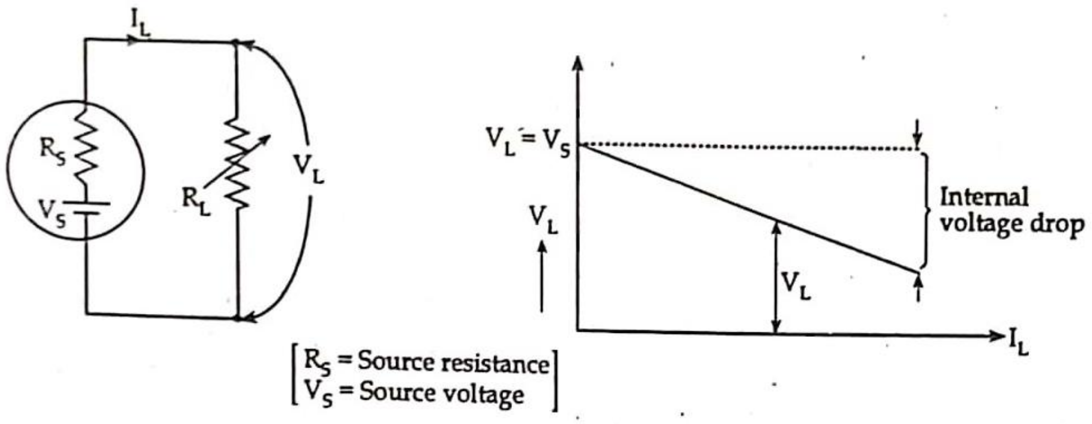
The current flowing through the terminal is,

$$I_L = \frac{V_L}{R_L} = \frac{V_S}{R_L} \qquad \text{[Here, load voltage = Source voltage]}$$

From figure (b), we can see that, the terminal voltage V_L remains constant and equal to the source voltage V_S irrespective of load current is small or large.

(ii) Practical voltage source

Practical voltage source do not exhibit characteristics similar to real voltage source. The circuit connection and V – I characteristics is shown in figure below:



From figure, we observed that as the load resistance R_L connected across the source is decreased, the corresponding load current I_L increases, while the terminal voltage across the source decreases. This voltage drop across terminals with increase in current can be realized from figure (b).

The terminal V – I characteristics of the practical voltage source can be described by an equation,

$$V_L = V_S - I_L R_S \quad (*)$$

In practice, when load resistance be 100 times larger than the source resistance R_S then it is approximately considered as an ideal source.

When, $R_S = 0$, then $V_L = V_S$

and when $R_S = R_L$

$V_L = 0$

B. Current Source

(i) An ideal current source

An ideal current source is that which delivers constant current to any load resistance connected across it. No matter what the terminal voltage is developed across the load *i.e.*, independent of the voltage across its terminal across the terminals.

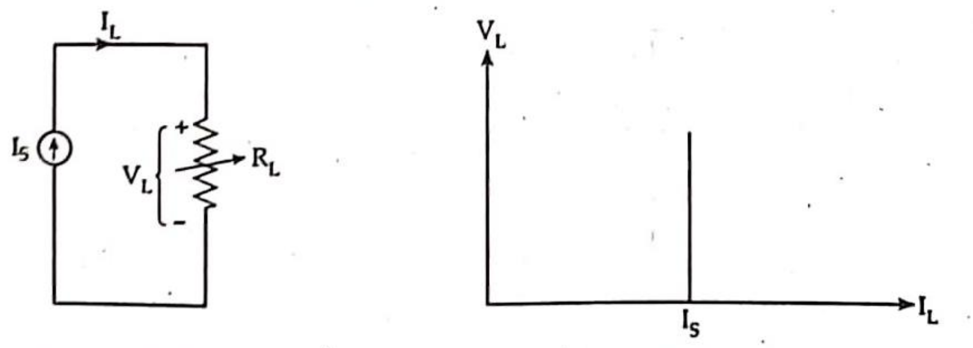
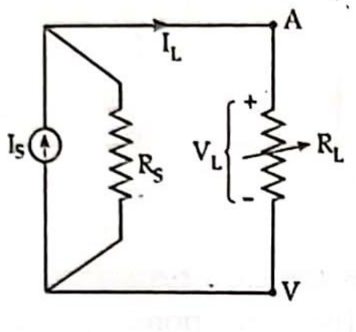


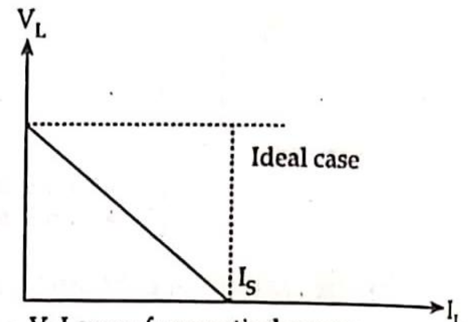
Figure: Circuit diagram I–V characteristics

From figure, it can be observed that the current flowing from the source to the load is always constant for any load resistance whether the R_L is small (V_L is small) or R_L is large (V_L is large).

(ii) **Practical current source**

When a load R_S is connected across a practical current source, we can observed that the current flowing in load resistance is reduced as the voltage across the current sources terminal is increased, by increasing the load resistance R_L.





V-I curve for practical source

Now, when a source resistance R_S is connected in parallel then, the distribution of source current in parallel paths, depends upon the value of R_S.

Now, applying KCL, we get

$$I_L = I_S - \frac{V_L}{R_S}$$

or, $V_L = I_S R_S - R_S I_L$

or, $V_L = V_{OC} - R_S I_L$

where, V_{OC} is the open circuit voltage.

Now; when,

$R_1 = 0$, $V_L = 0$ and $I_L = I_S$

when, $R_1 = \infty$, $I_L = 0$ then, $V_L = I_S R_S$

1.8.2 Source Transformation

(i) **Conversion of voltage source to current source**

The circuit diagram is shown in the figure below:

From figure, the load current is calculated as, $I_L = \frac{V_S}{R_L + R_S}$ from figure (a)

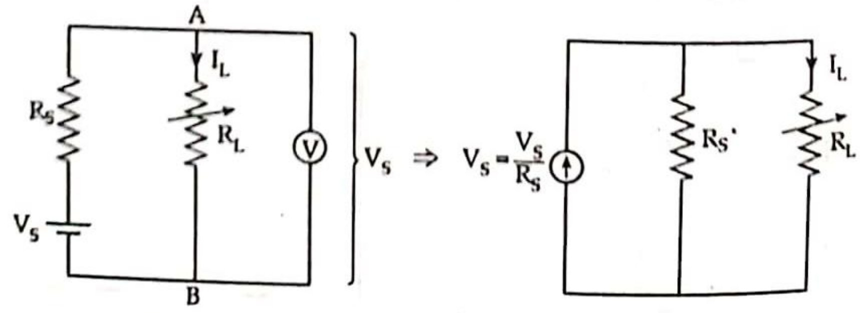


Figure (a) Figure (b)

Now, the maximum current delivered by source only when $R_L = 0$ (under short circuited condition) and is given as,

$$I_{max} = I_S = \frac{V_S}{R_S}$$

or, $V_S = I_S R_S$

Now, putting value of V_S in above equation

$$\therefore \quad I_L = \frac{I_S R_S}{R_S + R_L}$$

By this equation, a simple current divider circuit having two parallel branches as shown in fig (b) can be realized.

NOTE

A practical voltage source with a voltage V_S and an internal source resistance R_S can be replaced by an equivalent practical current source with a current $I_S = \frac{V_S}{R_S}$ and a source internal resistance R_S.

(ii) Current source to voltage source

For the circuit shown in figure below, the load voltage, V_L is given by

$$V_L = I_L R_L = \left(I_S \times \frac{R_S}{R_S + R_L} \right) \times R_L$$

or, $$V_L = I_S R_S \left(\frac{R_L}{R_S + R_L} \right)$$

or, $$\boxed{V_L = V_S \left(\frac{R_L}{R_S + R_L} \right)}$$

This equation represents the output from the voltage source across a load resistance and this act as voltage divider circuit figure (a).

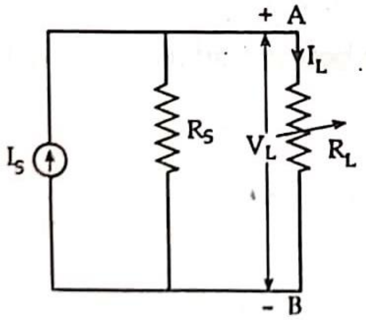


Figure (a)

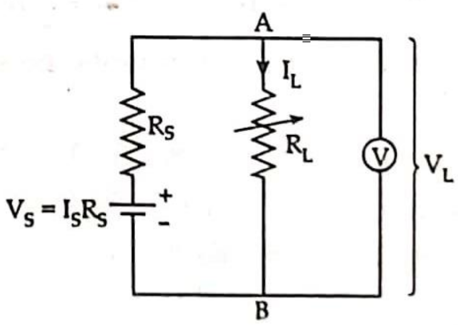


Figure (b)

From figure (b), it shows the situation that a voltage source with a voltage value $V_S = I_S R_S$ and an internal source resistance R_S has an equivalent effects on the same load resistor as the current source in figure (a).

NOTE

A current source with a magnitude I_S and a source internal resistance R_S can be replaced by an equivalent voltage source of magnitude $V_S = I_S R_S$ and an internal source resistance R_S.

Example 1.11

Find the resistance of 1000 meters of a copper wire 25 sq. mm in cross-section. The resistance of copper is 1/58 ohm per meter length and 1 sq. mm cross-section. What will be the resistance of another wire of the same material, three times as long and one-half area of cross-section?

Solution:

For the first case, R_1 ? ; $a_1 = 25$ mm^2; $l_1 = 1000$ m

For the second case, $R_2 = 1/58\ \Omega$; $a_2 = 1$ mm^2 ; $l_2 = 1$ m

$$R_1 = \rho\,(l_1/a_1)\ ;\ R_2 = \rho\,(l_2/a_2)$$

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{a_2}{a_1} = \left(\frac{1000}{1}\right) \times \left(\frac{1}{25}\right) = 40$$

or, $$R_1 = 40\ R_2 = 40 \times \frac{1}{58} = \frac{20}{29}\ \Omega$$

For the third case, $R_3 = ?$; $a_3 = a_1/2$; $l_3 = 3l_1$

$$\therefore \quad \frac{R_3}{R_1} = \left(\frac{l_3}{l_1}\right) \times \left(\frac{a_1}{a_3}\right) = (3) \times (2) = 6$$

or, $$R_3 = 6R_1 = 6 \times \frac{20}{29} = \frac{120}{29}\ \Omega$$

Example 1.12

A copper wire of diameter 1 cm had a resistance of 0.15 Ω. It was drawn under pressure so that its diameter was reduced to 50%. What is the new resistance of the wire?

Solution:

Area of wire before drawing, $a_1 = \frac{\pi}{4}(1)^2 = 0.785$ cm^2

Area of wire after drawing, $a_2 = \frac{\pi}{4}(0.5)^2 = 0.196$ cm^2

As the volume of wire remains the same before and after drawing,

$$\therefore \quad a_1 l_1 = a_2 l_2$$

or, $l_2/l_1 = a_1/a_2 = 0.785/0.196 = 4$

For the first case, $R_1 = 0.15\ \Omega$; $a_1 = 0.785$ cm^2 ; $l_1 = l$

For the second case, $R_2 = ?$; $a_2 = 0.196$ cm^2 ; $l_2 = 4l$

Now, $R_1 = \rho\frac{l_1}{a_1}$; $R_2 = \rho\frac{l_2}{a_2}$

$$\therefore \quad \frac{R_2}{R_1} = \left(\frac{l_2}{l_1}\right) \times \left(\frac{a_1}{a_2}\right) = (4) \times (4) = 16$$

or, $$R_2 = 16R_1 = 16 \times 0.15 = 2.4\ \Omega$$

Example 1.13

Two wires of aluminium and copper have the same resistance and same length. Which of the two is lighter? Density of copper is 8.9×10^3 kg/m^3 and that of aluminium is 2.7×10^3 kg/m^3. The resistivity of copper is $1.72 \times 10^{-8}\ \Omega$ m and that of aluminium is $2.6 \times 10^{-8}\ \Omega$ m.

Solution:

That wire will be lighter which has less mass. Let suffix 1 represent aluminum and suffix 2 represent copper.

$$R_1 = R_2 \quad \text{or,} \quad \rho_1 \frac{l_1}{A_1} = \rho_2 \frac{l_2}{A_2}$$

$$\text{or,} \quad \frac{\rho_1}{A_1} = \frac{\rho_2}{A_2} \qquad (\because l_1 = l_2)$$

$$\text{or,} \quad \frac{A_1}{A_2} = \frac{\rho_1}{\rho_2} = \frac{2.6 \times 10^{-8}}{1.72 \times 10^{-8}} = 1.5$$

Now,

$$\frac{m_1}{m_2} = \frac{(A_1 l_1)d_1}{(A_2 l_2)d_2} = \frac{A_1 d_1}{A_2 d_2} \quad (\because l_1 = l_2)$$

$$\text{or,} \quad \frac{m_1}{m_2} = \left(\frac{A_1}{A_2}\right) \times \left(\frac{d_1}{d_2}\right) = 1.5 \times \frac{2.7 \times 10^3}{8.9 \times 10^3} = 0.46$$

$$\text{or,} \quad \frac{m_1}{m_2} = 0.46$$

It is clear that for the same length and same resistance, aluminimum wire is lighter than copper wire. For this reason, aluminium wires are used for overload power transmission lines.

Example 1.14

A copper wire is stretched so that its length is increased by 0.1%. What is the percentage change in its resistance?

Solution:

$$R = \rho \frac{l}{a}; R' = \rho \frac{l'}{a'}$$

$$\text{Now,} \quad l' + \frac{0.1}{100} \times l = 1.001\, l$$

As the volume remains the same, $al = a'\,l'$.

$$\therefore \quad a' = a\frac{l}{l'} = \frac{a}{1.001}$$

$$\therefore \quad \frac{R'}{R} = \left(\frac{l}{l'}\right) \times \left(\frac{a}{a'}\right) = (1.001) \times (1.001) = 1.002$$

$$\text{or,} \quad \frac{R' - R}{R} \times 100 = 0.002 \times 100 = 0.2\%$$

Example 1.15

Two coils connected in series have resistances of 600 Ω and 300 Ω and temperature coefficients of 0.1% and 0.4% per °C at 20°C respectively. Find the resistance of combination at a temperature of 50°C. What is the effective temperature coefficient of the combination at 50°C?

Solution:

Resistance of 600 Ω coil at 50° C

$= 600 [1 + 0.001 (50 - 20)] = 618\ \Omega$

Resistance of 300 Ω coil at 50° C

$= 300 [1 + 0.004 (50 - 20)] = 336\ \Omega$

Resistance of series combination at 50° C is

$R_{50} = 618 + 336 = 954\ \Omega$

Resistance of series combination at 20° C is

$R_{20} = 600 + 300 = 900\ \Omega$

Now,

$R_{50} = R_{20} [1 + \alpha_{20} (t_2 - t_1)]$

or, $\alpha_{20} = \dfrac{\dfrac{R_{50}}{R_{20}} - 1}{t_2 - t_1} = \dfrac{\dfrac{954}{900} - 1}{50 - 20} = 0.002$

Now,

$$\alpha_{50} = \frac{1}{1/\alpha_{20} + (t_2 - t_1)} = \frac{1}{1/0.002 + (50 - 20)} = \frac{1}{530} /°C$$

Example 1.16

The coil of a relay takes a current of 0.12 A when it is at the room temperature of 15°C and connected across a 60 V supply. If the minimum operating current of the relay is 0.1 A, calculate the temperature above which the relay will fail to operate when connected to the same supply. Resistance temperature coefficient of the coil material is 0.0043 per °C at 0°C.

Solution:

Resistance of relay coil at 15° C, = 60/0.12 = 500 Ω

If the temperature increases, the resistance of relay coil increases and current in relay coil decreases. Let t° C be the temperature at which the current in relay coil becomes 0.1 A (= the minimum relay coil current for its operation). Clearly, $R_t = 60/0.12 = 500\ \Omega$.

Now,

$R_{15} = R_0 (1 + 15\ a_0) = R_0 (1 + 15 \times 0.0043)$

$R_t = R_0 (1 + \alpha_0 t) = R_0 (1 + 0.0043\ t)$

$\therefore \quad \dfrac{R_t}{R_{15}} = \dfrac{1 + 0.0043\ t}{1.0645}$

or, $\dfrac{600}{500} = \dfrac{1 + 0.0043\ t}{1.0645}$

On solving

$t = 64.5°C.$

Example 1.17

Two materials, A and B, have resistance temperature coefficients of 0.004 and 0.0004 respectively at a given temperature. In what proportion must A and B be joined in series to produce a circuit having a temperature coefficient of 0.001?

Solution:

Let the resistance of A be 1 Ω and that of B be x Ω *i.e.*, $R_A = 1\ \Omega$ and $R_B = x\ \Omega$.

Resistance of series combination = $R_A + R_B = (1 + x)\ \Omega$

Suppose the temperature rises by t°C.

Resistance of series combination at the raised temperature

$$= (1 + x) \times (1 + 0.001\ t) \qquad (1)$$

Resistance of A at the raised temperature = $1\ (1 + 0.004\ t)$ (2)

Resistance of B at the raised temperature = $x\ (1 + 0.0004\ t)$ (3)

As per the conditions of the raised problem, we have, **(2) + (3) = (1)**

or, $1(1 + 0.004\ t) + x\ (1 + 0.0004\ t) = (1 + x)\ (1 + 0.001\ t)$

or, $0.004\ t + 0.0004\ t\ x = (1 + x) \times 0.001\ t$

Dividing by t and multiplying throughout by 10^4, we have,

$40 + 4x = 10\ (1 + x)\ \therefore\quad x = 5$

$R_A : R_B = 1 : 5$ *i.e.*, R_B should be 5 times R_A.

Example 1.18

Two conductors, one of copper and the other of iron, are connected in parallel and carry equal currents at 25°C. What proportion of current will pass through each if the temperature is raised to 100°C? The temperature coefficients of resistance at 0°C are 0.0043/°C and 0.0063/°C for copper and iron respectively.

Solution:

Since copper and iron conductors carry equal currents at 25° C, their resistance are the same at this temperature. Let their resistance be R ohms at 25° C. If R_1 and R_2 are the resistances of copper and iron conductors respectively at 100° C, then,

$$R_1 = R\ [1 + 0.0043\ (100 - 25)] = 1.3225\ R$$

$$R_2 = R\ [1 + 0.0063\ (100 - 25)] = 1.4725\ R$$

If I is the total current at 100° C, then,

Current in copper conductor

$$= I \times \frac{R_2}{R_1 + R_2} = I \times \frac{1.4725\ R}{1.3225\ R + 1.4725\ R} = 0.5268\ I$$

Current in iron conductor

$$= I \times \frac{R_1}{R_1 + R_2} = I \times \frac{1.3225\ R}{1.3225\ R + 1.4725\ R} = 0.4732\ I$$

Therefore, at 100°C, the copper conduct will carry 52.68% of total current and the remaining 47.32% will be carried by iron conductor.

Example 1.19

A semi-circular ring of copper has an inner radius 6 cm, radial thickness 3 cm and an axial thickness 4 cm. Find the resistance of the ring at 50°C between its two end-faces. Assume specific resistance of copper at 20°C =1.724 × 10^{-6} W-cm and resistance temperature coefficient of copper at 0°C = 0.0043/°C.

Solution:

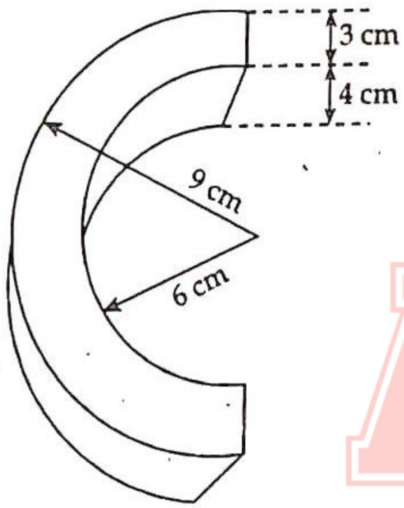


Figure shows the semi-circular ring.

Mean radius of the ring, $r_m = (6 + 9)/2 = 7.5$ cm

Mean length between end faces is

$l_m = \pi r_m = \pi \times 7.5 = 23.56$ m

Cross-sectional area of the ring is

$\alpha = 3 \times 4 = 12\ \text{cm}^2$

Now,

$$\alpha_{20} = \frac{\alpha}{1 + \alpha_0 t} = \frac{0.0043}{1 + 0.0043 \times 20} = 0.00396/°\text{C}$$

Also $\rho_{50} = \rho_{20}\,[1 + \alpha_{20}\,(t - 20)]$

$= 1.724 \times 10^{-6}\,[1 + 0.00396 \times (50 - 20)]$

$= 1.93 \times 10^{-6}\ \Omega\,\text{cm}$

$$\therefore \quad R_{50} = \frac{\rho_{50} l_m}{a} = \frac{1.93 \times 10^{-6} \times 23.56}{12} = 3.79 \times 10^{-6}\ \Omega$$

This example shows that resistivity of a conductor increases with the increase in temperature and vice-versa.

EXAMINATION QUESTION SOLUTIONS

1. An aluminium wire 7.5 m long is connected in parallel with a copper wire 6 m long. When a current of 5 A is passed through the parallel combination, it is found that the current in the aluminium wire is 3 A. The diameter of aluminium wire is 1 mm. Determine the diameter of copper wire. The resistivity of copper is 0.017 $\mu\Omega$ – m and that of aluminium is 0.028 $\mu\Omega$ – m.

Solution: See example 1.4.

2. 1 km of wire having a diameter of 11.7 mm and of resistance 0.031 Ω is drawn so that its diameter becomes 5 mm. What does its resistance becomes.

Solution: See example 1.3.

3. What are the ideal and practical voltage and current sources. Explain.

Solution: See definition part at 1.8.

4. The temperature rise of the machine field winding was determined by the measurement of winding resistance. At 20 °C the field resistance was 150 Ω. After running the m/c for 6 hours at full load, the resistance was found to be 175 Ω. If the temperature coefficient of resistance of the copper winding is 1.57×10^{-3} per °C, determine the temperature rise of the machine.

Solution:

Given that;

Resistance at 20 °C (R_{20}) = 150 Ω

Resistance at full load (R_f) = 175 Ω

Temperature coefficient of resistance (α_o) = 1.57×10^{-3} per °C

Temperature rise (Δt) = ?

We know that,

$$\alpha_{20} = \frac{\alpha_o}{1 + \alpha_o \Delta t}$$

or, $$\alpha_{20} = \frac{1.57 \times 10^{-3}}{1 + 1.57 \times 10^{-3} \times (20 - 0)}$$

or, $\alpha_{20} = 1.522 \times 10^{-3}$ per °C

Now,

$$R_f = R_{20} [1 + \alpha_{20} (\Delta t)]$$

or, $175 = 150 (1 + 1.522 \times 10^{-3} \Delta t)$

or, $\Delta t = 109.5$ °C

Now, increase in temperature is

$= 109.5 + 20 = 129.5$ °C

5. A coil is connected across a constant dc source of 120 V. It draws a current of 12 A at room temperature of 25 °C. After 5 hours of operation, its temperature rises to 65 °C and current reduces 8 A. Calculate;

(i) current when its temperature has increased to 80 °C

(ii) temperature coefficient of resistance at 30 °C

Solution:

Given that;

Supply voltage (V) 120 V

Current at 25 °C = 12 A

∴ Resistance at 25 °C

i.e., $R_{25} = \frac{V}{I} = \frac{120}{12} = 10\ \Omega$

Also,

At temperature 65 °C, current = 8 A

∴ Resistance $R_{65} = \frac{120}{8} = 15\ \Omega$

Now, we know that,

$R_{65} = R_{25}\ [1 + \alpha_{25} \times \Delta t]$

or, $15 = 10\ [1 + \alpha_{25} \times (65 - 25)]$

or, $1.5 = 1 + \alpha_{25} \times 40$

or, $\alpha_{25} = 0.0125$ per °C

Then, Resistance of coil at 80 °C is

$R_{80} = R_{25}\ [1 + \alpha_{25}\ (\Delta t)]$

or, $T_{80} = 10\ [1 + 0.0125 \times (80 - 25)$

or, $R_{80} = 16.875\ \Omega$

(i) So that, at 80 °C

Current (I) $= \frac{V}{R_{80}} = \frac{120}{16.875} = 7.11$ A

(ii) To get temperature coefficient of resistance at 30 °C,

$$\alpha_{30} = \frac{1}{\frac{1}{\alpha_{25}} + \Delta t} = \frac{1}{\frac{1}{0.0125} + (30 - 25)} = 0.01176 \text{ per °C}$$

∴ $\alpha_{30} = 0.01176$ per °C

6. The temperature rise of the machine field winding was determined by the measurement of the winding resistance. At 20 °C, the field resistance was 150 Ω. After running the m/c for 6 hours at full load, the resistance was 175 Ω. The temperature coefficient of resistance of the copper winding is 4.3×10^{-3} per K at 0 °C. Determine the temperature rise of m/c.

Solution: See solution of Q. No. 4.

7. The coil of relay takes a current of 0.12 A, when it is at the room temperature of 15 °C and connected across a 60 V supply. If the minimum operating current of the relay is 0.1 A, calculate the temperature above which the relay will fail to operate when connected to the same supply. Resistance temperature coefficient of the coil material is 0.0043 per °C at 6 °C.

Solution:

Given that;

Source voltage (V) = 60 V

Current at 15 °C = 0.12 A

Therefore, resistance at 15 °C,

$$R_{15} = \frac{V}{I} = \frac{60}{0.12} = 500\ \Omega$$

Operating minimum current = 0.1 A

So, operating resistance at operating temperature

$$t\ °C \text{ is, } R_t = \frac{60}{0.1} = 600\ \Omega$$

Temperature coefficient of coil at 6 °C is,

$$\alpha_6 = 0043 \text{ per } °C$$

Then, temperature coefficient of coil at 15 °C is,

$$\alpha_{15} = \frac{1}{\frac{1}{\alpha_6} + \Delta t} = \frac{1}{\frac{1}{0.0043} + (15 - 6)} = 4.1397 \times 10^{-3} \text{ per } °C$$

Again,

$$R_t = R_{15}\,[1 + \alpha_{15} \times \Delta t]$$

or, $600 = 500\,[1 + 4.1397 \times 10^{-3} \times \Delta t]$

or, $\Delta t = 48.13\ °C$

Now, $\theta - 15°C = \Delta t$

or, $\theta = 48.13 + 15 = 63.13°C$

Hence, when temperature exceeds 63.13°C, relay fails to operate.

8. A 230 V metal filament lamp has its filament 50 cm long with cross-sectional are of 3×10^{-6} cm². Specific resistance of the filament metal at 20 °C is $4 \times 10^{-6}\ \Omega$ – cm. If the working temperature of the filament is 2000°C, find the wattage of the lamp. Temperature coefficient of resistance of the filament material at 20 °C is 0.0055 per degree centigrade.

Solution:

Given that;

Operating supply voltage (V) = 230 V

Length of filament (l) = 50 cm

Cross-sectional area of filament (a) = 3×10^{-6} cm²

Specific resistance of filament at 20 °C = $4 \times 10^{-6}\ \Omega$ – cm

Therefore, Resistance of filament metal at 20 °C,

$$R_{20} = \frac{\rho l}{a} = \frac{4 \times 10^{-6} \times 10^{-2} \times 50 \times 10^{-2}}{(3 \times 10^{-6}) \times 10^{-4}}$$

or, $R_{20} = 66.667\ \Omega$

Temperature coefficient of resistance at 20 °C,

$\alpha_{20} = 0.0055$ per °C

Now; we know that,

$$R_{2000} = R_{20}\,[1 + \alpha_{20}\,(\Delta t)]$$

or, $R_{2000} = 66.667\,[1 + 0.0055 \times (2000 - 20)]$

or, $R_{2000} = 792.667\ \Omega$

Now, wattage of the lamp (power)

$$P = \frac{V^2}{R_{2000}} = \frac{(230)^2}{792.667} = 66.7368 \text{ Watt} = 66.74 \text{ watt.}$$

9. A lead wire and an iron wire are connected in parallel. Their specific resistance are in the ratio of 49 : 24. The former carries 80 % more current than the letter and the latter is 47 % longer than former. Determine the ratio of their cross-sectional area.

Solution: See example 1.5.

10. Explain the method for converting practical current source into practical voltage source.

Solution: See theory part 1.8.2.

11. The resistivity of metal alloy is $50 \times 10^{-8}\ \Omega$ – m. A sheet of metal 15 cm long, 6 cm wide and 0.014 cm thick. Calculate the resistance in the direction

(a) along the length and

(b) along the thickness

Solution:

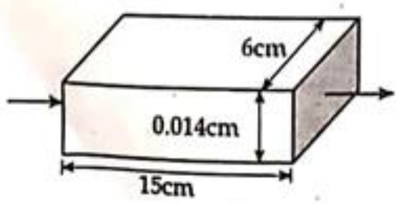
(a) Resistance along the length

Length = 15 cm

Width = 6 cm

Thickness = 0.014 cm

Therefore, cross-sectional area = $0.014 \times 0.06 = 8.4 \times 10^{-6}$ m²



Now,

$$R = \rho\frac{l}{a} = 50 \times 10^{-8} \times \frac{0.15}{8.4 \times 10^{-6}}$$

or, $R = 8.928 \times 10^{-3}\,\Omega$

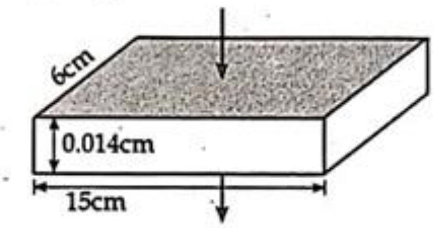
$= 8.93 \times 10^{-3}\,\Omega$

(b) Resistance along the thickness

$A = 0.06 \times 0.15 = 9 \times 10^{-3}\,m^2$

$\therefore$ $$R = \rho\frac{l}{a} = 50 \times 10^{-8} \times \frac{1.4 \times 10^{-4}}{9 \times 10^{-3}}$$

or, $R = 7.77 \times 10^{-9}\,\Omega$



12. A conductor material has free electron density of 10^{24} electrons per m^3. When a voltage is applied, a constant drift velocity of 1.5×10^{-2} m per sec is attained by the electrons. If the cross-sectional area of the material is 1 cm^2, calculate the magnitude of the current?

Solution:

Given that;

$n = 10^{24}$

$e = 1.6 \times 10^{-19}$ °C

$v = 1.5 \times 10^{-2}$ m per sec

$a = 1\,cm^2 = 1 \times 10^{-4}\,m^2$

Now, For magnitude of current; we know,

$i = vena$

$= 1.5 \times 10^{-2} \times 1.67 \times 10^{-19} \times 10^{24} \times 1 \times 10^{-4}$

$= 0.251$ A

13. A piece of resistance wire 15.6 m long of cross-section area 12 mm^2 at a temperature of 0 °C passes a current of 7.9 A. When connected to dc supply at 240 V. Calculate

(i) resistivity of the wire

(ii) The current when temperature rises to 55 °C. The temperature coefficient of wire is 0.00029 per °C.

Solution:

Given that;

Length of wire = 15.6 cm = 0.156 m

$a = 12\,mm^2 = 12 \times 10^{-6}\,m^2$

Resistance at 0 °C

$$R_o = \frac{V}{I} = \frac{240}{7.9} = 30.38\ \Omega$$

(i) Now, to get resistivity of wire, we know

$$R = \frac{\rho l}{q}$$

or, $$\rho = \frac{30.38 \times 12 \times 10^{-6}}{0.156} = 2.34 \times 10^{-5}\ \Omega - m$$

(ii) Again,

$$R_{55} = R_o [1 + \alpha_o \Delta t]$$

or, $$R_{55} = 30.38 \times [1 + 0.00029 \times (55 - 0)]$$

or, $$R_{55} = 30.86\ \Omega$$

Now, value of current when its temperature rises to 55°C is

$$\text{Current (I)} = \frac{V}{R_{55}} = \frac{240}{30.86} = 7.78\ \Omega.$$

14. Two resistor made of different material have temperature coefficients of resistance α_1 = 0.004 °C^{-1} and α_1 = 0.0005 °C^{-1} are connected in parallel and consume equal power at 15 °C. What is the ratio of power consumed in resistance R_2 to that in R_1 at 70 °C?

Solution:

For parallel combination, voltage remains same through two resistors.

Now,

$$\text{Power (P)} = \frac{V^2}{R}$$

At 15 °C, both resistor consumes equal powers, let P_1 and P_2 be the power consumed by resistor then,

$$P_1 = P_2$$

or, $$\frac{V^2}{R_1} = \frac{V^2}{R_2}$$

or, $$R_1 = R_2$$

But, we know that, when R_0' is resistance of 1st resistor at 0°C and α_1 its temperature coefficient of resistance, then at 15°C resistance is given as;

$$R_1 = R_0' (1 + \alpha_1 \times \Delta t)$$

or, $$R_1 = R_0' (1 + 15\alpha_1)$$

Similarly,

$$R_2 = R_0'' (1 + \alpha_2 \times \Delta t)$$

or, $$R_2 = R_0'' (1 + 15\ \alpha_2)$$

Since, $R_1 = R_2$;

$$R_0'(1 + \alpha_1 \times 15) = R_0''(1 + \alpha_2 \times 15)$$

or, $$\frac{R_0'}{R_0''} = \frac{1 + 15\,\alpha_2}{1 + 15\,\alpha_1}$$

Again, at 70 °C, similarly

$$R_1 = R_0'(1 + 70\,\alpha_1)$$

and, $$R_2 = R_0''(1 + 70\,\alpha_2)$$

Now, taking the ratio of P_1 and P_2, we get;

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{\frac{V^2}{R_2}}{\frac{V^2}{R_1}}\right) = \frac{R_1}{R_2} = \frac{R_0'}{R_0''} \times \frac{(1 + 70\,\alpha_1)}{(1 + 70\,\alpha_2)}$$

Substituting value of R_0' and R_0'', we get

or, $$\frac{P_2}{P_1} = \frac{(1 + 15\,\alpha_2)}{1 + 15\,\alpha_1} \times \frac{1 + 70\,\alpha_1}{1 + 70\,\alpha_2}$$

$\therefore$ $$\frac{P_2}{P_1} = \frac{1 + 15 \times 0.005}{1 + 15 \times 0.004} \times \frac{1 + 70 \times 0.004}{1 + 70 \times 0.005} = 0.962.$$

15. A coil connected to constant supply of 100 V draw a current of 13 A at room temperature of 25 °C. After sometime its temperature increased to 70 °C and current fell to 8.5 A. Find the current it will draw, when its temperature will further rise to 80 °C. Also find the temperature coefficient of resistance of the coil at 20 °C.

Solution: First part:

Supply voltage (V) = 100 V

At temperature 25 °C,

$$I = 13\text{ A}$$

$\therefore$ $$R_{25} = \frac{100}{13} = 7.692\ \Omega$$

After sometime; resistance at 70 °C,

$$R_{70} = \frac{100}{8.5} = 11.765\ \Omega$$

Second part: Same as Q. No. 5,

16. Why does the terminal voltage of a real voltage source decreases with increase in load current? Explain how a practical voltage source can be converted into a practical current source.

Solution: See definition part.

17. A coil has resistance of 18 Ω when its mean temperature is 20 °C and of 20 Ω when its mean temperature is 50 °C. Find its mean temperature rise when its resistance is 21 Ω and the ambient temperature is 15 °C.

Solution: See example 1.8.

18. Show that if α_1 is the resistance temperature coefficient of a conductor at temperature t_1 °C then resistance temperature coefficient at t_2 °C is given by $\frac{\alpha_1}{1 + \alpha_1(t_2 - t_1)}$.

Proof:

Suppose, a conductor of resistance Ro at 0 °C is heated to t °C. Its resistance R_t after heating is given by

$$R_t = R_o (1 + \alpha_o t) \quad (1)$$

where, α_o = temperature coefficient at zero °C.

Now,

Suppose, we have a conductor of resistance R_t at temperature t °C. Let this conductor be called from t °C to 0 °C. Then final resistance decreases from R_t to R_o, is given as

$$R_o = R_t [1 + \alpha_t (-t)]$$

or, $$R_o = R_t [1 - \alpha_t t] \quad (2)$$

From equation (2),

$$\alpha_t = \frac{R_t - R_o}{t \times R_t} \quad (3)$$

Substituting, value of R_t from equation (1) in (3),

$$\alpha_t = \frac{R_o(1 + \alpha_o t) - R_o}{t \times R_o(1 + \alpha_o t)} = \frac{\alpha_o t}{t(1 + \alpha_o t)}$$

$$\therefore \quad \alpha_t = \frac{\alpha_o}{1 + \alpha t} \quad (4)$$

Now, replacing

$\alpha_t = \alpha_2$ at $t = t_2$

$\alpha_o = \alpha_1$ at $t = t_1$

Then; we get,

$$\boxed{\alpha_2 = \frac{\alpha_1}{1 + \alpha_1(t_2 - t_1)}}$$

Hence, proved.

19. Derive the formula I = ven A. Where, the symbols have their usual meaning.

Solution: See definition part 1.1.

20. At room temperature of 20 °C, the current flowing at the instant of switching of a 40 Ω filament lamp with 220 V supply is 2 A. The filament has resistance temperature coefficient of 0.005 per °C, at 20 °C. Calculate the working temperature of filament and current taken by it during working condition.

Solution:

Here, operating temperature t °C

$$R_t = \frac{V^2}{P}$$

or, $$R_t = \frac{220^2}{40} = 1210\ \Omega$$

$$\alpha_{20} = 0.005 \text{ per °C}$$

Again, resistance at room temperature 20 °C,

$$I = \frac{V}{R}$$

or, $$R_{30} = \frac{220}{2} = 110\ \Omega$$

Now,

$$R_t = R_{20}\,[1 + \alpha_{20}\,\Delta t]$$

or, $$1210 = 110\,[1 + \alpha_{20}\,\Delta t]$$

or, $$11 = 1 + 0.005\,\Delta t$$

or, $$\Delta t = 2000\ °C$$

Therefore, final temperature = 2000 + 20 = 2020 °C

and; during working condition current is,

$$P = I^2 R$$

or, $$I = \sqrt{\frac{40}{1210}} = 0.1818\ A.$$

21. Explain ideal current and voltage sources.

Solution: See definition part.

22. Define temperature coefficient of resistance. The resistance of a certain length of wire is 4 at 20 °C. Determine

(a) the temperature coefficient of resistance of the wire at 0 °C

(b) the resistance of the wire at 60 °C

Solution: For first part: See theory portion.

For the second part: See example 1.7.

23. Discuss voltage and current source in brief. Also justify the statement "Terminal voltage goes on decreasing on increasing load current".

Solution: See theory part.

24. The field winding of dc motor connected across 230 V supply takes 1.15 A at room temperature of 20 °C. After working for some hours, the current falls to 0.26 A, the supply voltage remaining constant. Calculate the final working temperature of the field winding. Resistance temperature coefficient of copper at 20 °C is $\frac{1}{254.5}$.

Solution:

Here, Supply, voltage (V) = 230 V

Current at 20 °C = 1.15 A

Therefore, resistance at 20 °C

$$R_{20} = \frac{V}{I} = \frac{230}{1.15} = 200\ \Omega$$

Temperature coefficient of resistance at 20 °C

$$\alpha_{20} = \frac{1}{254.5} \text{ per } ^\circ C$$

Final working temperature (T_2) = ?

Here, α_{20} is given. So we don't need to calculate it.

Now,

$$\text{Final resistance of relay} = \frac{230}{0.26} = 884.6\ \Omega$$

Now; we know,

$$R_f = R_{20}\,[1 + a_{20}\,(T_f - T_{20})]$$

or, $8884.6 = 200\left[1 + \frac{1}{254.5}(T - 20)\right]$

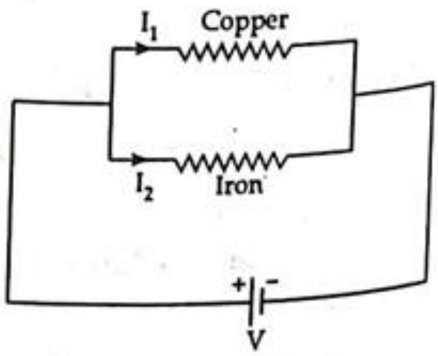
or, $3.423 = 3.929 \times 10^{-3}\,(T - 20)$

or, $T = (891.15)\ ^\circ C$

25. Two conductors one of copper and other of iron are connected in parallel and at 20 °C carry equal currents. What proportion of current will pass through if the temperature is raised to 100 °C. Assume temperature coefficient of resistance at 20 °C for copper as 0.0042 per degree and for iron as 0.006 per °C.

Solution:

Since, they carry equal currents at 20 °C, the two conductors have the same resistance at 20 °C. As temperature is raised their resistances increase through unequally for, Cu



$$R_{100} = R_{20}[1 + (100 - 20) \times 0.0042]$$

$$= 1.336\ R_{20}$$

For iron

$$R_{100} = R_{20}[1 + (100 - 20) \times 0.006] = 1.48\ R_{20}$$

Now; using current divider rule,

Current through copper conductor is

$$I_1 = I \times \frac{R_{100}}{R_{100} + R_{100}} = I \times \frac{1.48\ R_{20}}{1.336\ R_{20} + 1.48\ R_2} = \frac{1.48\ R_{20}}{2.816\ R_2} = 0.5256$$

or, $I_1 = 52.56\ \%$ of I

∴ $I_2 = I - 0.5256\ I = 0.4744\ I = 47.44\ I\ \%$

26. Define the terms of source transformation with suitable example.

Solution: See theory part 1.8.2.

27. The current in the field winding of a motor at 20 °C is 2 A. After running the motor for 6 hours at full load, the current falls to 1.75 A. If the voltage applied across the field winding is 240 V, determine the temperature rise of the winding. The temperature coefficient of resistance of the copper winding at 0 °C is 4.28×10^{-3} K.

Solution: See Q. No. 4.

Hint: Use, V = IR and get R_{20} and R_{final}.

28. Define the terms of source transformation with suitable example.

Solution: See theory part.

29. The current in the field winding of a motor at 20 °C is 2 A. After running the motor for 6 hours at full load, the current falls to 1.75 A. If the voltage applied across the field winding is 240 V, determine the temperature rise of the winding. The temperature coefficient of resistance of the copper winding at 0 °C is 4.28×10^{-3} K.

Solution: Same Q. No. 27.

30. Derive a relation between the known resistance R_1 at t_1 °C and the unknown resistance R_2 at t_2 °C_2, when α_0 is not known.

Solution: For deviation, see theory part.

$$\boxed{R_2 = R_1\,[1 + \alpha_0\,(t_1 - t_2)]}$$

31. Explain the process of source conversion. How is it helpful in solving electrical network?

Solution: For first part: See theory part.

For second part:

Source transformation is the process of simplifying a circuit solution, specially with mixed sources, by transforming voltage sources into current sources, and vice versa, using Thevenin's theorem and Norton's theorem. Source transformation methods are used for circuit simplification to modify the complex circuits by transforming independent current source into independent voltage sources and vice versa. After transformation, we can analyze the circuits by applying a simple voltage and current divider rules. This source transformation method can also be used to convert a circuit from Thevenin's equivalent into Norton's equivalent. So it is helpful in solving electrical networks.

32. Differentiate between electro motive force and potential difference.

Solution: See theory part 1.3.

33. The current in the filled winding of a motor at 20°C is 2A. After running the motor for 6 hours at full load the current falls to 1.75 A. If voltage applied across the field winding is 240 V, determine the temperature rise of the winding. The temperature coefficient of resistance of the copper winding 0°C is 0.00428/°C.

Solution:

Resistance at 20°C, $R_{20} = \frac{240}{2} = 120\ \Omega$

After running moter for 6 hrs,

Resistance at t°C, $R_t = \frac{240}{1.75} = 137.15\ \Omega$

∴ Temperature coefficient at 20°C

$$\alpha_{20} = \frac{1}{\frac{1}{\alpha_0} + (t - 0)} = \frac{1}{\frac{1}{0.00428} + (20 - 0)} = 0.00394/°C$$

∴ $\alpha_{20} = 0.0394/°C$

∴ $R_t = R_{20}\,[1 + \alpha_{20}\,(t - 20)]$

or, $137.15 = 120\,[1 + .00394\,(t - 20)]$

or, $1.143 = 1 + 0.00394\,(t - 20)$

or, $t - 20 = 36.27$

or, $\Delta t = 36.27$

∴ Temperature rise = 36.27°C

34. Explain the V-I characteristics of Ideal and practical current source.

Solution: See theory part 1.8.

35. A coil is connected across a constant dc source of voltage 120 V, draw a current of 12 A at room temperature. After running 6 hours temperature rises to 65°C and current reduces to 8A. Calculate the current when temperature increases to 80°C and the coefficient of resistance at 30°C. [Consider room temperature = 25°C].

Solution:

Resistance of coil at 25°C,

$$R_{25} = \frac{120}{12} = 10\ \Omega$$

Resistance of coil at 65°C,

$$R_{65} = \frac{120}{8} = 15\ \Omega$$

$\therefore$ $R_{65} = R_{25}\,[1 + \alpha_{25}\,(65 - 25)]$

or, $\alpha_{25} = 0.0125/°C$

i) Resistance of coil at 80°C

$$R_{80} = R_{25}\,[1 + \alpha_{25}\,(80 - 25)]$$
$$= 10\,[1 + 0.0125 \times 55]$$

$\therefore$ $R_{80} = 16.875\ \Omega$

$\therefore$ Current at temperature 80°C $= \frac{120}{16.875} = 7.111$ A

ii) Temperature coefficient at 30°C

$$\alpha_{30} = \frac{1}{\frac{1}{\alpha_{25}} + (30 - 25)} = \frac{1}{\frac{1}{0.0125} + 5} = 0.01176/°C$$

ADDITIONAL QUESTION SOLUTIONS

1. A current of 5 A flowing through a conductor for 4 minutes having resistance 10 Ω. Determine
   (i) How many columbs of charge flow?
   (ii) How many electrons pass through any section of this resistor?
   Take charge of electron e = 1.6×10^{-19} C

Solution:

Given that;

Current (I) = 5 A, time of flow = 4 min = 4 × 60 seconds.

Resistance (R) = 10 Ω

(e) = 1.6×10^{-19} C

Now,

(i) Charge (Q) $= i \times t = 5 \times 4 \times 60 = 1200$ C

(ii) For number of electrons, we know;

$$Q = n.e$$

or, $$n = \frac{Q}{e} = \frac{1200}{1.6 \times 10^{-19}}$$

or, $$n = 75 \times 10^{20} \text{ electrons}$$

2. A piece of silver wire has resistance of 1Ω. What will be the resistance of magnesium wire of one-third length and one-third the diameter? The specific resistance of magnesium is 30 times that of silver wire?

Solution:

<u>For silver wire</u>

length = l

Diameter = d

Resistance = 1 Ω

<u>For magnesium wire</u>

length = $\frac{l}{3}$

Diameter = $\frac{d}{3}$

Resistance = R

Now, we know that

$$R = \frac{\rho l}{A}$$

and so, $$R_1 = \frac{\rho_1 l_1}{A_1} \quad (1)$$

$$R_2 = \frac{\rho_2 l_2}{A_2} \quad (2)$$

Suffix 1 for silver and 2 for magnesium;

Now, $$\frac{R_2}{R_1} = \frac{A_1}{A_2} \times \frac{\rho_2 l_1}{\rho_1 l_1} = \frac{\frac{\pi(d_1)^2}{4} \times \rho_2 \times l_2}{\frac{\pi(d_2)^2}{4} \times \rho_1 \times l_1}$$

∴ $$\frac{R_2}{R_1} = \frac{d_1^2 \times P_2 \times l_2}{d_2^2 \times P_1 \times l_1}$$

Now putting all the values, we get;

$$R_2 = 1 \times \left(\frac{d}{d/3}\right)^2 \times \left(\frac{1}{30}\right) \times \left(\frac{1}{3}\right)$$

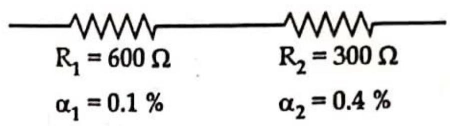
or, $$R_2 = 90\ \Omega$$

3. Two coils connected in series have resistance of 600 Ω and 300 Ω with temperature coefficient of 0.1% and 0.4 % respectively at 20°C. Find the resistance of the combination at a temperature of 50°C. What is the effective temperature coefficient of combination.

Solution:

Given, temperature = 20 %

at 50°C their resistance can be calculate as;



at 50°C,

The resistance of 600 Ω conductor

$$R_1^{50} = R_1^{20}\,(1 + \alpha_1\,\Delta t) = 600\left[1 + \frac{0.1}{100}(50 - 20)\right] = 618\ \Omega$$

and, Resistance of 300 Ω conductor

$$R_2^{50} = R_1^{20}\,(1 + \alpha_2\,\Delta t) = 300\left[1 + \frac{0.4}{100}(50 - 20)\right] = 336\ \Omega$$

Therefore, total combined resistance at 50°C = 618 + 336 Ω = 954 Ω

Again, combined resistance at 20°C = 600 + 300 = 900 Ω

If β be the combined effective temperature coefficient at 20°C, then,

$$R^{50} = R^{20}\,(1 + \beta\,\Delta t)$$

or, $$954 = 900\,[1 + \beta(50 - 20)]$$

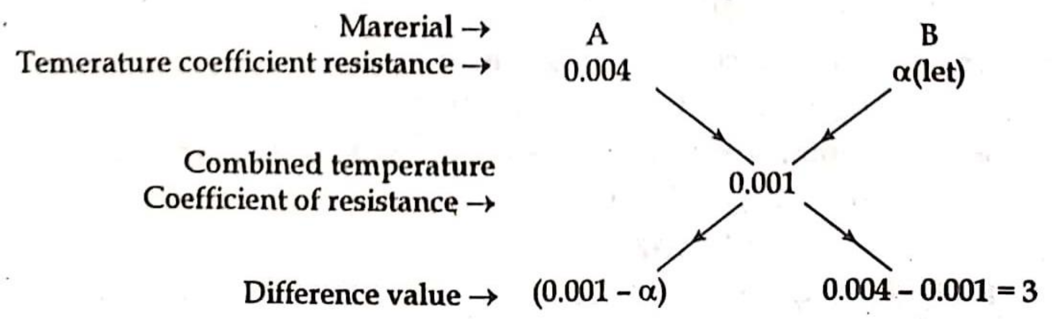
or, $$\beta = 0.002$$

4. Two wire A and B are connected in series at 0°C and resistance of B is 10 times that of A. The resistance temperature coefficient of A is 0.4% and that of combination is 0.1% find the resistance temperature coefficient of B?

Solution:

This question can be solved by using equation of resistance and temperature. But, if may be lengthy.

A quick solution is described here as;



Now, from question;

$$\frac{R_B}{R_A} = 10$$

Also, from figure;

$$\frac{R_B}{R_A} = \frac{0.003}{(0.001 - \alpha)}$$

$\therefore$ $$10 = \frac{0.003}{0.001 - \alpha}$$

or, $\alpha = 0.00143$

$\therefore$ $\boxed{\alpha = 0.143\%}$

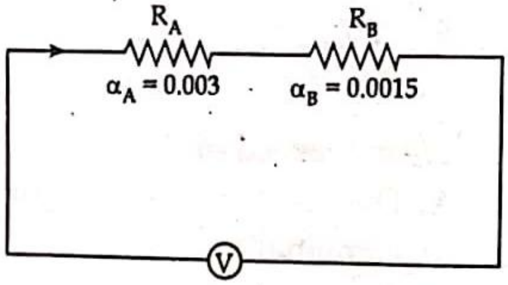
5. A Resistor of 80 Ω resistance having α = 0.0021/°C is to be constructed. Wires of two materials of suitable cross-sectional area are available. For material A, the resistance is 80 Ω per 100 m and α_A = 0.003/°C. For material B, the corresponding values are 60 Ω, per meter and 0.0015/°. Calculate suitable length of wires of materials A and B to be connected in series to construct the required resistor. Assume temperature to be same in each case.

Solution:

Let R_A and R_B be the two resistance values of materials A and B respectively. Also, the combination has,

combined resistance (R) = 80 Ω

combined temperature coefficient of resistance (α) = 0.0021/°C



Let, the final temperature be t °C.

Now, at final temperature; the resistance becomes;

$$R_A' = R_A(1 + \alpha_A t) = R_A\,(1 + 0.003\ t)$$

and that; of wire B,

$$R_B' = R_B(1 + 0.0015t)$$

Now, combined resistance at temperature t is,

or, $$R = R_A(1 + 0.003\ t) + R_B\,(1 + 0.0015\ t) \quad (1)$$

Now for combination, α = 0.0021, R = R_A + R_B and find temperature t

or, $$R = (R_A + R_B)\,(1 + 0.0021\ t) \quad (2)$$

Now, equation (1) = equation (2),

$$R_A(1 + 0.003\ t) + R_B(1 + 0.0015\ t) = (R_A + R_B)\,(1 + 0.0021\ t)$$

Now, simplifying the above, we get;

$$\frac{R_B}{R_A} = 1.5$$

Now,

$$R_A + R_B = 80$$

or, $R_A + 1.5R_A = 80$

or, $2.5\ R_A = 80$

or, $R_A = 32\ \Omega$

and, $R_B = 48\ \Omega$

Now, if L_A and L_B are their lengths, then

$$\left(\frac{L_A}{R_A}\right)_{individual} = \left(\frac{L_A}{R_A}\right)_{combination}$$

or, $\left(\frac{L_A}{32}\right)_{com} = \left(\frac{100}{80}\right)_{ind}$

or, $L_A = \left(\frac{100}{80}\right) \times 3$

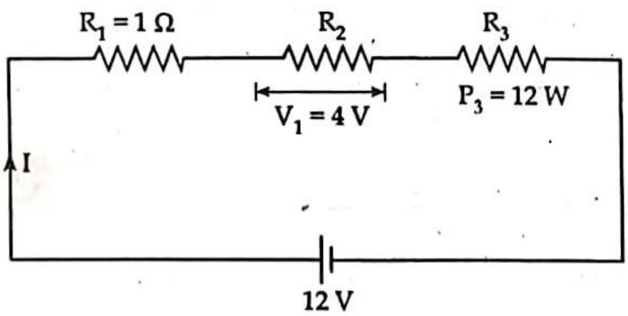
or, $L_A = 40\ m$

Similarly, $L_B = 80\ m$

Hence, length required for wire A = 40 m and wire B = 80 m.

6. Three resistor are connected in series across a 12 V battery. The first resistor has a value of 1 Ω. Second has voltage drop of 4 V and the third has power dissipation of 12 W. Calculate the value of circuit current.

Solution:



Now, from figure;

$$V = \Sigma IR$$

or, $12 = I(1 + R_2 + R_3)$ **(1)**

Again,

$$V_2 = IR_2$$

or, $I = \frac{V_2}{R_2} = \frac{4}{R_2}$ **(a)**

Also, $I^2 R_3 = 12$ **(b)**

From (a) and (b);

$$\frac{I^2(R_2)^2}{I^2R_3} = \frac{42}{12}$$

or, $R_3 = \frac{3}{4}R_2^2$ (1)

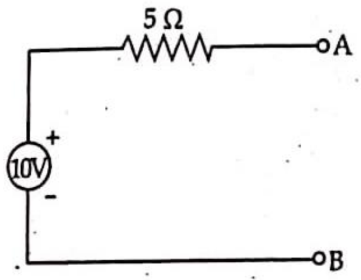
Now from equation (1)

$$12 = \frac{4}{R_2}\left(1 + R_2 + \frac{3R_2^2}{4}\right)$$ (2)

This is quadratic equation, by solving we get,

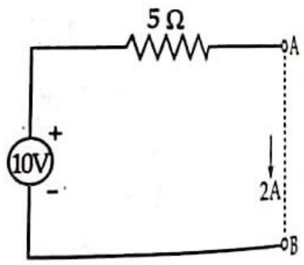
$I = 2A$ and $I = 6A$

7. Convert the voltage shown in the figure below into an equivalent current source:



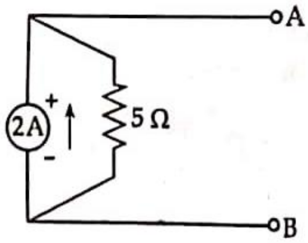
Solution:

Current following, if we joined terminal A and B is

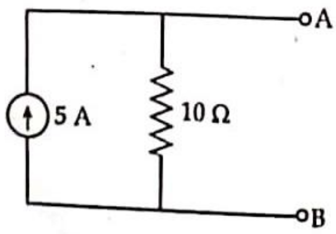


$$I = \frac{V}{R} = \frac{10}{5} = 2A$$

Now, to convert into current source, place 5 Ω resister in parallel with current 2A.



8. Find the equivalent source for the circuit shown in the figure below.



Solution:

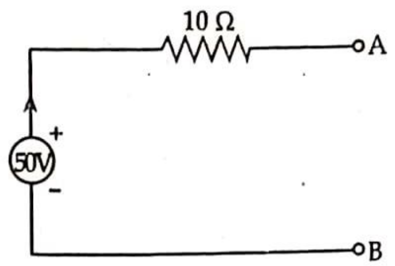
The open circuit voltage across A and B is

$V_{AB} = 5 \times 10 = 50$ V

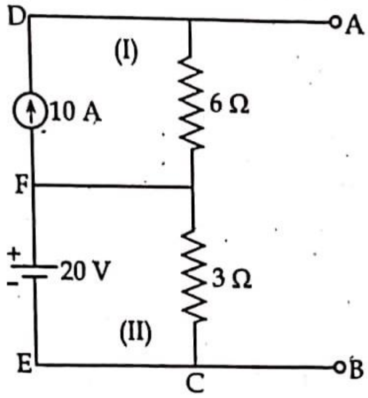
Now,

By adding a battery of 50 V, with a resistor of 10 Ω in series, the source can be converted into equivalent voltage source.

See the given figure.



9. Replace the given network by A single current source in parallel with resistance

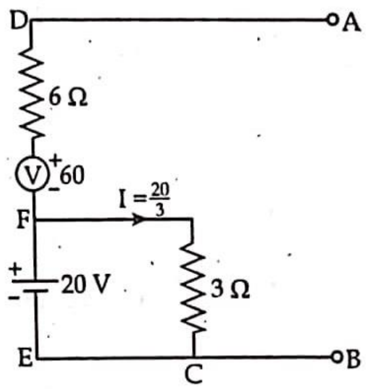


Solution:

For $loop_I$, equivalent voltage

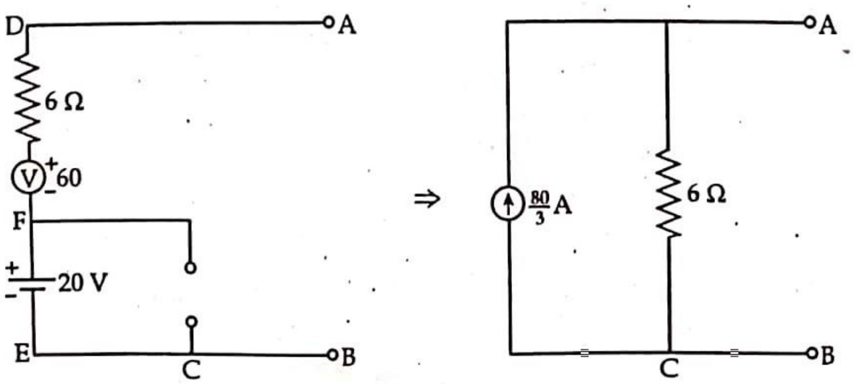
$$V = IR = 10 \times 6 = 60 \text{ V}$$

Now, re-drawing figure



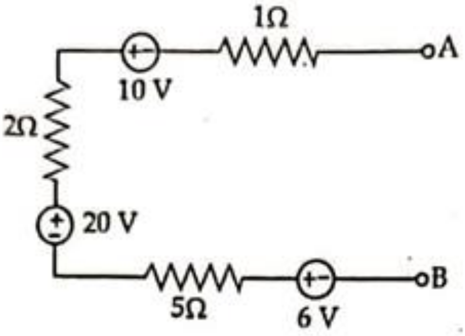
Here, the current of $\frac{20}{3}$ A is flowing inside the $loop_{II}$, which does not affect the terminal voltage between AB. So, we ommited this resistor.

Now,



where, $I = \frac{60 + 20}{6} = \frac{80}{6}$ amp.

10. Find the equivalent current source for the circuit shown in figure below?



Solution:

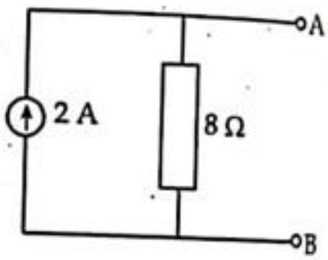
The total voltage source in the circuit is.

$$V = \Sigma v_i = 6 + 20 - 10 = 16\ V$$

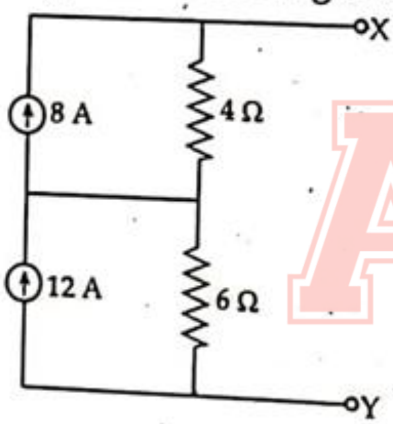
and, $R = 5 + 2 + 1 = 8\ \Omega$

Now, $I = \dfrac{V}{R} = \dfrac{16}{8} = 2\ A$

Now, to convert above circuit into equivalent current source, place a resistance of 8 Ω in parallel with 2 A current source.



11. Convert the following circuit into single voltage source.



Solution

The solution can be done as

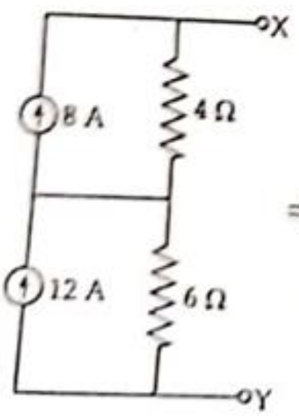


Fig (a)

⇒

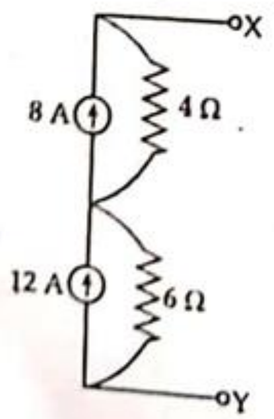


Fig (b)

⇒

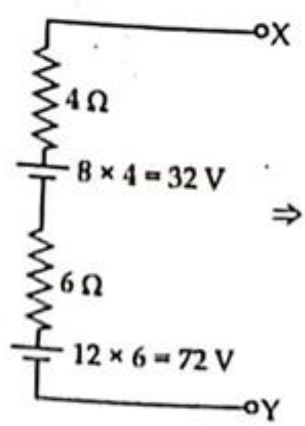


Fig (c)

⇒

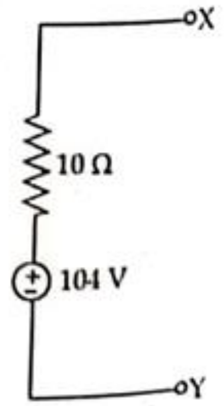


Fig (d)

CHAPTER 2

DC CIRCUITS

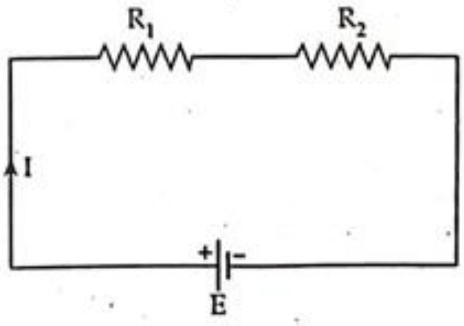
2.1 INTRODUCTION OF AN ELECTRICAL CIRCUIT

Electrical circuit is the closed path in which the current flows from the supply points through the load to complete path. It is interconnection of circuit elements. In this chapter we have to study about various types of electrical circuit. Following are three types of electrical circuit;

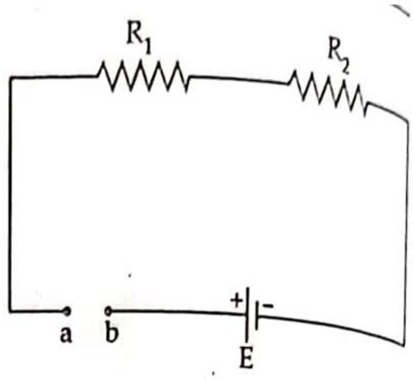
1. Closed circuit
2. Open circuit
3. Short circuit

1. Closed circuit

When a load is connected between two terminals of an electrical supply in such a way that the current should pass through the load is said to be closed circuit. In closed circuit current flows through and voltage drops across resistor are proportional to their resistance.



2. Open circuit



In a circuit, if there is no way to the flow of current due to disconnection of wire or if the switch is off state then the circuit is said to be open circuit. If circuit becomes open anywhere, following two effects are produced.

(i) Since, open offers infinite resistance, circuit current becomes zero and hence there is no voltage drop across resistors.

(ii) Whole of the apply voltage is felt across the open. *i.e.*, across the terminal a and b.

3. Short circuit

When the wire contacts each other and supply is on then short circuit occurs. Here, two terminals of the supply is connected directly without the load and the current flow in the circuit is infinite because it has no resistance.

Short in series circuit

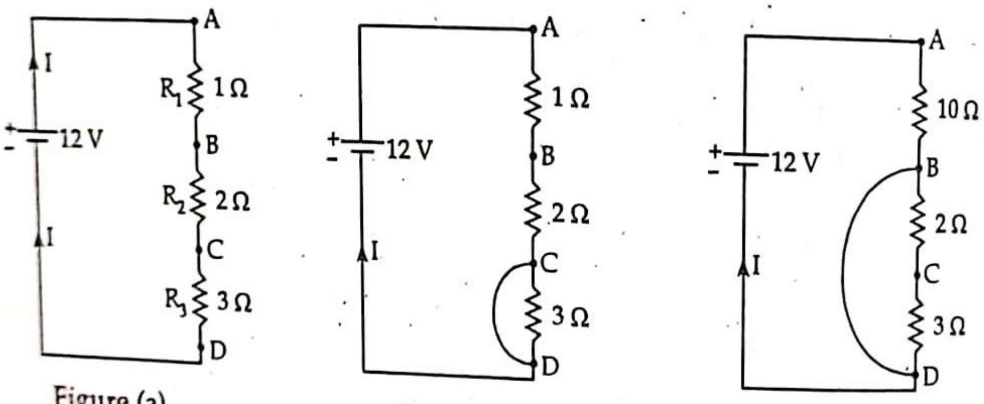


Figure (a) Figure (b) Figure (c)

Since, at short circuit, there is almost zero resistance and hence, it causes flow of excessive current to the loads. Due to this power dissipation by load increase many times and load burn out. For more clear, see example below;

Example 2.1

For above circuit diagram, what will be effect on power dissipation when (a) 3 Ω resistor is shorted (b) when both 3 Ω and 2 Ω resistor is shorted.

Solution:

From given figure (a), its normal circuit.

where, $V = 12\ V$, $R_{eq} = 1 + 2 + 3 = 6\ \Omega$

$$\therefore \quad I = \frac{V}{R_{eq}} = \frac{12}{6} = 2\ A$$

Now, Total power dissipated by loads,

$$P = I^2 R$$

or, $P = (2)^2 \times 6 = 24$ Watt

Now; from figure (b),

where, 3 Ω resistor has been shorted out by joining terminal C and D by copper coire.

∴ Resistance between CD terminal becomes zero

Now,

$$R_{eq} = 1 + 2 + 0 = 3\ \Omega$$

and, $I = \dfrac{V}{R_{eq}} = \dfrac{12}{3} = 4\ A$

Power dissipated (P) = $I^2 R = (4)^2 \times 3 = 48$ Watt.

From figure (c),

When both 2 Ω and three Ω resistor is shorted then,

$$R = 1 + 0 + 0 = 1\ \Omega$$

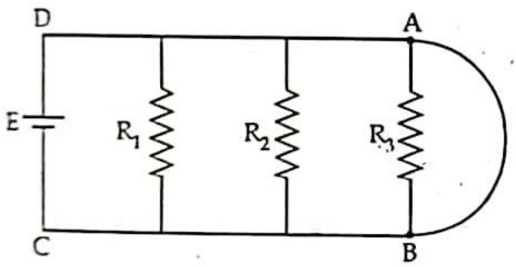
and, $I = \dfrac{V}{R} = \dfrac{12}{1} = 12\ A$

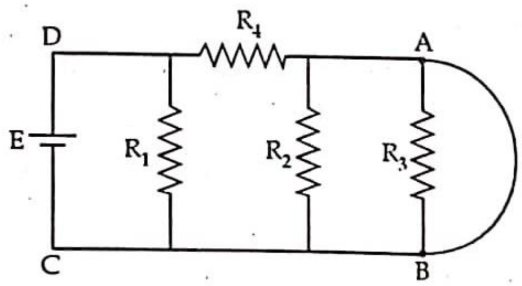
Power dissipated (P) = $I^2 R = (12)^2 \times 1 = 144$ Watt.

Comment:

Due to flow of this excessive current, connecting wires and other circuit components can become hot enough to ignite and burn out.

Short in parallel circuit





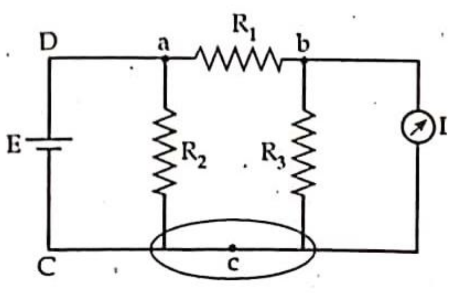
In figure (a); here, not only R_3 is shorted but also R_2 and R_3 are shorted out. *i.e.*, in parallel circuit, short across one branch means short across all branches. There is no current in the shorted resistors. If there were three bulbs then they will not glow.

- The shorted components are not damaged. If short circuit is removed, then they will work properly.
- In figure (b), a short circuit across R_3 may short out R_2 but not R_1 since it is protected by R_4.

Basic circuit parameters

i) Parameters: The various circuit elements are called parameter. *i.e.*, resistance, capacitance, etc.

ii) Linear circuit: It is one whose parameters are constant with times, they do not change with voltage and current and obeys ohm law.



iii) Non linear circuit: It's that circuit whose parameters are changed with voltage and current and do not obey ohm law.

iv) Bilateral circuit: This is one, whose characteristics are same in both directions. Transmission line is its example.

v) Unilateral circuit: It is that circuit whose properties change with their direction of operations. *For example*; diode, rectifier, etc.

vi) Electric network: A combination of various electric elements, connected in any manners is called an electric network.

vii) Passive network: It is one which contains no source of e.m.f. in it.

viii) Active network: It is one which contains one or more than one source of e.m.f. along with passive element.

ix) Active element: An active element is one which supplies electrical energy to the circuits. *For example*; voltage and current sources.

x) Passive element: It is one which receives electrical energy and then either converts it into heat or store in electric or magnetic field. *For example*; resistor, capacitor and inductor.

xi) Node: It is point in a circuit where two or more circuit elements are connected together at equipotential. In figure (a), (b) and (c) are node.

NOTE

A region whose every point has the same potential is called an equipotential region.

xii) Junction: It is a point in a network where three or more circuit elements are joined.

xiii) Branch: It is that path of network which lies between two nodes.

xiv) Loop: It is a closed path in a circuit in which no element or node is encountered more than once.

xv) Mesh: It is a loop that contains no other loop within it.

Classification of Electrical Circuits

An electrical circuit can be classified as;

i) Series circuit
ii) Parallel circuit
iii) Series parallel circuit
iv) Mesh or network

2.2 SERIES CIRCUIT

When resistance are so connected, such that same current passes each resistance, then they are said to be connected in series circuit. Here, the resistance R_1, R_2, and R_3 are connected in series with each other. Also, the current flows through each resistor is in same direction. Let, I be the amount of current, that flows in circuit. Then, by ohm's law, each resistor has voltage drop across it, given as

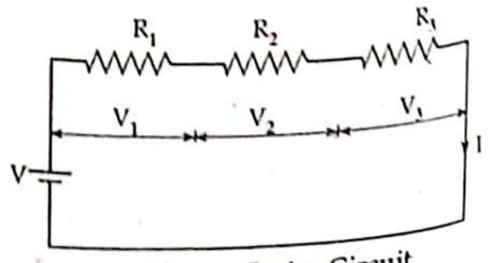


Figure: Series Circuit

$$V_1 = IR_1, V_2 = IR_2, V_3 = IR_3$$

The total drop in three resistors put together is,

$$V = V_1 + V_2 + V_3 \qquad \text{where, } V = IR$$

or, $$IR = IR_1 + IR_2 + IR_3$$

or, $$R = R_1 + R_2 + R_3 \qquad (1)$$

$\therefore$ $\boxed{R_{eq} = \Sigma R}$ (R_{eq} = Equivalent Resistance)

Hence, equivalent resistance of the circuit is the sum of individual resistance connected in series.

Voltage Divider Rule

Since, in series circuit, same current flows through each of the given resistor, voltage drop varies directly with its resistance in above figure, voltage across the individual resistor is,

$$V_1 = IR_1 = R_1 \times \left(\frac{V}{R_{eq}}\right) = R_1 \times \frac{V}{R_1 + R_2 + R_3}$$

$$\therefore \quad \boxed{V_1 = R_1 \times \frac{V}{R_1 + R_2 + R_3}}$$

Similarly,

$$V_2 = R_2 \times \frac{V}{R_1 + R_2 + R_3}$$

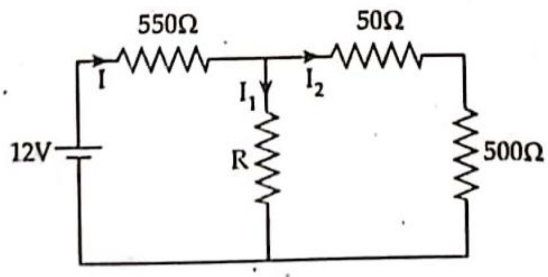
and, $$V_3 = R_3 \times \frac{V}{R_1 + R_2 + R_3}$$

If there is conductance (G) given in the circuit; then use,

$$\boxed{G = \frac{1}{R}}$$

Example 2.2

What is the value of unknown resistor R in the figure below; if voltage drop across 500 Ω resistor is 2.5 volts.



Solution:

Here, Current flowing through 500 W resistor is;

$$I_2 = \frac{V}{R}$$

or, $$I_2 = \frac{2.5}{500} = 0.005 \text{ A}$$

and, Voltage drop across 50 Ω

$$V_{50} = 50 \times 0.005 = 0.25 \text{ V}$$

Now, Voltage drop across 550 Ω is,

$$V_{550} = 12 - (2.5 + 0.25) = 9.25\ V$$

Hence, current flowing through 550 Ω is,

$$I = \frac{9.25}{550} = 0.0168\ A$$

So, current flowing through branch R is,

$$I_1 = I - I_2 = 0.0168 - 0.005 = 0.0118\ A$$

Now, using,

$$V = IR$$

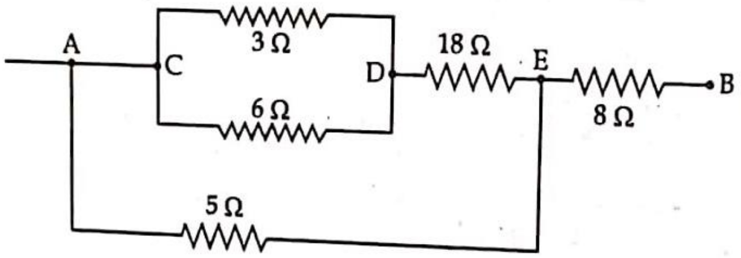
We get,

or, $2.75 = 0.0118\ R$

or, $R = 233\ \Omega$

Example 2.3

Calculate the effective resistance of the following circuit and the voltage drop across each resistance when a potential difference of 60 V is applies between point A and B.



Solution:

The equivalent resistance between A and B is

$$R_{AB} = [(3 \parallel 6) + 18] \parallel 5 + 8$$

$$= \left(\frac{3 \times 6}{3 + 6} + 18\right) \parallel 5 + 8$$

$$= \frac{20 \times 5}{20 + 5} + 8 = 12\ \Omega$$

Total current flowing in circuit,

$$I = \frac{V}{R} = \frac{60}{12} = 5\ A$$

Again, $R_{CDE} = (3 \parallel 6) + 18 = 20\ \Omega$

and, Current flowing in this branch $= 5 \times \frac{5}{20 + 5} = 1\ A$

∴ Current through 5 Ω

or, $5 - 1 = 4\ A$

and, Voltage drop across 5 Ω = 5 × 4 = 20 V

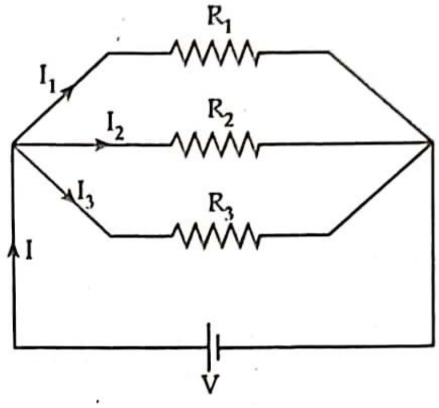
Voltage drop across 18 Ω = 18 × 1 = 18 V

Voltage drop across 8 Ω = 8 × 5 = 40 V

Voltage drop across 3 Ω and 6 Ω = 1 × 2 = 2 V

2.3 PARALLEL NETWORK

When resistors are connected across one another, so that the same voltage is applied between the end points of each then they are said to be in parallel circuit. The current in each resistor is different and the current taken from the supply is divided among the resistors.



From above parallel circuit,

$$I = I_1 + I_2 + I_3$$

Now, from ohm's law,

Using $V = I_1 R_1$, $V = I_2 R_2$ and $V = I_3 R_3$

[V is same for parallel circuit i.e. $V = V_1 = V_2 = V_3$]

We get,

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

or, $$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

or, $$\boxed{Geq = G_1 + G_2 + G_3}$$

Also, $$R = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

Hence, the equivalent resistance of three parallel resistors is equal to the product of their resistances divided by sum of products of two resistors taken in order.

Current Divider Rule

From above figure,

$$I_1 = \frac{V}{R_1} = \frac{I R_{eq}}{R_1} = \frac{I}{R_1} \times \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

$$\therefore \quad I_1 = I \times \frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

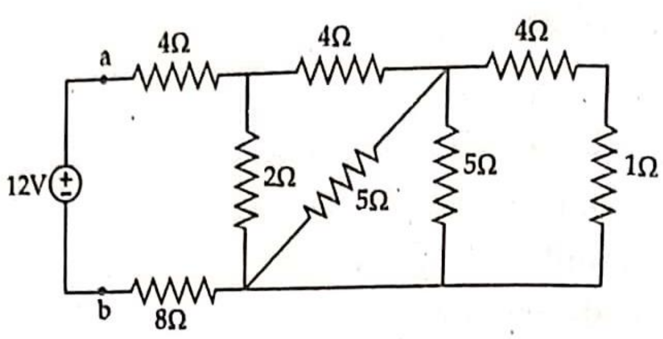
Similarly,

$$I_2 = I \times \frac{R_1 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

$$I_3 = I \times \frac{R_1 R_2}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

Example 2.4

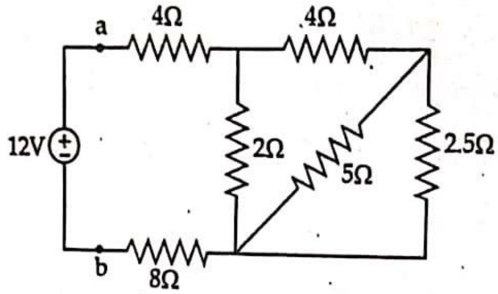
Calculate the equivalent resistance across ab and also current supplied by the source.



Solution:

In figure, 4 Ω and 1 Ω is in series. So, 4 + 1 = 5 Ω is in parallel with another 5 Ω. So its equivalent resistance

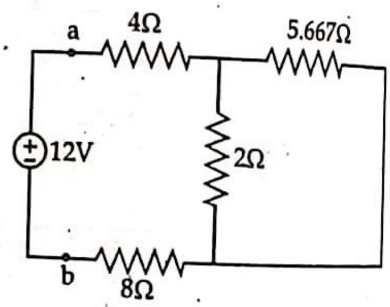
$$\frac{5 \times 5}{5+5} = 2.5\ \Omega$$



Here, $(5 \parallel 2.5) + 4\ \Omega = 5.667\ \Omega$

$\therefore \quad R = 4 + (5.667 \parallel 2) + 8 = 13.479\ \Omega$

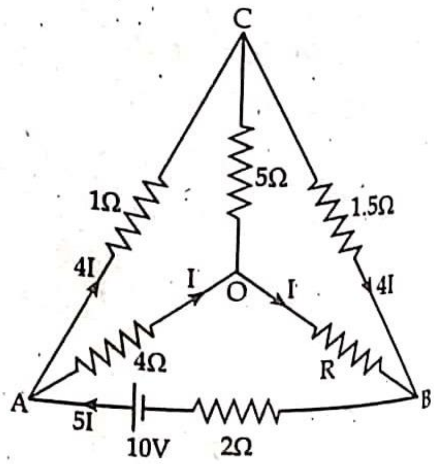
Now, $I = \frac{V}{R} = \frac{12}{13.479} = 0.89\ A$



Example 2.5

Calculate the value of unknown resistor R and the current flowing through it. When the current in the branch OC is zero.

Solution:



Let, Current through OB = I

so, $\quad I_{AO} = I_{OB} = I$

and, $\quad I_{OC} = 0$

Now; Equating voltage drop, we get

$$V_{AO} = V_{AC}$$

Current of 4I flows through AC and same through branch CB. So voltage drop in branch OB and CB are therefore equal.

Now,

$$V_{OB} = V_{BC}$$

or, $\quad I \times R = 1.5 \times 4\ I$

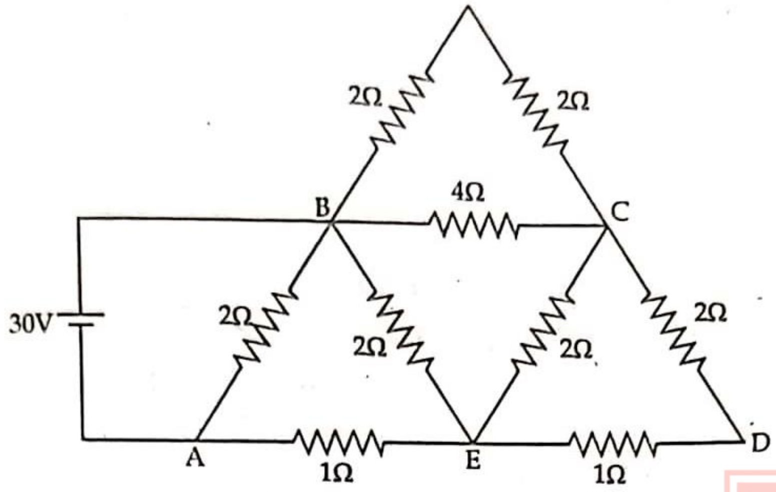
or, $\quad \boxed{R = 6\ \Omega}$

Now, at node A, applying KCL, a current of 5 I flows through branch B to A. Applying KVL around the loop BAOB

$\therefore \quad I = 0.5\ A$

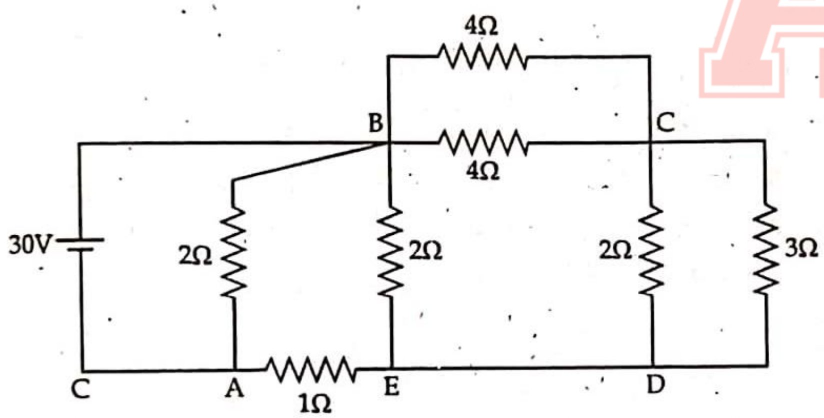
Example 2.6

Determine the current by the source in the circuit shown below.



Solution:

The figure can be re-drawn as,

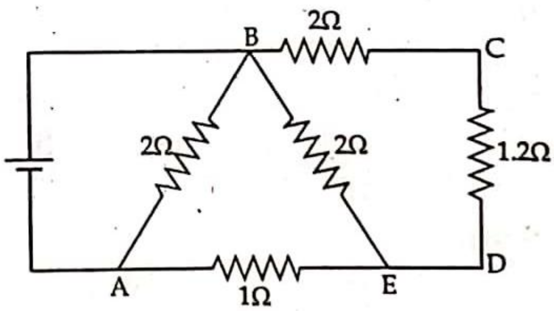


Here,

4 and 4 are parallel, 3 Ω and 2 Ω are parallel

$$R_{CD} = \frac{3 \times 2}{3 + 2} = 1.2\ \Omega$$

and, $$R_{BC} = \frac{4 \times 4}{4 + 4} = 2\ \Omega$$



Now, equivalent resistance is;

$$\begin{aligned} R &= [(2 + 1.2) \parallel 2 + 1] \parallel 2 \\ &= [(3.2 \parallel 2) + 1] \parallel 2 \\ &= (1.23 + 1) \parallel 2 = 1.054\ \Omega \end{aligned}$$

Now, current delivered by battery is;

$$I = \frac{V}{R} = \frac{30}{1.054} = 28.463\ \text{A}$$

Example 2.7

A 100 watt 250 V bulb is put in series with a 40 watt 250 V bulb across 500 V supply what will be the power consumed by each bulb will such combination work.

Solution:

Given that,

Bulb 1

$P_1 = 100$ watt

$V_1 = 250$ V

$\therefore \quad R_1 = \dfrac{V^2}{P_1} = \dfrac{(250)^2}{100} = 625\ \Omega$

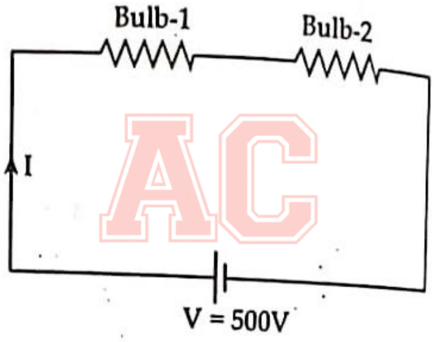
Bulb 2

$P_2 = 40$ watt

$V_2 = 250$ V

$\therefore \quad R_2 = \dfrac{(250)^2}{40} = 1562.5\ \Omega$

When, both bulbs are connected in series with 500 V then, current through the circuit is;



$$I = \frac{V}{R_1 + R_2} = \frac{500}{625 + 1562.5}$$

or, $\quad I = 0.2286$ A.

Now, Power consumed by bulb 1 $= I^2 R_1$

$= (0.2286)^2 \times 625 = 36.661$ watt

Power consume by bulb 2 $= I^2 R_2 = (0.2286) \times 1562.5 = 81.653$ watt

Also, Voltage across bulb 1 $\quad V_1 = IR_1 = 0.2286 \times 625 = 142.875$ V

Voltage across bulb 2 $\quad V_2 = 0.2286 \times 1562.5 = 357.187$ V

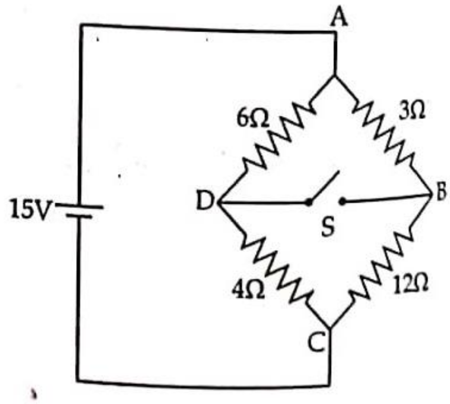
This combination will not work because voltage across 40 watt bulb is greater than its rated 250 V.

Example 2.8

In the unbalanced bridge circuit in fig;

i) find the potential difference across the open switch S.

ii) find the current which will flow through the switch when it is closed



Solution:

i) To get potential difference across A, when switch is open, the figure can be re-drawn as

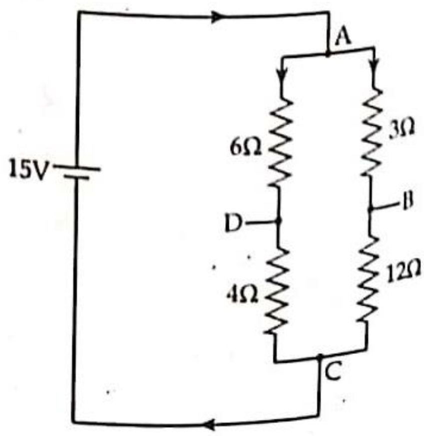
Here; equivalent resistance is

$$R = \frac{(6+4) \times (12+3)}{(6+4) + (12+30)}$$

or, $\quad R = 6\ \Omega$

and, $\quad$ Current through circuit is

$$I = \frac{15}{6} = 2.5\ \text{A}$$



Also, using CDR,

Current through ADC branch is

$$I_{ADC} = 2.5 \times \frac{15}{25} = 1.5\ \text{A}$$

∴ $I_{ABC} = 2.5 - 1.5 = 1$ A

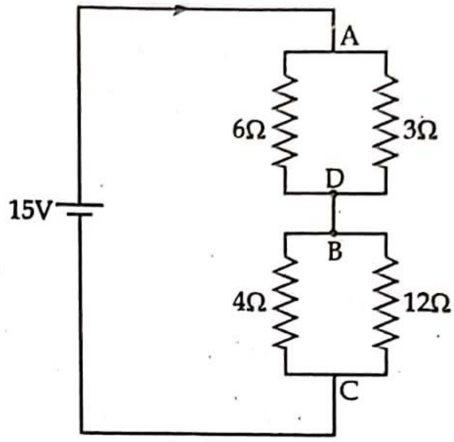
Now; using KVL, we get

$$V_D - 4 \times 1.5 - (-12 \times 1) = V_B$$

or, $V_D - 6 + 12 = V_B$

or, $V_D - V_B = V_{DB} = -6$ V

Now, again, when switch is closed; then, the switch acts as short circuit. Now the figure can be re-drawn as



Equivalent resistance is,

$$R = (6 \parallel 3) + (4 \parallel 12)$$
$$= 2 + 3 = 5\,\Omega$$

and current

$$I = \frac{15}{5} = 3 \text{ A}$$

Now, from figure

Current through Branch AB is $= 3 \times \frac{6}{3+6} = 2$A

Current through branch AD = 1A

Voltage drop across ADC = ABC = 15 V

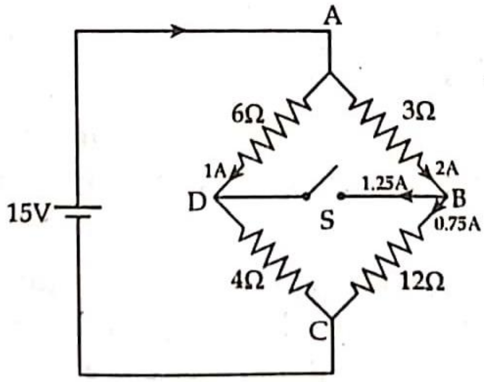
Voltage drop across AB = 3 × 2 = 6V

∴ Voltage drop across BC = 15 – 6 = 9 V

So, current flowing through arm BC $= \frac{9}{12} = 0.75$ A

Hence, remaining current that flowing through switch is

$$2 - 0.75 = 1.25\text{A}$$



2.4 KIRCHHOFF'S LAW

Kirchhoff's law is used to find out the current flow in the electrical circuit easily where ohm's law is not applicable. It is applicable both for AC and DC circuits. Those laws are also useful for calculating equivalent resistance of the circuit. The two Kirchhoff's laws are;

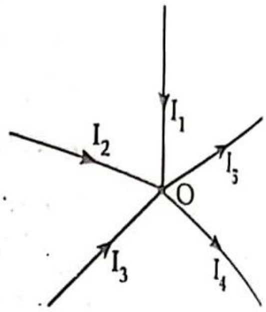
i) Kirchhoff's current or point law (KCL)

ii) Kirchhoff's voltage law or mesh law (KVL)

i) Kirchhoff's current or point law (KCL)

Statement of KCL: The sum of the currents flowing towards a junction is equal to the sum of the currents flowing away from the junction. This is known as Kirchhoff's current law.

In the figure, 'O' is the junction formed by five conductors. The currents in these conductors are I_1, I_2, I_3, I_4 and I_5. Some of these currents are flowing towards junction 'O' and some currents flowing away from it.



So, we can write

$$I_1 + I_2 + I_3 + I_4 + I_5 = 0$$

But, I_5 and I_4 flow away from 'O'

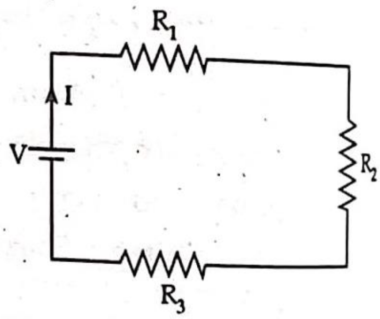
$\therefore \quad -I_1 + I_2 + I_3 - I_4 - I_5 = 0$

or, $\boxed{I_1 + I_2 + I_3 = I_4 + I_5}$

This is required KCL.

ii) Kirchhoff's voltage or mesh law (KVL)

Statement: The algebric sum of the products of current and resistances in each of the conductors in any closed path or mesh in a network is equal to the algebric sum of e.m.f.s in that circuit



Mathematically,

$$\boxed{\Sigma IR = \Sigma V}$$

This is Kirchhoff's 2nd law

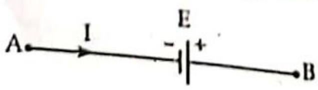
If we start from a particular junction and go around the mesh till we come back to the starting point. Then we must be at the same potential with which we are started.

Sign convention for KVL

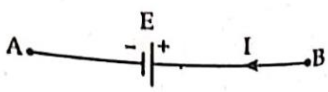
In applying Kirchhoff's law, to specific problems, particular attention should be taken. For this following sign convention is taken into mind:

(a) Sign of battery

For battery, rise in voltage is given as positive and fall in voltage is negative sign. So, when we go from the Negative terminal of the battery to its positive terminal, there is a rise in potential and hence this voltage should be given as positive sign. When we go from positive terminal to negative terminal then, there is fall of voltage and is denoted by negative sign.



Condition: Rise in voltage
sign of E = +ve



Condition in voltage
sign of E = –ve

(b) Sign of IR drop

For, resistor, if we go through in the same direction of current then there is a fall in potential because current flows from a higher to a lower potential. Hence, this voltage fall should be taken as negative. If we go in the opposite direction of current, then there is a rise in voltage. Hence, this voltage rise should be taken as positive.

2.5 POWER AND ENERGY

The potential difference applied across the coil causes the current to flow through it. This implies that there is an electrical workdone due to moving the charge in the circuit, this is called an electrical workdone. Its unit is joule. Therefore,

1 joule = volt × columb

or, $\boxed{W = Vq}$

or, $\boxed{W = V \times It}$

where, t = time taken to the charge in the circuit i.e. (rate of doing electrical workdone) is called electrical power. So,

$$\text{Power (P)} = \frac{\text{Work}}{\text{Time}}$$

or, $$P = \frac{W}{t} = \frac{Vq}{t} = V \times I$$

Now, from ohm's law,

We have,

$$V = IR$$

So, power is given as

$$P = (IR) \times R = I^2 R$$

and its unit is J/sec or watt.

Energy:

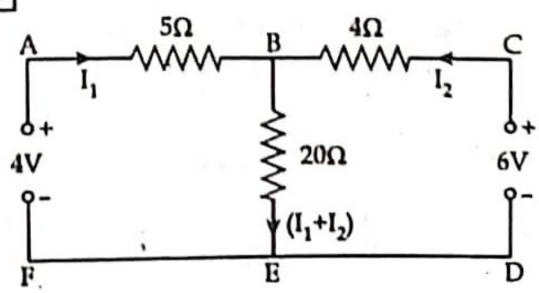
Electrical energy means the amount of workdone by a equipment during a time period of t. Its unit is joules. So,

$\boxed{\text{Energy} = \text{Power} \times \text{Time}}$

and, $\boxed{1 \text{ Horse Power} = 1\text{HP} = 746 \text{ watt}}$

Example 2.9

By using Kirchhoff's law, calculate the current flowing through each resistor as shown in the figure.



Solution:

Now, using KVL, from circuit ABEF,

$$4 - 5 I_1 - 20(I_1 + I_2) = 0$$

or, $5 I_1 + 20 I_1 + 20 I_2 = 4$

or, $25 I_1 + 20 I_2 = 4$

Again, from circuit BECD, we get

$$4 I_2 + 20(I_1 + I_2) = 6$$

or, $4 I_2 + 20 I_1 + 20 I_2 = 6$

or, $20 I_1 + 24 I_2 = 6$

Now, solving equation (1) and (2); we get,

$$I_1 = -0.12 \text{ A}$$

and, $I_2 = 0.35$ A

Here, the current $I_1 = -0.12$ is negative. It means that this current flo[illegible] through opposite direction, that we assumed.

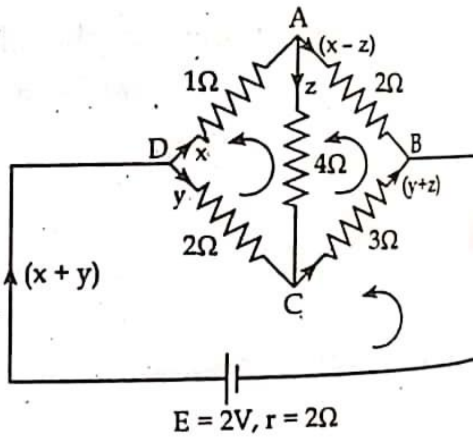
Now, Current through 5 Ω = 0.12 A

Current through 4 Ω = 0.35 A

Current through 20 Ω = $I_1 + I_2 = 0.35 - 0.12 = 0.23$ A

Example 2.10

Determine the currents in the unbalanced bridge circuit of given figure. Also determine the potential difference across BD and the resistance from B to D.



Solution:

Assume x and y currents flowing through branch DA and DC respectively. Now, using KVL to circuit DACD, we get

$$-x - 4z + 2y = 0$$

or, $x - 2y + 4z = 0$

Using KVL in circuit DABED; we get,

$$-x - 2(x - z) - 2(x + y) + 2 = 0$$

or, $5x + 2y - 2z - 2 = 0$

Using KVL in circuit ABCA,

$$-2(x - z) + 3(y + z) + 4z = 0$$

or, $2x - 3y - 9z = 0$

Now, solving equation (1), (2) and (3); we get

$$x = 0.33 \text{ A}, y = 0.187 \text{ A}, z = 0.011 \text{ A}$$

Hence, Current in branch DA = x = 0.33 A

Current in branch DC = y = 0.187 A

Current in branch AB = (x - z) = 0.318 A

Current in branch AC = z = 0.011 A

Current through branch CB = y + z = 0.1978 A

Total current flowing in circuit = x + y = 0.517 A

Internal voltage drop in cell $V_1 = I_r = 0.517 \times 2 = 1.034$ V

Now, for resistance across BD,

Using KVL,

$$V_D - 2 \times 0.187 - 3 \times (0.1978) + V_B$$

or, $$V_D - V_B = 0.97 \text{ V}$$

Also, equivalent resistance of the bridge between B and D is

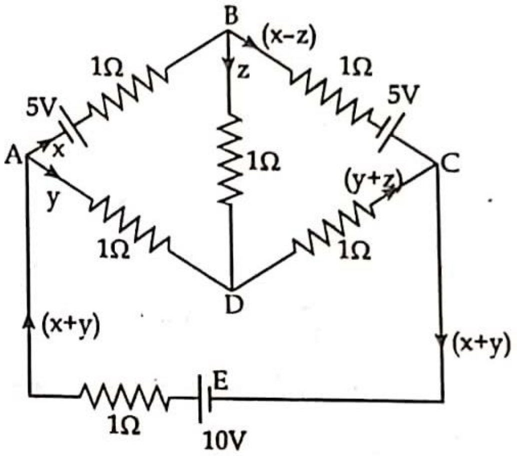
$$R = \frac{\text{Potential differences between B and D}}{\text{Current through B and D}} = \frac{0.97}{0.517} = 1.876\ \Omega$$

Example 2.11

Determine the branch currents in the given network below. Where, value of all resistance is in ohm.

Solution:

The directions of current is shown in figure.



Applying KVL in circuit ABDA

$$5 - x - z + y = 0$$

or, $$x - y + z = 5 \quad \text{(1)}$$

From circuit BCDB,

$$-(x - z) + 5 + (y + z) + z = 0$$

or, $$x - y - 3z = 5 \quad \text{(2)}$$

From circuit ADCEA, using KVL

$$-y - (y + z) + 10 - (x + y) = 0$$

or, $$x + 3y + z = 10 \quad \text{(3)}$$

Now, solving equation (1), (2) and (3); we get,

$$z = 0, x = 6.25 \text{ A and } y = 1.24 \text{ A}$$

Therefore,

Current through branch AB and BC = 6.25 A

Current through branch AD and DC = 1.24 A

Current through BD = 0

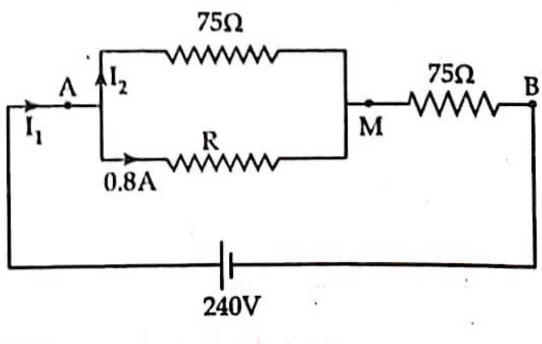
Total current supplied in CKT = 6.25 + 1.25 = 7.49 A

Example 2.12

A 384 watt, 240 V resistance coil AB is connected across 240 V supply, Calculate the value of resistance, which when connected between the midpoint of AB and end A will carry a current of 0.8 A.

Solution:

From given figure; resistance (R) is connected in midpoint of coil AB in parallel. Which divide resistance of coil into two equal half.



Resistance of coil

$$\left(\frac{V^2}{P}\right) = \frac{(240)^2}{284} = 150\ \Omega$$

$\therefore \quad \frac{150}{2} = 75\ \Omega$ of each two.

Now, $V_{BM} = I_1 \times R_{MB}$

or, $\boxed{V_{MB} = 75\ I_1}$

Also, $V_{AM} + V_{MB} = V_{AB}$

or, $V_{AM} = V_{AB} - V_{MB}$

$= 240 - 75\ I_1$

Using KCL at point A

$I_1 = I_2 + 0.8$

or, $I_1 = \frac{V_{AM}}{75} + 0.8$

or, $I_1 = \frac{240 - 75\ I_1}{75} + 0.8$

$\therefore \quad I_1 = \frac{300}{150} = 2\ A$.

and, From equation **(2)**; we get,

$V_{AM} = 240 - 75 \times 2 = 90$

and, $R = \frac{V_{AM}}{0.8} = \frac{90}{0.8} = 112.5\ \Omega$

Example 2.13

Determine the resistance and the power dissipation of resistor that must be placed in series with a 75-ohm resistor across 120 V source in order to limit the power dissipation in the 75-ohm resistor to 90 watts.

Solution:

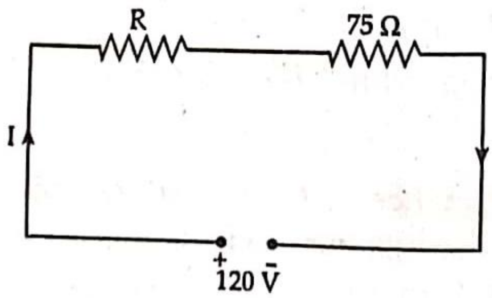


Figure represents the conditions of the problem.

$I^2 \times 75 = 90$

$I = \sqrt{90/75} = 1.095\ A$

Now, $I = \frac{120}{R + 75}$

or, $1.095 = \frac{120}{R + 75}\quad R = 34.6\ \Omega$

Power dissipation in $R = I^2 R = (1.095)^2 \times 34.6 = 41.5$ watts

Example 2.14

A generator of e.m.f. E volts and internal resistance r ohms supplies current to a water heater. Calculate the resistance R of the heater so that three-quarter of the total energy developed by the generator is absorbed by the water.

Solution:

$$\text{Current supplied by generator, } I = \frac{E}{R+r}$$

$$\text{Power developed by generator} = E\,I = \frac{E^2}{R+r}$$

$$\text{Power dissipated by heater} = I^2R = R \times \frac{E^2}{(R+r)^2} = \frac{E^2R}{(R+r)^2}$$

As per the conditions of the problem, we have,

$$\frac{E^2R}{(R+r)^2} = \frac{3}{4} \times \frac{E^2}{R+r^2} \quad \text{or} \quad \frac{R}{R+r} = \frac{4}{3} \quad \therefore R = 3\,r$$

Example 2.15

A current of 90 A is shared by three resistances in parallel. The wires are of the same material and have their lengths in the ratio 2 : 3 : 4 and their cross-sectional areas in the ratio 1 : 2 : 3. Determine current in each resistance.

Solution:

Conductance, $G = \sigma\frac{a}{l}$ so that $G \propto \frac{a}{l}$ ($\because \sigma$ is same)

$$G_1 : G_2 : G_3 :: \frac{a_1}{l_1} : \frac{a_2}{l_2} : \frac{a_3}{l_3}$$

or, $$G_1 : G_2 : G_3 :: \frac{1}{2} : \frac{2}{3} : \frac{3}{4}$$

or, $$G_1 : G_2 : G_3 :: 6 : 8 : 9$$

$$I_1 = I \times \frac{G_1}{G_1+G_2+G_3} = 90 \times \frac{6}{6+8+9} = 23.48 \text{ A}$$

$$I_2 = I \times \frac{G_2}{G_1+G_2+G_3} = 90 \times \frac{8}{6+8+9} = 31.30 \text{ A}$$

$$I_3 = I \times \frac{G_3}{G_1+G_2+G_3} = 90 \times \frac{9}{6+8+9} = 35.22 \text{ A}$$

Example 2.16

Two resistors R_1 = 2500 Ω and R_2 = 4000 Ω are joined in series and connected to a 100 V supply. The voltage drops across R_1 and R_2 are measured successively by a voltmeter having a resistance of 50000 Ω. Find the sum of two readings.

Solution:

When voltmeter is connected across resistor R_1 [See figure (i)], it becomes a series-parallel circuit and total circuit resistance decreases.

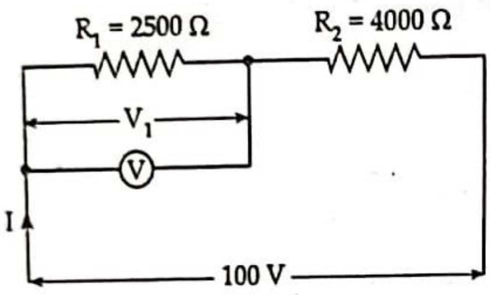
$$\text{Total circuit resistance} = 4000 + \frac{2500 \times 50000}{2500 + 50000} = 4000 + 2381 = 6381\ \Omega$$

Circuit current,

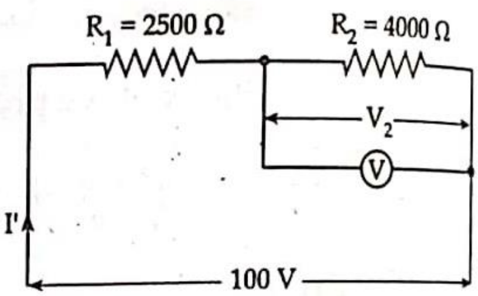
$$I = \frac{100}{6381}\ A$$

Voltmeter reading,

$$V_1 = I \times 2381 = \frac{100}{6381} \times 2381 = 37.3\ V$$



(i)



(ii)

When voltmeter is connected across R_2 [See figure (ii)], it becomes a series-parallel circuit.

$$\text{Total circuit resistance} = 2500 + \frac{4000 \times 50000}{4000 + 50000} = 2500 + 3703.7 = 6203.7\ \Omega$$

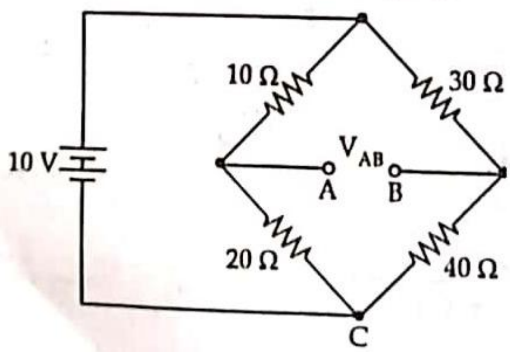
$$\text{Circuit current, } I' = \frac{100}{6203.7}\ A$$

$$\text{Voltmeter reading, } V_2 = I' \times 3703.7 = \frac{100}{6203.7} \times 3703.7 = 59.7\ V$$

$\therefore$ Sum of two readings $= V_1 + V_2 = 37.3 + 59.7 = 97\ V$

Example 2.17

Find the voltage V_{AB} in the circuit shown in figure.



Solution:

The resistors 10 Ω and 20 Ω are in series and voltage across this combination is 10 V.

$$V_{AC} = \frac{20}{10 + 20} \times 10 = 6.667\ V$$

The resistors 30 Ω and 40 Ω are in series and voltage across this combination is 10 V.

$$V_{BC} = \frac{40}{30 + 40} \times 10 = 5.714 \text{ V}$$

The point A is positive w.r.t. point B

$$V_{AB} = V_{AC} - V_{BC} = 6.667 - 5.714 = 0.953 \text{ V}$$

Example 2.18

A circuit consists of four 100 Ω lamps connected in parallel across a 230 V supply. Inadvertently, a voltmeter has been connected in series with the lamps. The resistance of the voltmeter is 1500 Ω and that of the lamps under the conditions stated is six times their value then burning normally. What will be the reading of the voltmeter?

Solution:

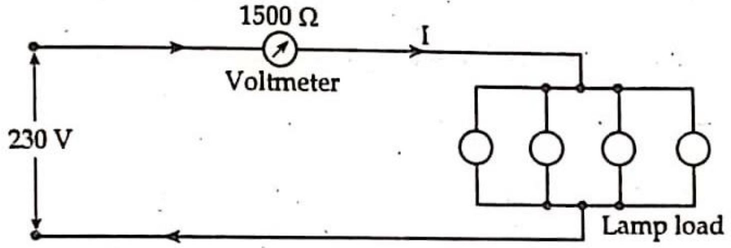


Figure shows the condiions of the problem. When burning normally, the resistance of each lamp is $R = V^2/P = (230)^2/100 = 529$ Ω. Under the conditions shown in figure above, resistance of each lamp = 6 × 529 = 3174 Ω.

∴ Equivalent resistance of 4 lamps under stated conditions is

$R_P = 3174/4 = 793$ Ω

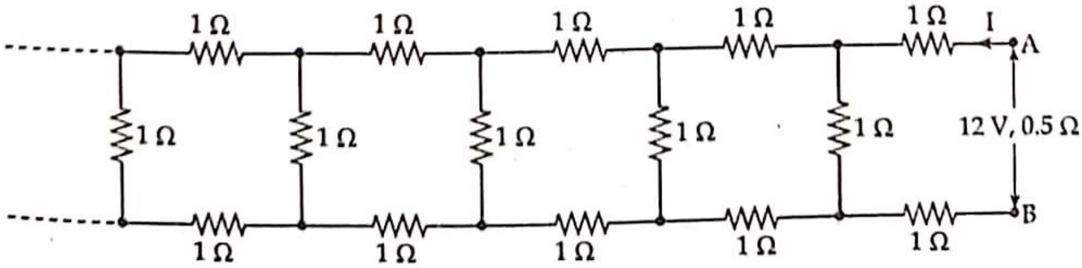
Total circuit resistance = $1500 + R_P = 1500 + 793.5 = 2293.5$ Ω

∴ Circuit current, $I = \frac{230}{2293.5}$ A

∴ Voltage drop across voltmeter = $I \times 1500 = \frac{230}{2293.5} \times 1500 \simeq 150$ V

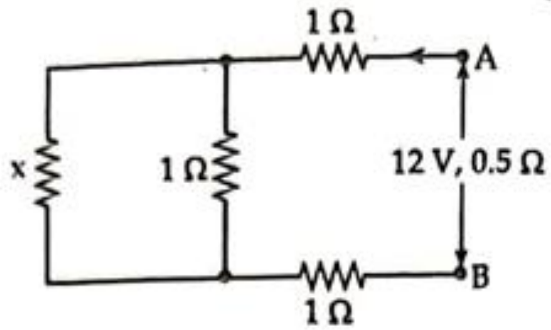
Example 2.19

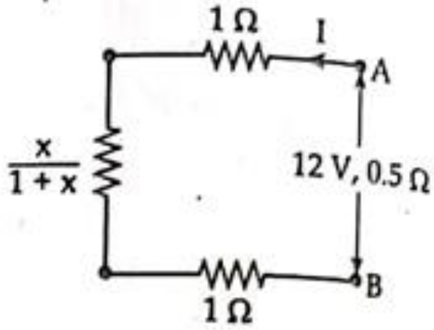
Determine the current drawn by a 12 V battery with internal resistance 0.5 Ω by the following infinite network (as shown in figure).



Solution:

Let x be the equivalent resistance of the network. Since the network is infinite, the addition of one set of three resistances, each of 1 Ω, will not change the total resistance, *i.e.*, it will remain x. The network would then become as shown in figure. The resistances x and 1 Ω are in parallel and their total resistance is R_P given by;





$$R_P = \frac{x \times 1}{x+1} = \frac{x}{1+x}$$

The circuit then reduces to the one showing in figure.

Total resistance of the network $= 1 + 1 + \frac{x}{1+x} = 2 + \frac{x}{1+x}$

But total resistance of the network is x as mentioned above

$\therefore \quad x = 2 + \frac{x}{1+x}$

or, $\quad x + x^2 = 2 + 2x + x$

or, $\quad x^2 - 2x - 2 = 0$

$\therefore \quad x = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2}$

or, $\quad x = 1 \pm \sqrt{3}$

As the value of the resistance cannot be negative,

$\therefore \quad x = 1 + \sqrt{3} = 1 + 1.732 = 2.732\ \Omega$

Total circuit resistance, $R_T = x +$ internal resistance of the supply

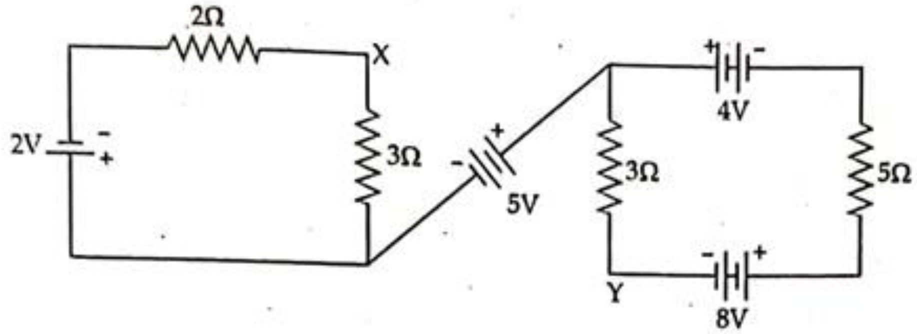
$= 2.732 + 0.5 = 3.232\ \Omega$

Therefore, current drawn by the network is

$$I = \frac{E}{R_T} = \frac{12}{3.232} = 3.71.\ A$$

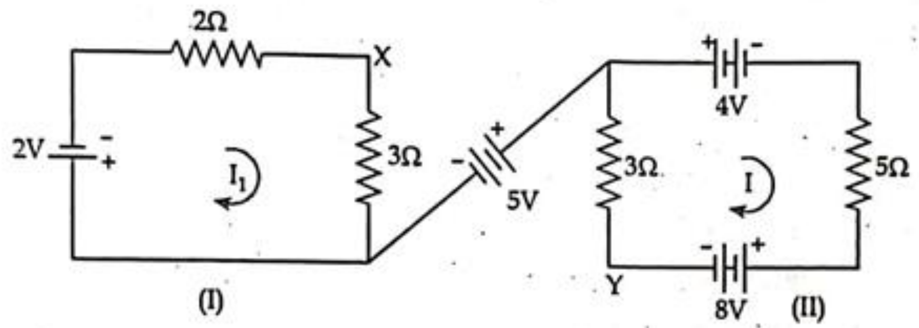
EXAMINATION QUESTION SOLUTIONS

1. What is the difference of potential between X and Y in the Network shown in the figure below?



Solution:

Taking the clockwise direction of current in each loop positive as shown below;



Now, applying KVL in loop I

$$-2-2I_1-3I_1=0$$

or, $\boxed{I_1=-\frac{2}{5}A}$

It means I, is anticlockwise direction.

Applying KVL in loop (ii)

$$-4-5I_2-8-3I_2=0$$

or, $-8I_2=12$

or, $\boxed{I_2=-\frac{3}{2}A}$

Anticlockwise I_2 in loop **(2)**

Now,

$$V_{XY}=V_X-V_Y$$

or, $V_x+3\times\frac{2}{5}+5-3\times\frac{3}{2}=V_Y$

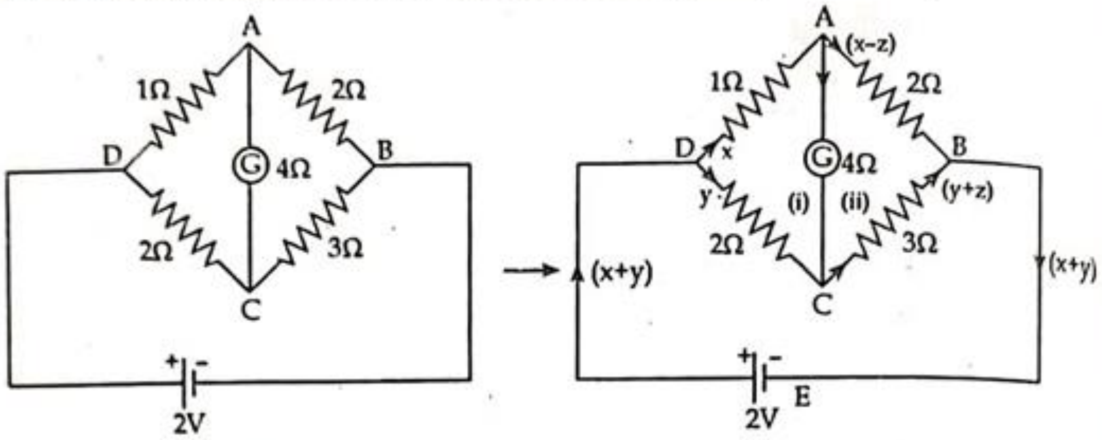
or, $V_X-V_Y=\frac{9}{2}-6.2$

or, $V_X-V_Y=-1.7\text{ V}$

2. Calculate the current through the galvanometer in the bridge circuit as shown in figure given below using Kirchhoff's laws.

Solution:

Assumed current direction is shown in the figure;



Now, applying Kirchhoff's law in mesh (1)

$$-x - 4z - 2 \times (-y) = 0$$

or, $x - 2y + 4z = 0$ (1)

Applying Kirchhoff's law in circuit DABED, we get

$$-x - 2(x - z) + 2 = 0$$

or, $-x - 2x + 2z + 2 = 0$

or, $-3x + 2z + 2 = 0$ (2)

Applying Kirchhoff's law in mesh (2)

$$+3(y + z) - 2(x - z) + 4z = 0$$

or, $3y + 3z - 2x + 2z + 4z = 0$

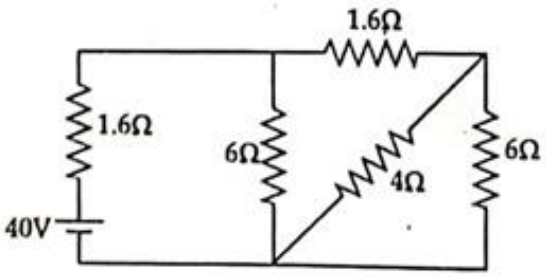
or, $2x - 3y - 9z = 0$ (3)

On solving from equation (1), (2) and (3); we get,

$$x = \frac{15}{22}\ y = \frac{17}{44} \text{ and } z = \frac{1}{44}$$

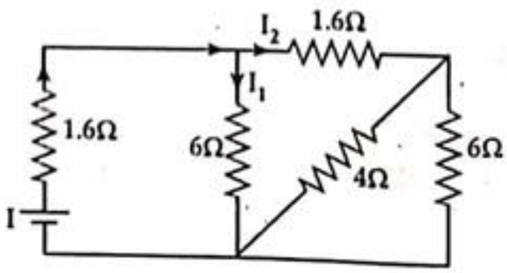
Therefore, current through Galvanometer $= \frac{1}{44}$ A

3. Find the current through 4 Ω resistance.



Solution:

Current direction is shown below;



The equivalent resistance of circuit is calculated by using series, parallel combination as

$$R = [(6 \parallel 4) + 1.6] \parallel 6 + 1.6 = \left(\frac{6 \times 4}{6+4} + 1.6\right) \parallel 6 + 1.6$$

$$= 4 \parallel 6 + 1.6 = \frac{4 \times 6}{4+6} + 1.6 = 4\ \Omega$$

Now, current supplied by battery

$$I = \frac{V}{R} = \frac{40}{4} = 10\ A$$

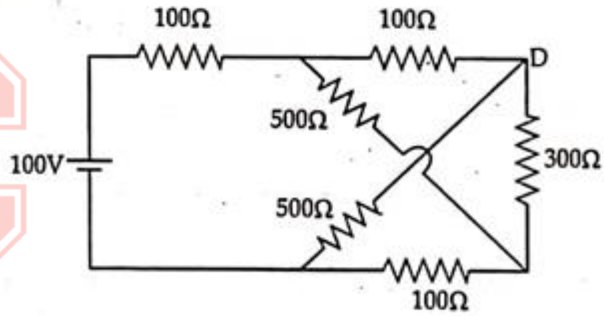
Now, using current divider rule; current through 1.6 Ω resistor is,

$$I_2 = \frac{10}{6 + \left(\frac{6 \times 4}{6+4} + 1.6\right)} \times 6 = \frac{10}{6+4} \times 6 = 6\ A$$

Now, current through 4 Ω resistor is

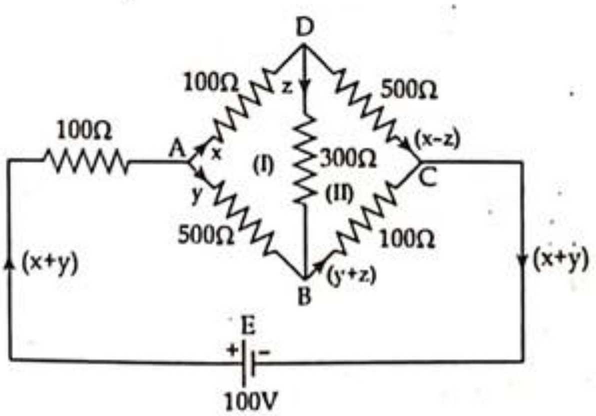
$$I_4 = \frac{6}{4+6} \times 6 = 3.6\ A$$

4. Determines the current supplied by the battery in the circuit shown in the figure below.



Solution:

This equation can also be solved by using star-delta transform. The given figure can be re-drawn as;



The direction of current and its distribution is shown in the figure.

Now, using Kirchhoff's voltage laws, we can write from loop (1)

$$-100x - 300z + 500y = 0$$

or, $x + 3z - 5y = 0$

or, $x - 5y + 3z = 0$ (1)

From loop (ii); writing KVL,

$$-500(x-z)+100(y+z)+300z=0$$

or, $-500x+500z+100y+100z+300z=0$

or, $-500x+100y+900z=0$

or, $5x-y-9z=0$ (2)

From loop EADCE,

$$-100(x+y)-100x-500(x-z)+100=0$$

or, $(x+y)-x+5(x-z)+1=0$

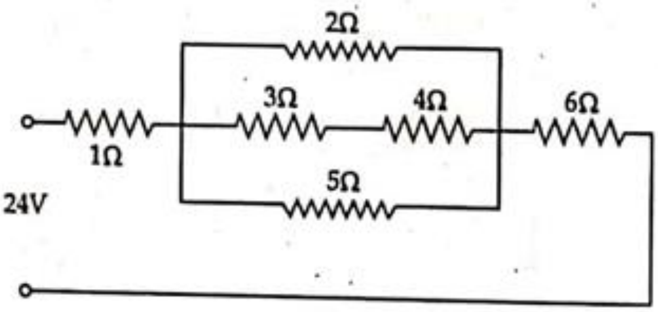
or, $7x+y-5z=-1$ (3)

Solving by calculator; we get,

$$x=\frac{1}{5},\quad y=\frac{1}{10} \text{ and } z=\frac{1}{10}$$

Therefore, current supplied by battery $=x+y=\frac{1}{5}+\frac{1}{10}=\frac{3}{10}$ A

5. Find the equivalent resistance in the figure below and power dissipated in the 5 Ω resistor.



Solution:

The Equivalent resistance between resistances between two terminals is given by using series-parallel combination as:

$$R=1+[2 \parallel (3+4) \parallel 5]+6=1+\left(\frac{2\times 7}{2+7}\parallel 5\right)+6$$

$$=7+\left(\frac{14}{9}\parallel 5\right)=7+\frac{\frac{14}{9}\times 5}{\frac{14}{9}+5}=8.186\ \Omega$$

Again, total current supplied by the battery is,

$$I=\frac{V}{R}=\frac{24}{8.186}=2.932\text{ A}$$

Now, using current division rule, we get,

Current through 5 Ω resistance is

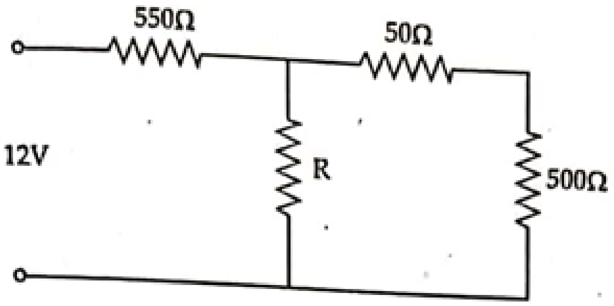
$$=\frac{R_2\times R_7}{R_2\times R_7+R_7\times R_5+R_5\times R_2}\times I=\frac{2\times 7}{2\times 7+2\times 5+7\times 5}\times 2.932$$

$$=0.6957\text{ A}$$

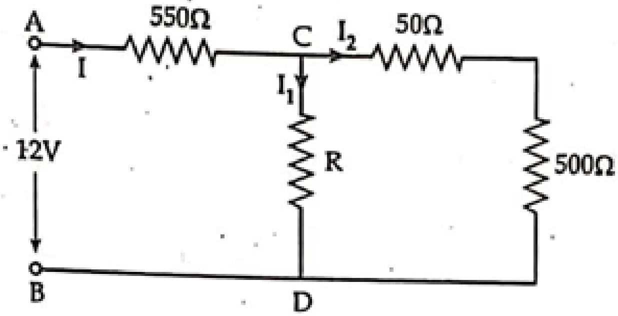
Now, power dissipated in 5 Ω resistor is

$$P=I^2R=(0.6957)^2\times 5=2.4197\text{ Watt}$$

6. Determine the value of unknown resistor 'R' in the circuit below, if the voltage drop across 500 Ω resistor is 2.5 volts.



Solution:



Using direct estimation method;

Current through 500 Ω resistance

$$I = \frac{V}{R} = \left(\frac{2.5}{500}\right) = 0.005 \text{ A}$$

∴ Voltage drop across 50 Ω

$$V_{50} = IR = 0.005 \times 50 = 0.25\text{V}$$

Therefore, total voltage drop across CD = 2.5 + 0.25 = 2.75 V

Now, total current supplied by battery

$$I = \frac{12 - 2.75}{550} = 0.0168 \text{ A}$$

Here, 12 – 2.75 is voltage drop across 550 Ω

And, current through resistance (R),

$$I_1 = I - I_2 = 0.0168 - 0.005 = 0.0118 \text{ A}$$

So, using

$$V = IR$$

We get,

$$2.75 = 0.0118 \times R$$

or, $R = 233\ \Omega$

7. What is the total cost of using the following at Rs. 7 per kilowatt hour?
   (i) A 1200 W toaster for 30 minutes
   (ii) Six-50 W bulbs for 4 hours
   (iii) A 400 W washing machine for 45 minutes
   (iv) A 4800 W electric clothes dryer for 20 minutes

Solution:

Total energy consumption is equal to energy consumed by A toaster + six bulbs + A washing machine + A electric clothes dryer.

$$= 1200 \times \frac{30}{60} + 6 \times 50 \times 4 + 400 \times \frac{45}{60} + 4800 \times \frac{20}{60}$$

$$= 3700 \text{ watt-hour}$$

$$= \frac{3700}{1000} \text{ kilowatt hour}$$

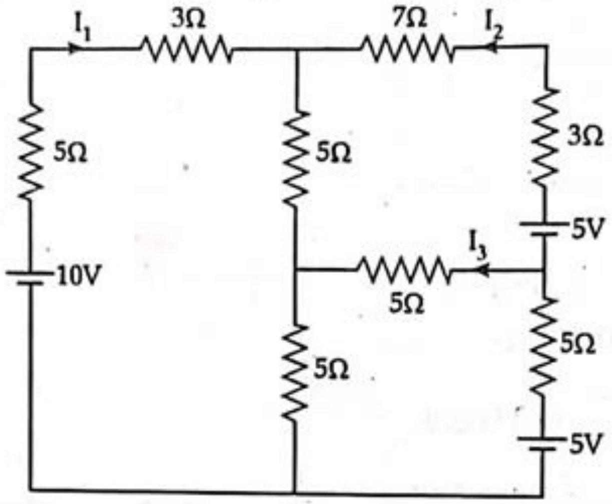
$$= 3.7 \text{ kilowatt hour}$$

> **NOTE**
> Energy = power × time
> where, time is in hour

Now, total cost for consumptions are

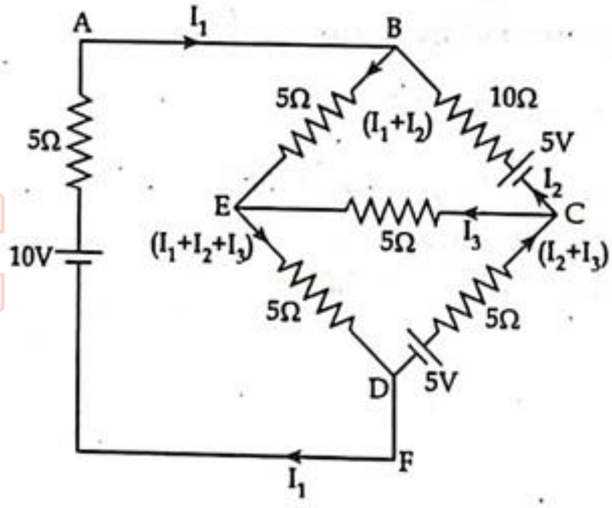
$= \boxed{\text{Total kw.hr} \times \text{cost per k.w.hr.}} = 7 \times 3.7 = \text{Rs. } 25.900$

8. Find the current I_1, I_2, I_3 using Kirchhoff's law and also find the power output of each voltage source of figure below.



Solution:

The figure can be redrawn as



Now, using KVL on mesh BECB, clockwise current

$$5(I_1 + I_2) - 5\, I_3 + 10\, I_2 - 5 = 0$$

or, $\quad 5\, I_1 + 15\, I_2 - 5\, I_3 = 5 \qquad$ (1)

Applying KVL on mesh ECDE, we get

$$5(I_1 + I_2 + I_3) + 5\, I_3 + 5\,(I_2 + I_3) - 5 = 0$$

or, $\quad 5\, I_1 + 10\, I_2 + 15\, I_3 = 5 \qquad$ (2)

Using KVL on mesh ABEDFA, we get,

$$10 - 5\, I_1 - 5(I_1 + I_2) - 5(I_1 + I_2 + I_3) = 0$$

or, $\quad -15\, I_1 - 10\, I_2 - 5\, I_3 = -10$

or, $\quad 15\, I_1 + 10\, I_2 + 5\, I_3 = 10 \qquad$ (3)

Now, solving equation (1), (2) and (3), we get,

$$I_1 = \frac{13}{24}\,\text{A},\ I_2 = \frac{1}{6}\,\text{A}$$

and, $\quad I_3 = \frac{1}{24}\,\text{A}$

To calculate power output of each voltage source,

For voltage source 10 V

$$P = I \times V = \frac{13}{24} \times 10 = 5.417 \text{ Watt}$$

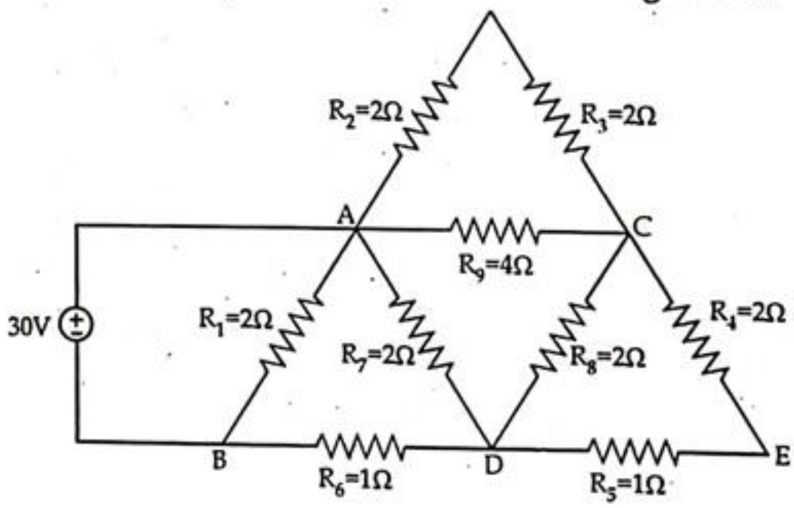
For 5 V source

$$DC = (I_2 + I_3) \times 5 = \left(\frac{1}{6} + \frac{1}{24}\right) \times 5 = 1.042 \text{ Watt}$$

For 5 V source

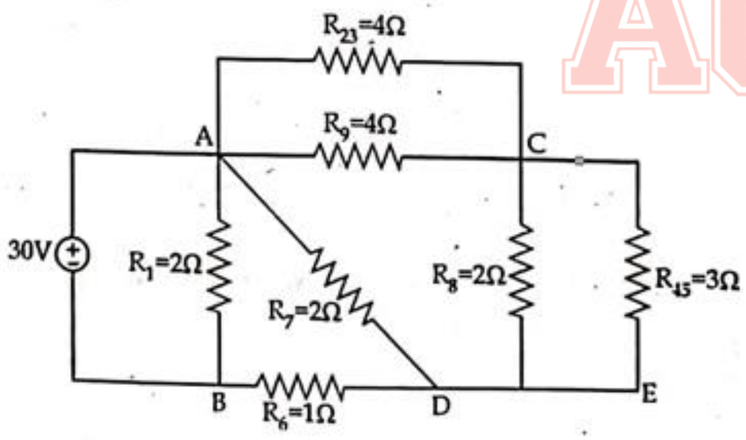
$$BC = I_2 \times 5 = \frac{1}{6} \times 5 = \frac{5}{6} \text{ Watt} = 0.833 \text{ Watt}$$

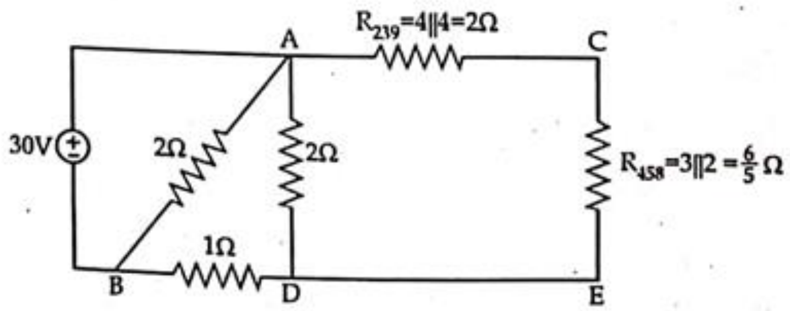
9. Using series parallel combination of the resistances, find the current delivered by the source in the following circuit.



Solution:

By using series and parallel combination,





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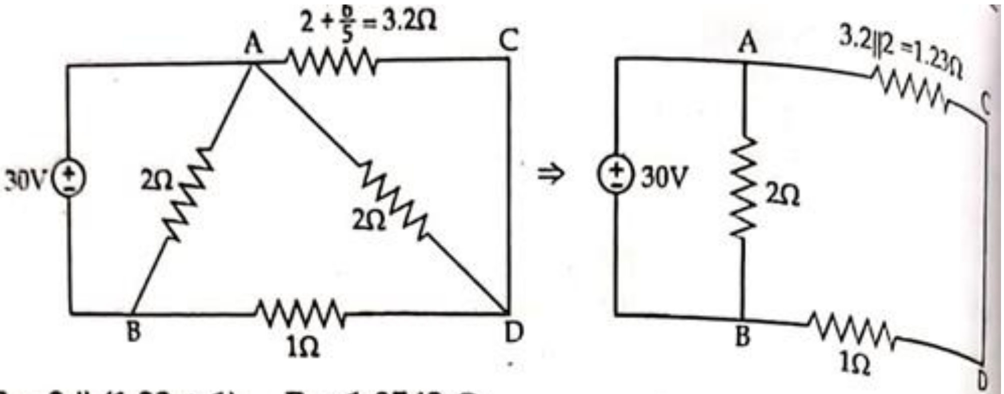
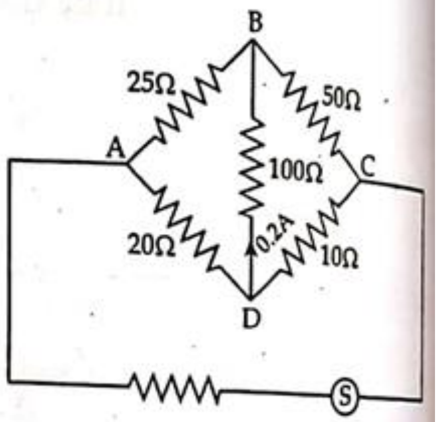


$R = 2 \parallel (1.23 + 1)$, $R = 1.0543\ \Omega$

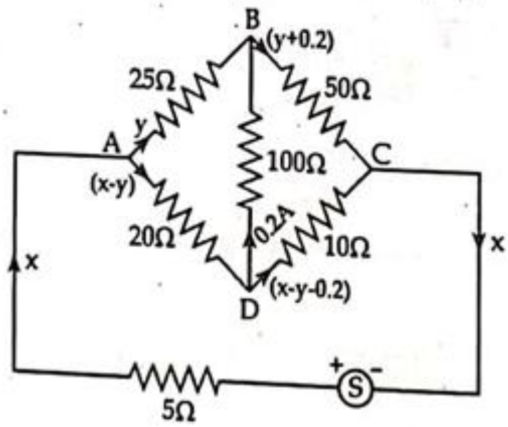
Therefore, delivered by battery $= \dfrac{30}{1.0543} = 28.453$ A

10. Using Kirchhoff's law, determine the magnitude of source current and polarity of the source S, if the current flowing through branch BD is 0.2 A. From D to B in the circuit shown in figure.



Solution:

The given circuit can be redrawn as shown in the figure below: Polarity of source, initially assigned as in figure.



Now, using Kirchhoff's voltage law to mesh ABDA, we get

$$20(x - y) - 25y + 0.2 \times 100 = 0$$

or, $\quad 20x - 45y + 20 = 0 \qquad (1)$

From mesh BCDB, using KVL, we get,

$$-50(y + 0.2) + 10(x - y - 0.2) - 0.2 \times 100 = 0$$

or, $\quad 10x - 60y - 32 = 0 \qquad (2)$

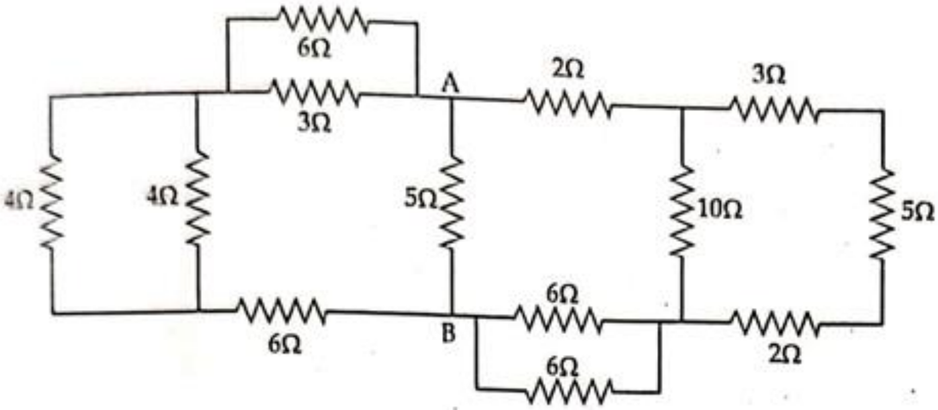
Now, solving equation **(1)** and **(2)**; we get,

$x = \dfrac{-88}{25} = -3.52$ A, $\qquad y = \dfrac{-28}{25} = -1.12$ A

For polarity of source;

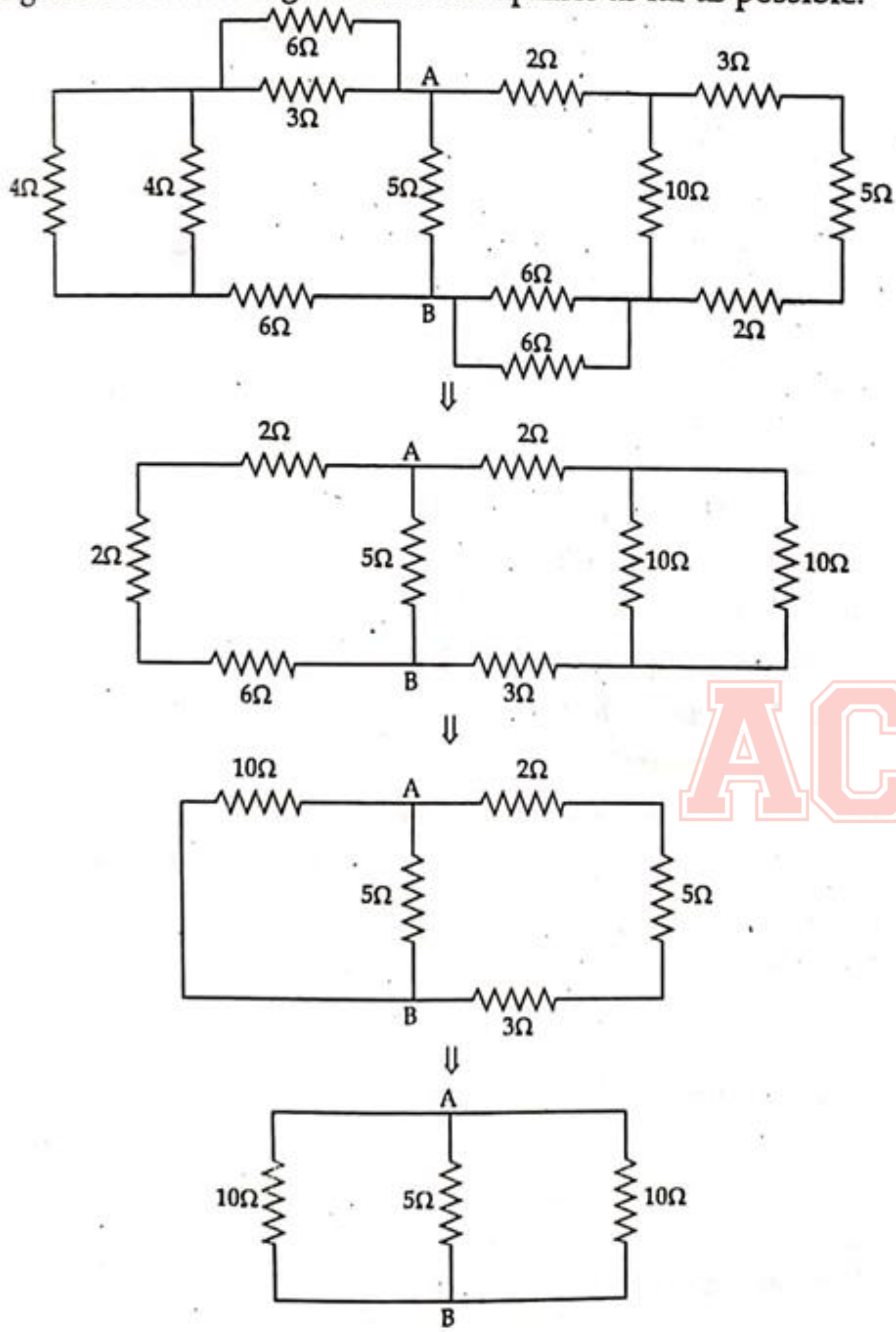
The direction of current x = –3.52 A implies that it flows just opposite direction of indicated direction.

11. Find the equivalent resistance across the terminals A and B, R_{AB}.



Solution:

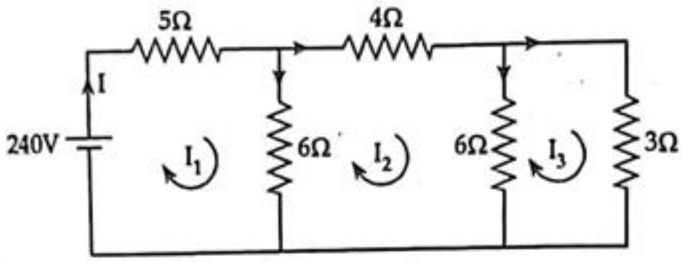
The solution can be done by using series parallel combination, as shown in the figure below. The figure can be simplifies as far as possible.



Now, from final reduced figure, 10 Ω, 5 Ω and 10 Ω are in parallel. So, using parallel combination

$$\therefore \quad R_{AB} = \frac{10 \times 10 \times 5}{10 \times 5 + 5 \times 10 + 10 \times 10} = 2.5\ \Omega$$

12. Find the circuit and current through each branch using branch current method.



Solution:

Applying KVL to mesh I, II and III.

Mesh I,

$240 - 5\,I_1 - 6(I_1 + I_2) = 0$

or, $-11\,I_1 + 6\,I_2 = -240$ (1)

Mesh II,

$-6(I_2 - I_1) - 4\,I_2 - 6(I_2 - I_3) = 0$

or, $6\,I_1 - 16\,I_2 + 6\,I_3 = 0$ (2)

Mesh III,

$-6(I_3 - I_2) - 3\,I_3 = 0$

or, $6\,I_2 - 9\,I_3 = 0$ (3)

Solving equation (1), (2) and (3); we get,

$I_1 = 30$ A, $I_2 = 15$ A and $I_3 = 10$ A

Current in resistor 5 Ω = $I_1 = 30$ A

Current in resistor 4 Ω = $I_2 = 15$ A

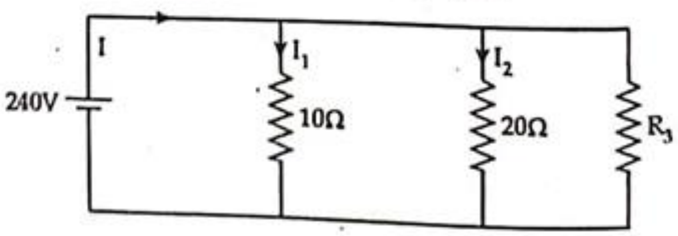
Current in resistor 6 Ω = $I_2 - I_3 = 5$ A

Current in resistor 3 Ω = $I_3 = 10$ A

Current in resistor 6 Ω = $I_1 - I_2 = 15$ A

Current through voltage source = $I_1 = 30$ A

13. Given the information provided in figure. Calculate R_3, E, I and I_2. Equivalent resistance of the circuit is 4 Ω.



Solution:

R_3 have to be determine in above circuit.

So, we know; for parallel growing of resistor, equivalent resistance,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{4} = \frac{1}{10} + \frac{1}{20} + \frac{1}{R_3}$$

$$\therefore \quad R_3 = 10\ \Omega$$

Using current divider rule,

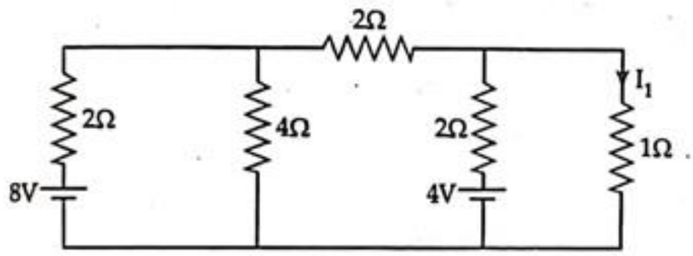
$$\therefore \quad I_2 = \frac{\frac{1}{20}}{\frac{1}{10} + \frac{1}{20} + \frac{1}{10}} \times I$$

$$\therefore \quad I_2 = 0.2\ I$$

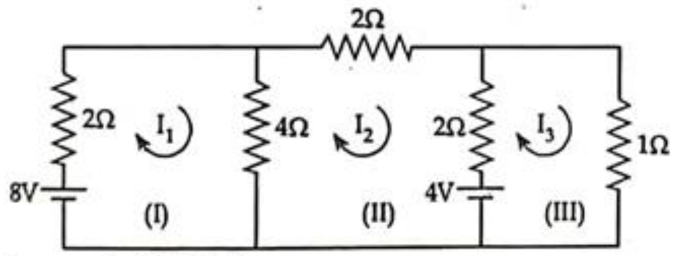
Then, $I = \frac{E}{R} = \frac{E}{4}$ A

$$I_2 = 0.2\ I = 0.2 \times \frac{E}{4} = 0.5\ E\ A$$

14. Apply KVL and KCL to determine current I_2 through 1 Ω resistor in the network shown below.



Solution:



Applying KVL to mesh I, II, III

Mesh I,

$$8 - 2\ I_1 - 4(I_1 - I_2) = 0$$

or, $\quad -6\ I_1 + 4\ I_2 = -8 \quad$ (1)

Mesh II,

$$-2\ I_2 - 2(I_2 - I_3) - 4 - 4(I_2 - I_1) = 0$$

or, $\quad 4\ I_1 - 8\ I_2 + 2\ I_3 = 4 \quad$ (2)

Mesh III,

$$4 - 2(I_3 - I_2) - 1 \times I_3 = 0$$

or, $\quad 2\ I_2 - 3\ I_3 = -4 \quad$ (3)

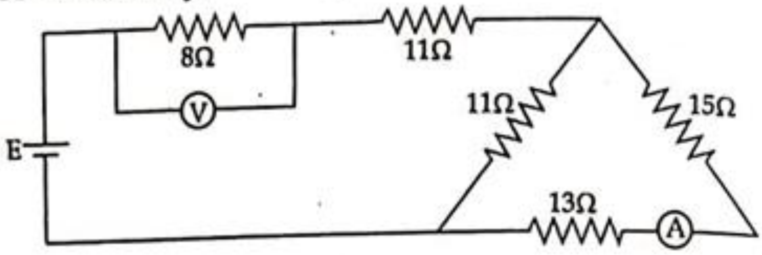
Solving equation (1), (2) and (3); we get,

$$I_1 = 2\ A,\ I_2 = 1\ A \text{ and } I_3 = 2A$$

$$\therefore \quad I_1 = I_3 = 2\ A$$

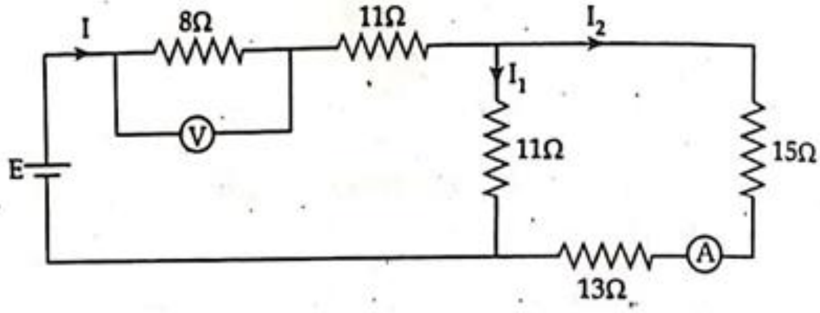
Hence current through 1Ω resistor is 2A flows in clockwise direction.

15. A battery of unknown e.m.f. is connected across resistances as shown in figure below. The voltage drop across the 8 Ω resistor is 20 V. What will be the current reading in the ammeter? What is the e.m.f. of the battery?



Solution:

From question, it is given that, Voltage drop across 8 Ω resistor = 20 V



Now, current through 8 Ω resistor is

$$I = \frac{20}{8} = 2.5$$

and, $R_{equivalent}$ is calculated as

$$R_{eq} = (8 + 11) + [11 \parallel (15 + 13)] = (8 + 11) + (11 \parallel 28) = 26.897\ \Omega$$

∴ $E = IR = 2.5 \times 26.897 = 67.2435$ V

Ammeter reading

$$I_2 = \frac{I}{11 + 13 + 15} \times 11 = \frac{2.5}{39} \times 11 = 0.705 \text{ A}$$

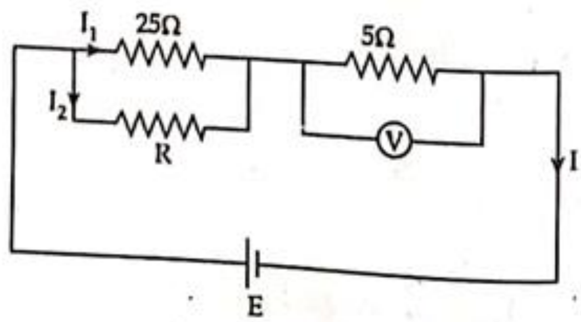
16. A direct current circuit comprises two resistors A of value 25 Ω and B of unknown value, connected in parallel together with a third resistor C of value 5 Ω connected in series C is found to 90 V. If the total power in the circuit is 4320 Ω calculate;

i) the value of resistor B

ii) the voltage applied to the ends of the whole circuit

iii) the current in each resistor

Solution:



Given, Voltage drop across 5 Ω resistor V = 90 V

Power (P) = 4320 Watt

$$\text{Current in circuit (I)} = \frac{V}{5} = \frac{90}{5} = 18\text{ A}$$

$$\text{Voltage of circuit } E = \frac{P}{I} = \frac{4320}{18} = 240\text{ V}$$

$$R_{eq} = \frac{E}{I} = \frac{240}{18} = 13.333\ \Omega$$

Also, $R_{eq} = (R \parallel 25) + 5$

or, $13.33 = \dfrac{R \times 25}{R + 25} + 5$

or, $8.33 = \dfrac{25R}{R + 25}$

or, $16.67R = 208.33$

$\therefore$ $R = 12.5\ \Omega$

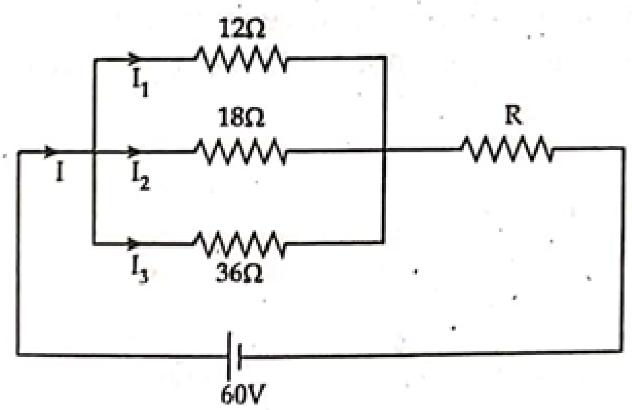
$$\text{Current in resistor A} = I_1 = \frac{18}{12.5 + 25} \times 12.5 = 6\text{ A}$$

Current in resistor B = $I - I_1 = 18 - 6 = 12$ A

Current in resistor C = $I = 18$ A

17. A circuit containing three resistor with resistance 12 Ω, 18 Ω and 36 Ω respectively. Joined in parallel is connected in series with a fourth resistance. The whole circuit is supplied at 60 V and it is found that power dissipated in 12 Ω resistance is 36 watt. Determine the value of fourth resistance and total power dissipated in the group.

Solution:



Given that;

Power dissipated in 12 Ω resistor $P_{12} = 36$ Watt

Then, $P_{12} = I_1^2 R_{12}$

or, $36 = I_1^2 \times 12$

$\therefore$ $I_1 = 1.732$ A

$V_1 = I_1 R$

$V_1 = 1.732 \times 12 = 20.78$ V

and, $V_1 = V_2 = V_3 = 20.78$ V

Now, $I_2 = \frac{V_2}{R_2} = \frac{20.78}{18} = 1.154$ A

$I_3 = \frac{V_3}{R_3} = \frac{20.78}{36} = 0.577$ A

so, $I = I_1 + I_2 + I_3 = 1.732 + 1.154 + 0.577 = 3.463$ A

Then, equivalent resistance of the circuit is calculated as;

$$R = \frac{V}{I} = \frac{60}{3.463} = 17.32\ \Omega$$

or, $R = R + (12 \parallel 18 \parallel 36)$

or, $17.32 = R + 6$

$\therefore$ $R = 11.32\ \Omega$

Now, total power dissipated in the group is

$$P = I \times V = 3.463 \times 60 = 207.780 \text{ Watt}$$

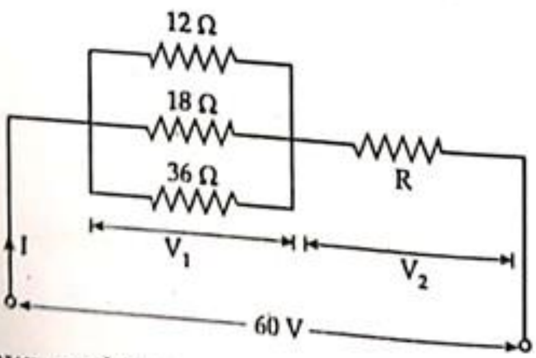
18. A direct current circuit comprise two resistors, A value of 25 Ω and B of unknown in series with parallel group. The potential difference across C is found to 90 V. If the total power in the circuit is 4320 W. Calculate value of unknown resistor B, the voltage applied to the end of the whole circuit and the current in each resistor.

Solution: See Q. No. 16.

19. A circuit, containing of three resistance 12 Ω, 18 Ω and 36 Ω respectively jointed in parallel, is connected in series with a fourth resistance. The whole is supplied at 60 V and it is found that the power dissipated in the 12 Ω resistance is 36 W. Determine the value of the fourth resistance and the total power dissipated in the group.

Solution:

The figure can be drawn as;



Let, R be the unknown resistance and V_2 be the voltage drop across it.

Power dissipated in the 12 Ω resistor = 36 watt.

so, $P = I \times V = \frac{V^2}{R}$

or, $V^2 = P \times R$

or, $V = \sqrt{36 \times 12} = 20.785$ V

Since, all three resistors are in parallel, voltage drop should be same for all resistors.

Again, equivalent resistance of parallel combination is

$$R_{eq} = (12 \parallel 18 \parallel 36)\ \Omega = \left(\frac{12 \times 18}{12 + 18}\right) \parallel 36 = \frac{7.20 \times 36}{7.20 + 36} = 6\ \Omega$$

and equivalent current through network is

$$I = \frac{V}{R} = \frac{20.785}{6} = 3.464\ A$$

The same current flows through resistor R.

Voltage drop across

$R = 60 - 20.785 = 39.215$ V

and using ohm's law,

$V = IR$

or, $39.215 = 3.40 \times R$

or, $\boxed{R = 11.32\ \Omega}$

Hence, value of unknown resistance is 11.32 Ω

Again, total equivalent resistance of circuit is

$$= (12 \parallel 18 \parallel 36) + 11.32 = 6 + 11.32 = 17.32\ \Omega$$

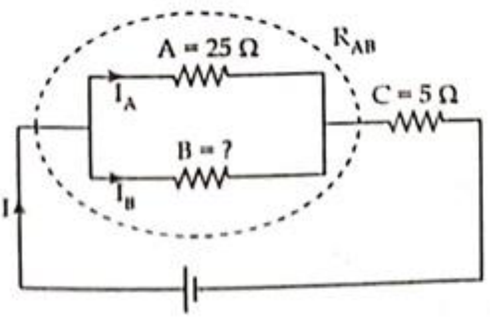
and current in circuit = 3.464 A

Therefore, total power disspated in group is

$$P = i^2R = (3.464)^2 \times 17.32 = 207.827 \text{ watt}$$

20. A dc circuit comprises two resistors, A of value 25 ohms, and B of unknown value, connected in parallel, together with a third resistor C of value 5 ohms connected in series with the parallel group. The potential difference across C is found to 90 V. If the total power in the circuit is 4320 watt. Calculate (a) the value of resistor B, (b) the voltage applied to the ends of the whole circuit, (c) the current in each resistor.

Solution:



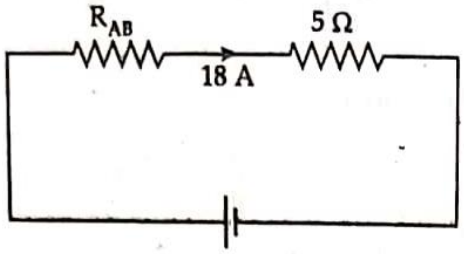
Given that;

Potential difference across C is = 90 V

Total power in the circuit = 4320 watt

Current through the 5 Ω resistor

$$I_C = \frac{90}{5} = 18\text{ A}$$

i) 

$\therefore \quad R_{AB} = \frac{25 \times B}{25 + B}$

and, $\quad R_{eq} = \frac{25 \times B}{25 + B} + 5$

Current flowing in series is same

Now,

$P = I^2 R_{eq}$

or, $\quad 4320 = (18)^2 \times R_{eq}$

or, $\quad R_{eq} = \frac{4320}{18^2} = 13.33\ \Omega$

$\because \quad \frac{25 \times B}{25 + B} + 5 = 13.33$

or, $\quad \frac{25 \times B}{25 + B} = 8.33$

or, $\quad 16.67\ B = 208.25$

$\therefore \quad B = 12.49\ \Omega$

$\therefore$ Resistance at B, $R_B = 12.5\ \Omega$

ii) $R_{eq} = (24 \parallel 12.5) + 5$

$\therefore \quad R_{eq} = 13.33\ \Omega$

$\therefore \quad P = \frac{V^2}{R_{eq}}$

or, $\quad V^2 = 4320 \times 13.33$

or, $\quad V^2 = 57585.6$

or, $\quad V = 239.969$

$\therefore \quad V \approx 240$ volt.

iii) Current flowing through whole circuit is 18 A.

$\therefore \quad I_A = \frac{18 \times 12.5}{25 + 12.5} = 6\text{ A}$

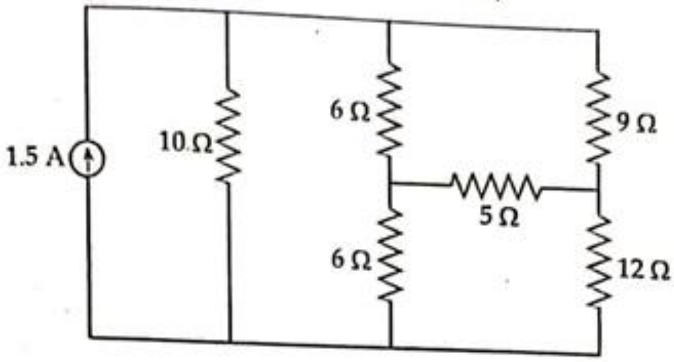
and, $\quad I_B = \frac{18 \times 25}{25 + 12.5} = 12\text{ A}$

Current through resistor A, $I_A = 6$ A

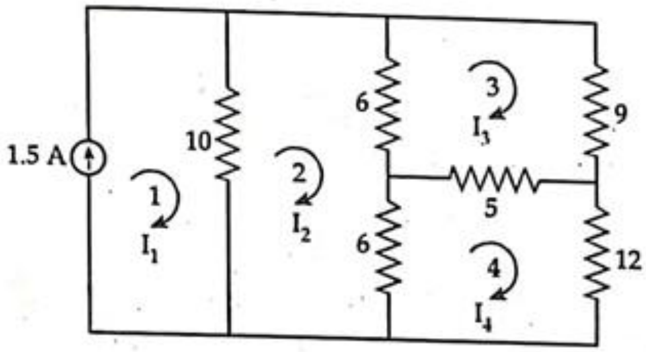
Current through resistor B, $I_B = 12$ A

Current through resistor C, $I_C = 18$ A

21. Use loop current method to calculate the current through the 5 Ω resistance for the Network shown below;



Solution:



Consider loop current be as shown in figure.

$I_1 = 1.5$ A

Applying KVL on loop 2, 3 and 4, we get;

Loop 2:

$$-10(I_2 - I_1) - 6(I_2 - I_3) - 6(I_2 - I_4) = 0$$

or, $-10I_2 + 15 - 6I_2 + 6I_3 - 6I_2 + 6I_4 = 0$

or, $-22I_2 + 6I_3 + 6I_4 = -15$ (1)

Loop 3:

$$-6(I_3 - I_2) - 9I_3 - 5(I_3 - I_4) = 0$$

or, $-6I_3 + 6I_2 - 9I_3 - 5I_3 + 5I_4 = 0$

or, $-6I_2 - 20I_3 + 5I_4 = 0$ (2)

Loop 4:

$$-6(I_4 - I_2) - 5(I_4 - I_3) - 12I_4 = 0$$

or, $-6I_4 + 6I_2 - 5I_4 + 5I_3 - 12I_4 = 0$

or, $6I_2 + 5I_3 - 23I_4 = 0$ (3)

Solving equation (1), (2), and (3); we get;

$I_2 = 0.85$ A

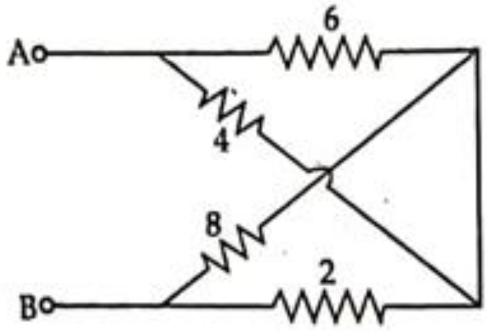
$I_3 = 0.33$ A

$I_4 = 0.29$ A

Therefore, current flowing through 5 Ω resistors is

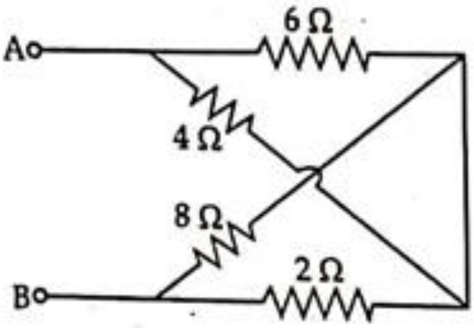
$= I_3 - I_4 = 0.33 - 0.29 = 0.04$ A

22. Find the current through 2 Ω resistor in the given circuit if 24 V is applied from AB terminals. (All resistances are in Ω).

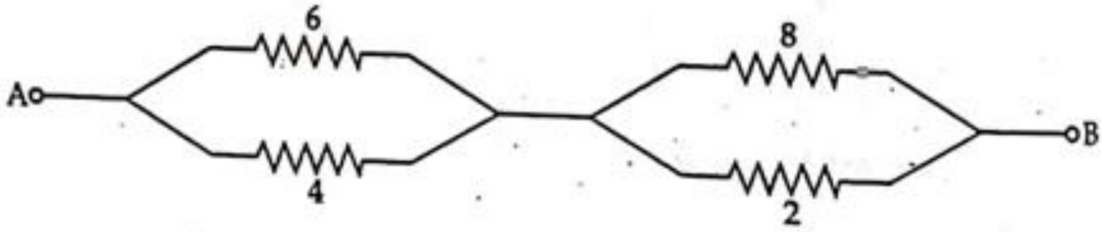


Solution:

Given circuit is



To calculate equivalent resistance between A and B



$\therefore \quad R_{eq} = (6||4) + (8||2) = \dfrac{6 \times 4}{6+4} + \dfrac{8 \times 2}{8+2} = 2.4 + 1.6 = 4\ \Omega$

Total current flowing in the circuit AB is

$$I = \frac{24}{4} = 6\ A$$

Current flowing in 2 Ω resistance is

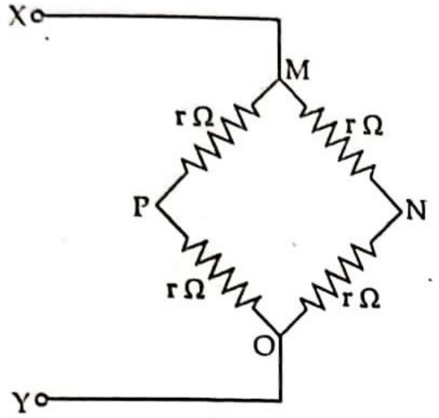
$$I_2 = I \times \frac{8}{(8+2)}$$

$$\therefore \quad I_2 = \frac{8 \times 6}{8+2} = 4.8\ A$$

ADDITIONAL QUESTION SOLUTIONS

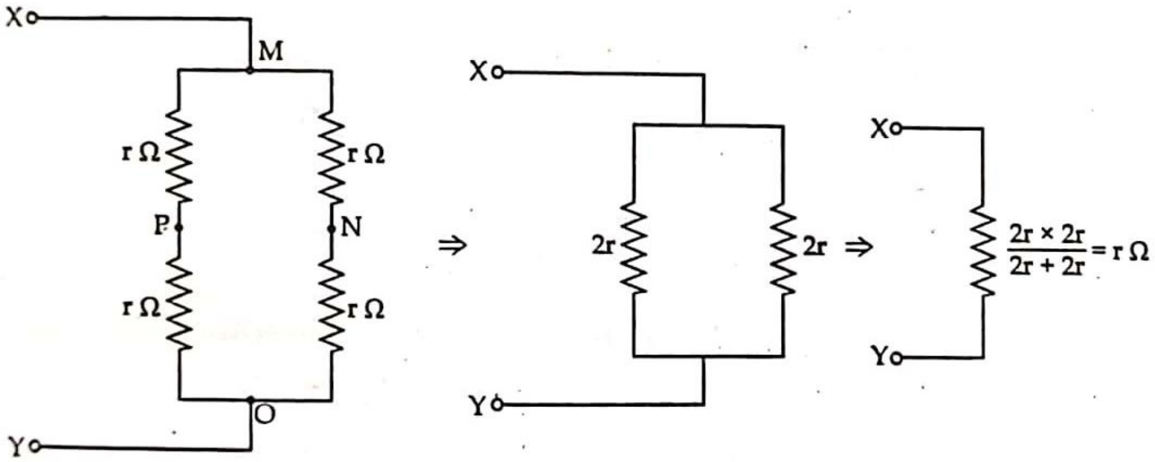
1. Determine the equivalent resistance R across xy in the figure

(i)

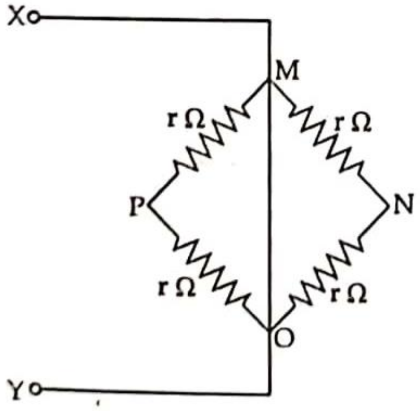


Solution:

The given figure can be re-drawn as;



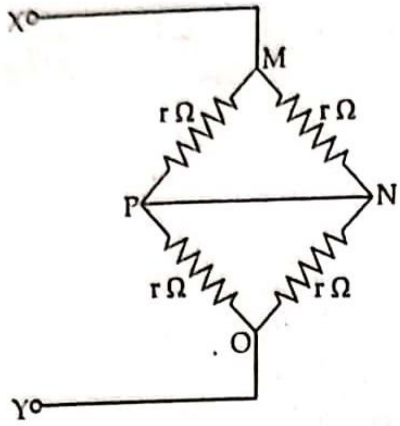
(ii)



Solution:

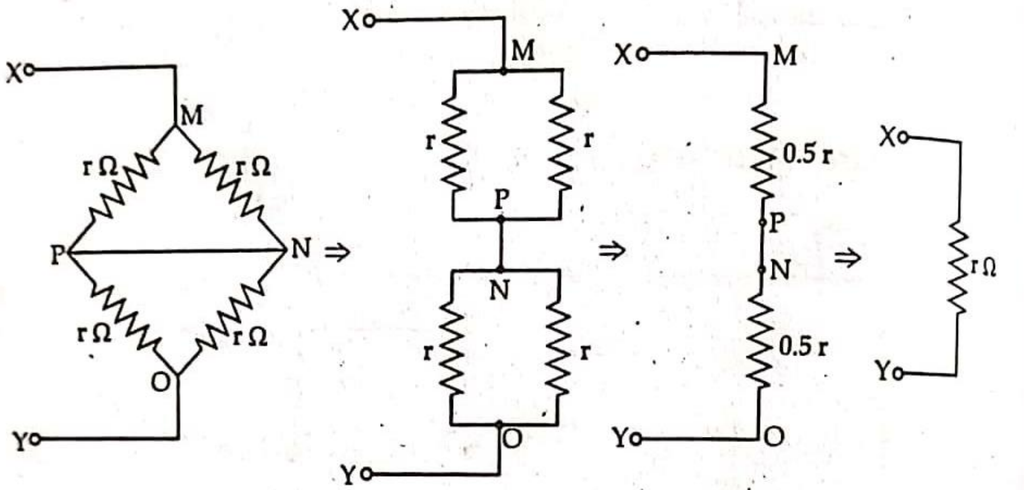
In this case, the equivalent resistance of the combination is zero because, out of three parallel branches. *i.e.,* Branch MNO, Branch MPO and Branch MO, the Branch MO has no resistance, Further, it act as short circuit and hence, total resistance of the combination becomes zero.

(iii)

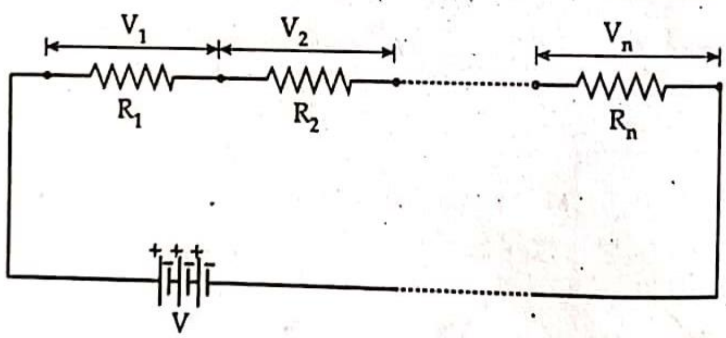


Solution:

The figure can be redrawn as;



2. A series circuit is shown in the figure below. Determine the voltage drop of the nth resistor in terms of the other resistance.



Solution:

V be the supply voltage

V_n = voltage drop across at nth resistor

Then,

$V_1 = IR_1$, $V_2 = IR_2$, ... , $V_n = IR_n$

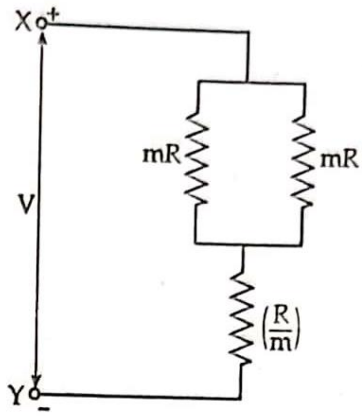
Now, using voltage divider rule, w get,

$$V_n = \frac{V}{\Sigma R} \cdot R_n$$

or, $$V_n = \frac{V}{R_1 + R_2 + \ldots + R_n} \times R_n$$

or, $V_n = \frac{VR_n}{R}$

or, $\boxed{V_n = \frac{R_n}{R} . V}$



3. Find the condition when the circuit current is maximum as shown in the figure.

Solution:

The equivalent resistance across xy is;

$$R_{xy} = \left(\frac{mR}{2} + \frac{R}{m}\right) = \frac{m^2R + 2R}{2m}$$

It may be noted that I will be maximum when,

$$\frac{\partial R_{xy}}{\partial m} = 0$$

or, $\frac{\partial}{\partial m}\left(\frac{m^2 R + 2R}{2m}\right) = 0$

or, $2m(2mR) - 2(m^2R + 2R) = 0$

∴ $\boxed{m = \sqrt{2}}$

Hence, if the resistance are selected in such a way that $m = \sqrt{2}$ in the figure. The net resistance of circuit will be minimum, giving the circuit current to maximum.

Now, the current in the circuit is

$$I = \frac{V}{R_{eq}} = \frac{V}{\frac{mR}{2} + \frac{R}{2}} = \frac{V}{\sqrt{2}R}$$

at $m = \sqrt{2}$

We get $R_{eq} = \sqrt{2} . R$

For power, across terminal x – y;

$$P = I^2 R_{xy} = \left(\frac{V}{\sqrt{2} \times R}\right)^2 \times \sqrt{2} . R$$

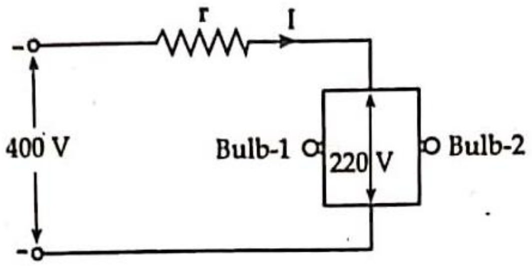
or, $P = \frac{V^2}{\sqrt{2}R}$

4. Two bulbs of 100 W, 220 V are required to be connected across a 400 V supply. Find the value of the resistance to be connected in the line so that the voltage across the bulbs does not exceed 220V.

Solution:

Total power drawn from the circuit is $(2 \times 100) = 200$ W

Hence, Supply current $I = \frac{P}{V} = \frac{200}{220} = 0.91$ A



Let r be the series resistance to be connected in the circuit so that the voltage across the bulb is 220 V. From figure;

Supply voltage = V.D. across (r + bulb)

or, $400 = Vr + 220$

or, $400 - 220 = Vr$

or, $Vr = 180$ V

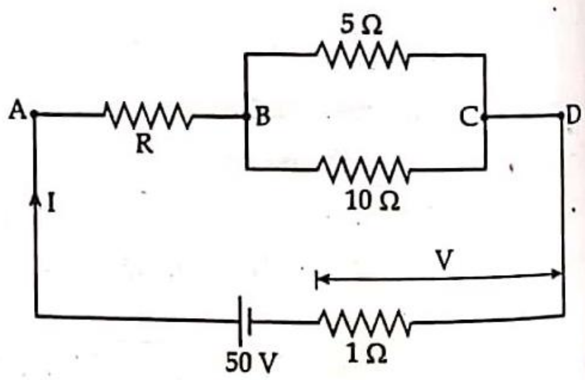
Again, $Vr = I.r.$

or, $180 = 0.91\ r$

or, $r = 197.80 \cong 198\ \Omega$

Hence, series resistance of 198 Ω is to be connected.

5. Find the value of R such that power dissipated the 5 Ω resistor is 100 watt. Assume the internal resistance of the battery of 50 V to be 1 Ω.



Solution:

Given that;

Source supply = 50 V

Internal resistance (r) = 1 Ω

Now,

Power loss in 5 Ω resistor $= \frac{V^2}{R}$

or, $100 = \frac{V^2}{5}$

or, $V = 22.36$ volt across BC.

Now; Current through 5 Ω resistor $= \frac{22.36}{5} = 4.47$ A

Current through 10 Ω resistor $= \frac{22.36}{10} = 2.236$ A

Therefore total current flowing through circuit is

$I = I_1 + I_2$

$= 2.236 + 4.47 = 6.7$ A

Now, using series parallel combination, we get;

$$R = r + 1\ (5 \| 10)$$

$$= r + 1 + \frac{5 \times 10}{5 + 10}$$

$\therefore$ $\boxed{R = 4.333 + r}$

Now, using ohm's law

$$V = IR$$

or, $I = \frac{V}{R}$

or, $6.7 = \frac{50}{4.333 + r}$

or, $4.333 + r = 7.463$

or, $r = 3.1296$

$\therefore$ $\boxed{r = 3.13\ \Omega}$

6. A repeating function current of $i = 10^3 \cdot t$ is applied in a resistor of $5\ \Omega$. Find the expression for power and its magnitude between 0 to T. Take T = 2 sec.

Solution:

We know that

$V = IR$

and $P = IV$

$\therefore$ $P = I^2R$

or, $P = (10^3\ t)^2 \times 5$

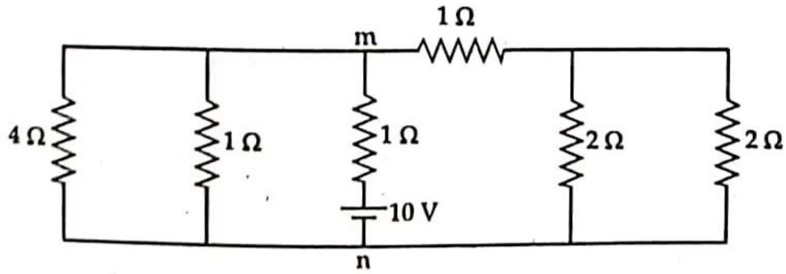
or, $P = 5 \times 10^6\ t^2$ watt

This is required expression of power. Where, V, i and P all are instantaneous values.

Again,

$$P = \frac{1}{T}\int_0^T p\,dt = \frac{1}{2}\int_0^2 5 \times 10^6 \times t^2\,dt = 6.667 \times 10^6 = 6.667\ \text{MW}$$

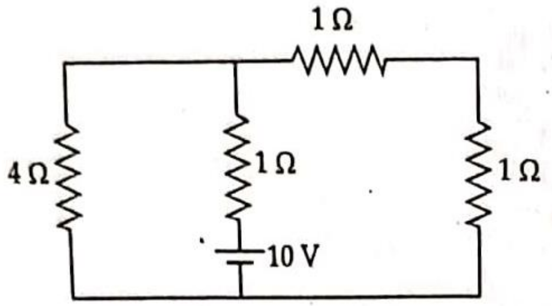
7. What is magnitude of current drained from source 10 V as shown in the figure below?



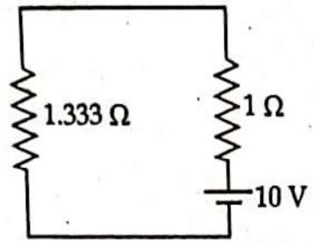
Solution:

First of all, calculating equivalent resistance across mn terminal we get reducing

(4 ∥ 1) and (2 ∥ 2) = 0.8 Ω and 1 Ω

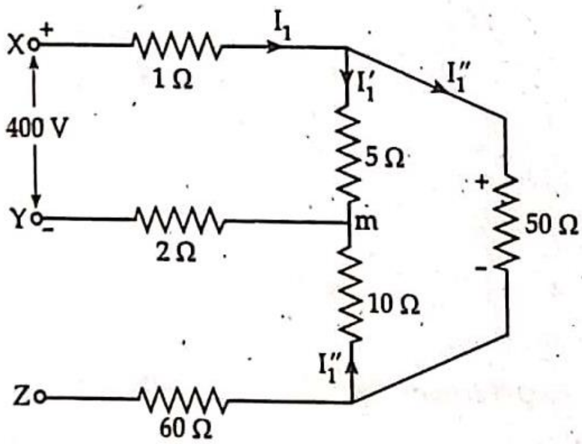


Here, (1 + 1) ∥ 4 = 1.333 Ω



Now current through 10 V source $= \dfrac{10}{1 + 1.333} = 4.286.$

8. Determine the voltage appearing across terminal y – z if a dc voltage of 100 V is applied across x – y terminal.



Solution:

Let, current through 1 Ω resistor is I_1 and that through 5 Ω and 50 Ω are I_1' and I_2'' respectively.

Now, equivalent resistance across terminal mn is

$$R_{mn} = \frac{5(50 + 10)}{5 + 10 + 50} = 4.62\ \Omega$$

and, Net resistance across xy = $R_{xy} = 1 + 2 + 4.62 = 7.62\ \Omega$

Hence,

$$I_1 = \frac{100}{7.62} = 13.12\ \text{A}$$

By current divider rule;

$$I_1' = I_1 \times \frac{60}{5 + 10 + 50} = 13.12 \times \frac{60}{65} = 12.11\ \text{A}$$

Also, V_{nm} = 100-drop across 1 Ω – drop across 2Ω

$V_{nm} = 100 - 13.12 \times 1 - 13.12 \times 2 = 60.64$ V

and, $I_1'' = I_1 \times \dfrac{5}{50 + 10 + 5} = 13.12 \times \dfrac{5}{65} = 1.01$ Amp

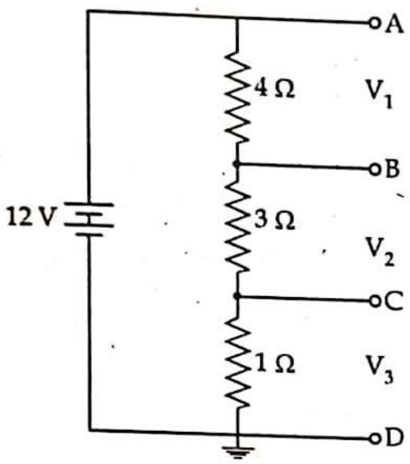
Now, To calculate voltage across 10 Ω resistor, we have

$$V_{10} = I_2'' \times 10\ \Omega = 1.01 \times 10 = 10.1\ V$$

The voltage drop across 2 Ω resistor is obviously.

$$= I_1 \times 2 = 13.12 \times 2 = 26.24\ V$$

9. Find the value of different voltages that can be obtained from 12 V battery with the help of voltage divider circuit.



Solution:

Total resistance (R) $= R_1 + R_2 + R_3 = 4 + 3 + 1 = 8\ \Omega$

Now, Voltage drop across $R_1 = R_1 \times \dfrac{V}{R} = 4 \times \dfrac{12}{8} = 6$ V

Voltage drop across $R_2 = R_2 \times \dfrac{V}{R} = 3 \times \dfrac{12}{8} = 4.5$ V

Voltage drop across $R_3 = 12 \times \dfrac{1}{8} = 1.5$ V

Now,

Voltage at D (VD) = OV. Since it is grounded.

Voltage at A = $V_A = 12$ V

Voltage at B = V_A – V.D. across R_1

$V_B = 12 - 6 = 6V$

Voltage at C = VC = Voltage at B – V.D. across R_2

$= 6 - 4.5 = 1.5$ V

$V_{AB} = V_1 = V_A - V_B = 12 - 6 = 6$ V

$V_{BC} = V_B - V_C = 6 - 15 = 4.5$ V

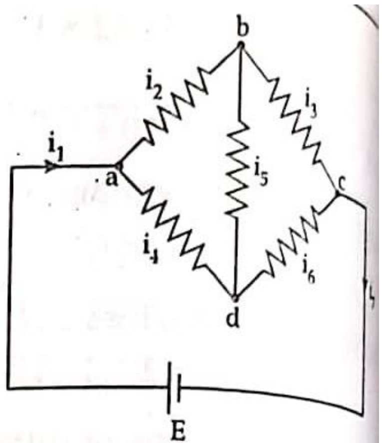
$V_{CD} = V_C = V_D = 1.5 - 0 = 1.5$ V

Again;

$V_{AC} = V_A - V_C = 12 - 1.5 = 10.5$ V

$V_{AD} = V_A - V_D = 12 - 0 = 12$ V

10. Find the magnitude and direction of the unknown currents in the figure. Where $i_1 = 10$ A, $i_2 = 6$ A and $i_3 = 4$ A.



Solution:

This question can be solved by observation and using KCL.

Given that;

$i_1 = 10$ A, $i_2 = 6$ A, $i_3 = 4$ A

Then;

$i_7 = i_1 = 10$ A [Total in = Total out]

and, that of current i_3 is given as 4 A

$\therefore$ $i_6 = 10 - 4 = 6$ A from d to c.

$i_2 = 6$ A is given

So that, $i_4 = i_1 - i_2 = 10 - 6 = 4$ A from a to d

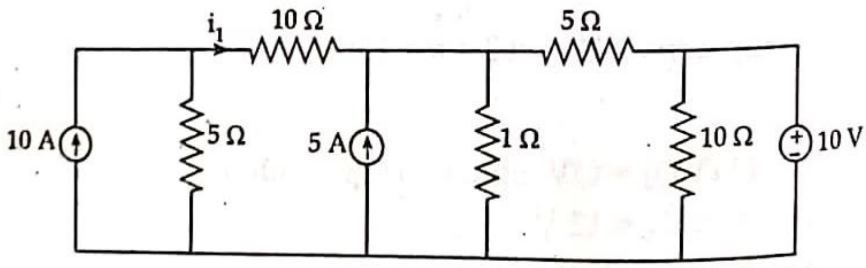
Since, current through $i_3 = 4$ A flowing from b to c

$\therefore$ Current $i_5 = i_2 - i_3 = 6 - 4 = 2$ A from b to d

Hence,

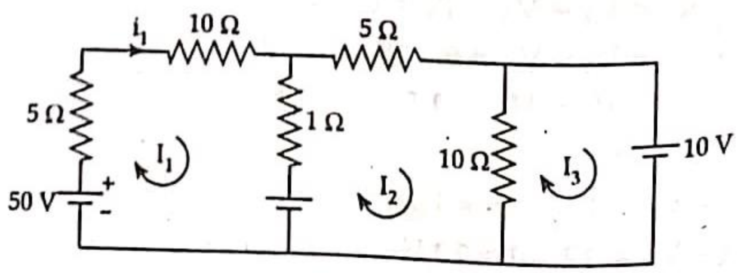
$i_1 = 10$ A	
$i_2 = 6$ A	(a to b)
$i_3 = 4$ A	from b to c
$i_4 = 4$ A	from a to d
$i_5 = 2$ A	from b to d
$i_6 = 6$ A	from d to c
$i_7 = 10$ A	

11. Obtain the current i_1 using KVL as shown in the figure below.



Solution:

Using source conversion and converting 10 A and 5 A current source into their voltage source as 50 V and 5 V respectively and re-drawing the figure again, we get



The assumed direction of current is shown in the figure above. Now,
Using KVL,
Loop I,

$$50 - I_1 (5 + 10) - 1(i_1 - I_2) - 5 = 0$$

or, $16 I_1 - I_2 = 45$ (1)

Loop II

$$-5 I_2 - 10(I_2 - I_3) + 5 - 1(I_2 - I_1) = 0$$

or, $16I_2 - I_1 - 10 I_3 = 5$ (2)

From loop III

$$- 10 - 10 (I_3 - I_2) = 0$$

or, $I_3 - I_2 = -1$ (3)

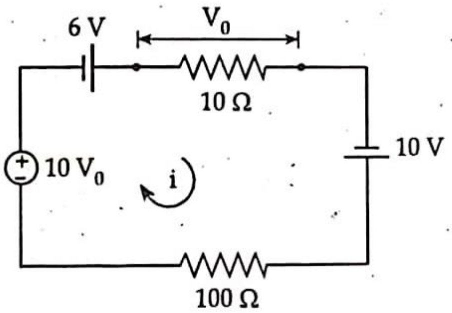
Now, solving above three equation, we get the values of loop currents as;

$I_1 = 2.79$ A

$I_2 = -0.37$ A

$I_3 = -0.63$ A

12. Find the value of current i in the circuit and also check for the power balance condition.



Solution:

Applying KVL equation in the above loop, we get;

$$6 - V_0 + 10 - 10 V_0 + i(-100) = 0$$

or, $16 - 11 V_0 - 100 i = 0$ (1)

Also,

Current through 10 Ω resistor is i. So,

$$V_0 = i \times R = 10 i$$

Again,

From equation (1), we get;

$$16 - 11 \times 10 i - 100 i = 0$$

or, $i = \frac{16}{210} = 0.0762$ A

Voltage across dependent source $= 10 \times (10 \times i) = 100 \times 0.0762 = 7.62$ V

in the given power circuit, power is supplied by only 6 V and 10 V source where as power is absorbed by resistor and dependent source

Thus,

Power supplied by 10 V source = IV = 10 × 0.0762 = 0.762 W

Power supplies by 6 V source = 6 × 0.0762 = 0.4572 watt

Total power supplied (W) = 0.4572 + 0.762 = 1.219 watt

Again

Power absorbed by dependent source is IV

$= 7.62 \times 0.0762$

$= 0.5806$ watt

Power absorbed by 100 Ω resistor is

$= 100 \times (0.0762)^2$

$= 0.5806$ watt

Also

Power absorbed by 10 Ω resistor is

$= 10 \times (0.0762)^2$

$= 0.05806$ watt

Total power absorbed $= 0.5806 + 0.5806 + 0.05806$

$= 1.219$ watt

Since; absorbed power = supplied power

So that, the given circuit is balanced.

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CHAPTER 3

NETWORK THEOREMS

3.1 INTRODUCTION

There are certain theorems, which applied to the solution of electric network and circuit. As we know, a network is said to be completely solved or analyzed when all voltages and all currents in the different elements in it are determined. There are two general approach of network analysis:

(i) Direction method

In direction method, the network is left in its original form while determining its voltage and current and basically Kirchhoff's laws are used. *For examples;* Loop analysis, Nodal analysis, superposition theorem, reciprocity theorem, etc.

(ii) Network reduction method

In this method, original network is converted into a much similar equivalent circuit for ease of calculation. Following are examples of this method; Star-delta and Delta-star transformation, Norton theories, Thevenin's theorem, etc.

3.2 APPLICATION OF KIRCHHOFF'S LAW IN NETWORK SOLUTION

Kirchhoff's laws are more suitable than ohm's law and are used for solving electrical networks, which cannot be easily solved by other practice of ohm's law. Following are two most common application of Kirchhoff's law.

3.2.1 Nodal Analysis

This method determine branch currents in the circuit and voltage at individual nodes. This method is based on the Kirchhoff's current law.

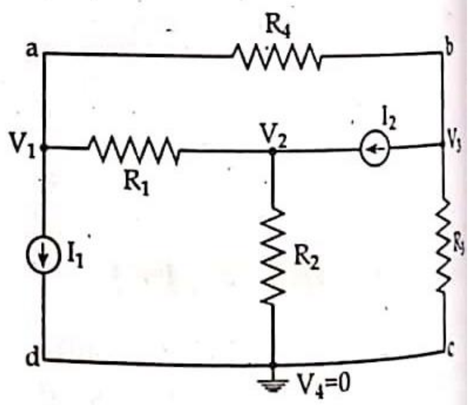
Following points should be taken into mind;

- First of all, identify all of the node in the network.
- Take one of the nodes as a reference node. The potential of reference node (V = 0) is taken. The node having maximum connection is taken as reference nod, because it simplify the problem.
- The node voltage is measured with respect to the reference node.
- Write equation of each node using KCL.
- The number of simultaneous equations to be solved becomes (n – 1), where, n = number of independent node.

Following are two cases, for analysis of nodal methods;

(i) When there is only current sources in the circuit

Consider a circuit, containing only current sources. Now,



- First, find number of nodes in the circuit and take any one node as reference node.

  Here 1, 2, 3 and 4 are nodes.
- Assign value of voltage as V_1, V_2, V_3 and V_4 for each nodes and reference node is grounded. So, V_4 = 0 volt.
- Now, apply KCL to each node. The direction of current flowing is either incoming or outgoing to the node.

  At node 1,

$$I_1 + \frac{V_1 - V_2}{R_1} + \frac{V_1 - V_3}{R_4} = 0$$

$$\boxed{I = \frac{V}{R} = \frac{V_1 - V_2}{R}}$$

where, $V_1 - V_2$ = relative potential difference

or, $$V_1\left(\frac{1}{R_1} + \frac{1}{R_4}\right) - \frac{V_2}{R_1} - \frac{V_3}{R_4} = -I_1 \qquad (1)$$

At node 2, Writing KCL

$$\frac{V_2 - V_1}{R_1} + \frac{V_2 - V_4}{R_2} - I_2 = 0$$

or, $$\frac{V_2}{R_1} - \frac{V_1}{R_1} + \frac{V_2}{R_2} - \frac{V_4}{R_2} = I_2$$ [Here, I_2 is coming to the junction.]

or, $$V_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_1\left(\frac{1}{R_1}\right) = I_2 \quad (2)$$

where, $V_4 = 0$, since it is grounded.

At node 3, writing KCL

$$I_2 + \frac{V_3 - V_1}{R_4} + \frac{V_3 - V_4}{R_3} = 0$$

or, $$\frac{V_3}{R_4} - \frac{V_1}{R_4} + \frac{V_3}{R_3} - \frac{V_4}{R_3} = -I_2$$

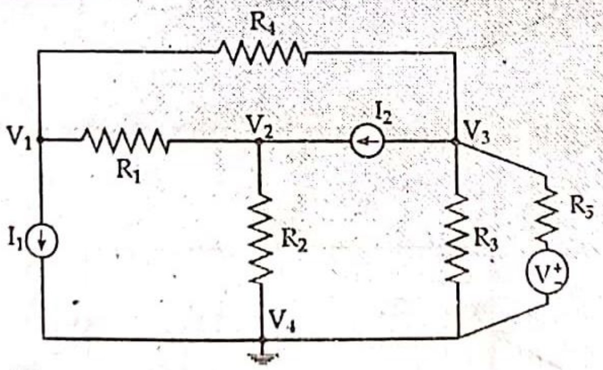
or, $$V_3\left(\frac{1}{R_3} + \frac{1}{R_4}\right) - \frac{V_1}{R_4} = I_2 \quad (3)$$

Now, by solving equation (1), (2) and (3); we get required values of node voltage. Then we can also find current in between any two nodes. *For example*, current flowing in ab is;

$$\boxed{I_{ab} = \frac{V_1 - V_3}{R_4}}$$

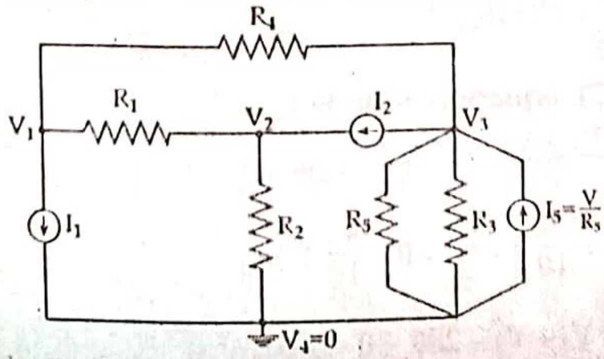
NOTE

If there is voltage source, which can be transferred into current source, then transfer them into current source as;



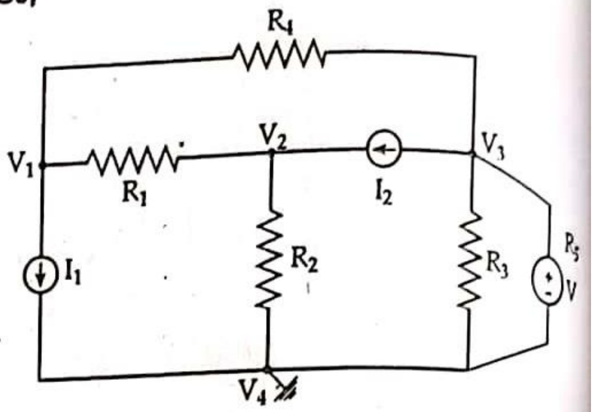
Then, we do; $I_S = \frac{V}{R_S}$

Add a resistance R_5 in parallel between that nodes



Then use similar above process.

(ii) When voltage sources also present in the circuit and cannot converted into current sources;



Then, we can write a equation for this problem, as follows;

Here,

$$\boxed{V_3 - V = V_4}$$

But, here (V_4 = 0) because it is reference node.

$\therefore$ $V_3 - V = 0$

or, $\boxed{V_3 = V}$

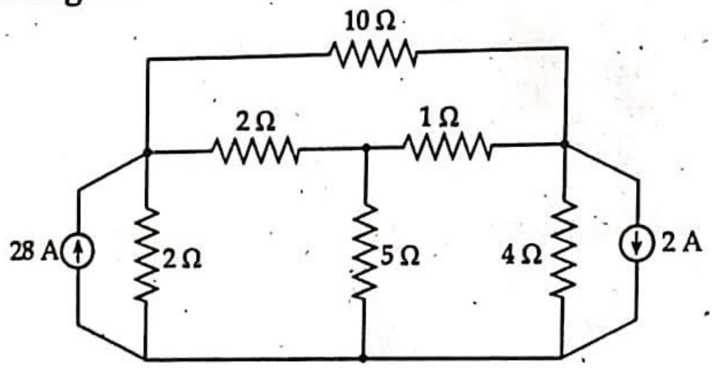
If V_4 has certain magnitude or it is unknown then,

$$\boxed{V_3 - V = V_4} \quad (1)$$

After this new equation, again do similar method, as before.

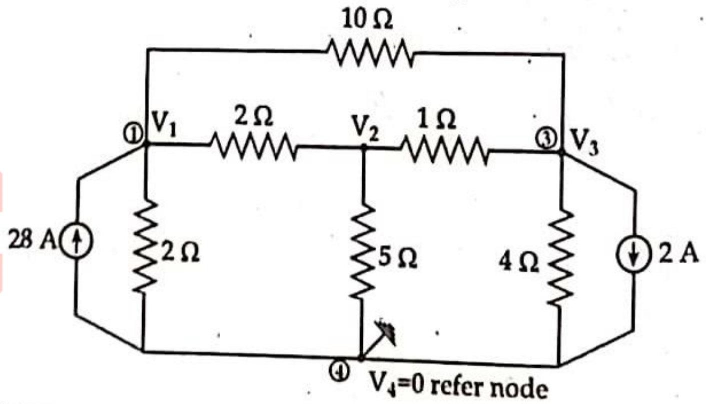
Example 3.1

Use nodal analysis method to find currents in the various resistors of the circuit shown in figure.



Solution:

The given circuit can be redrawn with various nodes and node voltages as below:



Now, writing KCL equation at node 1,

$$\frac{V_1 - V_2}{2} + \frac{V_1 - V_4}{2} + \frac{V_1 - V_3}{10} = 28$$

or, $$V_1\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{10}\right) - \frac{V_2}{2} - 0 - \frac{V_3}{10} = 28$$

or, $$11\,V_1 - \frac{10}{2}V_2 - V_3 - 280 = 0 \quad (1)$$

At node 2, we get;

$$V_2\left(\frac{1}{2}+\frac{1}{5}+1\right)-\frac{V_1}{2}-\frac{V_3}{1}=0$$

or, $\frac{17}{10}V_2-\frac{V_1}{2}-\frac{V_3}{1}=0$

or, $5\,V_1-17\,V_2+10\,V_3=0$ (2)

At node 3

$$V_3\left(\frac{1}{4}+1+\frac{1}{10}\right)-\frac{V_2}{1}-\frac{V_1}{10}=-2$$

or, $\frac{27}{20}V_3-V_2-\frac{V_1}{10}=-2$

or, $V_1+10\,V_2-13.5\,V_3-20=0$ (3)

Now, solving equation (1), (2) and (3), we get;

$V_1 = 36$ V

$V_2 = 20$ V

$V_3 = 10$ V

Now, currents in various resistors are;

At 2 Ω,

$$I_2=\frac{V_1-V_4}{2}=\frac{36-0}{2}=18\text{ A}$$

At 2 Ω,

$$I_2=\frac{V_1-V_2}{2}=\frac{36-20}{2}=8\text{ A}$$

At 5 Ω,

$$\frac{V_2-V_4}{5}=\frac{20-0}{5}=4\text{ A}$$

At 1 Ω,

$$\frac{V_2-V_3}{1}=\frac{20-16}{1}=4\text{ A}$$

At 4 Ω,

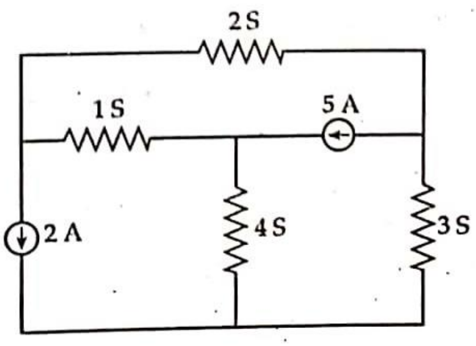
$$\frac{V_3-V_4}{4}=\frac{16}{4}=4\text{ A}$$

At 10 Ω,

$$\frac{V_1-V_3}{10}=\frac{36-16}{10}=2\text{ A}$$

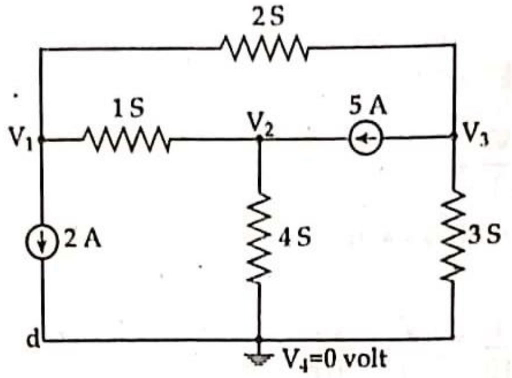
Example 3.2

Using nodal analysis find different branch currents in the circuit of figure below. All branch conductance are in simens.



Solution:

Using the procedure;



Since $\left(\frac{1}{R} = S\right)$. So, write directly $I = \frac{V}{R} = V \cdot S$

Now, at node 1,

$$(V_1 - V_2) \times 1 + (V_1 - V_3) \times 2 = -2$$

or, $\quad 3\,V_1 - V_2 - 2\,V_3 + 2 = 0 \qquad (1)$

At node 2,

$$(V_2 - V_4) \times 4 + (V_2 - V_1) \times 1 = 5$$

or, $\quad V_1 - 5\,V_2 - 5 = 0 \qquad (2)$

At node 3,

$$(V_3 - V_1) \times 2 + (V_3 - V_4) \times 3 = -5$$

or, $\quad 2\,V_1 - 5\,V_3 + 5 = 0 \qquad (3)$

Solving equation (1), (2) and (3); we get,

$V_1 = -1.5$ V

$V_2 = 0.7$

$V_3 = -1.6$ V

Now, current through

$1S \Rightarrow (V_1 - V_2) \times 1 = (-1.5 - 0.7) = -2.2$ A

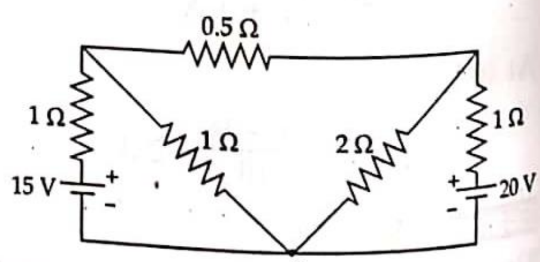
$2S \Rightarrow (V_1 - V_3) \times 2 = (-1.5 + 1.6) \times 2 = 0.2$ A

$3S \Rightarrow (V_3 - V_4) \times 3 = (-1.6 - 0) \times 3 = -4.8$ A

$4S \Rightarrow (V_2 - V_4) \times 4 = (0.7 - 0) \times 4 = 2.8$ A

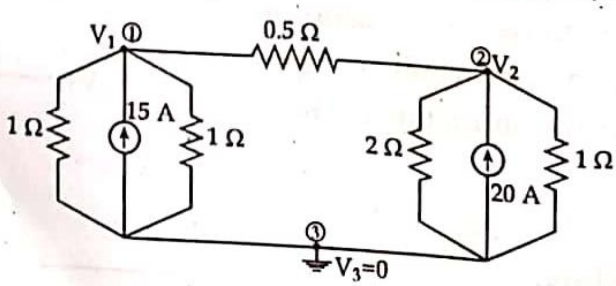
Example 3.3

Find the current flowing through 2 Ω and 0.5 Ω resistor shown in the figure.



Solution:

This circuit can be re-drawn as;



Here, V_3 is reference node.

Now, at nod 1,

$$\frac{V_1 - V_2}{0.5} + \frac{V_1 - V_3}{1} + \frac{V_1 - V_3}{1} = 15$$

or, $2(V_1 - V_2) + V_1 + V_1 = 15$ $[\because V_3 = 0]$

or, $4 V_1 - 2 V_2 = 15$ (1)

At node 2

$$\frac{V_2 - V_1}{0.5} + \frac{V_2 - V_3}{2} + \frac{V_2 - V_3}{1} = 20$$

or, $2(V_2 - V_1) + \frac{3V_2}{2} = 20$

or, $4 V_1 - 7 V_2 = 40$ (2)

Solving equation **(1)** and **(2)**; we get,

$V_1 = 9.25$ V

$V_2 = 11$ V

Now, Current through 2 Ω resistor is,

$$I_2 = \frac{V_2 - V_3}{2} = \frac{11 - 0}{2} = 5.5 \text{ A}$$

Current through 0.5 Ω resistor is,

$$I = \frac{V_1 - V_2}{0.5} = \frac{9.25 - 11}{0.5} = -3.5 \text{ A}$$

3.2.2 Mesh Analysis

This method is used to determine branch currents and voltages across the element of the network. In mesh analysis, KVL is used. This method is applicable only for planar circuits.

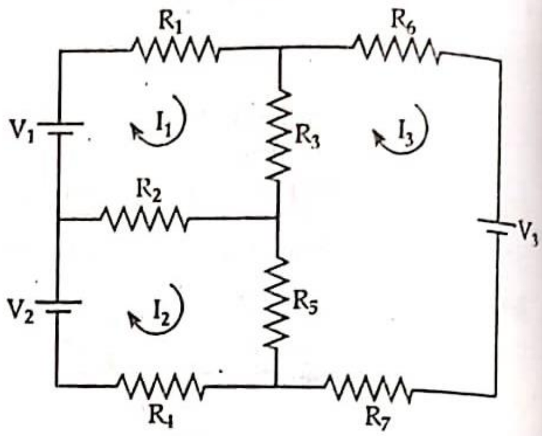
Following steps are followed in this method;

- Loop currents are taken and initially assumed to be flow in the clockwise direction.
- Branch currents, then find in term of loop currents.
- Sign convention for potential difference drop at resistor is taken as negative and for battery (→|⁺→) is taken as positive.
- This method is easier, when all the sources are given as voltage source. If there is any current source, present in a network then convert it into equivalent voltage source as,

  $$V = I \times R$$

  and place same value of resistor in series with voltage source.
- When current source is exists between two meshes, then super mesh is formed.

(i) **When there is only voltage sources present in the system**



The given circuit system consist of three meshes as shown in figure. Let I_1, I_2 and I_3 be three meshes currents, and all are assumed to be flow in the clockwise direction for obtaining symmetry in mesh equations.

Now, applying KVL to mesh (1), we have,

$$V_1 - I_1 R_1 - R_3(I_1 - I_3) - R_2(I_1 - I_2) = 0$$

or, $(R_1 + R_2 + R_3) I_1 - R_2 I_2 - R_3 I_3 = V_1$ (1)

Applying KVL to mesh (2), we have

$$-V_2 - R_2(I_2 - I_1) - R_5(I_2 - I_3) - I_2 R_4 = 0$$

or, $-R_2 I_1 + (R_2 + R_4 + R_5) I_2 - R_5 I_3 = V_2$ (2)

Applying KVL to mesh (3), we have,

$$V_3 - I_3 R_7 - R_5(I_3 - I_2) - R_3(I_3 - I_1) - I_3 R_6 = 0$$

or, $-R_3 I_1 - R_5 I_2 + (R_3 + R_5 + R_6 + R_7) I_3 = -V_3$ (3)

Now, solving these three equations, we obtain values of I_1, I_2 and I_3. But, sometime matrix method is also used to solve mesh-current method.

Now, writing equation (1), (2) and (3) in matrix from, we get

$$\begin{bmatrix} R_1 + R_2 + R_3 & -R_2 & -R_3 \\ -R_2 & R_2 + R_4 + R_5 & -R_5 \\ -R_3 & -R_5 & R_3 + R_5 + R_6 + R_7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ -V_3 \end{bmatrix}$$

Here, we can see that, at place of R_{11}, R_{22} and R_{33} have sum of resistances available in that mesh. So, we can draw following conclusion and can write matrix equation directly.

Hence,

R_{11} = Self resistance of mesh (1). [In mesh (1) R_1, R_2 and R_3 are present.]

R_{22} = Self resistance of mesh (2)

R_{33} = Self resistance of mesh (3)

$R_{12} = -R_{21}$ [Sum of all resistance common to mesh (1) and (2)]

$R_{23} = -R_{32}$ [Sum of all resistance common to mesh (2) and (3)]

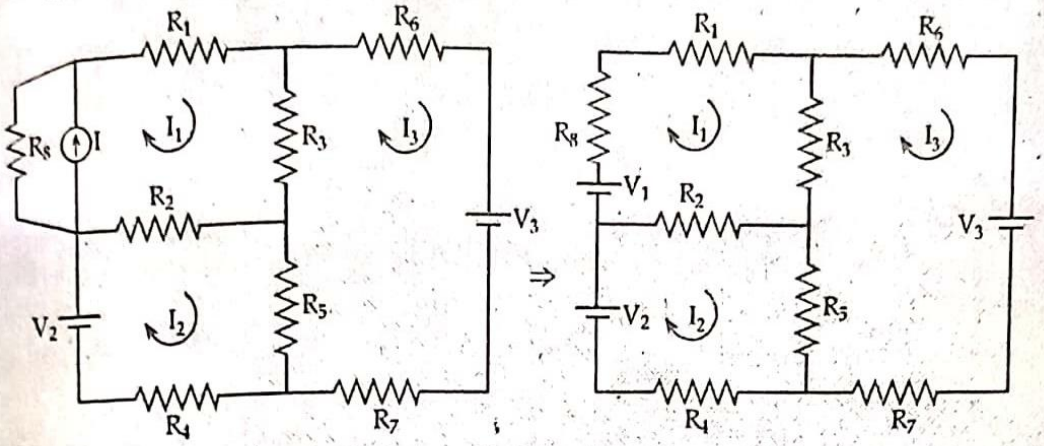
NOTE

All self resistance are always positive and all mutual resistance are always negative.

Now, solving above matrix by Cramer's rule, we get

$$I_1 = \frac{\Delta}{\Delta_1} \qquad I_2 = \frac{\Delta}{\Delta_2} \qquad \text{and} \qquad I_3 = \frac{\Delta}{\Delta_3}$$

When, current source present in the circuit, which can be converted into voltage source then,



Here,

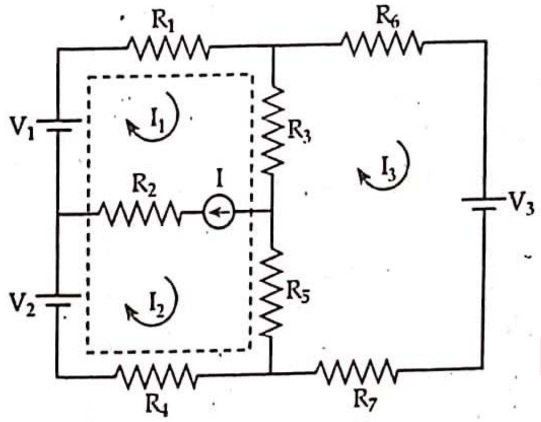
$V_1 = IR_8$

Then, place a resistance R_8 in series with that V_1 as shown in right side figure.

Now, we follow above described method to obtain equations.

(ii) When current source is present and cannot converted into voltage source then

Here, current source I is common between two mesh (1) and (2). So, super mesh is formed here.



Now, from KVL in super mesh,

$$I + I_2 = I_1 \qquad (1)$$

and applying KVL for super mesh (1) and (2); we get,

$$V_1 - I_1 R_1 - R_3(I_1 - I_3) - R_5(I_2 - I_3) - I_2 R_5 + V_2 = 0$$

or, $I_1 R_1 - I_1 R_3 - I_3 R_3 + I_2 R_5 - I_3 R_5 + I_2 R_5 = V_2 + V_1$

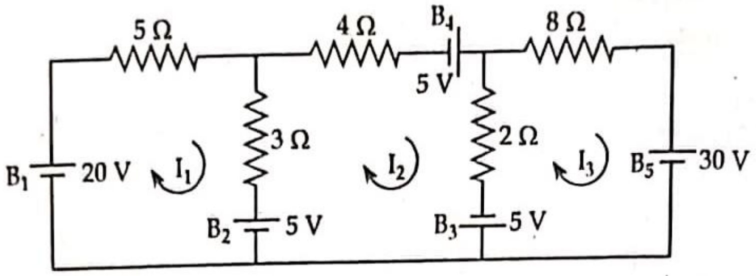
or, $I_1 (R_1 + R_3) + I_2(R_2 + R_5) - I_3 R_3 - I_3 R_5 = V_2 + V_1 \qquad (2)$

Also, equation (3) for mesh (3) is same as first case.

Now, solving these equations, we get required value of currents.

Example 3.4

Determine the current supplied by each batteries in the circuit shown in the figure.



Solution:

Let, I_1, I_2 and I_3 are the current following in loops (1), (2) and (3) respectively.

Writing KVL for loop 1,

$$20 - 5\,I_1 - 3(I_1 - I_2) - 5 = 0$$

or, $8\,I_1 - 3\,I_2 = 15$ (1)

For loop 2,

$$-4\,I_2 + 5 - 2(I_2 - I_3) + 5 + 5 - 3(I_2 - I_1) = 0$$

or, $3\,I_1 - 9\,I_2 + 2\,I_3 = -15$ (2)

For loop 3,

$$-8\,I_3 - 30 - 5 - 2(I_3 - I_2) = 0$$

or, $2\,I_2 - 10\,I_3 = 35$ (3)

Now, solving equation (1), (2) and (3); we get,

$\therefore$ $I_1 = 2.558$ A

$\therefore$ $I_2 = 1.823$ A

$\therefore$ $I_3 = -3.135$ A

Now, currents from battery

$B_1 = I_1 = 2.558$ A

$B_2 = I_2 - I_1 = 2.558 - 1.823 = 0.735$ A

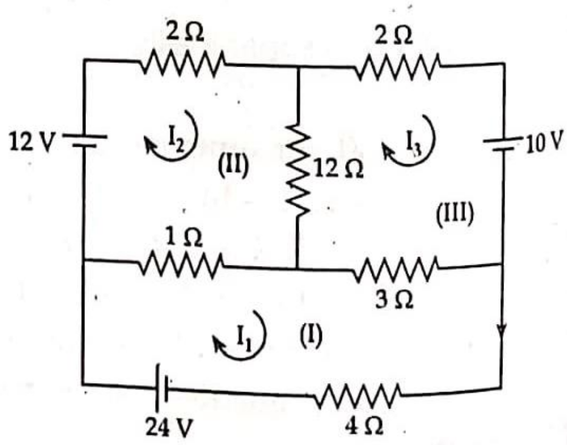
$B_3 = I_2 - I_3 = 1.823 + 3.135 = 4.958$ A

$B_4 = I_2 = 1.823$ A

$B_5 = I_3 = -3.135$ A

Example 3.5

Find the current in the 4 Ω resistor in the circuit shown in figure, by matrix method.



Solution:

Writing matrix from given circuit using concept of self and mutual resistance, we get,

$$R_{11} = 4 + 1 + 3 = 8\ \Omega$$

$$R_{22} = 2 + 1 + 12 = 15\ \Omega$$

$$R_{33} = 2 + 3 + 12 = 17\ \Omega$$

$$R_{12} = R_{21} = -1\ \Omega$$

$R_{23} = R_{32} = -12\ \Omega$

$R_{13} = R_{31} = -3\ \Omega$

and, $V_1 = 24$ V, $V_2 = 12$ V, $V_3 = -10$ V

Therefore, writing matrix equation, we get

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

or,
$$\begin{bmatrix} 8 & -1 & -3 \\ -1 & 15 & -12 \\ -3 & -12 & 17 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 12 \\ -10 \end{bmatrix}$$

Now,
$$\Delta = \begin{bmatrix} 8 & -1 & -3 \\ -1 & 15 & -12 \\ -3 & -12 & 17 \end{bmatrix}$$

$$= 8(255 - 144) + 1(-17 - 36) - 3(12 + 45) = 664$$

and,
$$\Delta_1 = \begin{bmatrix} 24 & -1 & -3 \\ 12 & 15 & -12 \\ -10 & -12 & 17 \end{bmatrix}$$

$$= 24(255 - 144) - 12(-17 - 36) - 10(12 + 45) = 2730$$

Now,
$$I_1 = \frac{\Delta_1}{\Delta} = \frac{2730}{664} = 4.1 \text{ A}$$

3.3 STAR DELTA AND DELTA STAR TRANSFORMATION

When network containing large number of branches, then it is very difficult to solve the network by using only Kirchhoff's laws, since there is large number of simultaneous equations. Such type of complicated networks can be simplified by successively replacing delta connected loop to star equivalent system and vice-versa. In delta (Δ) network, three resistor are connected in delta faishon (Δ) and in star network, three resistors are connected in wye (Y) faishon.

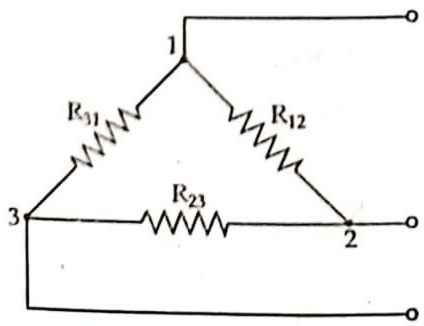


Figure: Delta connection (π)

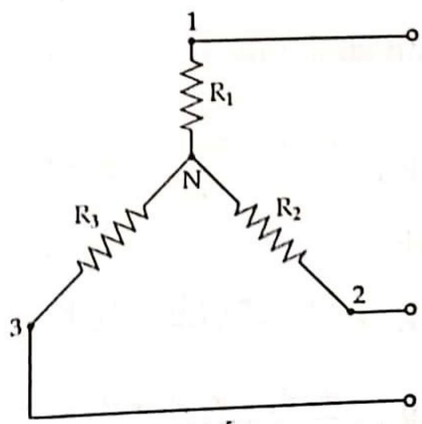


Figure: Star connection (T or Y)

(i) Delta to star conversion

From figure (a), between terminal 1 and 2, R_{12} is in parallel with (R_{23} + R_{32}). So, equivalent resistance between 1 and 2 is

$$= \frac{R_{12} \times (R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}}$$

Now, from figure (b), (star connection) between terminal 1 and 2. There are R_1 and R_2 two resistance in series. So, terminal resistance is = $R_1 + R_2$

Now, for both the case, terminal resistance between terminal 1 and 2 should be same.

$$\therefore \quad R_1 + R_2 = \frac{R_{12} \times (R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \tag{1}$$

Similarly, for terminal 2 and 3 and terminal 3 and 1

$$R_2 + R_3 = \frac{R_{23} \times (R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \tag{2}$$

$$\text{and,} \quad R_3 + R_1 = \frac{R_{31} \times (R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \tag{3}$$

Now, equation **(1)** – equation **(2)** + equation **(3)**; we get,

$$\boxed{R_1 = \frac{R_{12}\, R_{31}}{R_{12} + R_{23} + R_{31}}} \quad \boxed{R_2 = \frac{R_{23}\, R_{12}}{R_{12} + R_{23} + R_{31}}} \quad \boxed{R_3 = \frac{R_{31}\, R_{23}}{R_{12} + R_{23} + R_{31}}}$$

How to remember

Resistance of each arm of the star is given by the product of the resistances of the two delta sides that meet at its end divide by the sum of the three delta resistances.

$$\text{Any arm of star} = \frac{\text{Product of two adjacent arm of delta}}{\text{Sum of the three } \Delta\text{resistance}}$$

(ii) Star to delta conversion

Now, multiplying equations **(1)** × **(2)**, **(2)** × **(3)** and **(3)** × **(1)** and adding we get,

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_{23}\, R_{31}\, R_{12}}{R_{12} + R_{23} + R_{31}} \tag{4}$$

Now, dividing this equation with value of R_3, from above derivation we get,

$$R_{12} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

Similarly,

$$R_{23} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

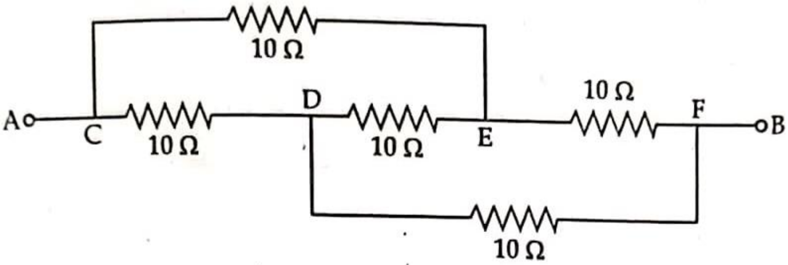
$$\text{and,} \quad R_{31} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = R_3 + R_1 + \frac{R_3 R_1}{R_2}$$

How to remember formula

Resistance between two terminals of delta = sum of star resistance connected to those terminals + products of the same two resistance divided by third resistance.

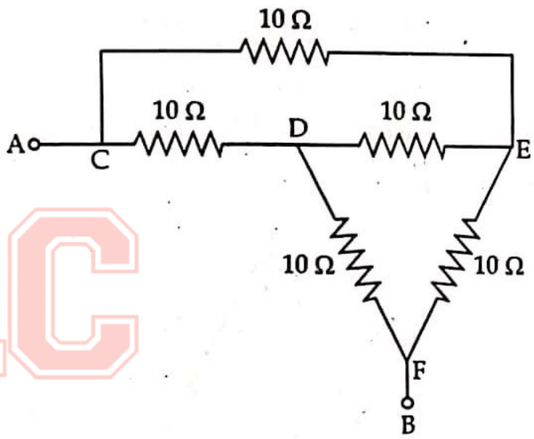
Example 3.6

Calculate the equivalent resistance between the terminals A and B in the network shown in figure.



Solution:

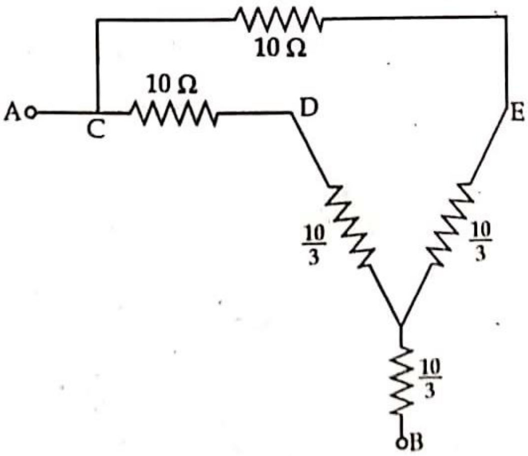
The figure can be redrawn as



For $\Delta \rightarrow Y$ Transformation in Mesh DEF

$$R = \frac{10 \times 10}{10 + 10 + 10} = \frac{100}{30} = \frac{10}{3}$$

Then, replacing Δ by star value and re-drawing figure.

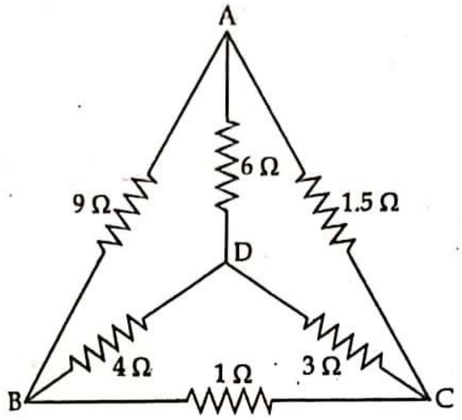


Equivalent resistance is,

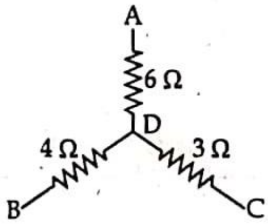
$$R_{AB} = \left[\left(10 + \frac{10}{3}\right) \parallel \left(10 + \frac{10}{3}\right)\right] + \frac{10}{3} = 10\ \Omega$$

Example 3.7

Calculate terminal resistance between A and B for given figure.



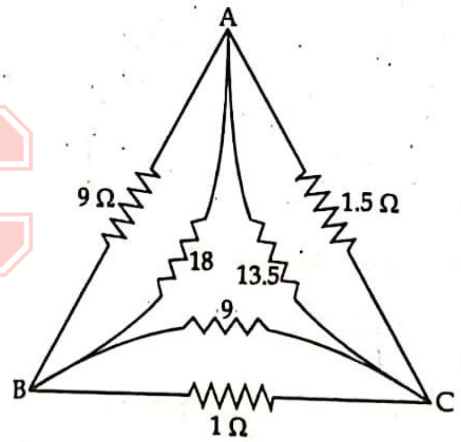
Solution:

First, this part of figure. 

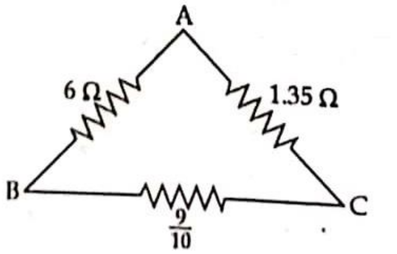
is converted into Δ.

We get, $R_{AB} = 18$, $R_{BC} = 9$ and $R_{AC} = 13.5$

Then, figure can be drawn as



Now, using parallel combination of 9 ||18, 9 || 1 and 13.5 || 1.5, we get,

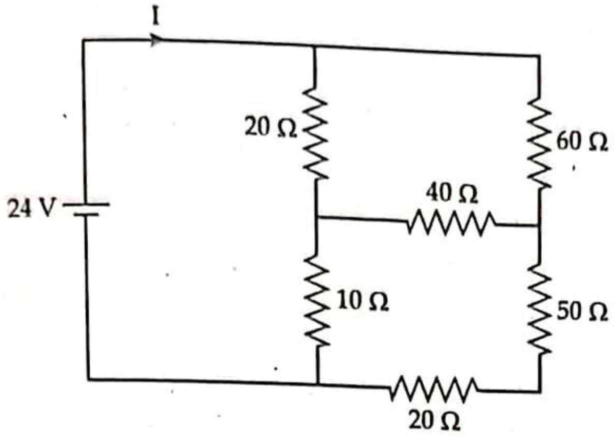


Now,

$$R_{AB} = 6 \parallel (1.35 + 0.9) = 6 \parallel 2.25 = \frac{6 \times 2.25}{6 + 2.25} = 1.6363\ \Omega$$

Example 3.8

Obtain the equivalent resistance of the given network and find value of



Solution:

Here, 10 Ω, 40 Ω and (20 + 50) = 70 Ω are in delta connection. So, transforming delta to star we get,

$$R_1 = \frac{40 \times 70}{40 + 70 + 10}$$

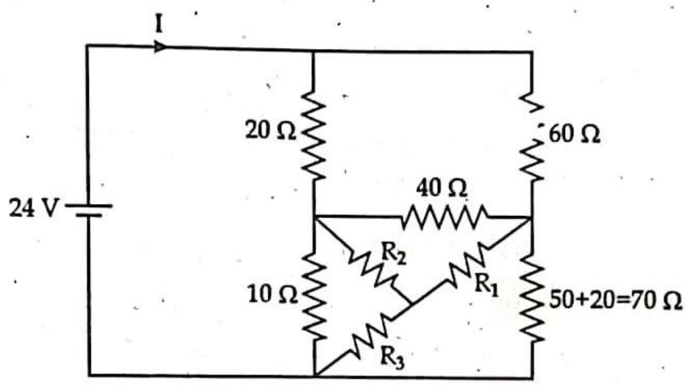
$$= 23.33\ \Omega$$

$$R_2 = \frac{10 \times 40}{40 + 70 + 10}$$

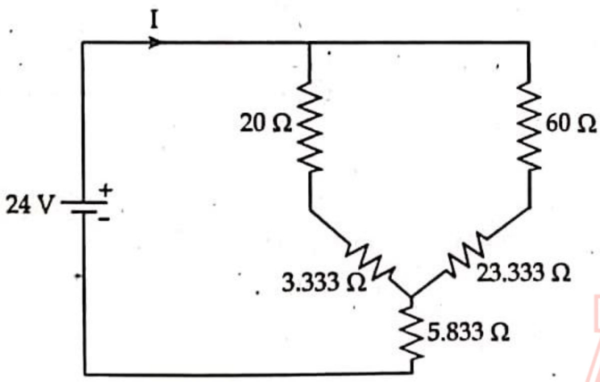
$$= 3.333\ \Omega$$

$$R_3 = \frac{10 \times 70}{40 + 70 + 10}$$

$$= 5.8333\ \Omega$$



Now, redrawing the figure again with star connection, we get,



Now, $R = [(20 + 3.333) \parallel (60 + 23.333)] + 5.833$

$$= \frac{23.333 \times 83.333}{23.333 + 83.333} + 5.833 = 24.06\ \Omega$$

and, $I = \frac{V}{R} = \frac{24}{24.06} = 0.998 \cong 1\ \text{A}$

3.4 SUPERPOSITION THEOREM

Statement: When number of voltage or current sources are acting simultaneously in a linear network, then the voltage across (or current through) an element is the algebric sum of the voltages across (or current through) that element due to each independent source acting alone.

The principle of superposition help us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately. Two points should be taken in mind while using superposition theorem.

(i) **Only consider one independent source at a time while all other sources are turned off.**

For this, we remove the voltage source and short circuited the respective terminal. Also, for current sources, just delete the current from the branch and left the terminals be open.

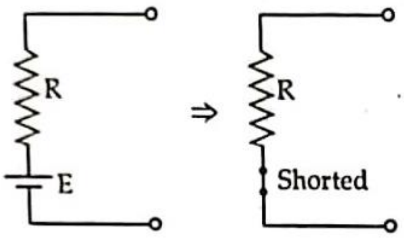


Figure: Voltage source

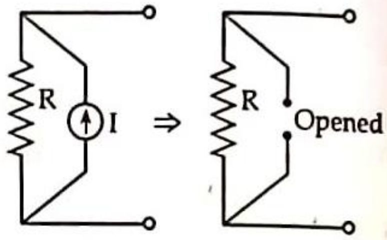


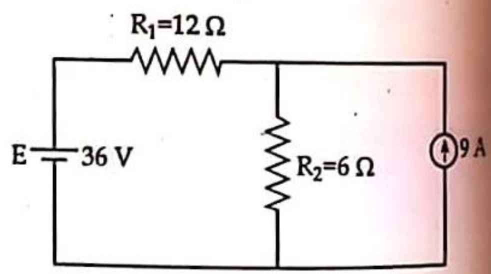
Figure: Current source

(ii) **Dependent source are left intact because they are controlled by circuit variables**

<u>Step to apply super position principle;</u>

Example 3.9

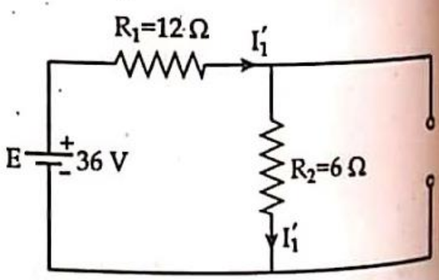
Using superposition theorem, determine the current through resistor R_2 and show that superposition theorem is not applicable for power determination.



Following steps should be used to solve the problem.

Step-1

Here, total two sources are present. One voltage source and one current source. Turn off any one source first. Let take voltage source only.



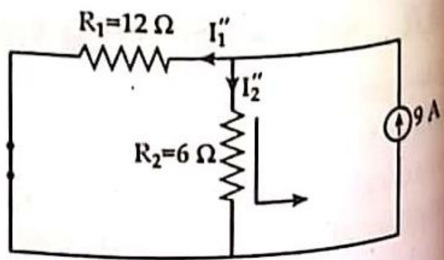
Step 2

Then, use nodal or mesh method to find current flowing in resistor due to voltage source alone. Let it be I_1'.

$$\therefore \quad I_1' = \frac{E}{R_1 + R_2} = \frac{36}{12+6} = \frac{36}{18} = 2\text{ A}\downarrow$$

Step 3

Now, consider current source only and short circuited voltage source and then determine current following in the resister with their direction.



Using CDR

$$I_2'' = I \times \frac{R_1}{R_1 + R_2} = 9 \times \frac{12}{12 + 6} = 6\ A\downarrow$$

Step 4

To determine net current in resistor; since, current through R_2 due to voltage source and current source acting alone are in same direction.

So, net current through R_2 is,

$$I_2 = I_1' + I_2'' = 2\ A + 6\ A = 8\ A$$

For power, taking R_2 resistor

Then, we have two values of current 2 A and 6 A

Now,

$$P_{2A} = I^2 R = 2^2 \times 6 = 24\ \text{Watt}$$

$$P_{6A} = I^2 R = 6^2 \times 6 = 216\ \text{Watt}$$

Total,

$$P = 24 + 216 = 240\ \text{Watt}$$

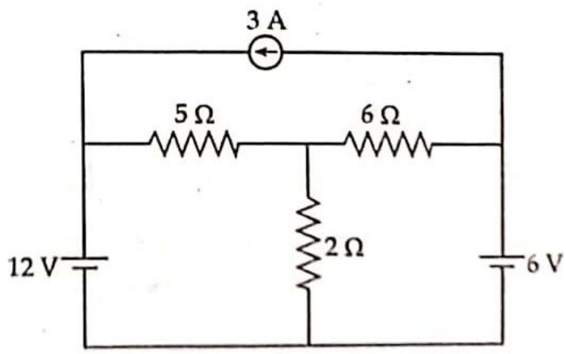
Again, we have net current through $R_2 = 8$ A

$$\therefore\quad P_{8A} = 8^2 \times 6 = 384\ \text{Watt}$$

It is clear that power delivered to 6 Ω resistor, using total current is not equal to sum of power delivered due to each source independently.

Example 3.10

Use superposition theorem to find the potential drop across 2 Ω resistor.



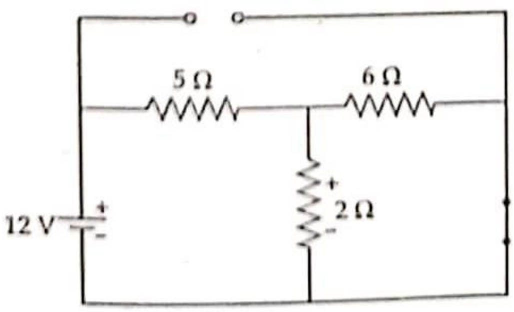
Solution:

Keep 12 V source only, then

Here,

$$I = \frac{E}{R} = \frac{12}{6 \parallel 2 + 5}$$

$$= \frac{12}{\frac{6 \times 2}{6 + 2} + 5} = 1.846\ A$$

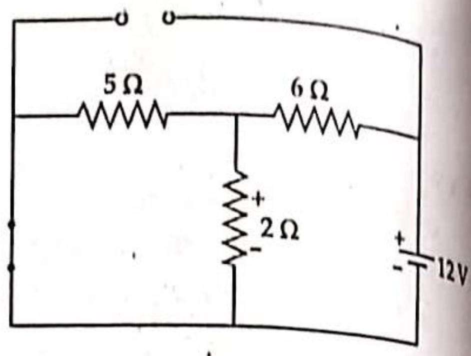


Now, current through 2 Ω resistor is,

$$I_2 = I \times \frac{6}{6 + 2} = 1.846 \times \frac{6}{8} = 1.385\ A \circlearrowright$$

and, Voltage across 2 Ω

$$V_1 = I R = 1.385 \times 2 = 2.77 \text{ V}$$



Now, keeping 6 V only

$$\text{Source current (I)} = \frac{6}{(5 \parallel 2) + 6} = 0.807 \text{ A}$$

and, $I_2 = I \times \frac{5}{5+2} = 0.807 \times \frac{5}{7} = 0.576 \text{ A}$

$$V_2 = I_2 R = 0.576 \times 2 = 1.152 \text{ V}$$

Keeping current source only, then

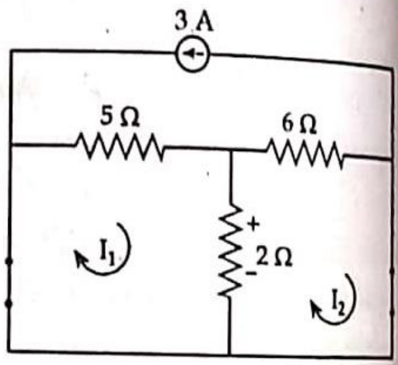
Now, using mesh analysis,

$$I_3 = -3 \text{ A}$$

loop 1

$$-5(I_1 - I_3) - 2(I_1 - I_2) = 0$$

or, $-7 I_1 + 2 I_2 = 15$ (1)



and from loop 2,

$$-6(I_2 - I_3) - 2(I_2 - I_1) = 0$$

or, $-8 I_2 + 2 I_1 = 18$ (2)

Solving both equations, we get,

$$I_1 = -3 \text{ A and } I_2 = -3 \text{ A}$$

Therefore, current through 2 Ω

$$I_2 - I_1 = -3 - (-3) = 0 \text{ A}$$

$\therefore$ $V_3 = 0$

So, total voltage across 2 Ω

$$V_1 + V_2 + V_3 = 2.77 + 1.152 + 0 = 3.922 \text{ V}$$

3.5 THEVENIN'S THEOREM

Statement: Any two output terminals (A and B) of an active linear network containing independent sources (voltage and current sources) can be replaced by a simple voltage source of magnetude V_{th} in the series with a single resistor R_{th}, where R_{th} is the equivalent resistance of the network, when looking from the output terminals A and B, with all sources (voltage and current) removed and replaced by their internal resistance and the magnitude of V_{th} equal to the open circuit voltage across the A and B terminals.

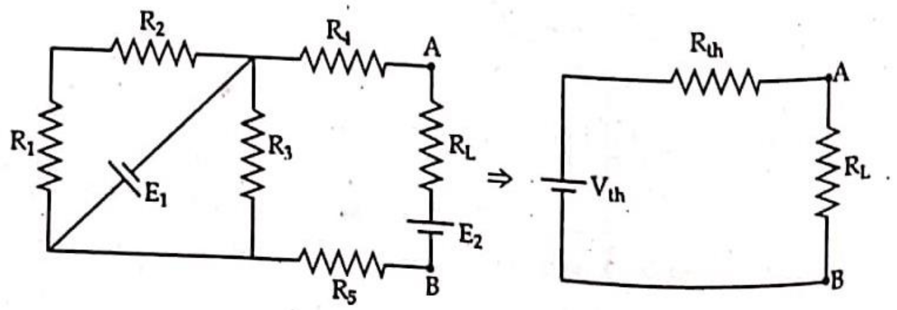


Figure: Any circuit

Thevenin's equivalent circuit

Steps to analyze electric circuit through Thevenin's theorem

i) Remove the load resistance, (current through that resister is need to determine) and make open circuit terminal A and B.

ii) Then find open circuit voltage, which is Thevenin's voltage (V_{th})

iii) Then short voltage sources and open current sources and find open circuit resistance. This is Thevenin's resistance (R_{th}).

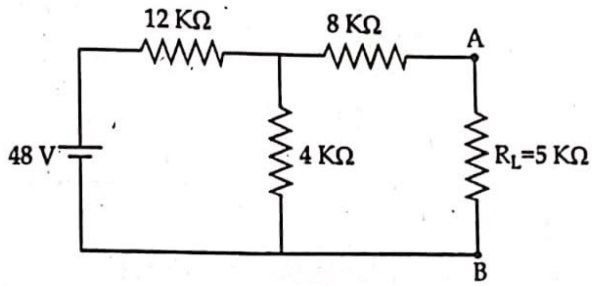
iv) Then redraw the circuit by placing R_{th} and V_{th} in series and connect to the load resistance R_L.

v) Now, find total current using ohm's law.

For clear concept see example below;

Example 3.11

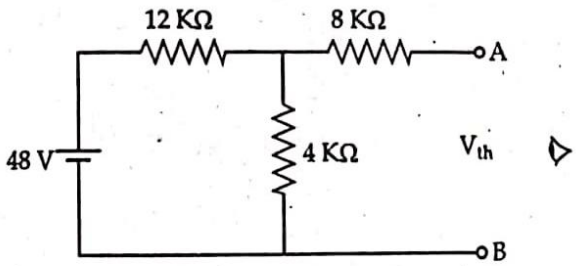
Find V_{th}, R_{th} and the load current flowing through and load voltage across the load resistor by using Thevenin's theorem.



Solution:

Step 1

Open 5 KΩ resistor and redraw figure.



Step 2

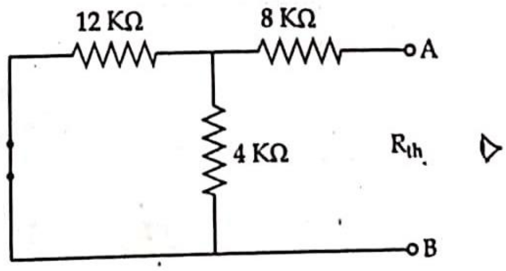
To find open CKt voltage

$$V_{th} = V \times \frac{4}{4+12} = 48 \times \frac{4}{4+12}$$

$$V_{th} = 12\text{ V} \qquad \text{[using VDR]}$$

Step 3

To get R_{th}, open current sources and short the voltage sources)



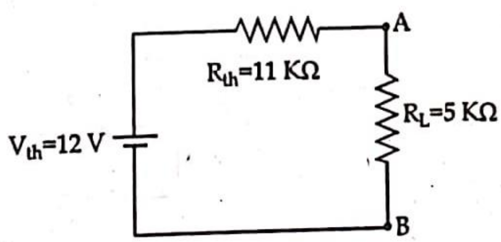
$$R_{th} = (12 \parallel 4) + 8$$

$$= \frac{12 \times 4}{12+4} + 8$$

$$= 11\text{ K}\Omega$$

Step 4

Thevenin's equivalent circuit can be redrawn as;



Step 5

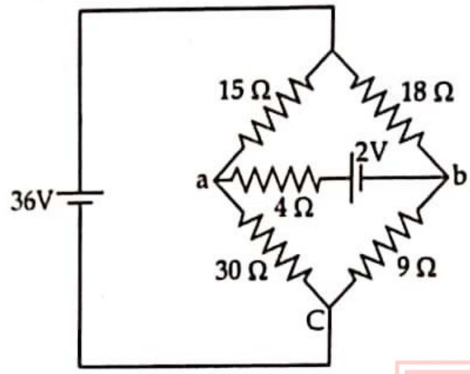
Load current,

$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{12}{(11 + 5) \times 1000} = 0.75 \text{ mA}$$

And load voltage $V_L = I_L \times R_L = 0.75 \times 10^{-3} \times 5 \times 10^3 = 3.75$ Volt

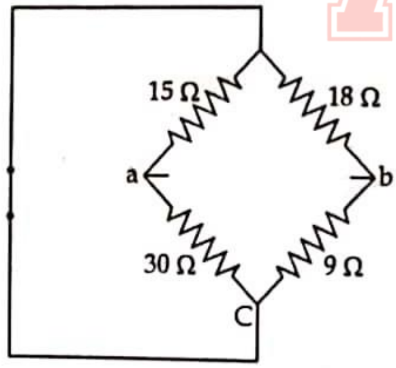
Example 3.12

Find the current flowing through the 4 Ω resistors in given figure.



Solution:

To find R_{th}, ab terminal is removed

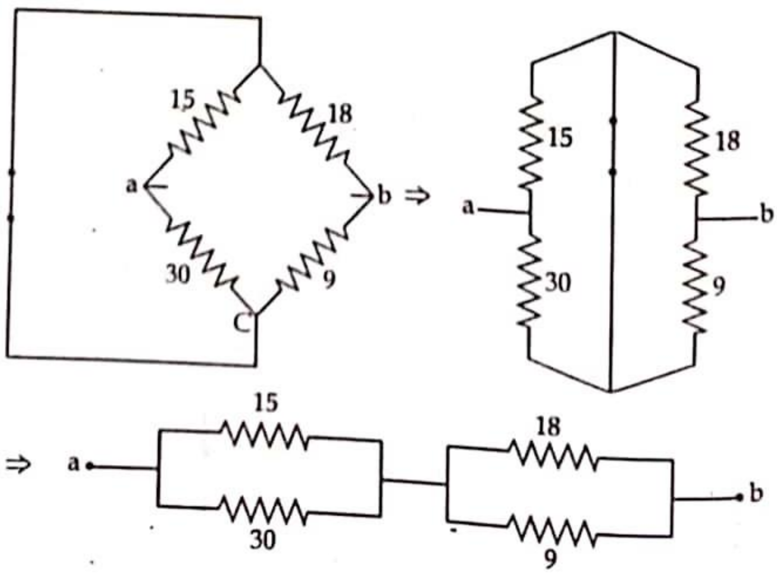


∴ $R_{th} = (15 \parallel 30) + (18 \parallel 9) = 16\ \Omega$

Again, to get V_{th}, figure can be drawn as using VDR,

Voltage at A is,

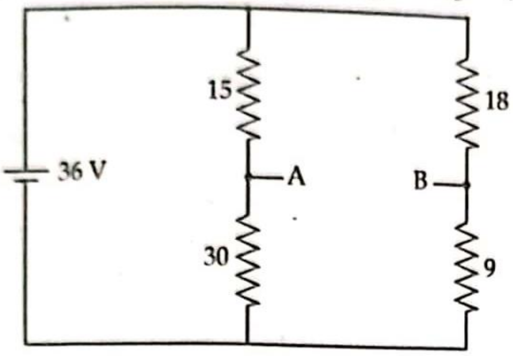
$$V_A = 36 \times \frac{15}{15 + 30} = 12 \text{ V}$$



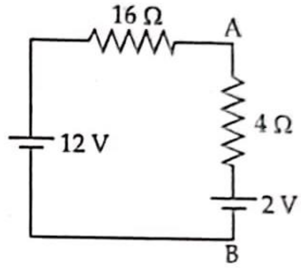
And at terminal B

$$V_B = 36 \times \frac{18}{18+9} = 24\text{ V}$$

And V_{th} (Thevenin's voltage) = $V_{AB} = V_B - V_A = 24 - 12 = 12$ V



Now, drawing, Thevenin's equivalent circuit, we get



Now,

$$I_4 = \frac{12-2}{16+4} = \frac{10}{20} = 0.5\text{ A}$$

3.6 NORTON'S THEOREM

This is another useful theorem to analyze electric circuits like Thevenin's theorem, which reduce linear active circuits and complex networks into a simple equivalent circuit. The main difference between Thevenin's and Norton's theorem is that, Thevenin's theorem provides an equivalent voltage source and an equivalent series resistance, while Norton's theorem provides an equivalent current source and an equivalent parallel resistance.

Statement of Norton theorem

It may be stated as any linear electric network or complex circuit with current and voltage sources can be replaced by an equivalent circuit containing of a single independent current source I_N and parallel resistance R_N.

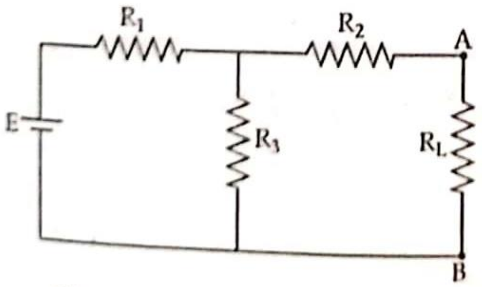
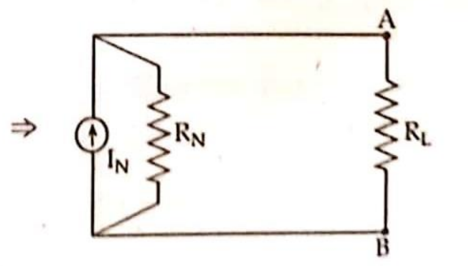


Figure: Any electric circuit



Norton's equivalent circuit

Steps used to analyse electric circuit using Norton theorem

i) Short the terminals at which Norton equivalent is required.

ii) Calculate the short circuited current. This is the Norton current I_N.

iii) Then, to get R_N, open current sources, short the voltage sources and remove load resistance. Then measure open circuit resistance, which is Norton's resistance (R_N).

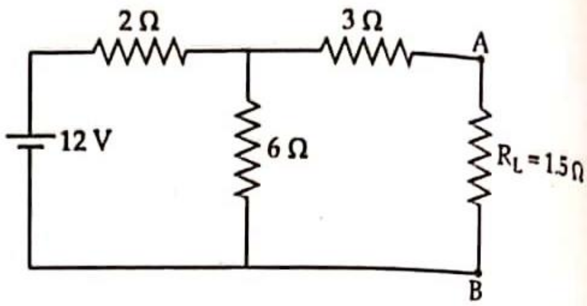
iv) Now, redraw the circuit with Norton's current and resistance in parallel and add load resistance at terminals.

v) Then use any rules to get current/voltage at load

See example below for clear concept.

Example 3.13

Find I_N, R_N current flowing through load and voltage across the load resistor using Norton theorem.

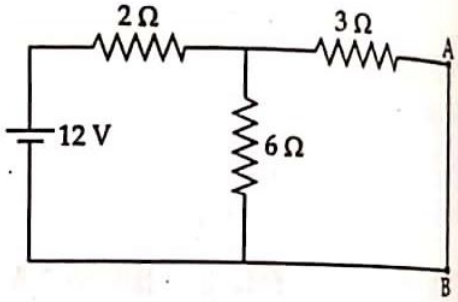


Solution:

Step 1, To find I_N,

Short the load resistor.

Step 2, To get I_N,



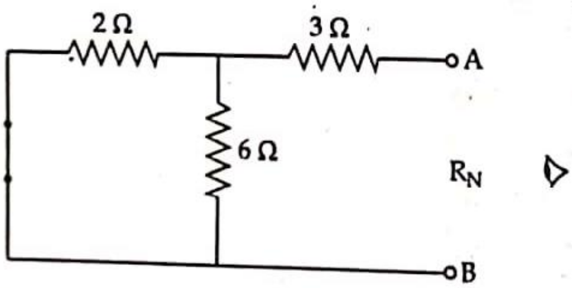
$$R = 2 + \frac{3 \times 6}{3 + 6} = 4\,\Omega$$

$$I = \frac{12}{4} = 3\text{ A}$$

Step 3, To get R_N and I_N

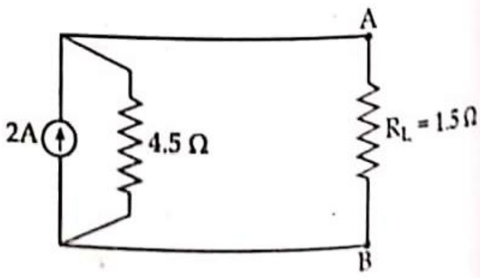
$$R_N = (2 \parallel 6) + 3 = \frac{2 \times 6}{2 + 6} + 3 = 4.5\,\Omega$$

$$I_N = 3 \times \frac{6}{3 + 6} = \frac{18}{9} = 9\text{ A}$$



Step IV, Draw Norton's equivalent circuit

Step 5, To get current through, R_L, using CDR



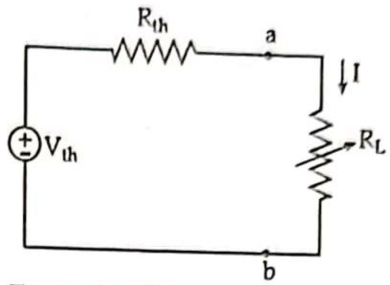
$$I_L = 2 \times \frac{4.5}{4.5 + 1.5} = 1.5\text{ A}$$

and, $V_L = I_L \times R_L = 1.5 \times 1.5 = 2.25$

3.7 MAXIMUM POWER TRANSFER THEOREM

In many practical situations, a circuit is designed to provide power to the load. While for electric utilities, minimizing power losses in the process of transmission and distribution is critical for efficiency and economic reasons. There are other applications such as communication, where it is desirable to maximize power delivered to the load.

The Thevenin equivalent circuit is useful in finding the maximum power that a linear circuit deliver to the load. So, from equivalent Thevenin circuit,



Equivalent Thevenin Circuit

Here,

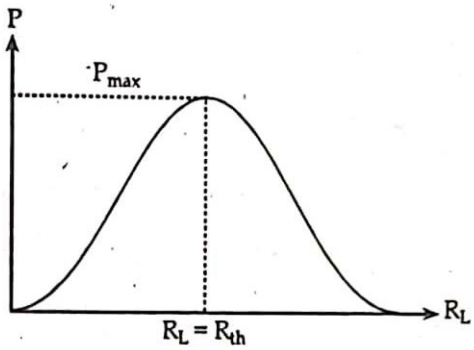
$$I = \frac{V_{th}}{R_{th} + R_L}$$

Now, power can be given as

$$P = I^2 R_L$$

or, $$P = \left(\frac{V_{th}}{R_{th} + R_L}\right)^2 \times R_L \qquad \textbf{(1)}$$

Here, V_{th} and R_{th} are fixed. Now varying the load resistance R_L, the power delivered to the load is also varies, which is sketched in the figure.



We can notice from the figure that the power is small for small or very large value of R_L and it is maximum at some values for R_L between (0 to ∞). And, now, we want to show that this power is maximum when R_L is equal to R_{th}. This is known as maximum power transfer theorem.

Proof of MPT

To proof maximum power transfer theorem, differentiating equation (1), wrt R_L we get,

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\left(\frac{V_{th}}{R_{th} + R_L}\right)^2 \times R_L$$

$$= V_{th}^2 \frac{d}{dR_L}\left[\frac{R_L}{(R_{th} + R_L)^2}\right]$$

$$= V_{th}^2 \times \frac{[(R_{th} + R_L)^2 - 2R_L(R_{th} + R_L)]}{[(R_{th} + R_L)^2]^2}$$

$$\therefore \quad \frac{dP}{dR_L} = (V_{th})^2 \times \frac{[(R_{th} + R_L) - 2R_L]}{(R_{th} + R_L)^3} \qquad \textbf{(2)}$$

Now, for maximum power,

$$\frac{dP}{dR_L} = 0$$

so, $$(V_{th})^2 \times \frac{(R_{th} + R_L) - 2R_L}{(R_{th} + R_L)^3} = 0$$

or, $R_{th} + R_L - 2R_L = 0$

or, $\boxed{R_{th} = R_L}$ (3)

Hence, the maximum power takes place when the load resistance R_L equals to the Thevevin resistance R_{th}.

Now, from equation (1), maximum power is given as,

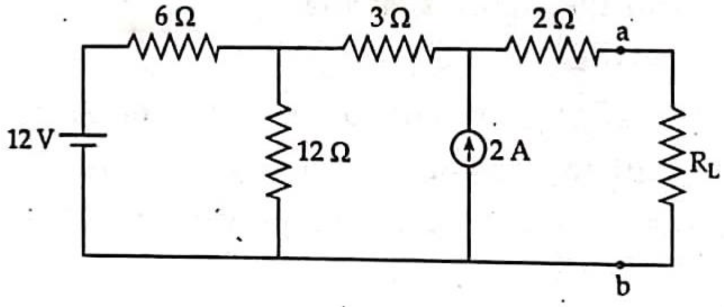
$$P_{max} = \left(\frac{V_{th}}{R_{th} + R_L}\right)^2 \times R_{th}$$

or, $$P_{max} = \frac{V_{th}^2}{(R_{th} + R_{th})^2} \times R_{th}$$

or, $$\boxed{P_{max} = \frac{V_{th}^2}{4\,R_{th}}}$$

Example 3.14

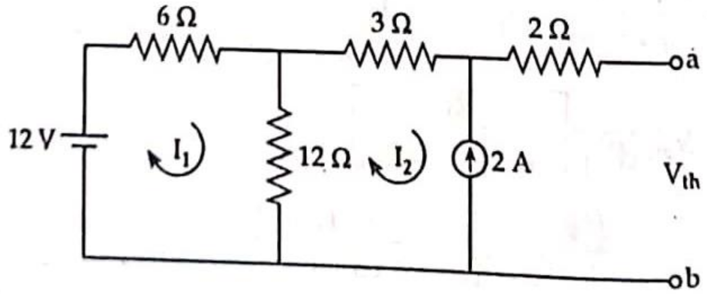
Find the value of R_L for maximum power transfer and find also the maximum power for given circuit.



Solution:

Using Thevenin theorem to solve the network.

For V_{th};



Writing KVL for loop 1,

$$12 - 6I_1 - 12(I_1 - I_2) = 0$$

or, $18\,I_1 - 12\,I_2 = 12$ (1)

From loop 2,

$$I_2 = -2\text{ A}$$

$\therefore$ $$I_1 = -\frac{2}{3}\text{ A}$$

Now, to get Vth, using KVL in outer loop,

$$V_a - 2 \times 0 - 3 \times (-2) - 6 \times \left(-\frac{2}{3}\right) + 12 = V_b$$

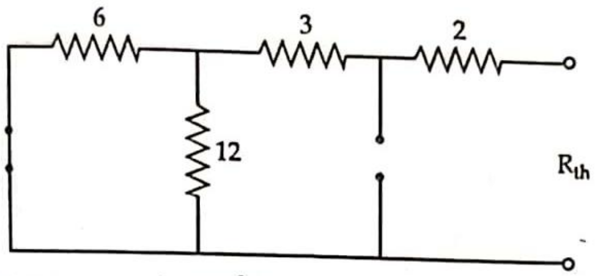
or, $V_a + 6 + 4 + 12 = V_b$

or, $V_a - V_b = V_{th} = 22$ Volt

Also, to get R_{th}

$R_{th} = (6 \parallel 12) + 3 + 2$

or, $R_{th} = 4 + 5 = 9\ \Omega$



Now, for maximum power transfer.

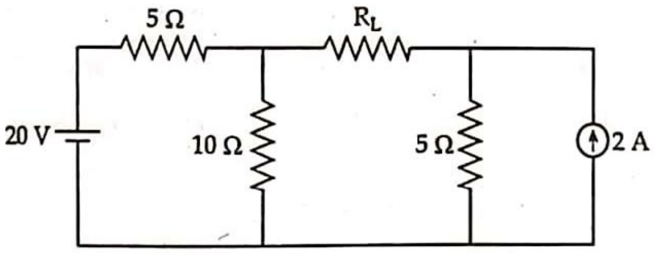
$R_{th} = R_L = 9\ \Omega$

and, Maximum power is

$$P_{max} = \frac{V_{th}^2}{4 R_{th}} = \frac{(22)^2}{4 \times 9} = 13.44 \text{ Watt}$$

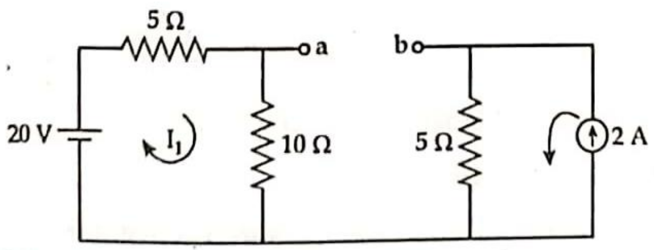
Example 3.15

Find the value of R_L for maximum power transferred to the load. Also find the maximum power.



Solution:

To get V_{th};



Now, using VDR, voltage across 10 Ω resistance is,

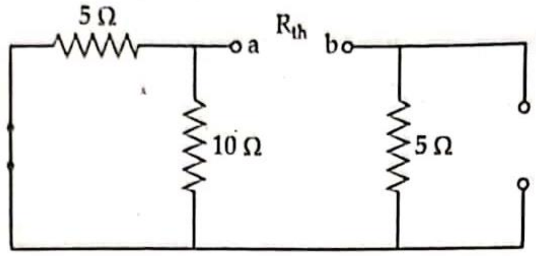
$$V_{10} = 20 \times \frac{10}{10 + 5} = 13.333 \text{ V}$$

Now, using KVL

$V_a - 13.333 - 5 \times (-2) = V_b$

or, $V_a - V_b = V_{th} = 13.333 - 10 = 3.333$ V

Also, to get R_{th}



Here,

$$R_{th} = (5 \parallel 10) + 5 = 8.333\ \Omega$$

For maximum power

$$R_{th} = R_L = 8.333\ \Omega$$

and, Maximum power,

$$P_{max} = \frac{V_{th}^2}{4 R_{th}} = \frac{(3.333)^2}{4 \times 8.333} = 0.333 \text{ Watt.}$$

3.8 RECIPROCITY THEOREM

Statement: The current I in any branch of a network due to a single voltage source E, anywhere, else in the network will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current I was originally measured.

In other words, the location of the voltage source and the resulting current may be interchanged without a change in current. The theorem required that the polarity of the voltage source have the same correspondence with the direction of the branch current in each position.

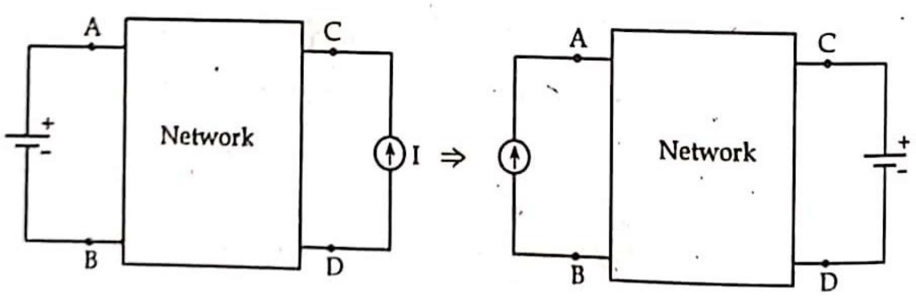


Figure: Reciprocity theorem

Steps for using reciprocity theorem

i) Mark the branch in which reciprocity theorem is to be applied.

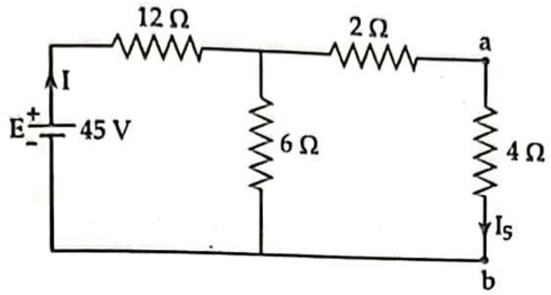
ii) Then find current in that branch by any method discussed previously.

iii) Now, the voltage source, for which reciprocity theorem is to be verify is interchanged with I.

iv) Then calculate the current in that branch.

Example 3.16

Verify reciprocal theorem for branch ab.



Solution:

First determining current in branch ab due to

$E = 45$ V

Equivalent resistance (R_T) $= (4 + 2) \parallel 6 + 12$

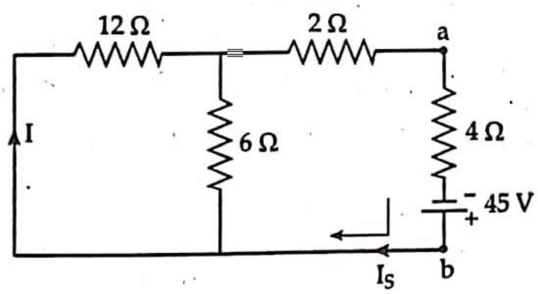
$= 6 \parallel 6 + 12 = 3 + 12 = 15\ \Omega$

and current delivered (I) $= \dfrac{45}{15} = 3$ A

Also, current flowing in branch ab is,

$$I_S = \frac{I}{2} = \frac{2}{3} = 1.5 \text{ A} \downarrow$$

Again, interchanging voltage source, and redrawing above circuit network, we get,



Now, finding equivalent resistance, we get,

$$R_T = (12 \parallel 6) + 6 = 10\ \Omega$$

and, $I_S = \dfrac{45}{16} = 4.5$ A

Using CDR

$$I = I_S \times \frac{6}{12+6} = 4.5 \times \frac{6}{12+6} = 1.5 \text{ A} \downarrow$$

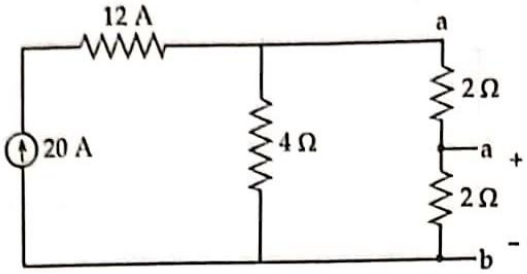
Hence, reciprocity theorem is verified.

Disadvantage of Reciprocity Theorem

The reciprocity theorem is applicable only to a single source networks. It is therefore, not a theorem used in the analysis of multisource networks.

Example 3.17

Verify the reciprocity theorem for figure below.



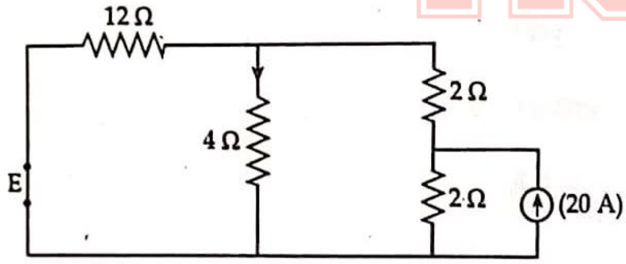
Solution:

For given figure

Current through 2 Ω resistor is using KVL

$$I_2 = 20 \times \frac{4}{4+4} = 10 \text{ A}$$

and, $V_{ab} = 2 \times 10 = 20$ V



Again, interchanging the source and redrawing the figure, using CDR

$$I_2 = 20 \times \frac{2}{(4+2)+2}$$

or, $I_2 = 5$A

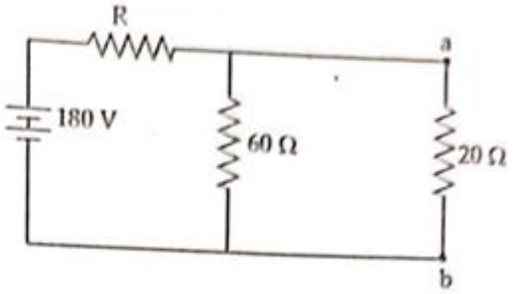
Now, E = V at 4 Ω resistor

$$I_2 \times R = 5 \times 4 = 20 \text{ V}$$

Hence, reciprocity theorem is verified.

EXAMINATION QUESTION SOLUTIONS

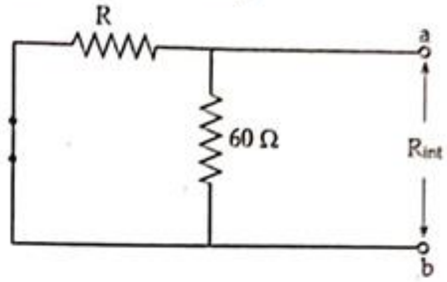
1. Consider the circuit shown in figure. Determine;
   i) the value of R so that the load of 20 Ω should draw maximum power
   ii) the value of maximum power drawn by the load.



Solution:

Since for maximum power drawn from load 20 Ω if $R_{int} = R_{th}$

i) To calculate R_{int} short circuiting the voltage source, then we find R and 60 Ω are in parallel combination.



So,

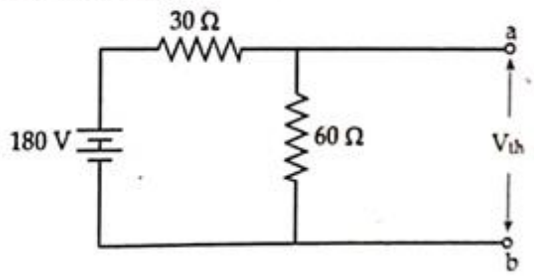
$\therefore \quad R_{int} = R \parallel 60$

or, $\quad 20 = \dfrac{60\,R}{60 + R}$

$\therefore \quad R = 30\,\Omega$

ii) Maximum power, using Thevenin's theorem

$$P_{max} = \frac{V_{th}^2}{4\,R_{th}}$$



So, V_{th} can calculated as,

$\therefore \quad V_{th} = \dfrac{180}{30 + 60} \times 60$

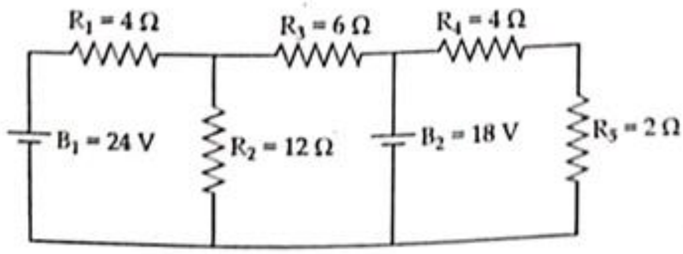
[using voltage divider rule]

$V_{th} = 120$ V

or, V_{th} also calculated using KVL

$\therefore \quad P_{max} = \dfrac{(120)^2}{4 \times 20} = 180$ Watt

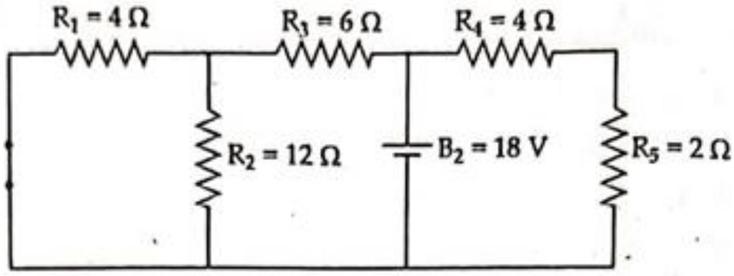
2. Using superposition theorem, find the current in resistor R_3 in the circuit shown in figure below.



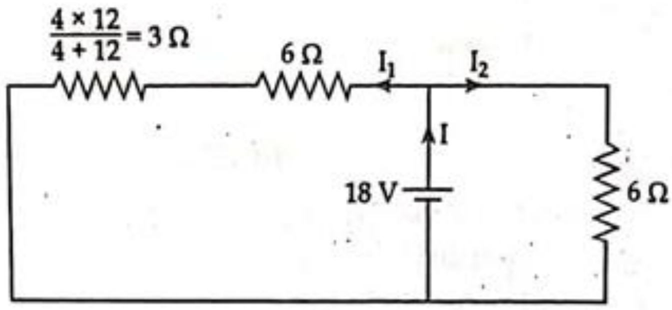
Solution:

There are two source, so first of all we calculate current through each source alone and then apply superposition theorem.

Consider B_2 and making B_1 short circuit.



$\cong$

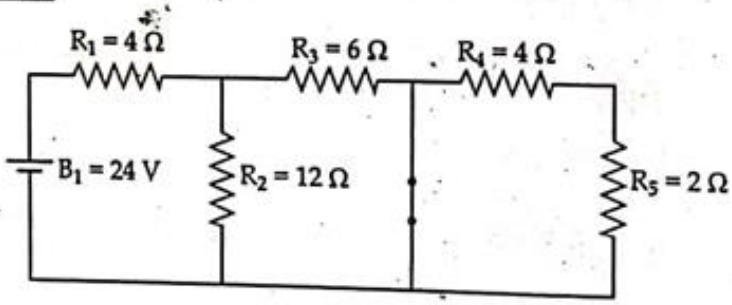


Now, current flowing in circuit,

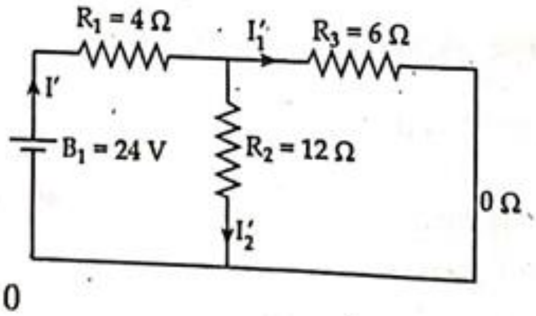
$$I = \frac{18}{(6+3) \parallel 6} = 5\text{ A}$$

and, $I_1 = \frac{5}{(6+3)+6} \times 6 = 2\text{ A} \leftarrow$

<u>Taking B_1 source:</u>



$\cong$



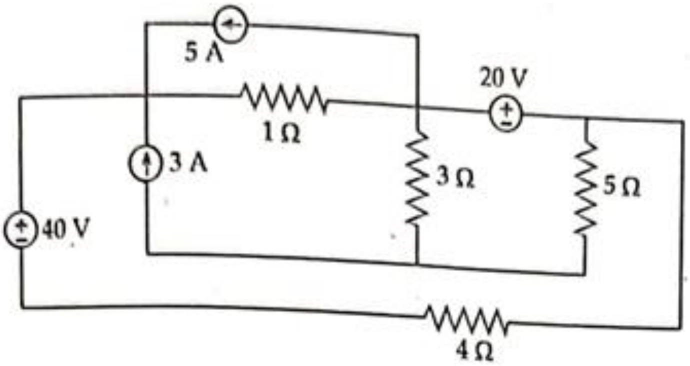
Since $[0 \parallel (4+2)] = 0$

$\therefore \quad I = \frac{24}{4 + (6 \parallel 12)} = 3\text{ A}$

$\therefore \quad I_1' = \frac{3}{6+12} \times 12 = 2\text{ A} \rightarrow$

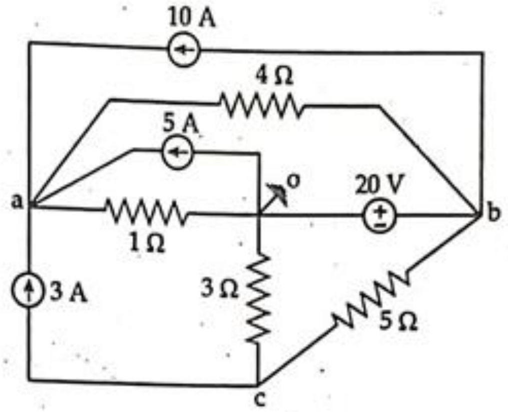
Therefore, net current through $6\,\Omega = I_1' - I_1 = 2 - 2 = 0$ A

3. For the circuit shown below, determine the power dissipated by $5\,\Omega$ resistor in the circuit using nodal analysis.



Solution:

Here 0 be reference node and voltage at a, b, c be V_a, V_b, V_c and $V_b = 20$ V.



Applying KCL at node c we get,

or, $$0 = 3 + \frac{V_c - 0}{3} + \frac{V_c - V_b}{5}$$

or, $$0 = 3 + \frac{V_c}{3} + \frac{V_c}{5} - \frac{20}{5}$$

$\therefore$ $$V_c = \frac{15}{8}\text{ V} = 1.875\text{ V}$$

To calculate, power dissipated by 5 Ω resistor;

Current through 5 Ω is

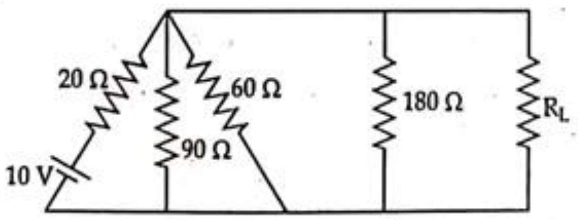
$$I_5 = \frac{V_b - V_c}{5} = \frac{20 - 1.875}{5}$$

$$I_5 = 3.625\text{ A}$$

And power is

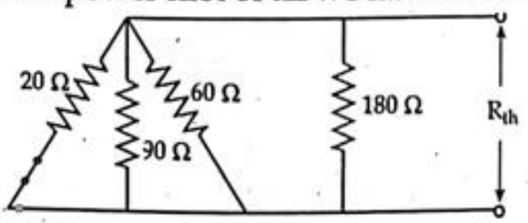
$$P = I_5^2 \times R_5 = (3.625)^2 \times 5 = 65.703\text{ Watt}$$

4. For the circuit shown below, what should be the value of R_L to get maximum power. What is the maximum power delivered to the load?



Solution:

To calculate maximum power first of all we have to find R_{th} and V_{th}



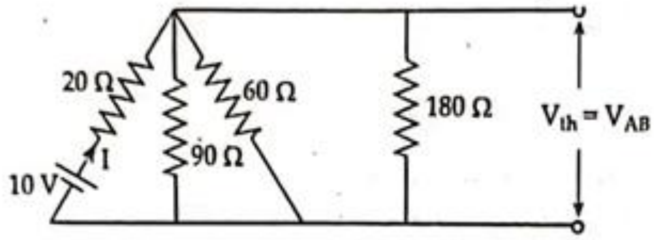
$$R_{th} = (180 \,||\, 60 \,||\, 90 \,||\, 20) = 12\ \Omega$$

To deliver maximum power

$$R_{th} = R_L = 12\ \Omega$$

and, $$P_{max} = \frac{V_{th}^2}{4\ R_{th}}$$

Then,



Now, current flowing through the battery is

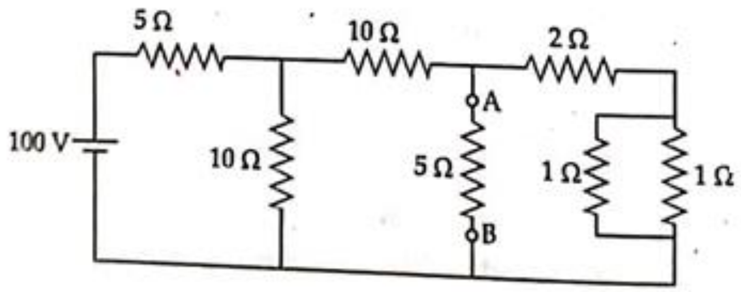
$$I = \frac{E}{R_{eq}}$$

$$I = \frac{10}{20 + (90 \parallel 60 \parallel 180)} = 0.2\ A$$

and, $V_{th} = 10 - I \times 20 = 10 - 0.2 \times 20 = 6\ V$ [∴ Using ohm's law]

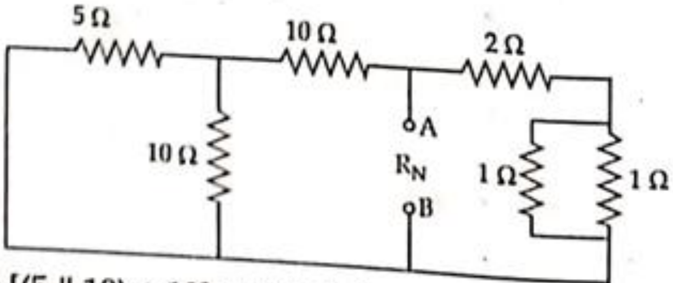
so, $$P_{max} = \frac{(6)^2}{4 \times 12} = 0.75\ \text{Watt.}$$

5. Determine the current flowing through the 5 Ω resistor connected between AB in the circuit shown below using Norton's theorem.



Solution:

To calculate current through 5 Ω resistor using Norton's theorem, firstly find R_N, I_N.

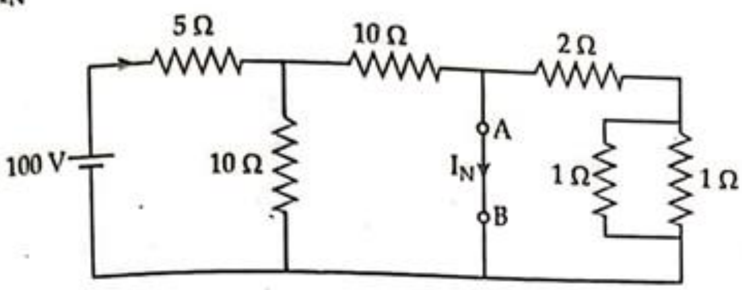


$$R_N = [(5 \parallel 10) + 10] \parallel [2 + (1 \parallel 1)]$$

$$= \left[\frac{5 \times 10}{5 + 10} + 10\right] \parallel \left[2 + \frac{1 \times 1}{1 + 1}\right]$$

$$= 2.105\ \Omega$$

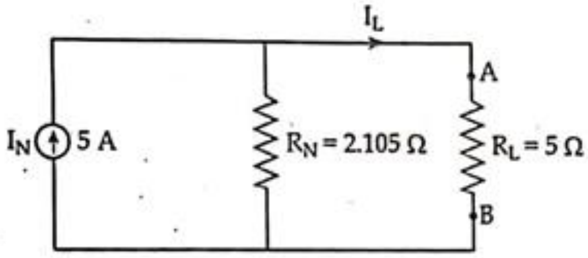
Also, to find I_N



$$\therefore \quad I = \frac{100}{(10 \parallel 10) + 5} = 10$$

$$\therefore \quad I_N = \frac{I}{10 + 10} \times 10 = 5 \text{ A}$$

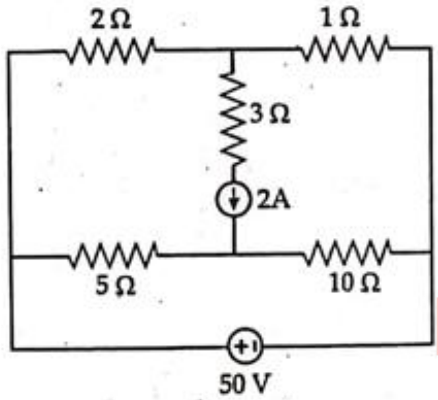
Norton's equivalent circuit



$$\therefore \quad I_1 = \frac{5}{2.105 + 5} \times 2.105$$

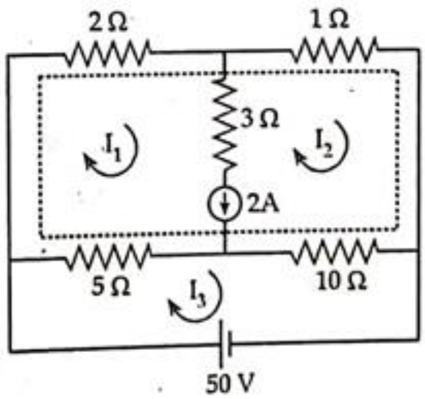
$$= 1.481 \text{ A}$$

6. Determine the current in 5 Ω resistor in the network shown below, using loop formulation method.



Solution:

Direction of flow of current is shown below in figure. Here, loop I and II form super mesh.



Applying KVL to super mesh.

$$-2 I_1 - I_2 - 10(I_2 - I_3) - 5(I_1 - I_3) = 0$$

or, $\quad -7 I_1 - 11 I_2 + 15 I_3 = 0 \qquad (1)$

Applying KVL to III

$$50 - 5(I_3 - I_1) - 10(I_3 - I_2) = 0$$

$\therefore \quad 5 I_1 + 10 I_2 - 15 I_3 = -50 \qquad (2)$

And super mesh condition

$I_1 - I_2 = 2 \qquad (3)$

Solving equation (1), (2) and (3); we get,

$$I_1 = 17.33 \text{ A}$$

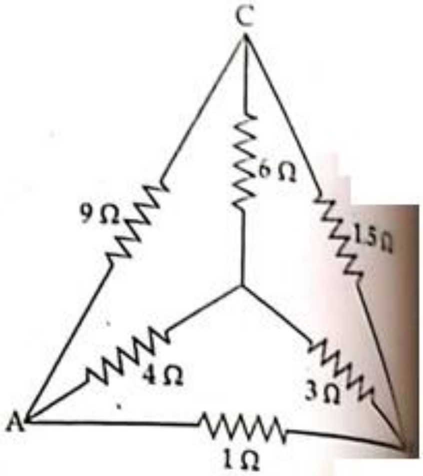
$$I_2 = 15.33 \text{ A}$$

$$I_3 = 19.33 \text{ A}$$

So, current through 5 Ω

$= I_3 - I_1 = 19.33 - 17.33 = 2$ A

7. In the network shown, using star delta transformation, calculate the network resistance between terminal A and B. [2066 Kartik]



Solution:

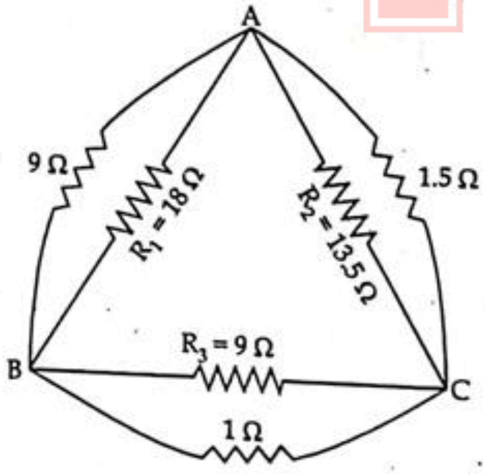
Convert star connection into delta equivalent, then resistance of each arm is given as

$$R_3 = \frac{4 \times 3 + 3 \times 6 + 6 \times 4}{6} = 9\ \Omega$$

$$R_2 = \frac{4 \times 3 + 3 \times 6 + 6 \times 4}{4} = 13.5\ \Omega$$

$$R_1 = \frac{4 \times 3 + 3 \times 6 + 6 \times 4}{3} = 18\ \Omega$$

Now, simplified figure is shown;



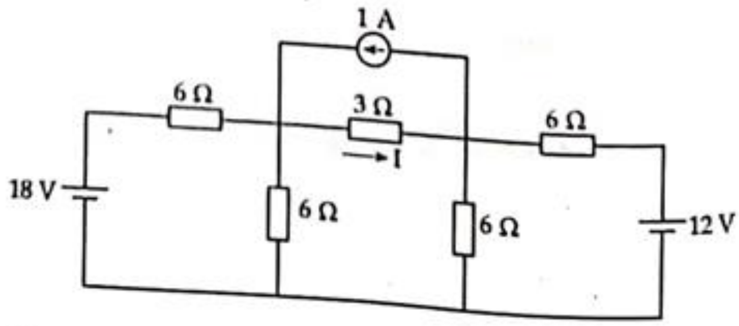
Now, from figure, we can calculate equivalent resistance using series parallel combination as;

$\therefore \quad R_{AB} = [9 \parallel 18 + (1.5 \parallel 13.5)] \parallel [9 \parallel 1]$

$= (7.35) \parallel (0.9) = 0.8\ \Omega$

Hence, equivalent resistance between AB is 0.8 Ω.

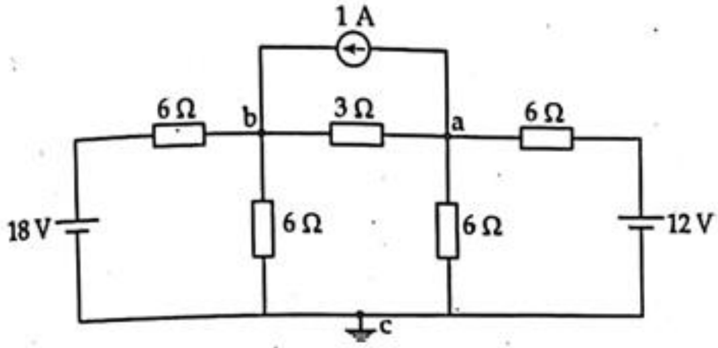
8. Find the current I in the circuit of figure given applying nodal analysis.



Solution:

Let us consider a and b as two node and c as reference node.

We apply KCL at node a, b equating the incoming current at node a with outgoing current



$$\frac{V_a - V_b}{3} + \frac{V_a - 12}{6} + \frac{V_a - 0}{6} + 1 = 0$$

or, $0.667\ V_a - 0.333\ V_b = -1$ (1)

Similarly, KCL at node b,

$$\frac{V_b - 0}{6} + \frac{V_b - 18}{6} + \frac{V_b - V_a}{3} = 1$$

or, $0.667\ V_b - 0.333\ V_a = 4$ (2)

Solving (1) and (2), we get;

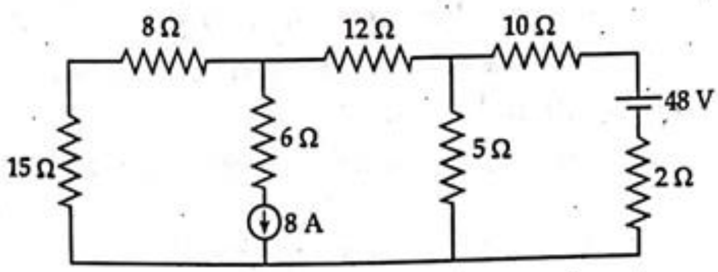
$V_a = 5.985$ V

and, $V_b = 8.985$ V

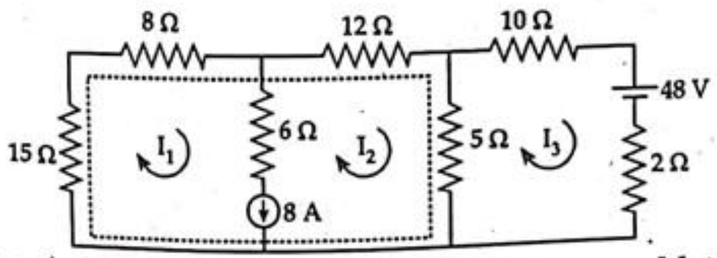
Therefore, current through 3 Ω resistor

$$= \frac{V_a - V_b}{3} = \frac{8.985 - 5.985}{3} = 1\ A$$

9. Calculate the current through 15 Ω resistor in figure given below.



Solution:



Direction of flow of current is shown in above figure and let I_1 be current through 15 Ω

Now, applying KVL to each loop

Loop I,

$$-15\,I_1 - 8\,I_1 - 5(I_2 - I_3) - 12\,I_2 = 0$$

or, $-23\,I_1 - 17\,I_2 + 5\,I_3 = 0$ (since loop I and II form super mesh) (1)

Due to super mesh formation,

$$I_1 - I_2 = 8 \quad (2)$$

Loop II

$$-5\,(I_3 - I_2) - 10\,I_3 - 48 - 2\,I_3 = 0$$

or, $5\,I_2 - 17\,I_3 = 48$ (3)

Solving I, II and III, we get;

$$I_1 = 2.858\text{ A}$$

$$I_2 = -5.142\text{ A}$$

$$I_3 = -4336\text{ A}$$

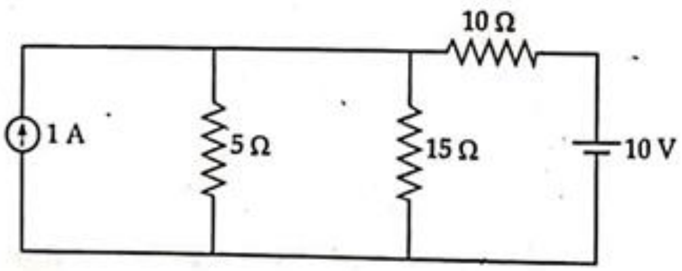
NOTE

Here, I_2 and I_3 are negative.

Negative sign shows current flow in opposite direction of assumption.

Therefore, current through 15 Ω = I_1 = 2.858 A

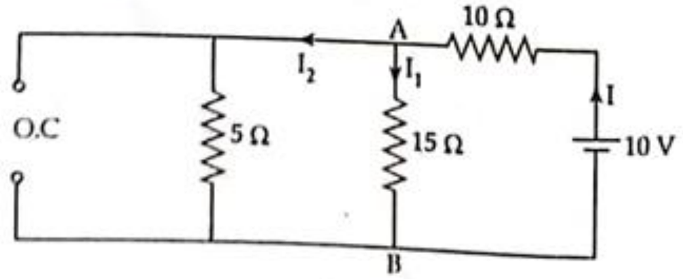
10. Calculate the current in 15 Ω resistor in the network shown in figure below using superposition theorem.



Solution:

There are two source in above circuit, current and voltage source. So firstly, we should calculate current through 15 Ω from individual one and then apply superposition theorem.

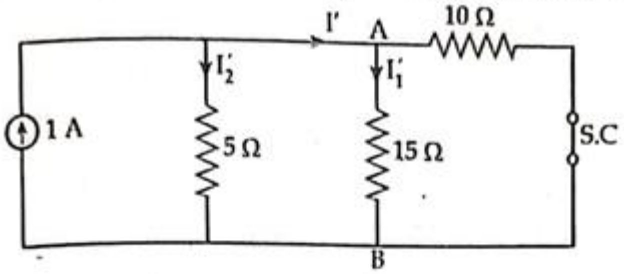
Taking 10 V source only and making current source open circuit as



$$\text{Total current I} = \frac{10}{10 + (5 \parallel 15)} = 0.7272\text{ A}$$

$$\text{and,} \quad I_2 = \frac{0.7272}{5+15} \times 5 = 0.1818\text{ A (A to B)}$$

Now, taking current source and making voltage source short circuit.

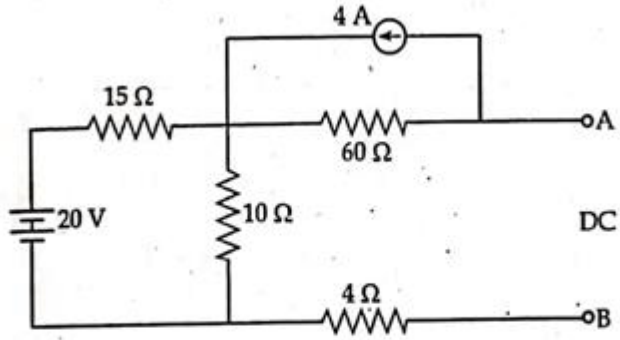


$$\text{Current } (I') = \frac{1}{5 + (10 \parallel 15)} \times 5 = 0.4545 \text{ A}$$

so, $$I_1' = \frac{0.4545}{15 + 10} \times 10 = 0.1818 \text{ A (A to B)}$$

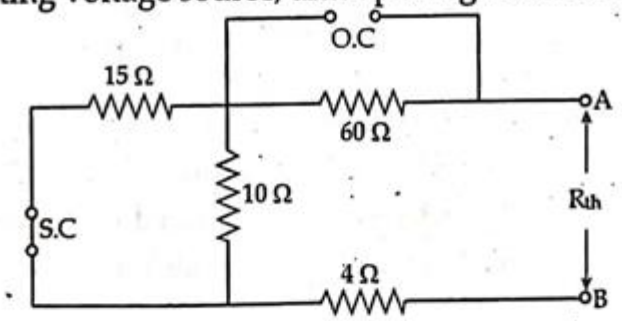
Therefore, net current through 15 Ω, $I_{15\Omega} = 0.1818 + 0.1818 = 0.3636$ A.

11. Find the Thevenin's equivalent circuit for terminal pair AB of the network shown in figure given below.



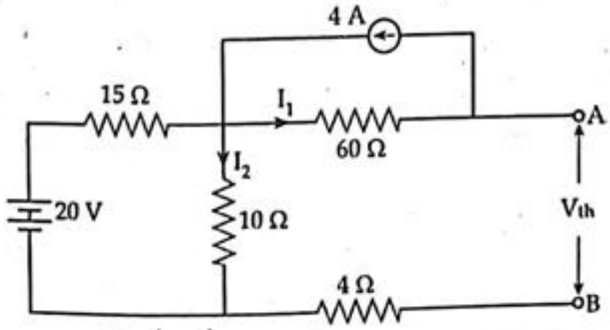
Solution:

To get R_{th}, shorting voltage source, and opening current source;



∴ $R_{th} = 6 + (15 \parallel 10) + 4 = 16\ \Omega$

Also, to find V_{th}



Using KVL, the current from battery is;

$$I = \frac{20}{15 + 10} = 0.8 \text{ A}$$

Now, writing KVL between terminal AB, we get

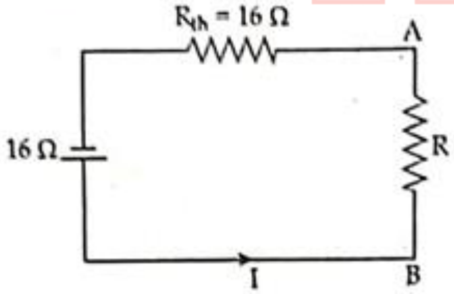
$$V_A - 6 \times (-4) - 10 \times 0.8 = V_B$$

or, $V_A + 24 - 8 = V_B$

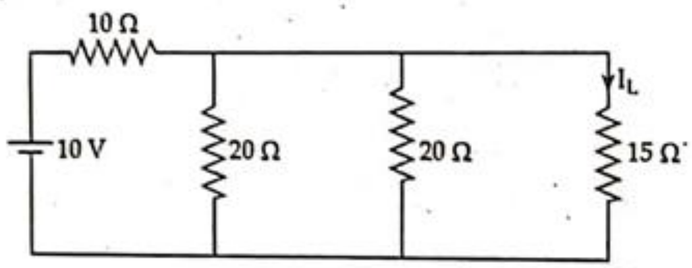
or, $V_A - V_B = -16$

$\therefore$ $V_{th} = -16$ V

Therefore Thenvenin's equivalent circuit

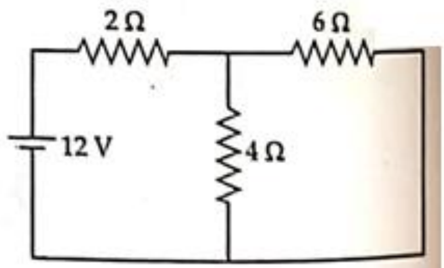


12. Determine the current I_L, through 15 Ω resistance in the network by Norton's theorem.



Solution: See Q. No. 5.

13. Verify the reciprocity theorem in the network given below. **[2067 Mangsir]**

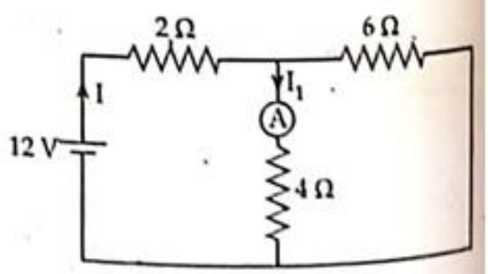


Solution:

To verify reciprocity theorem, firstly we calculate current through 4 Ω resistor. Again, we remove voltage source and is placed in the arm containing 4 Ω resistor and current in 2 Ω resistor is then calculated.

Now, supply current

$$I = \frac{12}{(6 \parallel 4) + 2} = 2.7272 \text{ A}$$

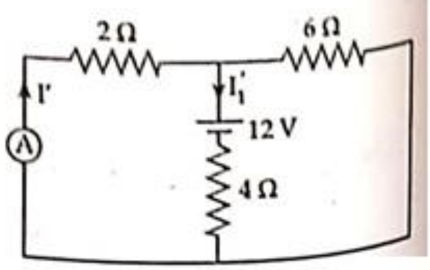


and, $I_1 = \frac{2.727}{4+6} \times 6 = 1.636$ A

Again, from figure (2)

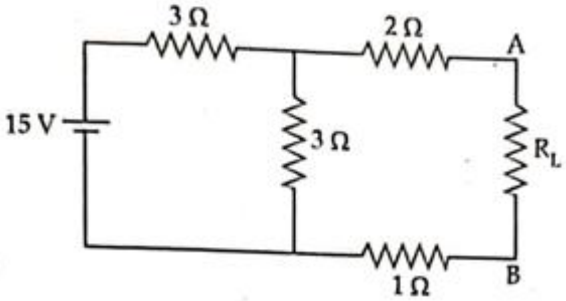
Supply current $I_1' = \frac{12}{(6 \parallel 2) + 4} = 2.1818$ A

and, $I' = \frac{2.1818}{2+6} \times 6 = 1.636$ A



Hence, in both condition ammeter current remain same which proves reciprocity theorem.

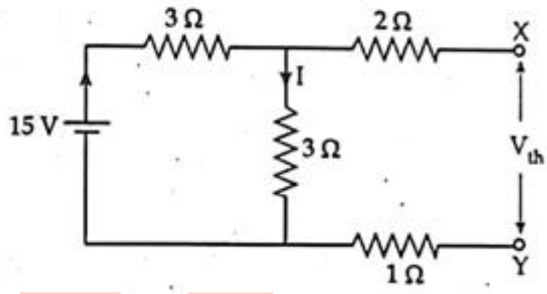
14. Find the value of R_L such that maximum power will be transferred of R_L. Find the value of the maximum power.



Solution:

For maximum power delivered in the network, the internal resistance must be equal to the load resistance. For this, we have to calculate equivalent resistance and terminal voltage between terminal of load resistance. Generally, Thevenin's theorem is used, maximum power is given as,

$$P_{max} = \frac{V_{th}^2}{4\,R_{th}}$$



Now, to get V_{th}

The current $(I) = \frac{15}{3+3} = 2.5$ A

Now, writing KVL between terminal X and Y,

$$V_X - 3 \times 2.5 = V_Y$$

or, $V_X - V_Y = 7.5$ V

∴ $V_{th} = 7.5$ V

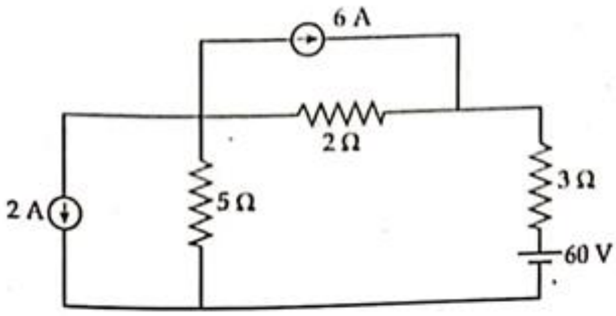
Again, to get R_{th}, using series parallel combination, also shorting voltage source, we get;

$$R_{th}\,(3 \parallel 3) + 2 + 1 = 4.5\,\Omega$$

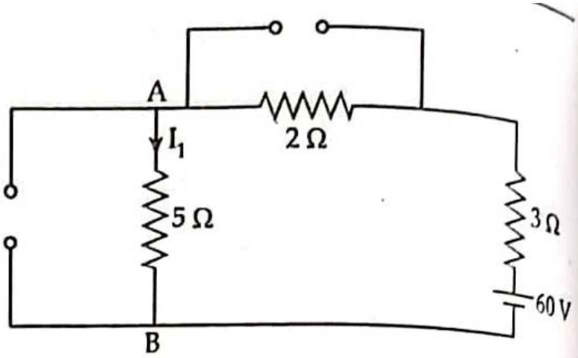
Therefore, for maximum power transfer, $R_{th} = R_L = 4.5\,\Omega$.

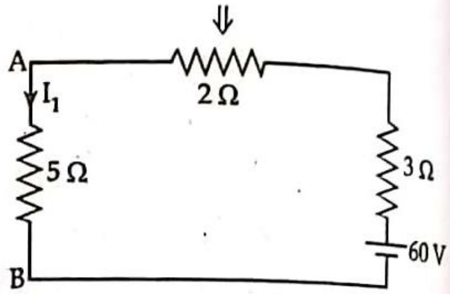
And, maximum power $= \frac{(7.5)^2}{4 \times 4.5} = 3.125$ Watt.

15. Apply superposition theorem to the circuit shown below to find the voltage drop V across 5 Ω resistor.



Solution:





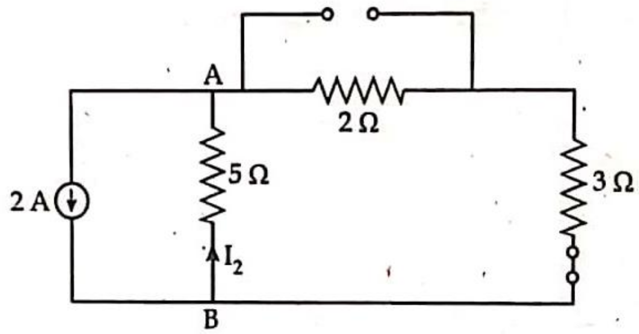
Current due to individual source is calculated and finally superposed

Considering 60 V only. Then current flowing through 5 Ω resistor is

$$I_1 = \frac{\Sigma V}{\Sigma R}$$

$$I_1 = \frac{60}{5+3+2} = 6\ A\downarrow$$

Considering 2 A only



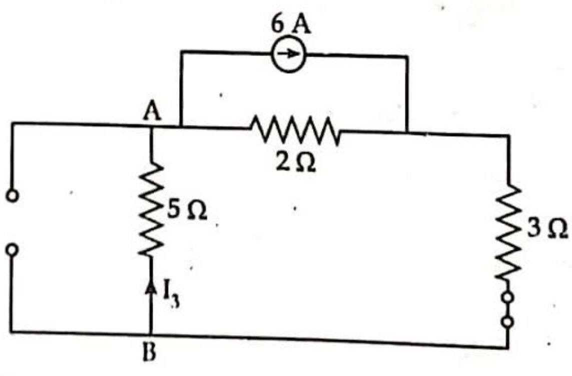
Shorting voltage and opening current, then

$$R_{eq} = (2+3) \parallel 5$$

Using CDR,

$$\therefore \quad I_2 = \frac{2}{5+2+3} \times (2+3) = 1\ A\uparrow$$

Considering 6 A only



$$\therefore \quad I_3 = \frac{6}{2+(3+5)} \times 2 = 1.2\ A\ (B\ to\ A)$$

Using superposition theorem

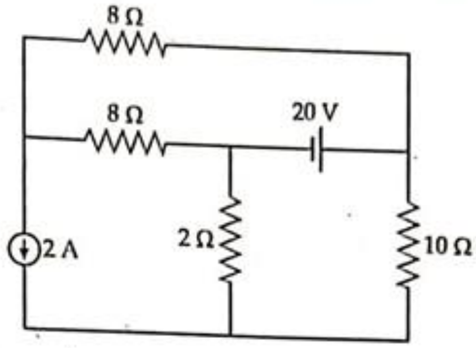
Current through 5 Ω, after combining all 3 cases, we get

$$I = I_1 - I_2 - I_3$$

$$= 6 - 1 - 1.2 = 3.8\ A\ (A\ to\ B)$$

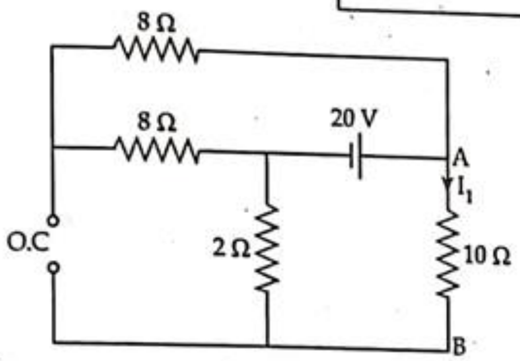
$\therefore$ Voltage drop across 5 Ω, $V = 3.8 \times 5 = 19\ V$

16. Use superposition theorem to find the current flowing through 10 Ω resistor as shown in figure.



Solution:

Here, considering one source at a time, then

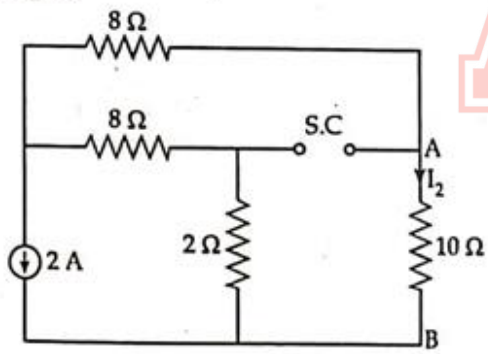


Considering 20 V supply only, then current through 10 Ω is

$$I_1 = \frac{\Sigma V}{\Sigma R} = \frac{20}{2+10} = 1.667 \text{ A} \downarrow$$

Considering 2 A source only

Then, using CDR, we get,



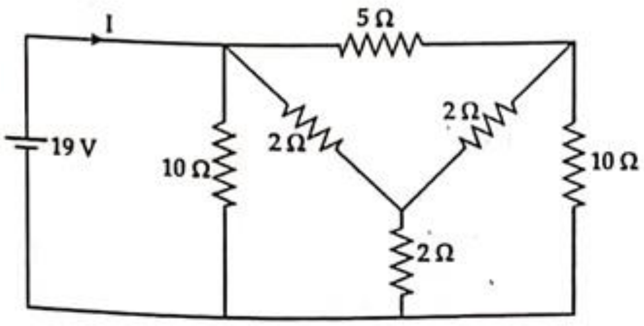
$$I_2 = \frac{2}{2+10} \times 2 = \frac{1}{3} \text{ A} = 0.333 \text{ A} \uparrow$$

Therefore, using superposition theorem;

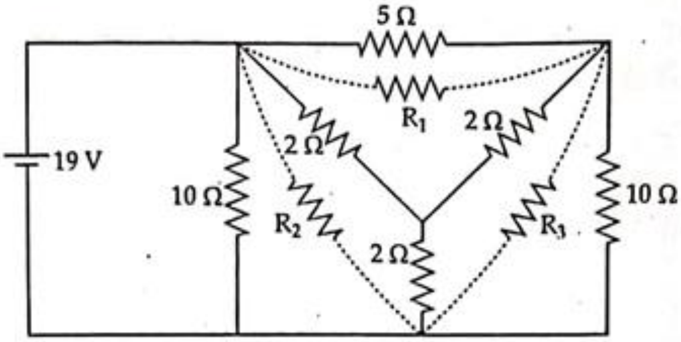
Current through 10 Ω resistor = $I_1 - I_2 = 1.667 - 0.333 = 1.333$ A.

Hence, 1.333 A following from A to B in clockwise direction.

17. Find the current I as shown in figure below using star-delta transformation.



Solution:



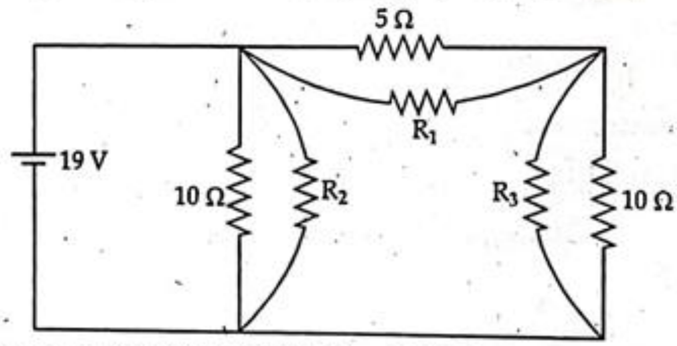
Converting 2Ω, 2Ω, 2Ω star connection to delta

$$R_1 = \frac{2\times2+2\times2+2\times2}{2} = 6\ \Omega$$

$$R_3 = \frac{2\times2+2\times2+2\times2}{2} = 6\ \Omega$$

$$R_3 = \frac{2\times2+2\times2+2\times2}{2} = 6\ \Omega$$

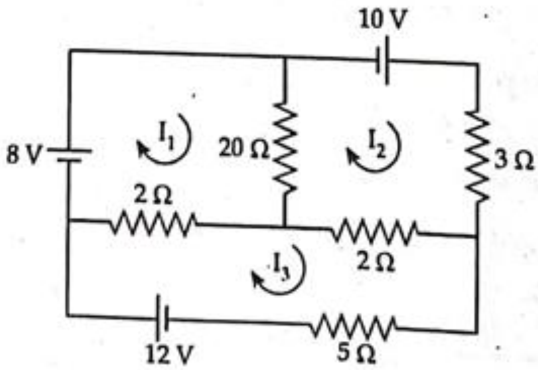
Then, simplifying the figure below, by applying R_1, R_2, R_3 values;



$\therefore$ $R = [(10 \parallel 6) + (5 \parallel 6)] \parallel (10 \parallel 6) = 2.37\ \Omega$

Therefore, current supply through battery $= I = \frac{19}{2.37} = 8.01\ A$

18. Determine current in 5 Ω resistor by mesh analysis in figure below.



Solution:

From figure; there are three loops, the direction of current in each loop is shown.
Now, applying KVL on loop 1, we get,

$$-20(I_1 - I_2) - 2(I_1 - I_3) + 8 = 0$$

$\therefore$ $-22\, I_1 - 20\, I_2 + 2\, I_3 = -8$ (1)

Applying KVL on loop 2, we get,

$$10 - 3I_2 - 2(I_2 - I_3) - 20(I_2 - I_1) = 0$$

$$\therefore \quad 20I_1 - 25I_2 + 2I_3 = -10 \qquad (2)$$

Applying KVL on loop 3, we get,

$$-2(I_3 - I_1) - 2(I_3 - I_2) - 5I_3 + 12 = 0$$

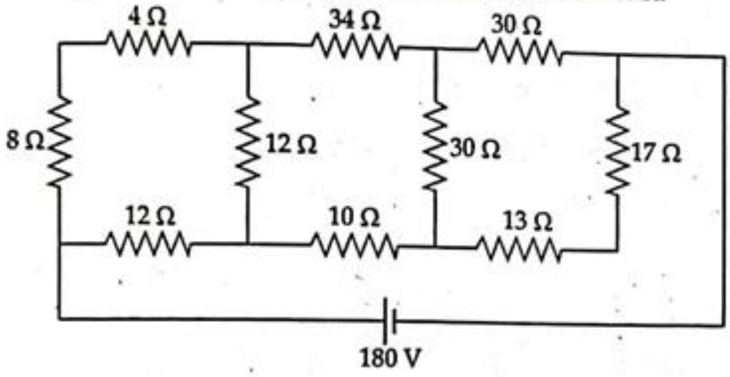
$$\therefore \quad 2I_1 + 2I_2 - 9I_3 = -12 \qquad (3)$$

Solving equation (1), (2) and (3); we get,

$\therefore \quad I_1 = 4.68$ A $\quad \therefore \quad I_2 = 4.41$ A $\quad \therefore \quad I_3 = 3.35$ A

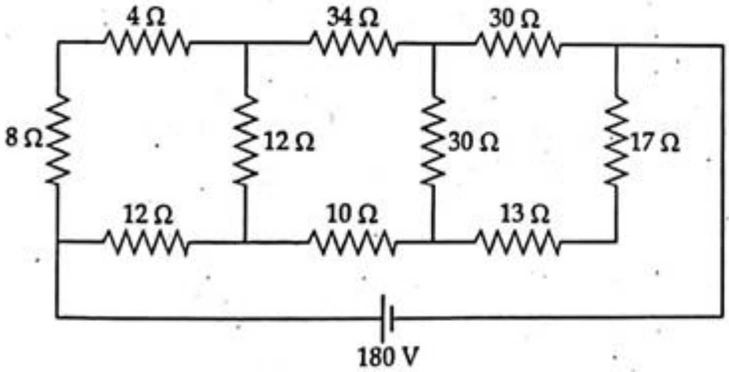
Hence current through 5 Ω resistor = 3.35 A

19. Determine the value of current in 10 Ω resistance in the network shown in figure below using star delta conversion.



Solution:

Using star delta transformation



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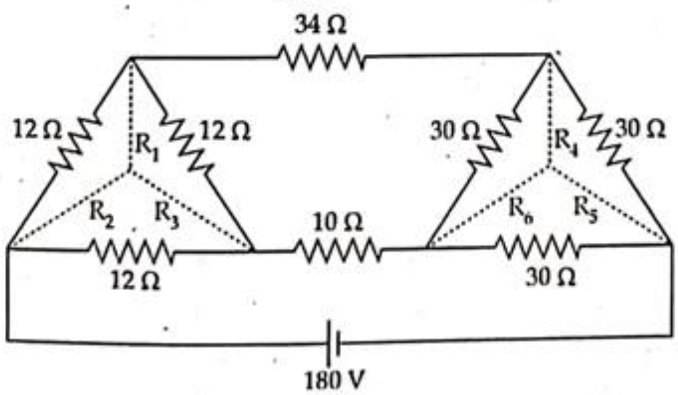
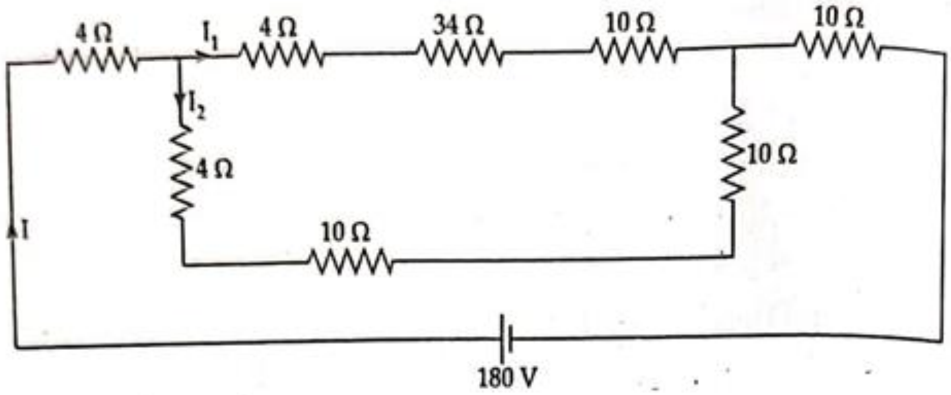


$$R_1 = R_2 = R_3 = \frac{12 \times 12}{12 + 12 + 12} = 4\ \Omega$$

Also, $R_4 = R_5 = R_6 = \dfrac{30 \times 30}{30 + 30 + 30} = 10\ \Omega$



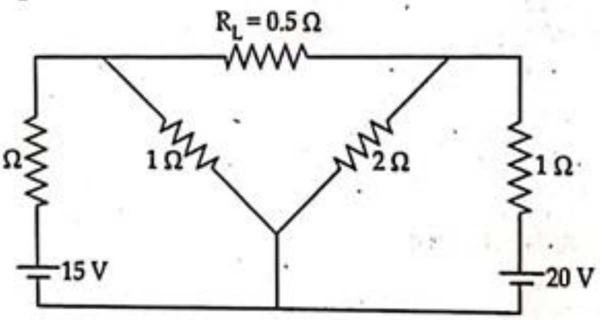
Total current

$$I = \frac{180}{4 + [(4+34+10) \parallel (4+10+10)] + 10} = 6\text{ A}$$

Then, current through 10 Ω

$$I_2 = \frac{6}{(10+4+10) + (4+34+10)} \times (4+34+10) = 4\text{ A}$$

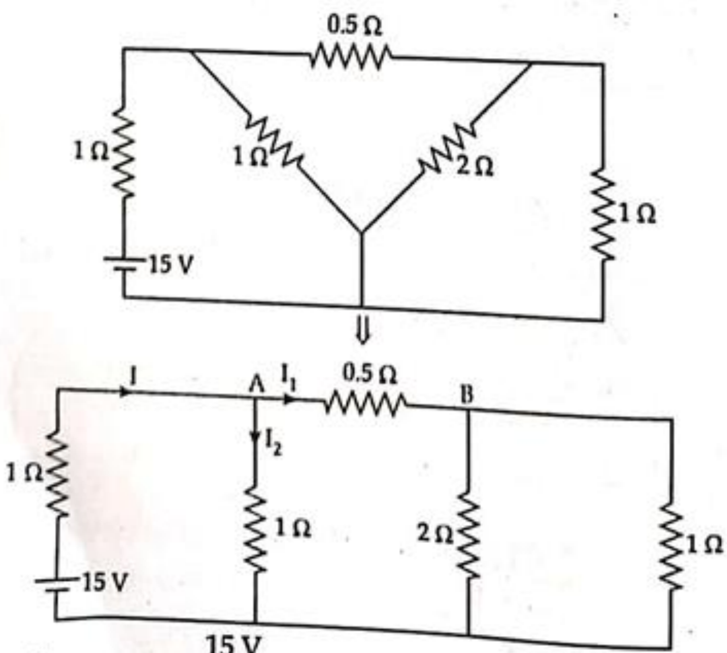
20. Find the current in 0.5 Ω resistor in the following network shown, by using superposition.



Solution:

Since there are two voltage source. So, we should calculate current through 0.5 Ω by each one and applying superposition theorem.

Considering 15 V source



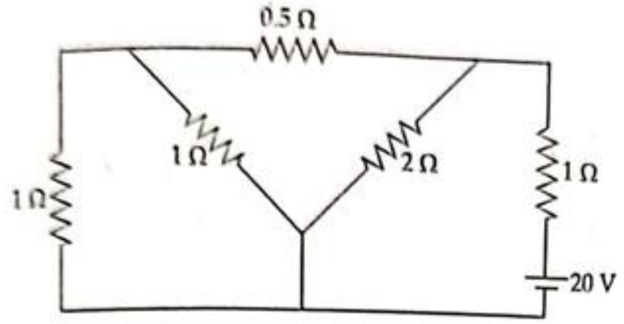
$$\text{Total current } (I) = \frac{15\text{ V}}{1 + [1 \parallel \{0.5 + (2 \parallel 1)\}]} = 9.75\text{ A}$$

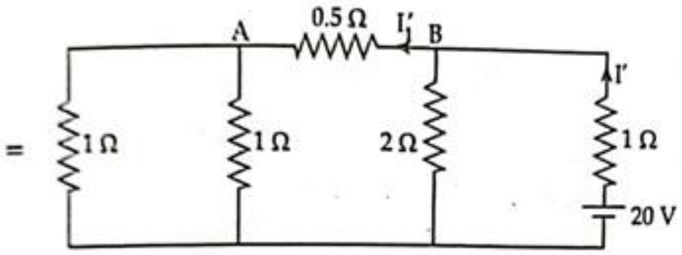
Now, current through 0.5 Ω resistor, using CDR,

We get,

$$I_1 = \frac{9.75}{1 + [0.5 + (2 \parallel 1)]} \times 1 = 4.5 \text{ A (A to B)}$$

Considering 20 V source





Current through supply (I') = $\frac{20}{[\{(1 \parallel 1) + 0.5\} \parallel 2] + 1} = 12$ A

Current through 0.5 Ω resistor

$$I_1' = \frac{12}{[(1 \parallel 1) + 0.5] + 2} \times 2$$

∴ $I_1' = 8$ A (B to A)

Hence net current through 0.5 Ω resistor

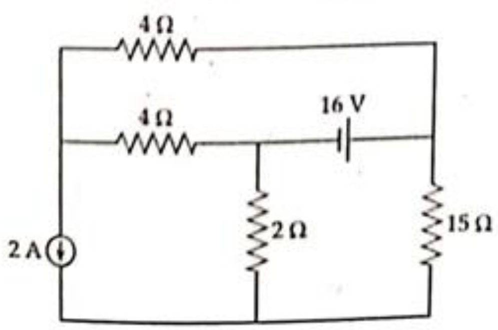
$= I_1 - I_1'$

$= 4.5 - 8$

$= -3.5$ (A to B)

$= 3.5$ A (B to A)

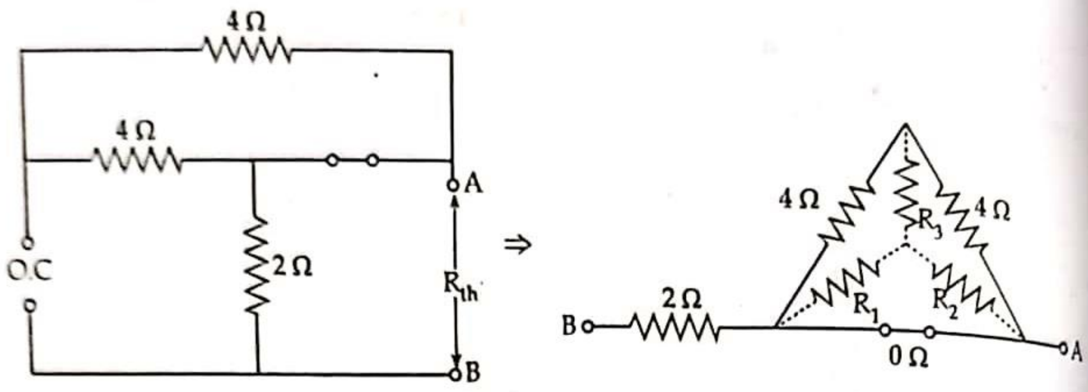
21. Using Thevenin's theorem to find the current following through 15 Ω resistor of the network of given figure.



Solution:

First of all, we have to calculate V_{th} and R_{th}.

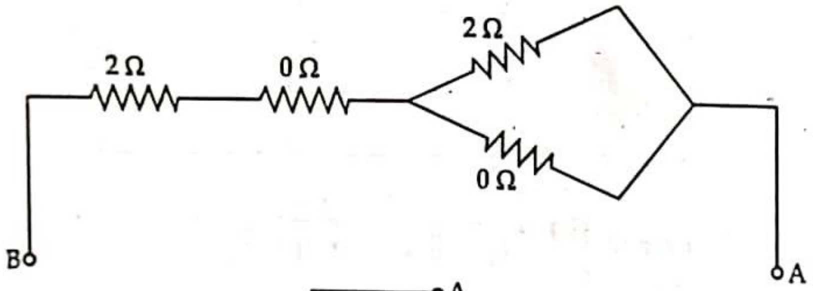
Calculating R_{th}

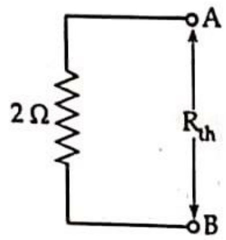


$$R_1 = \frac{4 \times 0}{4 + 0 + 4} = 0$$

Similarly,

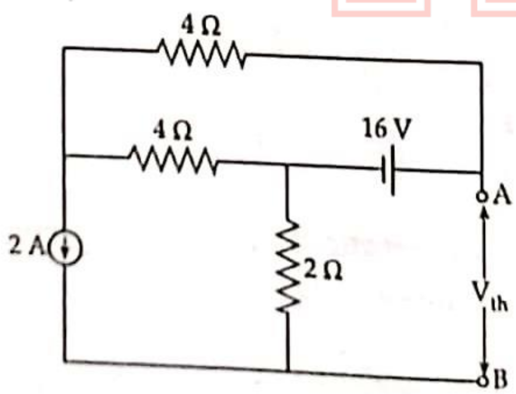
$R_3 = 2,$ $\quad R_2 = 0$





$\therefore$ $\quad R_{th} = 2\,\Omega$

Calculating V_{th}

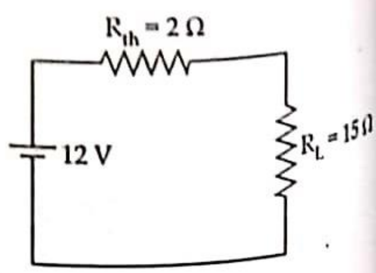


Applying KVL,

$$V_A - 16 + 2 I_1 = V_B$$

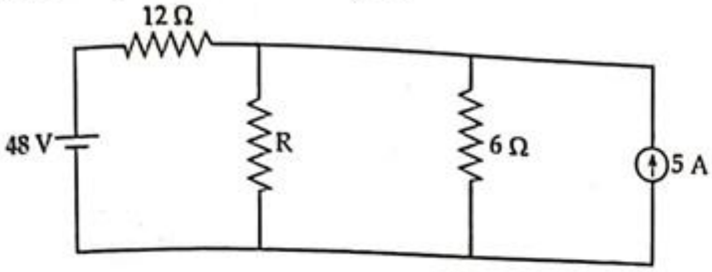
$$V_A - V_B = -2 \times 2 + 16$$

$\therefore$ $\quad V_{th} = 12\text{ V}$



$$\therefore \quad I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{12}{2 + 15} = 0.71\text{ A}$$

22. Use Norton's theorem to calculate the value of R that will absorb maximum power from the circuit shown in figure. Also calculate the maximum power drawn by it.

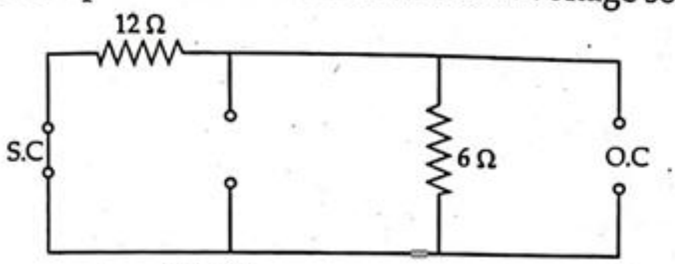


Solution:

To absorb maximum power through R, it should be equal to Norton's equivalent resistance.

Calculating R_N

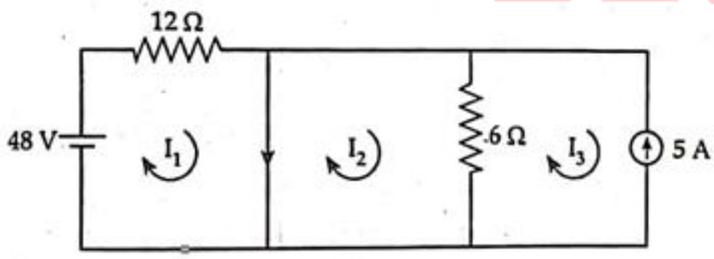
For this we have to open current source and short voltage source



$$\therefore \quad R_N = (12\,\Omega \parallel 6\,\Omega) = \frac{12 \times 6}{12 + 6} = 4\,\Omega$$

Hence, value of $R = R_N = 4\,\Omega$

Calculating I_N;



In loop I using KVL,

$$48 - 12\,I_1 = 0$$

$$\therefore \quad I_1 = 4\text{ A}$$

In loop II, using KVL

$$(I_2 - I_3) \times 6 = 0$$

$$\therefore \quad I_2 = I_3$$

In loop III, using KVL,

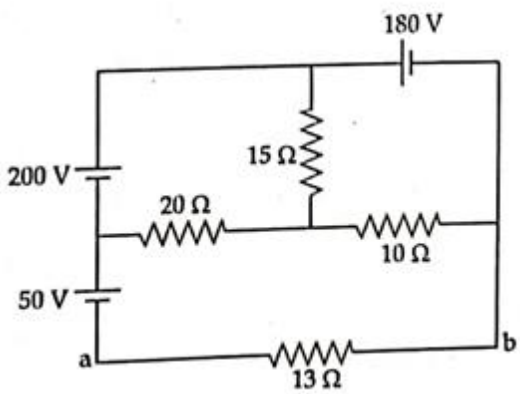
$$I_3 = -5\text{ A}$$

$$\therefore \quad I_N = I_1 - I_2 = 4\text{ A} - (-5\text{ A}) = 9\text{ A}$$

Hence, maximum power is given by,

$$P_{max} = \frac{I_N^2 R_N}{4} = \frac{(9)^2 \times 4}{4} = 81\text{ watt}$$

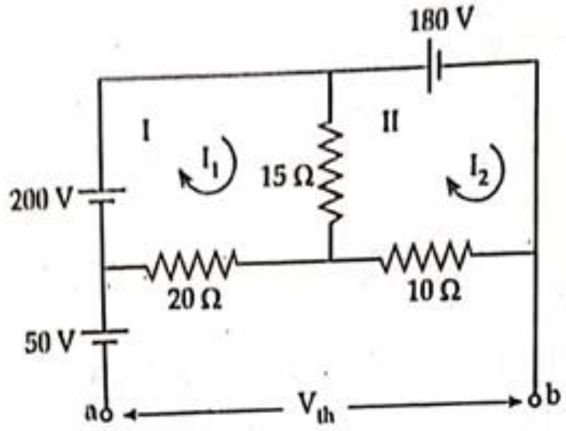
23. Applying Thevenin theorem, calculate the magnitude and direction of current in the 13 Ω resistor in the circuit in the given figure.



Solution:

For, Thevenin's theorem, we have to calculate V_{th} and R_{th}.

Now; Calculating V_{th}



Applying KVL to mesh I, we get

$$200 - 15(I_1 - I_2) - 20 I_1 = 0$$

$$\therefore \quad -35 I_1 + 15 I_2 = -200 \quad (1)$$

Applying KVL to mehs 2,

$$-180 - 10 I_2 - 15(I_2 - I_1) = 0$$

$$15 I_1 - 25 I_2 = 180 \quad (2)$$

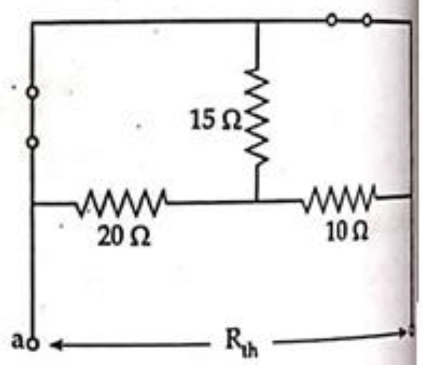
Solving equation (1) and (2); we get,

$$I_1 = 3.54 \text{ A}, \qquad I_2 = -5.07 \text{ A}$$

Now, $V_a + 50 + 20 \times 3.54 - 10 \times 5.07 = V_b$

or, $V_a + 50 + 70.8 - 50.7 = V_b$

or, $V_a - V_b = 70.1$ V

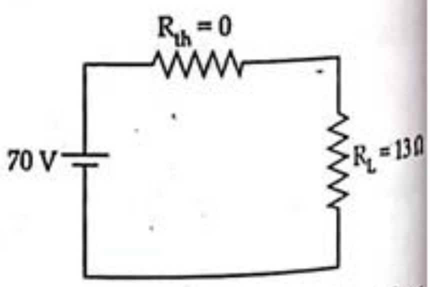


Calculating R_{th}

$$R_{th} = 0$$

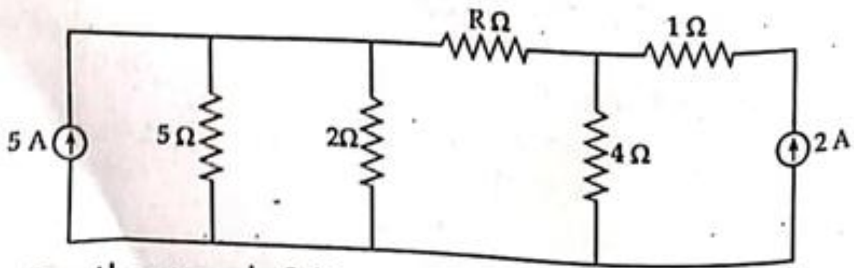
To calculate R_{th}, we need to short 200 V and 180 V source. Due to which 0 Ω resistance becomes parallel with other using star delta, we get $R_{th} = 0$.

Thevenin's equivalent circuit,



$$\therefore \quad I_1 = \frac{V_{th}}{R_{th} + R_L} = \frac{70.1}{0 + 13} = 5.392 \text{ A}$$

24. Find the value of R such that maximum power transfer take place from the current sources to the load R in figure below. Obtain the amount of power supplied.



Solution: See the example 3.15.

CHAPTER 5

ALTERNATING QUANTITIES

5.1 AC SYSTEM

Alternating current may be generated by rotating a coil in a magnetic field or by rotating a magnet within a stationary coil. Alternating current flows in one direction of flow. And the magnitude changes at every time. The magnitude depends upon the position of the coil.

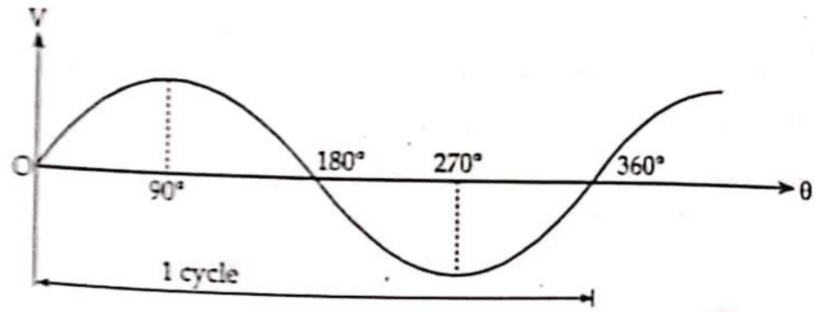


Figure: AC current, showing change in direction and magnitude.

Advantages of AC:

i) Easy to conduct AC from one place to another place.
ii) In AC current, easy to develop high voltage.
iii) Equipment costs is low.
iv) Can be converted into DC.
v) Easy to step up and step down the voltage by transformer.
vi) AC motors are cheapest.

Disadvantages of AC:

i) Cannot able to store in battery

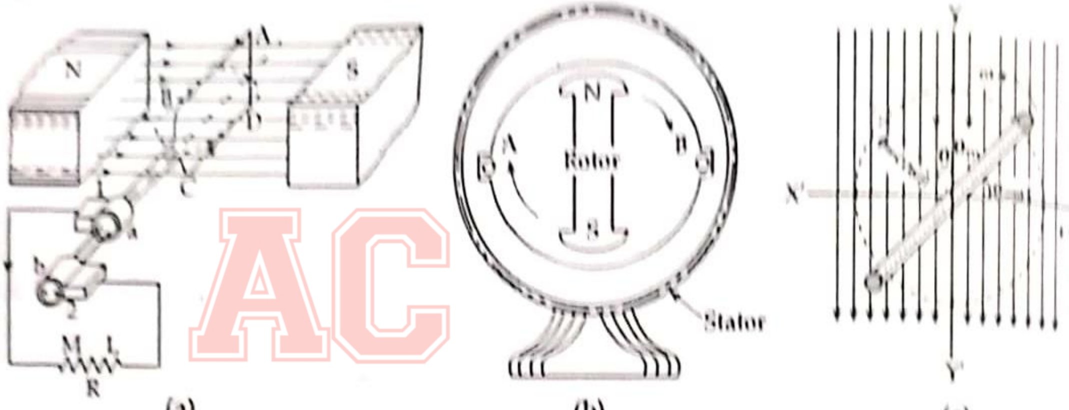
ii) Compared to DC high electric shock, AC circuit should have good insulation and earthing

iii) Voltage drop occur due to high starting current

iv) According to induction load, power factor may get low

5.2 WAVEFORM, TERMS AND DEFINITIONS

(i) Wavefom



(a) (b) (c)

In figure (a) A coil fixed as to rotate in magnetic field. In figure (b) A magnetic field fixed as to rotate inside the coil.

If coil rotate in magnetic field or magnet rotate inside the coil, there is an alternating e.m.f. generated in the coil. The generated e.m.f. is proportional to the number of turns of coil, magnetic field strength and the angle between the coil and magnetic field.

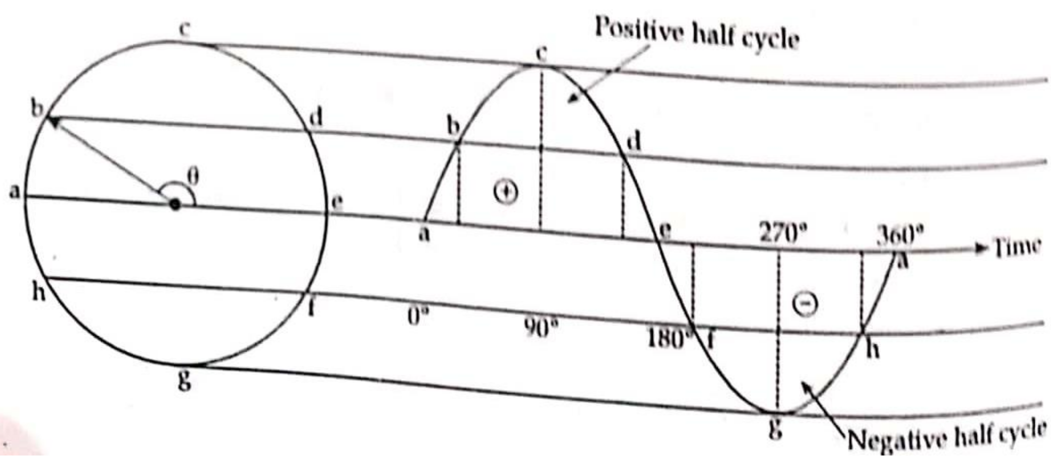
According to Faraday's law of electromagnetic induction, the induced e.m.f. in the coil is

$$E = -N\frac{d}{dt}(\phi_m \cos \omega t) = N\phi_m \omega \sin \omega t$$

$$\therefore \quad \boxed{E = E_m \sin \omega t}$$

where, N = number of turns and $E_m = N\phi_m \omega$ is the maximum e.m.f. induced in the coil.

The generated AC e.m.f. values depends upon the sine value of the angle between the magnetic field and conductor.



Let, the coil rotates in clockwise direction. First at point 'a' the angle between the conductor and magnetic field is 0°. At that time e.m.f. is zero {sin ωt = 0}. The conductor is then moves from position b, c, d and e. At position c, e.m.f. is maximum {sin 90° = 1} and at point e, e.m.f. is again zero {sin 0° = 0}. Then another $\frac{1}{2}$cycle is start, which is opposite in direction of first half cycle.

(ii) Cycle

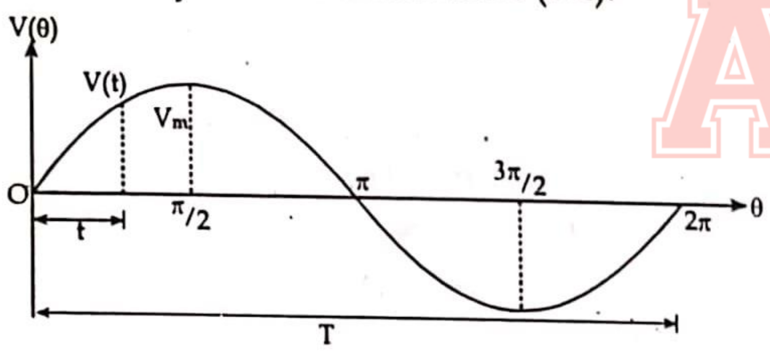
In AC, complete set of one positive half cycle and one negative half cycle is called one cycle.

(iii) Time period

The time taken by an alternating quantity to complete one cycle is called its time period. It is denoted by 'T'. *For example*; in the AC frequency, 50 Hz, the time period for one cycle is $\frac{1}{50}$ seconds.

(iv) Frequency

The number of cycle per second is called the frequency of the alternating quantity. It is denoted by f and its unit is hertz (Hz).



Also, frequency is reciprocal of time period and given as;

$$\text{Frequency (f)} = \frac{1}{\text{Time period (T)}}$$

(v) Instantaneous value

The alternating quantity changes at every time. At any particular time, its value is called instantaneous value at time t, it is given as;

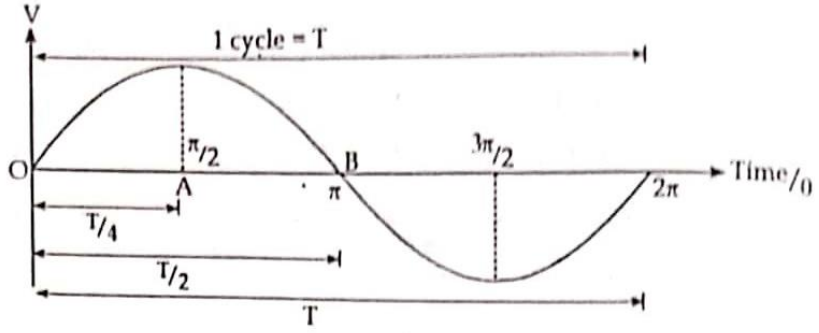
$$V(t) = V_m \sin \omega t \qquad (1)$$

(vi) Peak value

The maximum value (positive or negative) of an AC quantities is known as its peak value. It is also known as amplitude of the wave form.

(vii) Phase

Phase of an alternating current means the fraction of time period of that alternating current, which has elapsed since the current last passed through the zero position of reference. *For example*; the phase of current at point A is $\frac{T}{4}$ seconds (where T = Time period) and at point B, it is $\frac{T}{4}$ in seconds and π in radians.



(Viii) Phase difference

When two conductors are rotated in the magnetic field with same speed but at different angles and their sine wave is drawn, then the AC quantities never gets maximum or zero at same time. This different AC quantities between these two conductors are called phase difference and denoted by θ.

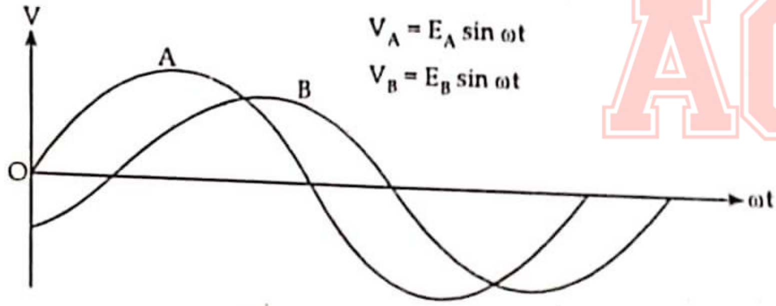


Figure: Lagging phase

In the figure, A sine wave gets the maximum value first than that sine wave B. So, A sine wave is leading than the sine wave B or sine wave B is lagging the sine wave A.

If wave form of two AC quantities get maximum and zero at same time, then they are said to be 'in phase'.

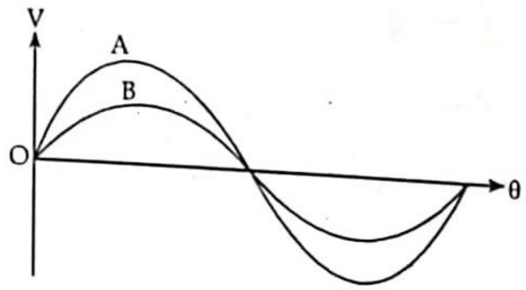


Figure: In phase

If waveform of two AC quantities get maximum and zero at two different time then they are said to be out of phase.

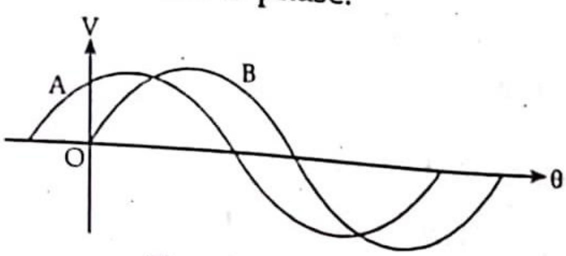


Figure: Out of phase

NOTE

$$\text{Phase} = \frac{\text{Time elapsed from zero refernce position}}{\text{Time period}} = \frac{t}{T}$$

$\therefore$ Phase angle (θ) = $2\pi \times$ phase

$$\theta = \frac{t}{T} \times 2\pi$$

Example 5.1

Find the amplitude, phase, time period and frequency of the sinusoidal waveform $V(t) = 15 \cos (40 t + 20°)$.

Solution:

Given that,

$$V(t) = 15 \cos (40 t + 20°) \qquad (1)$$

Comparing it with $V(t) = V_m \cos (\omega t + \theta)$, we get

Amplitude (V_m) = 15 V

Phase angle (θ) = 20°

Angular frequency (ω) = 40 rad/sec

Time period (T) = $\frac{2\pi}{\omega} = \frac{2\pi}{40}$ = 0.157 sec

Frequency (f) = $\frac{1}{T} = \frac{40}{2\pi}$ = 6.366 Hz

Example 5.2

Calculate the phase angle between $V_1 = -10 \cos (\omega t + 50)$ and $V_2 = 12 \sin (\omega t - 10)$. State which wave is leading?

Solution:

To compare two waveform, first, we have to express both function in same form *i.e.*, either sine form or cosine form.

Then, $V_1 = -10 \cos (\omega t + 50)$ (1)

$\therefore$ $\theta_1 = 50°$

$V_2 = 12 \sin (\omega t - 10°)$

or, $V_2 = -12 \cos [90° + (\omega t - 10°)]$

or, $V_2 = -12 \cos (\omega t + 80°)$ (2)

$\therefore$ $\theta_2 = 80°$

Now, phase difference = $\theta_2 - \theta_1 = 80 - 50 = 30°$

and, V_2 leads V_1 by 30°.

Example 5.3

The current in an AC circuit at any time 't' second is given as $i = 120 \sin (100 \pi t + 0.36)$ A. Find;

i) (a) the peak value (b) the periodic time (c) frequency and phase

ii) The value of current when t = 0 sec and t = 8 milli sec

iii) The time when the current is first a maximum

Solution:

Given that; $i = 120 \sin(100\pi t + 0.36)$ A

i) (a) Peak value = 120 Amp

(b) For time, we get, $\omega = 100\pi$ and $\theta = 0.36$ rad.

Then, $\omega = 2\pi f = 100\pi$

or, $f = \dfrac{100\pi}{2\pi}$

or, $\dfrac{1}{T} = \dfrac{100\pi}{2\pi}$

or, $T = \dfrac{1}{50}$ sec

(c) Frequency (f) $= \dfrac{1}{T} = 50$ Hz

and, phase angle $(\theta) = 0.36$ rad $= 0.36 \times \dfrac{180°}{\pi} = (20.63)°$

ii) When, time (t = 0) sec

$i = 120 \sin(100\pi \times 0 + 0.36) = 42.3$ Amp

When t = 8 milli sec = 8×10^{-3} sec

$i = 120 \sin(100\pi \times 8 \times 10^{-3} + 0.36) = 31.8$ Amp

iii) The maximum value of current at V = 120 V;

$120 = 120 \sin(100\pi t + 0.36)$

or, $t = 3.85 \times 10^{-3} = 3.85$ milli sec

5.3 AVERAGE AND RMS VALUE OF VOLTAGE AND CURRENT

Average value of AC

The average value of an AC quantity is that steady value which transfers the same charge across the circuit, as it is transferred by that alternating quantity during the same time.

The arithmatic average of all the value of an alternating quantity over one cycle is called its average value.

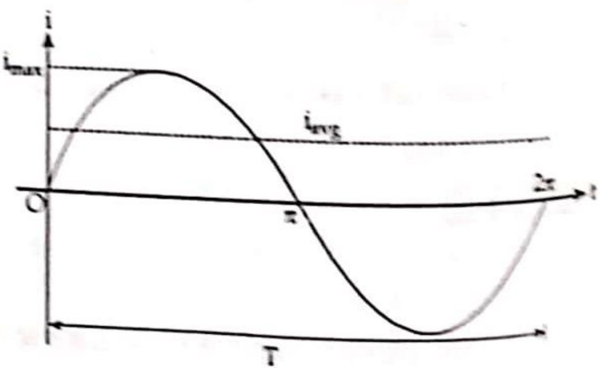
$$\text{Average value} = \frac{\text{Area under one cycle}}{\text{Base}}$$

$$\therefore \quad V_{avg} = \frac{1}{T}\int_0^T V\,dt$$

at $T = 2\pi$

$$\therefore \quad \boxed{V_{avg} = \frac{1}{2\pi}\int_0^{2\pi} V(dt)}$$

$$\text{and,} \quad \boxed{i_{avg} = \frac{1}{2\pi}\int_0^{2\pi} i.dt}$$



NOTE

For symmetrical wave form, the average value calculated over one cycle becomes equal to zero because the positive area cancels the negative area. Hence for symmetrical waveforms, the average is calculated for half cycle.

i.e., $$V_{avg} = \frac{1}{\pi}\int_0^{\pi} V\,dt$$

RMS or Effective/Virtual value of AC

The effective or RMS value of an alternating quantity is that steady current (dc) which when flowing through a given resistance for a given time period, produces the same amount of heat, that produced by the alternating current flowing through the same resistance for the same time.

$\therefore$ $$\text{RMS value} = \sqrt{\frac{1}{T}\int_0^{T} V^2\,dt} \quad \text{for voltage}$$

and, RMS value for current

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^{T} I^2\,dt}$$

NOTE

RMS value of a waveform is greater than the average value of the particular waveform.

Form factor

The ratio of RMS value to the average value of an alternating quantity is called form factor.

Mathematically,

$$FF = \frac{\text{RMS value}}{\text{Average value}}$$

Peak factor or crest factor

It is the ratio of maximum value to the RMS value of an alternating quantity.

Mathematically,

$$PF = \frac{\text{Maximum value}}{\text{RMS value}}$$

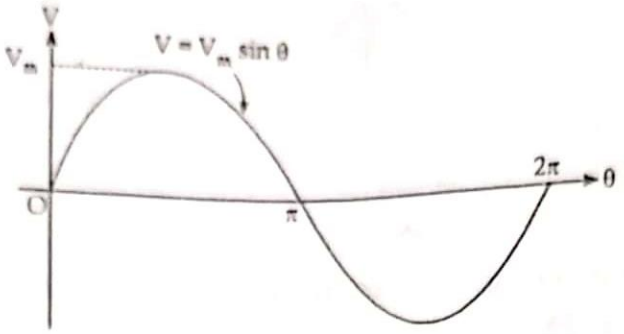
Example 5.4

Find the average value, RMS value, peak factor and form factor of the sine wave.

Solution:

As we know, sine wave is symmetrical, its average value over a complete ... zero. So, we have to take only half cycle to find the average value.

So, for half cycle,



Time period (T) = π

Equation

$$V(\theta) = V_m \sin\theta$$
$$\text{for } (0 < \theta < \pi)$$

Then, to get V_{avg}, we know,

$$V_{avg} = \frac{1}{T}\int_0^T V(\theta)\, d\theta = \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta\, d\theta = \frac{V_m}{\pi}[-\cos\theta]_0^{\pi} = \frac{V_m}{\pi}[-(-1-1)]$$

$$\therefore \quad \boxed{V_{avg} = \frac{2V_m}{\pi}}$$

Again, $$V_{rms}^2 = \frac{1}{T}\int_0^T V^2(\theta)\, d\theta = \frac{1}{\pi}\int_0^{\pi} (V_m \sin\theta)^2\, d\theta = \frac{V_m^2}{\pi}\int_0^{\pi}\left[\frac{1-\cos 2\theta}{2}\right] d\theta$$

$$= \frac{V_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi} = \frac{V_m^2}{2\pi}\left[\left(\pi - \frac{0}{2}\right) - 0\right] = \frac{V_m^2}{2}$$

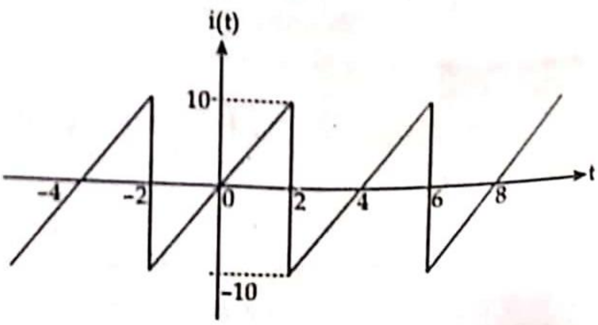
$$\therefore \quad \boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$$

Again, Form factor (FF) $= \dfrac{\text{RMS value}}{\text{Average value}} = \dfrac{\frac{V_m}{\sqrt{2}}}{\frac{2V_m}{\pi}} = \dfrac{\pi}{2\sqrt{2}} = 1.11$

and, Peak factor (PF) $= \dfrac{\text{Maximum value}}{\text{RMS value}} = \dfrac{V_m}{\frac{V_m}{\sqrt{2}}} = \sqrt{2} = 1.414$

Example 5.5

Determine the RMS value and average value for the given waveform. If the current is passed through 2 Ω resistor, find the average power absorbed by the resistor.



Solution:

The given waveform is symmetrical. So only half cycle is taken for average value.

We know,

$$I_{avg} = \frac{\text{Area under load}}{\text{Total base lenght}} = \frac{\frac{1}{2} \times 2 \times 10}{2} = 5\text{ A}$$

Now, writing the equation

$$I(t) = \frac{10 - (-10)}{2 - (-2)}\, t = \frac{20\, t}{4} = 5t \qquad [y = mx]$$

Now, $$I_{rms}^2 = \frac{1}{T}\int_0^T i^2(t)\, dt = \frac{1}{2}\int_0^2 (5t)^2\, dt = \frac{1}{2} \times 5^2 \int_0^2 t^2\, dt = \frac{25}{2} \times \left[\frac{t^3}{3}\right]_0^2$$

or, $$I_{rms}^2 = \frac{25}{2 \times 3} \times [8 + 0] = 33.333$$

∴ $$I_{rms} = \sqrt{33.333} = 5.7735\text{ Amp}$$

Again, $$I_{avg} = \frac{1}{T}\int_0^T i(t)\, dt = \frac{1}{2}\int_0^2 5\, t\, dt = \frac{5}{2}\left[\frac{t^2}{2}\right]_0^2 = \frac{5}{4}[4 + 0] = 5\text{ Amp.}$$

5.4 PHASOR REPRESENTATION AND PHASOR DIAGRAM

An alternating quantity can be represented by a rotating line, called phasor. A phasor is a line of definite length rotating in anticlockwise direction at a constant angular velocity. The diagram in which phasor represents current, voltage and their phase difference is known as phasor diagram.

The waveform and equation representation by using phasor is as shown below.

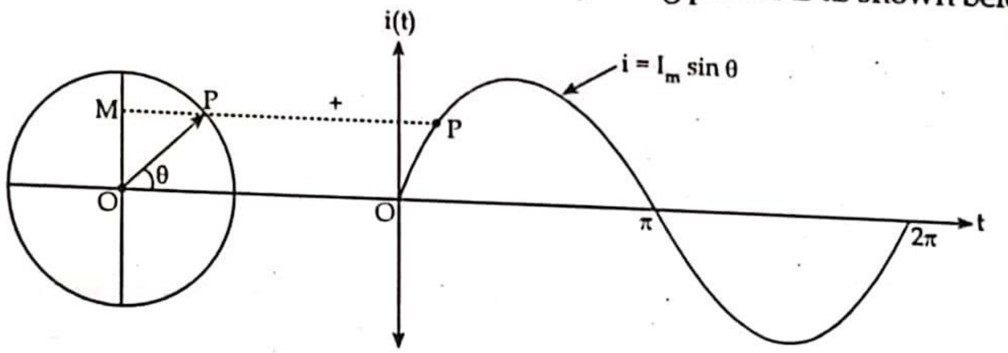


Figure: Phasor diagram

In phasor form, the above waveform is written as

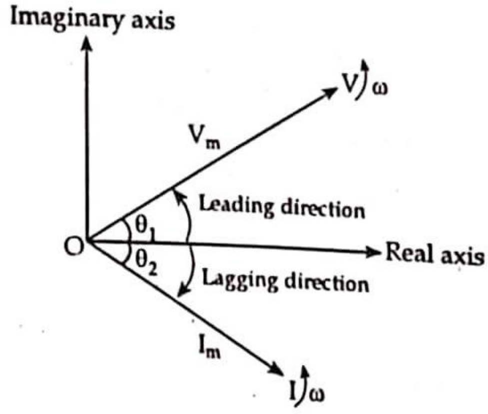
$$I = I_m \angle\theta°$$

Phasor can also be represented using complex number. If an alternating quantity is represented by $V = V_m \sin(\omega t + \theta)$ then its phasor representation is,

$$\boxed{V = V_m \angle\theta = V_m e^{j\theta}}$$

where, j is complex number.

Since, a phasor has magnetude and phase, it behaves as a vector. *For*



example: phasor $V = V_m \angle\theta$ and $I = I_m \angle\theta$ are graphically represented in given figure.

Table: Sinusoidal phasor transformation:

Time domain representation	Phasor representation
$V_m \cos(\omega t + \theta)$	$V_m \angle\theta$
$V_m \sin(\omega t + \theta)$	$V_m \angle(\theta - 90°)$
$I_m \cos(\omega t + \theta)$	$I_m \angle\theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle(\theta - 90°)$

Example 5.6

Transfer these sinusoid to phasor:

(a) $V(t) = -4 \sin(30t + 80°)$ (b) $i(t) = 6\cos(50t - 20°)$

Solution:

(a) $V(t) = -4\sin(30t + 80°) = 4\cos(30t + 80° + 90°) = 4\cos(30t + 170°)$

Therefore, phasor value

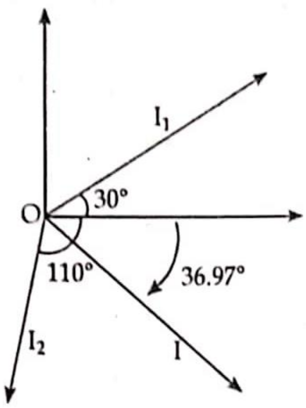
$V = 4\angle 170°$

(b) $i(t) = 6\cos(5t - 20°) = 6\angle{-20°}$

Example 5.7

Find the sum of $I_1(t) = 4\cos(\omega t + 30°)$ and $I_2(t) = 5\sin(\omega t - 20°)$ using phasor method:

Solution:



Here, $I_1(t) = 4\cos(\omega t + 30°)$

or, $I_1 = 4\angle 30°$

and, $I_2(t) = 5\sin(\omega t - 20°)$

$= 5\cos(\omega t - 20° - 90°)$

$= 5\cos(\omega t - 110°)$

$\therefore$ $I_2 = 5\angle{-110°}$

As, we know that, the sum of the phasor is again a phasor. So,

$I = I_1 + I_2$

$= 4\angle 30° + 5\angle{-110°}$

$= 3.218\angle{-36.97°}$ A

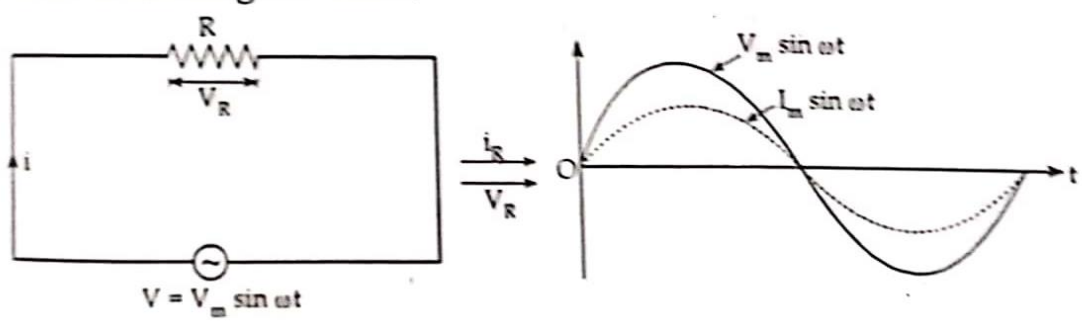
Transforming this phasor quantity to time domain, we get,

$I(t) = 3.218\cos(\omega t - 36.97°)$ A

SINGLE PHASE AC CIRCUITS

6.1 AC IN RESISTIVE CIRCUITS

A circuit without inductance and capacitance, containing resistors only is called pure resistive circuit. A resistive circuit with a pure resistance R is as shown in the figure below.



Instantaneous value of e.m.f. (V) = $V_m \sin \omega t$

According to Ohm's law, the applied voltage has to supply voltage drop at resistor only. *i.e.*, $V_R = V$. So,

$$V_R = iR$$

or, $i = \dfrac{V}{R}$ [$\because V = V_m \sin \omega t$ for sine wave]

or, $i = \dfrac{V_R}{R}$

or, $i = \frac{V_m \sin \omega t}{R}$

or, $i = \left(\frac{V_m}{R}\right) \sin \omega t$

or, $i = I_m \sin \omega t$ (1)

where, $I_m = \frac{V_m}{R}$

So, in purely resistive circuit, voltage and current are in the same phase because there is no any difference in their equational value.

6.2 CURRENT AND VOLTAGE IN AN INDUCTIVE CIRCUIT

Consider an AC circuit containing inductor only. When AC voltage is applied to the circuit, back e.m.f. is induced due to the self inductance of the coil. Since, there is no any resistor in the circuit, the induced e.m.f. has to overcome through this applied voltage.

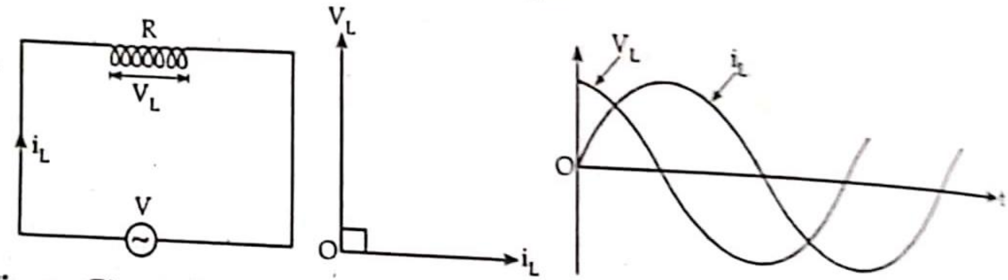


Figure: Circuit diagram   Phasor diagram   V-i waveforms

Now,

$$V = L\left(\frac{di}{dt}\right) \quad \text{[Also, we have } V = V_m \sin \omega t\text{]}$$

or, $V_m \sin \omega t \, dt = L\left(\frac{di}{dt}\right)$

or, $\left(\frac{V_m}{L}\right) \sin \omega t \, dt = di$

Integrating both sides, we get,

$$\int \frac{V_m}{L} \sin \omega t \, dt = \int di$$

or, $\frac{V_m}{L} \int \sin \omega t \, dt = \int di$

or, $i = \frac{V_m}{\omega L} [-\cos \omega t]$

or, $i = \frac{V_m}{\omega L} \sin (\omega t - 90°)$

$\therefore$ $\boxed{i = \frac{V_m}{X_L} \sin (\omega t - 90°)}$ (2)

where, $X_L = \omega L$ is known as inductive reactance. Also, value of current is given as,

$$i = \frac{V_m}{X_L}\sin(\omega t - 90°)$$

or, $i = I_m \sin(\omega t - 90°)$

for $i = I_{max}$, $\sin(\omega t - 90°)$ to be Unity. (3)

We can see that, current in inductive circuit lags behind the voltage by 90° or the phase difference between voltage and current is 90°.

NOTE

We see that, $X_L = \omega_L = 2\pi fL$, which is called inductive reactance of circuit and it opposes the flow of current in the circuit.

6.3 CURRENT AND VOLTAGE IN CAPACITIVE CIRCUIT

When an AC voltage is applied to the capacitor, the capacitor is charged first and then current flow further.

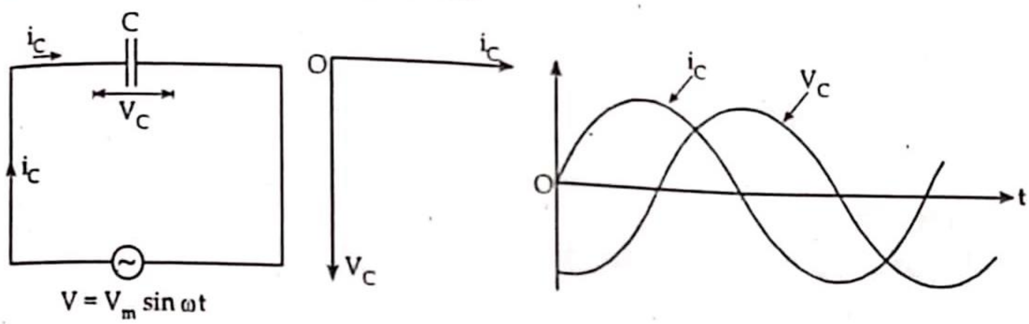


Figure: Circuit diagram Phasor diagram i–V waveforms

Here,

V_C = Potential difference developed between plates of capacitor at any time

q = Charge on plate at that time

Then, we know that,

$q = CV = CV_m \sin\omega t$ $[\because V = V_m \sin\omega t]$

For capacitor, current i is given by

$$i = \frac{dq}{dt}$$

Putting value of q we get,

$$i = \frac{d}{dt}(CV_m \sin\omega t)$$

or, $i = CV_m \frac{d}{dt}(\sin\omega t)$

or, $i = CV_m \times \omega \times [\cos\omega t]$

$\therefore$ $i = \dfrac{V_m}{\dfrac{1}{C\omega}}\sin(\omega t + 90°)$

or, $\boxed{i = \frac{V_m}{X_C} \sin(\omega t + 90°)}$ (4)

$\therefore \quad X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

where, $X_C = \frac{1}{\omega C}$ is called capacitive reactance of the capacitor which oppose the flow of alternating current replacing $\frac{V_m}{X_C} = I_m$, we get

$$i = I_m \sin(\omega t + 90°) \quad (5)$$

Here, we can see that, in purely capacitive circuit current leads the applied voltage by 90°.

Example 6.1

An AC quantity (200 V, 50 Hz) is applied to a coil of pure inductor having inductance 4.3 H. Determine the reactance of the coil and current flowing through it.

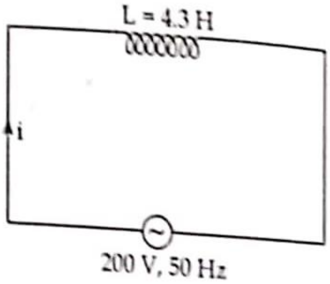
Solution:

Given that,

Supply voltage (V_s) = 200 V

Frequency (f) = 50 Hz

Inductance of coil (L) = 4.3 H



Now, the reactance of the coil is

$$X_L = 2\pi fL = 2 \times \pi \times 50 \times 4.3 = 1350.885\ \Omega$$

and, Current flowing through the coil is

$$i = \frac{V}{X_L} = \frac{200}{1350.885} = 0.148 \text{ Amp}$$

Example 6.2

A 50 μF capacitor is connected across a 230 V, 50 Hz supply. Calculate

(a) The reactance offered by the capacitor

(b) The maximum current and

(c) RMS value of the current drawn by the capacitor

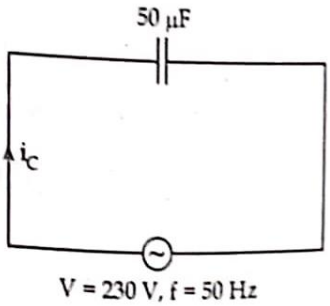
Solution:

Given that,

Supply AC voltage (V) = 230 V

Frequency (f) = 50 Hz

Capacitance of the capacitor (C) = 50 μF



Now,

(a) Reactance (X_C) $= \frac{1}{\omega C} = \frac{1}{2\pi fC}$

$$= \frac{1}{2 \times \pi \times 50 \times 10^{-6} \times 50} = 63.6\ \Omega$$

(b) Since, given supply voltage 230 V represents RMS value,

$$V_{max} = V_{rms} \times \sqrt{2}$$

$$= 230 \times \sqrt{2} = 325.27 \text{ V}$$

$$\therefore i_{max} = \frac{V_{max}}{X_C} = \frac{325.27}{63.6} = 5.114 \text{ A}$$

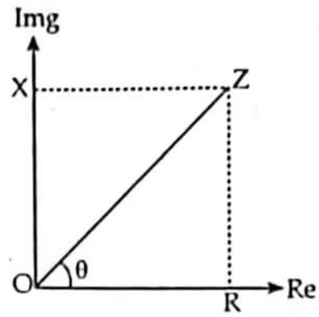
(c) $$I_{rms} = \frac{V_{rms}}{X_C} = \frac{230}{63.6} = 3.62 \text{ A}$$

6.4 CONCEPT OF COMPLEX IMPEDANCE AND ADMITTANCE

Electrical impedance describes the measure of opposition to alternating current. Impedance is defined as the frequency domain ratio of the voltage to the current. It is denoted by capital alphabet 'Z'. In general, impedance is a complex number but its unit is same as resistance. *i.e.*, ohm. The impedance may be represented either in the polar form or complex number. Complex number representation is more powerful for circuit analysis purpose.

Representation of impedance

Here, X and Y axes are real and imaginary axes respectively. Real axis represent resistance, where as imaginary axis represent either inductance or capacitance.



(i) Now, equation in complex from can be written as

$$Z = R + JX \quad (6)$$

where, J = complex number.

* Now, for inductor, inductive impedance is;

$$Z = R + JX_L$$

where, X_L = inductive inductance.

* For capacitor capacitive impedance is,

$$Z = R - JX_C$$

where X_C = capacitive inductance

(ii) In polar form, impedance is given as

$$Z = |Z| < \theta$$

where, $|Z| = \sqrt{R^2 + X^2}$

and $\tan\theta = \frac{X}{R}$

where, $X = |Z| \sin\theta$ and $R = |Z| \cos\theta$

Admittance

The reciprocal value of impedance is called admittance. It is denoted by 'Y' and measured in opposite of ohm. *i.e.*, ohm^{-1} or simen,

$$\therefore \quad Y = \frac{1}{Z}$$

or, $\quad Y = G + JB$

where, G = Real part (Y), also called conductance

B = I_m imaginary part (Y) called susceptance

Now, as Y is reciprocal of Z, we have

$$Y = \frac{1}{R \pm JX} = \frac{1}{R \pm JX} \times \frac{R \pm JX}{R \pm JX} = \frac{R \pm JX}{R^2 \pm X^2} = \frac{R}{R^2 \pm X^2} \pm J\frac{X}{R^2 \pm X^2}$$

Now, comparing this result with $Y = G \pm JB$, we get;

$$\therefore \quad G = \frac{R}{R^2 \pm X^2} \text{ and } B = \pm\frac{X}{R^2 \pm X^2}$$

6.5 RL, RC AND RLC CIRCUIT ANALYSIS

(i) R–L in AC circuit

When the resistance and inductance are connected in series, there is no phase difference in resistance circuit but in the inductance circuit, the voltage is lead by 90° to the current. The voltage across the resistor is V_R and voltage across inductor is V_L.

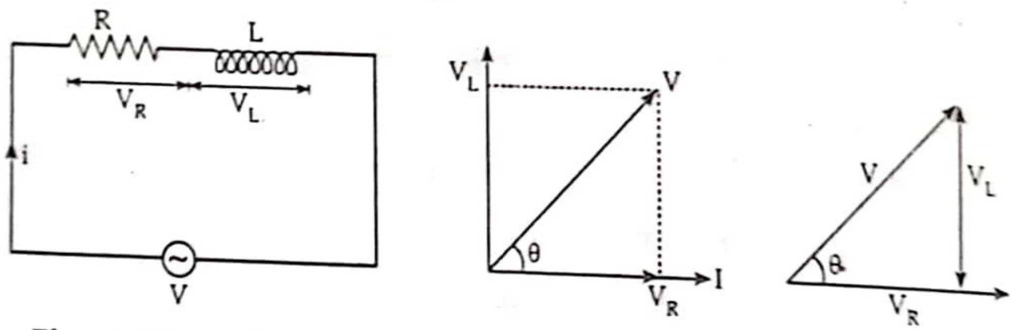


Figure: Circuit diagram Phasor diagram Voltage triangle

From phasor diagram, and voltage triangle, we can write

$$V = V_R + JV_L = \sqrt{(V_R)^2 + (V_L)^2} \qquad (7)$$

Now, dividing both side by current, we get,

$$\frac{V}{i} = \frac{V_R}{i} + J\frac{V_L}{i}$$

$$\therefore \quad \boxed{Z = R + JX_L} \qquad (*)$$

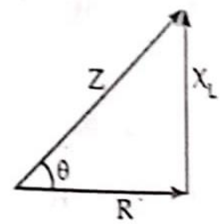


Figure: Impedance triangle

Which gives impedance of R–L circuit.

Again, from impedance triangle,

$$\tan\theta = \frac{X_L}{R}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Example 6.3

A coil of inductance 318.3 mH and negligible resistance is connected in a series with a 200 Ω resistor to a 240 V, 50 Hz supply. Calculate

(a) inductive reactance of the coil

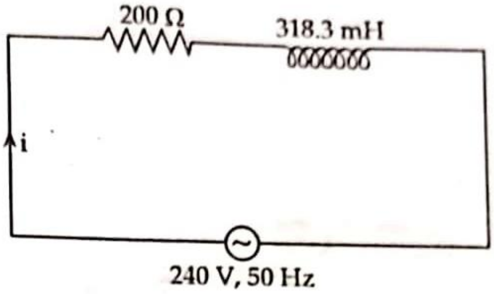
(b) impedance of the circuit

(c) current in the circuit and its phase

(d) potential difference across each component

Solution:

For this types of question, it is better to solve in complex number format.



Given that,

Supply voltage (V) = 240 V

Frequency (f) = 50 Hz

Resistance (R) = 200 Ω

Inductance (L) = 318.3 mH

Now,

(a) Inductance (X_L) = $2\pi fL = 2 \times 3.14 \times 50 \times 318.3 \times 10^{-3} = 100\ \Omega$

(b) Impedance (Z) = $R + JX_L = 200 + J\,100 = 223.6\ \angle 26.56°\ \Omega$

(c) Current (i) = $\dfrac{V}{Z} = \dfrac{240\ \angle 0}{223.6\ \angle 26.56°} = 1.0733\ \angle -26.565°$ A

and its phase angle is 26.565°

(d) Potential difference across the resistance is

$$V_R = i \times R = 1.0733 \times 200 = 214.66\ \angle -36.565°$$

and potential difference across the inductance is

$$V_L = i \times X_L = 1.0733\ \angle -26.565° \times 100 = 107.33\ \angle -26.565°$$

(ii) R-C in AC Circuit

In the figure, the resistance R and capacitor C is connected in series with the circuit. The voltage between capacitor and resistance are V_C and V_R respectively. The total supply voltage is V and current is i. For this CKT, current leads voltage by 90°.

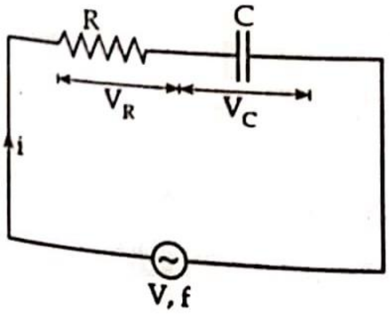
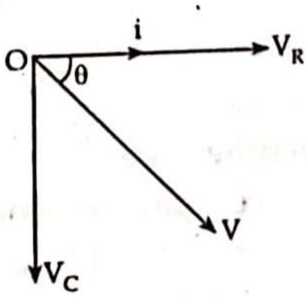
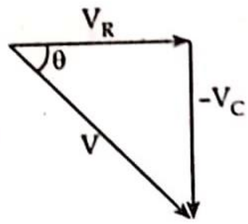


Figure: Circuit diagram



Phasor diagram



Voltage triangle

Now, from voltage triangle, we can write

$V = V_R - jV_C$ [which is in vector form]

or, $Vi = V_R \cdot i - jV_C\, i$ [Multiplying both sides by i]

Since, $Z = Vi$ we get

$$\boxed{Z = R - jX_C}$$

Also, from impedance triangle, we can write

$Z = \sqrt{R^2 + (-X_C)^2}$

or, $Z = \sqrt{R^2 + (-X_C^2)^2}$ (*)

And, $\tan\theta = \frac{-X_C}{R}$

or, $\theta = \tan^{-1}\left(\frac{-X_C}{R}\right)$

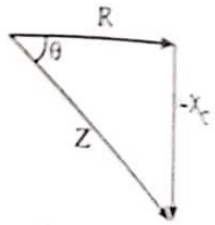


Figure: Impedance triangle

Example 6.4

A capacitor C is connected in series with a 40 Ω resistor across a supply of frequency 60 Hz. A current of 3 A flows and the circuit impedance is 50 Ω. Calculate

(a) The value of capacitance

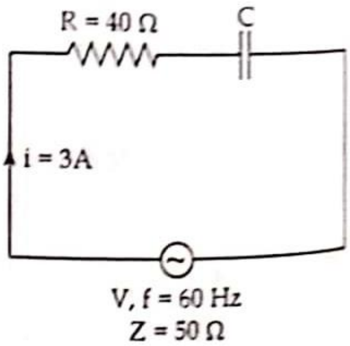
(b) The supply voltage

(c) Potential difference across the resistor and capacitor

(d) Draw the phasor diagram for RC series circuit

Solution:

(a) For impedance,



$Z = \sqrt{R^2 + (-X_C)^2}$

or, $(50)^2 = (40)^2 + (X_C)^2$

or, $X_C^2 = 900$

∴ $X_C = 30\ \Omega$

Now, capacitance

$X_C = \frac{1}{2\pi fC}$

or, $C = \frac{1}{2\pi \times 60 \times 30} = 8.842 \times 10^{-5}$

∴ $C = 88.42\ \mu F$

(b) For supply voltage

$V = iZ = 3 \times 50 = 150\ V$

(c) To get potential difference

Potential difference across resistor,

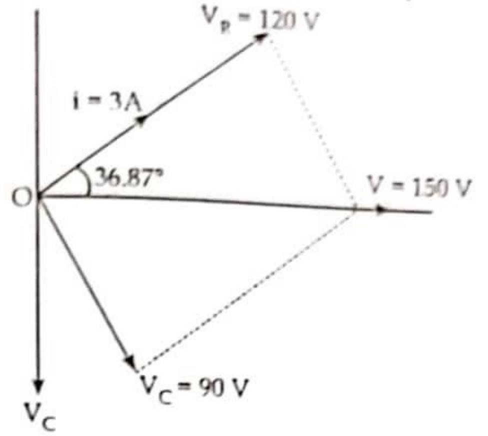
$V_R = iR = 3 \times 40 = 120\ V$

Potential difference across capacitor

$V_C = iX_C = 3 \times 30 = 90\ V$

(d) For phasor diagram

Phase angle $\theta = \tan^{-1}\left(\frac{-X_C}{R}\right) = \tan^{-1}\left(\frac{-30}{40}\right) = (-36.87°)$



(iii) **L-R-C in series AC circuit**

In this circuit, the resistance, capacitor and inductor are connected in series. In this circuit, current is same but voltage is differed as per resistance offered by L, C and R.

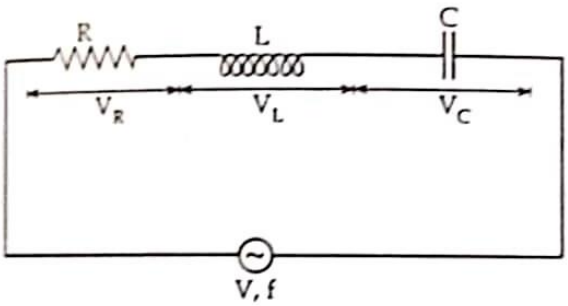
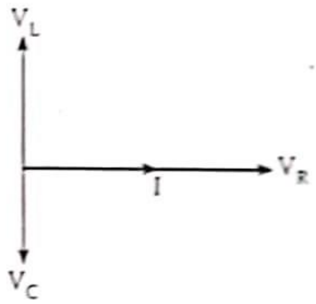


Figure: Circuit diagram



Phasor diagram

Here, V_R = in phase with current i

V_L is leading with current at 90°

V_C is lagging with current at 90°

Case-I When $V_L > V_C$

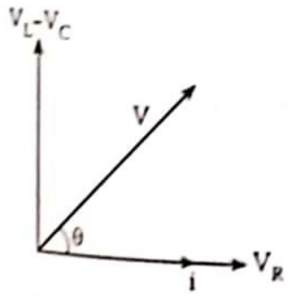
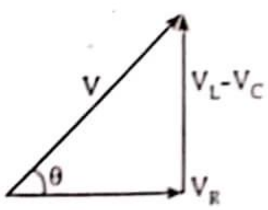
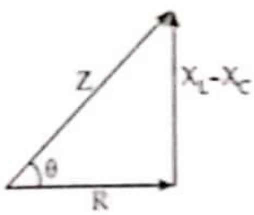


Figure: Phasor diagram



Voltage triangle



Impedance triangle

Now, vector sum of voltage is

$$V = V_R + j(V_L - V_C)$$

Dividing both sides by i, we get;

$$\frac{V}{i} = \frac{V_R}{i} + j\,\frac{V_L - V_C}{i}$$

$$\therefore \quad \boxed{Z = R + j(X_L - X_C)}$$

Since, $\frac{V}{i} = Z, \frac{V_R}{i} = R$, etc.

Now, from impedance triangle,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

and, $\tan\theta = \left(\frac{X_L - X_C}{R}\right)$

Case-II When $V_L < V_C$; then, vector sum of voltage is

$$V = V_R + j(V_C - V_L)$$

Dividing both sides by i

$$\frac{V}{i} = \frac{V_R}{i} + j\left(\frac{V_C - V_L}{i}\right)$$

or, $Z = R + j(X_C - X_L)$ (10)

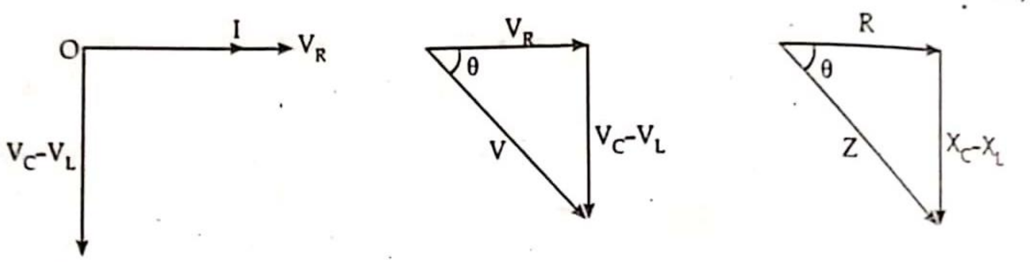


Figure: Phasor diagram Voltage triangle Impedance triangle

Again from impedance triangle

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

and, $\tan\theta = \left(\frac{X_C - X_L}{R}\right)$

$\therefore$ $\boxed{\theta = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)}$

From equation **(10)**.

When the voltage V applied to the electrical network containing resistance, inductance and capacitance, is in phase with resulting current i, the circuit is said to be in resonance. So, for resonance

$$|V_L| = |V_C|$$

or, $|iX_L| = |iX_C$

or, $|X_L| = |X_C|$

or, $2\pi fL = \frac{1}{2\pi fC}$

or, $f^2 = \frac{1}{4\pi^2} \times \frac{1}{LC}$

or, $\boxed{f = \frac{1}{2\pi} \times \sqrt{\frac{1}{LC}}}$ (11)

Equation **(11)** gives resonance frequency of L-C-R circuit.

6.6 AC PARALLEL CIRCUIT

In series AC circuit, there is current common, but in parallel AC circuit, voltage is common and thus voltage is taken as reference phasor while solving AC parallel circuit and drawing phasor diagram. There are four AC parallel circuits;

(a) R-L parallel circuit
(b) R-C parallel circuit
(c) L-C parallel circuit
(d) LRC parallel circuit

(a) R–L Parallel Circuit

Consider a parallel circuit containing resistance and inductance. For resistance, voltage drop V_R and current flowing through resistor i_R lies in the same phase. But for inductance V_L leads i_L by 90°. For parallel combination, voltage remains same. So, voltage is taken as reference phasor.

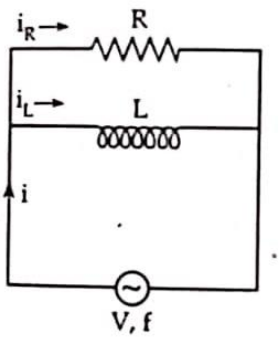
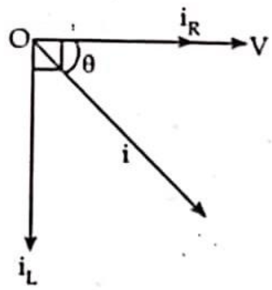
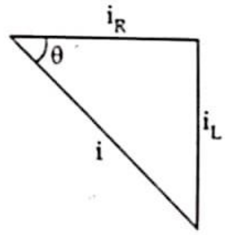


Figure: Circuit diagram



Phasor diagram



Current triangle

Now, from phasor diagram, we can write

$$i = i_R - j i_L \qquad (*)$$

or, $$i = \sqrt{(i_R)^2 + (i_L)^2}$$

Again, dividing the equation by V, we get

$$\frac{i}{V} = \frac{i_R}{V} - j\frac{i_L}{V}$$

or, $$Y = Y_R - jY_L \qquad (**)$$

Here, $\frac{i}{V} = Y$ is admittance of parallel circuit.

Again, from admittance triangle

$$Y = \sqrt{Y_R^2 + V_L^2}$$

and, $$\tan\theta = \frac{-Y_L}{Y_R}$$

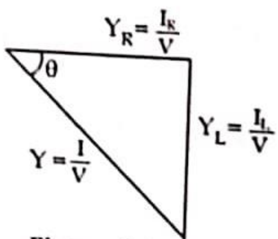


Figure: Admittance triangle

Similarly, you can analyse other remaining parallel circuit in the same way.

Example 6.5

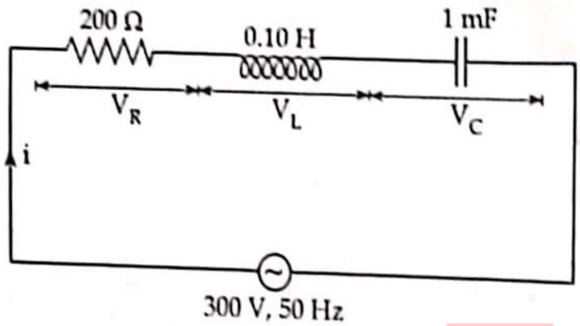
A resistor having resistance 100 Ω and capacitor of capacitance 1 mF is connected in series with an inductance of 0.10 H. If supply voltage is 300 V, 50 Hz, then calculate

(a) impedance of the circuit

(b) current, that flowing in the circuit

(c) voltage across the inductor and capacitor

Solution:



(a) For impedance

$$Z = R + j(X_L - X_C)$$

Now,

$$R = 100\ \Omega$$

$$X_L = 2\pi fL = 2 \times \pi \times 50 \times 0.10 = 31.416\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 1 \times 10^{-3}} = 3.183\ \Omega$$

So, the circuit is inductive because, $X_L > X_C$

$\therefore$ $Z = 100 + j\,(31.416 - 3.183)$

or, $Z = 100 + j\,28.233$

or, $Z = 103.909\ \angle 15.766°$

(b) Current (i) $= \dfrac{V}{Z} = \dfrac{300\ \angle 0°}{103.909\ \angle 15.766°} = 2.887\ \angle{-15.766}$ A

(c) Voltage across inductor,

$$V_L = i \times X_L$$

$$= (2.88\ \angle{-15.766}) \times (j \times 3.183)$$

$$= 9.1897\ \angle{-105.76°}$$

NOTE

To fix calculator in complex mode:

i) Press $\boxed{\text{Mode}}$ then press 2. $\boxed{\text{Complx}}$

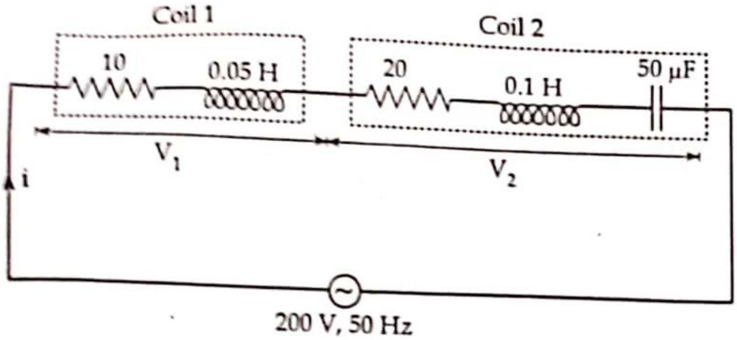
ii) Again press $\boxed{\text{Shift}}$ $\boxed{\text{Mode}}$ then scroll down and select 3. $\boxed{\text{Complx}}$

iii) Select required format, we need r ∠θ format. So press 2.

iv) Then, for input (5 + j 10), enter 5 from calculator then press $\boxed{+}$ again then press $\boxed{10}$ and then $\boxed{i}$. Now press $\boxed{=}$, to get required value.

Example 6.6

Draw a vector for a given figure indicating the resistance and reactance drop, then terminal voltage V_1 and V_2 and the current. Find the values of (i) current (ii) V_1 and V_2.



Solution:

Here,

$L = 0.05 + 0.1 = 0.15$ H, $R = 20 + 10 = 30\ \Omega$, $C = 50\ \mu F$

$\therefore \quad X_L = 2\pi fL$

$= 2 \times \pi \times 50 \times 0.15 = 47.123\ \Omega$

and, $\quad X_C = \dfrac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.7\ \Omega$

$R = 30\ \Omega$

Now,

$Z = \sqrt{30^2 + (63.7 - 47.1)^2} = 34.3\ \Omega$

(i) The current flowing through the circuit is,

$$i = \frac{V}{Z} = \frac{200}{34.3} = 5.83\ A$$

(ii) To get V_1

$R_1 = 10\ \Omega \qquad L_1 = 0.05$ H

$\therefore \quad V_L = 2\pi fL_1 = 2\pi \times 50 \times 0.05 = 15.7\ \Omega$

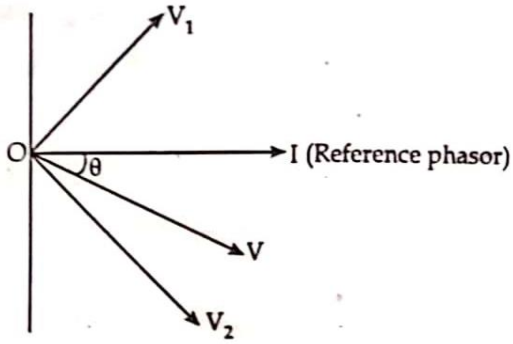
$\therefore \quad Z_1 = 10 + 15.7\ j\Omega$

$= \sqrt{10^2 + (15.7)^2} = 18.6\ \Omega$

$\therefore \quad V_1 = i \times Z_1$

$= 5.83 \times 18.6$

$= 108.4$ V



Again, to get V_2

$Z_2 = 20 + j\,(63.7 - 31.4)$

$= 20 + j\,32.3$

$= \sqrt{20^2 + (32.3)^2}$

$= 37.990\ \Omega$

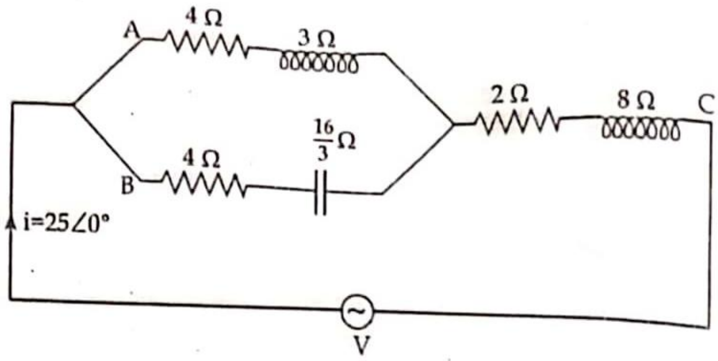
and, $\quad V_2 = R \times Z_2$

$= 5.83 \times 37.99$

$= 221.485$ V

Example 6.7

In a series parallel circuit, the parallel branches A and B are in series with C. If current $i_C = 25 + 0j$ flowing in the circuit, find the branch currents and the total voltage.



Solution:

Here, $Z_A = 4 + j3\ \Omega = 4 + j3 = 5\ \angle 36.56°$

$$Z_B = 4 - j\frac{16}{3} = \frac{20}{3}\ \angle 53.13°$$

$$Z_C = 2 + 8j = 8.25\ \angle 76.963°\ \Omega$$

The impedance A and B are in parallel

$$\therefore \quad Z_{AB} = Z_A \parallel Z_B = \frac{Z_A \times Z_B}{Z_A + Z_B} = \frac{(4 + j3) \times \left(4 - j\frac{16}{3}\right)}{(4 + j3) + \left(4 - j\frac{16}{3}\right)} = 4 + J0 = 4\ \angle 0°$$

Now, $V_{AB} = i_C \times Z_{AB} = (25 + j0) \times (4 + j0) = (100 + j0) = 100\ \angle 0°$ V

and, Currents in branches, A and B are

$$i_A = \frac{V_{AB}}{Z_B} = \frac{100\ \angle 0°}{5\angle 36.86°} = 20\ \angle -36.86°\ \text{A}$$

$$i_B = \frac{V_{AB}}{Z_B} = \frac{100\ \angle 0°}{\frac{20}{3}\ \angle 53.13°} = 15\ \angle -53.13°\ \text{A}$$

Also, $V_C = i_C Z_C$

$= 25\ \angle 0° \times 8.25\ \angle 75.963°$

$= 206.25\ \angle 75.963°$ V

and, $Z = Z_{AB} + Z_C$

$= 4\angle 0° + 8.25\ \angle 75.963°$

$= 10\ \angle 53.13°\ \Omega$

and, $V = i_C \times Z$

$= 25\ \angle 0° \times 10\ \angle 53.13°$

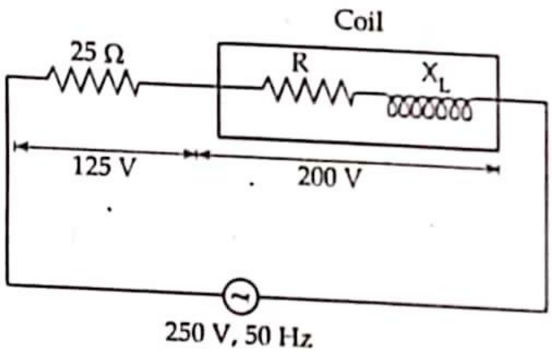
$= 250\ \angle 53.13°$ V

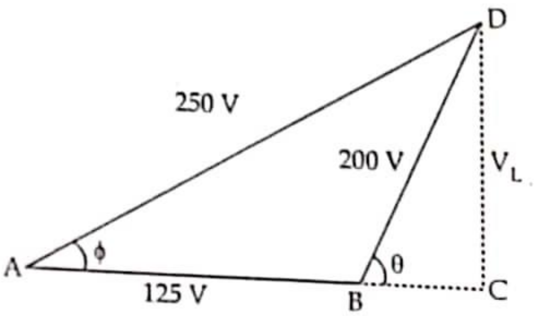
ADDITIONAL QUESTION SOLUTIONS

1. A current of 5 A flows through a non inductive resistance in series with a choking coil when supplied at 250 V - 50 Hz. If the voltage across the resistance is 125 V and across the coil is 200 V. Calculate
   (a) Impedance, resistance and reactance of the coil.
   (b) The power absorbed by the coil.
   (c) The total power
   (d) Draw the vector diagram

Solution:

For solution of this types of problem, first draw a circuit diagram and its vector diagram.





From vector diagram;

$$BC^2 + CD^2 = 200^2 \qquad (1)$$

Again $(125 + BC)^2 + (CD)^2 = (250)^2 \qquad (2)$

Solving equation (1) and (2), we get;

$$BC = 27.5 \text{ V and } CD = 198.1 \text{ V}$$

Now,

(a) Impedance of the coil

$$Z = \frac{V_C}{I} = \frac{200}{5} = 40\ \Omega$$

Again, for resistance, we know;

$$V_R = I \cdot R$$

NOTE: $BC = BR = 27.5$, $DC = V_L = 198.1$

or, $5R = 27.5$

or, $\boxed{R = 5.5\ \Omega}$

And, for reactance of the coil

$$V_L = I\,X_L$$

or, $198.1 = 5\,X_L$

or, $\boxed{X_L = 39.62\ \Omega}$

(b) To get power absorbed by the coil,

$$P = I^2 R$$

or, $P = 5^2 \times 5.5$

or, $P = 137.5$ watt

NOTE

$$\cos\phi = \frac{AC}{AD},$$

$$\cos\phi = \frac{152.5}{250} = 0.61$$

(c) Total power absorbed

$$P_T = IV \cos \phi$$

or, $P_T = 250 \times 5 \times 0.61$

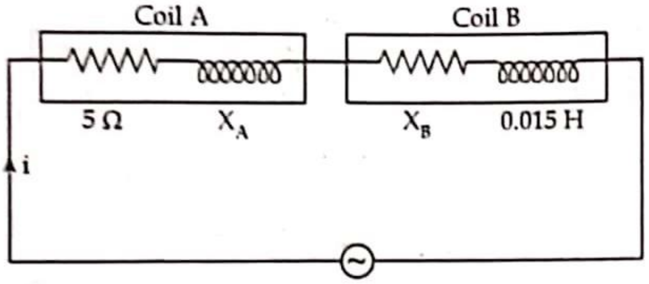
or, $P_T = 762.5$ watt

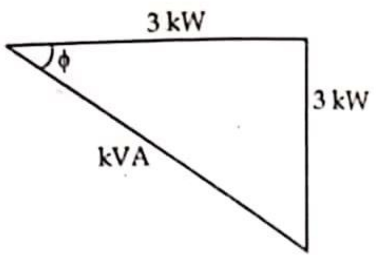
(d) Vector diagram is shown above.

2. 2 coils A and B are connected in series across a 240 V, 50 Hz supply. The resistance of A is 5 Ω and the inductance of B is 0.015 H. If the input from the supply is 3 kW and 2 kVAR, find the inductance of A and the resistance of B. Calculate the voltage across each coil

Solution:

First of all, drawing circuit diagram and vector diagram.





Now, kVA of circuit is given as

$$kVA = \sqrt{3^2 + 2^2} = 3.606 \text{ or } 3606 \text{ VA}$$

Now, current flowing in the circuit is

$$\frac{3606}{240} = 15.03 \text{ A}$$

Now, power across resistor

$$P = I^2 R$$

or, $3 \times 10^3 = (15.3)^2 \times (R_A + R_B)$

or, $R_A + R_B = 13.280$

or, $R_B = 13.280 - 5$

or, $\boxed{R_B = 8.28 \ \Omega}$

Also

$$X_B = 2\pi f L_B = 2\pi \times 50 \times 0.015 = 4.713 \ \Omega$$

Now, power across inductor

$$P_L = I^2 X_L$$

or, $200 = (15.03)^2 (X_A + X_B)$

or, $X_A = 8.84 - 4.713 = 4.13 \ \Omega$

∴ $X_A = 2\pi f L_A$

or, $L_A = \dfrac{4.13}{2\pi \times 50}$

or, $\boxed{L_A = 0.0132 \text{ H}}$

Also,

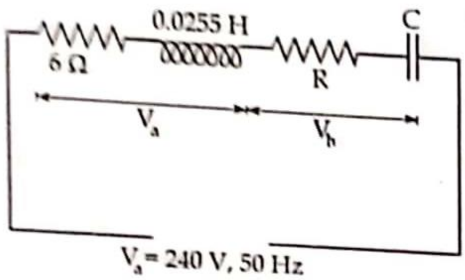
$$Z_A = \sqrt{R_A^2 + X_A^2} = \sqrt{5^2 + 4.13^2} = 6.485\ \Omega$$
$$Z_B = \sqrt{R_B^2 + X_B^2} = \sqrt{8.3^2 + 4.713^2} = 6.545\ \Omega$$

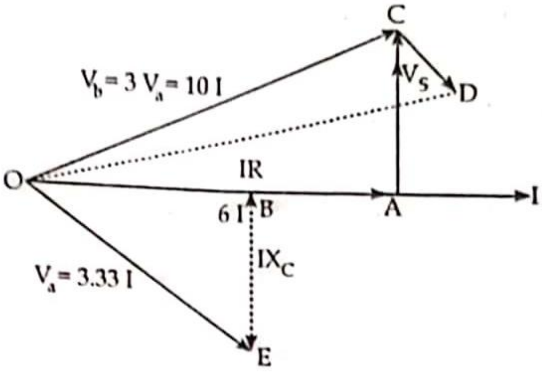
Now,

Potential differential across coil A = $I Z_A$ = 15.03 × 6.485 = 97.5 V

Potential differential across coil B = $I Z_B$ = 15.03 × 9.545 = 143.5 V

3. For the circuit shown in figure below. Find the value of R and C so that $V_b = 3V_a$ and V_b and V_a are in phase quadrature. Find also the phase relationship between V_s and V_b and V_b and I.





Solution:

Given that;

$\angle COA = \phi = 53.13°$

$\angle BOE = 90° - 53.13° = 36.87°$

$\angle DOA = 34.7°$ angle between V and I.

Angle between V_s and V_b = 18.43°

Now, $X_L = 2\pi f L = 2\pi \times 50 \times 0.0255 = 8\Omega$

∴ $Z_B = 6 + j8 = 10\ \angle 53.13\ \Omega$

$V_b = 10\ I = 3\ V_a$

∴ $V_a = 3.33\ I$

In above phasor diagram, I is taken as reference. V_b is in first quadrant. Hence V_a must be in the fourth quadrant. Since Z_a consist of R and X_c, angle between V_a and I is then 36.87. Since Z_a and Z_b are in series, V is represented by phasor OD, which is at angle of 34.7°

Now,

$$|V| = \sqrt{10}\ V_a = 10.53\ \Omega$$

or, $$\frac{I}{V} = Z = 10.53\ \Omega$$

Thus the circuit have total effective impedance of 10.53 Ω.

In the phasor diagram

$OA = 6\ I$

$AC = 8\ I$

$OC = 10\,I$

$V_b = 3 \cdot V_a$

Hence, $V_a = OE = 3.33I$

Since, $\angle BOE = 36.87°$

$OB - RI = OE \times \cos 36.87° = 3.33 \times 0.8 \times I = 2.66\,I$

$\therefore \quad R = 2.66\,\Omega$

Again, $\quad BE = OE \sin 36.86 = 3.33\,I \times 0.6 = 2\,I$

Hence, $\quad X_C = 2\,\Omega$

For $\quad C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{2\pi \times 50 \times 2} = 1593\,\mu F.$

Horizontal component of OD = OB + OA = 8.66 I

Vertical component of OD = AC – BE = 6 I

$$OD = 10.54\,I = V_s = \frac{V_s}{I} = 10.54$$

Hence the total impedance = $10.54\,\Omega = (8.66 + j6)\,\Omega$

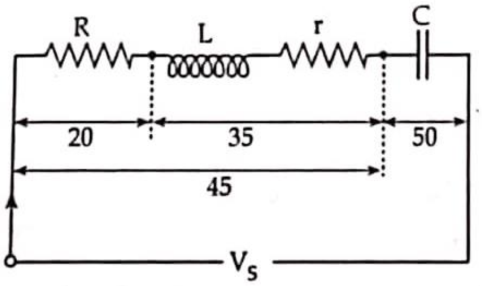
Angle between V_s and I $\angle DOA = \tan^{-1}\left(\dfrac{6}{8.60}\right) = 34.7°$

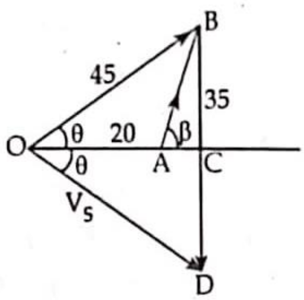
4. Resistor R, check coil (r, L) and a capacitor of 25.2 μF are connected in series. When supply from an AC source, it takes 0.4 A. If the voltage across the choke is 35 volts, and across the capacitor is 50V, find

(a) The values of r, L

(b) Applied voltage and its frequency

(c) PF of total circuit and active power consumed. Also draw phasor





Solution:

Given that;

Current, $I = 0.4$ A

$$\therefore \quad R = \frac{V}{I} = \frac{20}{0.4} = 50\,\Omega$$

Impedance of the coil $Z_1 = \dfrac{35}{0.4} = 87.5\,\Omega$

Capacitive reactance $X_C = \dfrac{50}{0.4} = 125\,\Omega$

Now,

(b) We know that,

$$X_C = \frac{1}{2\pi fC}$$

or, $f = \frac{1}{2\pi . C. X_C} = \frac{1}{2\pi \times 25.2 \times 10^{-6} \times 125}$

or, $f = 50$ Hz

(c) Now, from the phasor diagram, taking I as reference and solving triangle OAB,

$$\cos\theta = \frac{45^2 + 20^2 - 35^2}{2 \times 45 \times 20} = 0.667$$

$\therefore \quad \theta = 48.25°$

Again, $\cos(180° - \beta) = \frac{400 + 1224 - 2025}{2 \times 20 \times 35} = -0.2857$

or, $180° - \beta = 106.6$

or, $\beta = 180° - 106.6 = 73.4°$

Now,

$$OC = OA + AC = 20 + 35\cos\beta = 30$$

$$BC = 35\sin\beta = 35\sin(73.4) = 33.54$$

Also, BD = 50, CD = 16.46 volts

We get

$$V_s = \sqrt{(OC)^2 + (CD)^2}$$

$$= \sqrt{30^2 + (16.46)^2}$$

$$= 34.22 \text{ volts}$$

and $\angle COD = \phi = \cos^{-1}\left(\frac{OC}{OD}\right) = 28.75°$

Now, the power factor of total circuit

$$= \cos\phi = \cos(28.75)°$$

$$= 0.877 \text{ lead}$$

(Since, I leads V)

(d) Now, again for the coil ACB part of the phasor diagram is to be observed;

$$\gamma = \frac{AC}{I} = \frac{10}{0.4} = 25\ \Omega$$

$$X_L = \frac{BC}{I} = \frac{33.54}{0.4} = 83.85\ \Omega$$

Hence coil inductance, $L = \frac{X_L}{2\pi f} = \frac{83.85}{314} = 267$ mH

P = Active power consumed $= V_s . I\cos\phi = 12$ watt

or, $P = (0.4)^2 \times (R + r) = 12$ watts

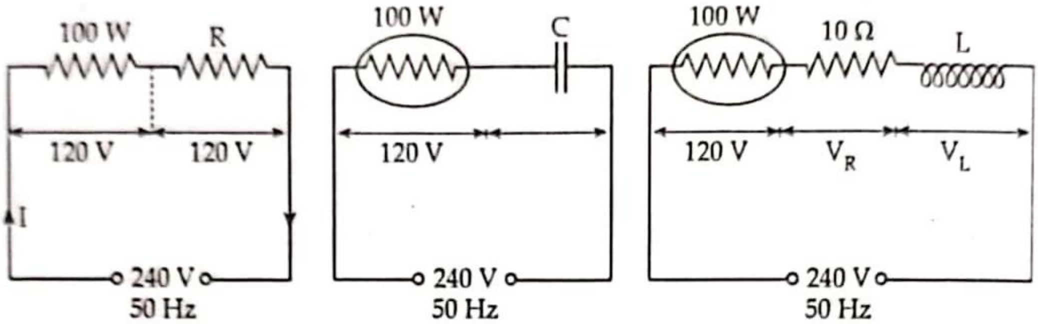
5. It is desired to operate a 100 – W, 120 V electric lamp at its current rating from a 240 – V, 50 Hz supply. Give details of the simplest manner in which this could be done using

(a) a resistor

(b) a capacitor

(c) An inductor having resistance of 10 Ω

What power factor would be presented to the supply in each case and which method is the most economical of power.



Solution:

(a) Rated current of the bulb is

$$I = \frac{P}{V} = \frac{100}{120} = \frac{5}{6}$$

The bulb can be run at its correct rating by any one of the three methods, shown in figure above;

We have

Potential difference across R = 240 – 120 = 120 V

$$R = \frac{120}{\frac{5}{6}} = 144\ \Omega$$

The power factor of circuit is unity

$$\text{Power consumed} = 240 \times \frac{5}{6} = 200\ \text{W}$$

(b) Referring figure (b);

$$V_C = \sqrt{240^2 - 120^2} = 207.5\ \text{V}$$

$$\therefore \quad X_C = 207.5 \times \frac{5}{6} = 249\ \Omega$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 249} = 12.8\ \mu\text{F}$$

and, Power factor

$$\cos\phi = \frac{120}{240} = 0.5\ \text{(led)}$$

and, Power consumed $= 240 \times \left(\frac{5}{6}\right) \times 0.5 = 100$ W

(c) The circuit connections are shown in figure (c)

$$V_R = \left(\frac{5}{6}\right) \times 10 = \frac{25}{3}\ \text{V}$$

$$\therefore \quad V_L = \sqrt{(240)^2 - \left(120 + \frac{25}{3}\right)^2}$$

or, $\quad V_L = 230$ V

And, $\quad 314L \times \left(\frac{5}{6}\right) = 203$

or, $\quad L = 0.775$ H

Total resistive drop $= 120 + \left(\frac{25}{3}\right) = 128.3$ V

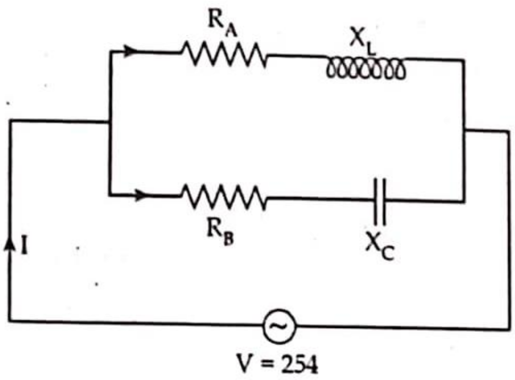
$$\cos\phi = \frac{128.3}{240} = 0.535 \text{ (leg)}$$

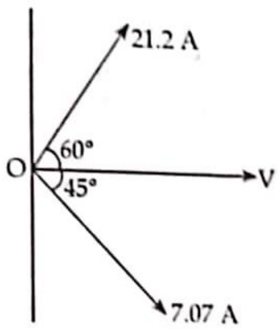
and, power consumed $= 240 \times \left(\frac{5}{6}\right) \times 0.535 = 107$ W

Hence method (b) is most economical because it involves least consumption of power.

6. The currents in each branch of two branched parallel circuit are given by the expression $i_A = 7.7 \sin\left(314t - \frac{\pi}{4}\right)$ and $i_B = 21.2 \sin\left(314t + \frac{\pi}{3}\right)$. The supply voltage is given by the expression $v = 354 \sin 314t$. Derive a similar expression for the supply current and calculate the ohmic value of the component. Assuming two pure components in each branch. State whether the reactive components are indicative or capacitive.

Solution:





From the given equation of voltage and currents,

$$V = 354 \sin 314t \qquad (1)$$

$$i_A = 7.07 \sin\left(314t - \frac{\pi}{4}\right) \qquad (2)$$

$$i_B = 21.2 \sin\left(314t + \frac{\pi}{3}\right) \qquad (3)$$

Hence, I_A lags the voltage by $\frac{\pi}{4}$ and I_B leads the voltage by $\frac{\pi}{3}$. So that, branch A contains resistance in series with pure capacitive reactance which is shown in above figure.

Now, (I_{max}) in branch A = $(i_A)_{max}$ = 7.07 A

(I_{max}) in branch B = $(i_B)_{max}$ = 21.2 A

Now, from vector diagram, the components of current are;

X-component = 21.2 cos 60° + 7.07 cos 45° = 15.6 A

Y-component = 21.2 sin 60° - 7.07 sin 45° = 13.36 A

Now, Maximum value of current = $\sqrt{(15.6)^2 + (13.36)^2}$ = 20.55 A

And, $\phi = \tan^{-1}\left(\frac{13.36}{15.6}\right) = \tan^{-1}(0.856) = 40.5°$ (lead)

Hence, the expression for the supply current is i = 22.55 sin (314t + 40.5°)

And, again

$$Z_A = \frac{V}{I_A} = \frac{354}{7.07} = 50\ \Omega$$

Also, $\cos\phi_A = \cos(45°) = 0.707$

And $\sin\phi_A = (\sin 45°) = 0.707$

Now, $R_A = Z_A \cos\phi = 50 \times 0.707 = 35.4\ \Omega$

And, $X_L = Z_A \sin\phi = 50 \times 0.707 = 35.4\ \Omega$

Again, for coil B,

$$Z_B = \frac{V}{i_B} = \frac{354}{20.2} = 17.5\ \Omega$$

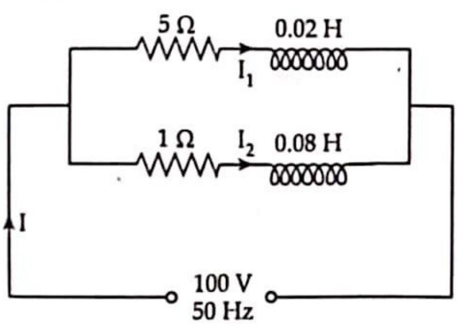
And, $R_B = Z_B \cos\phi = 17.5 \cos 60° = 8.75\ \Omega$

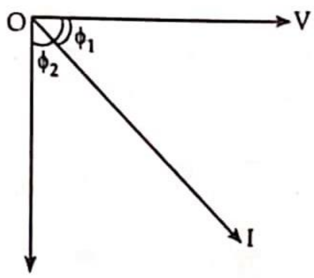
And, $X_C = Z_B \sin\phi = 17.5 \sin 60° = 15.16\ \Omega$

7. A coil having resistance of 5 Ω and an inductance of 0.02 H is arranged in parallel with another coil having a resistance of 1 Ω and an inductor of 0.08 H. Calculate the current through the combination and the power absorbed when a voltage of 100 V at 50 Hz is applied. Estimate the resistance of a single coil which will take the same power factor.

Solution:

From the information given in question. The circuit diagram and its phasor diagram can be drawn as;





For branch 1

$$X_1 = 2\pi fL = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$$

$$Z_1 = \sqrt{R^2 + X_1^2} = \sqrt{5^2 + (6.28)^2} = 8\ \Omega$$

$$I_1 = \frac{V}{Z_1} = \frac{100}{8} = 12.5\ \text{A}$$

$$\cos\phi_1 = \left(\frac{R_1}{Z_1}\right) = \left(\frac{5}{8}\right) = 0.625$$

$$\sin\phi_1 = \left(\frac{X_1}{Z}\right) = \left(\frac{6.28}{8}\right) = 0.785$$

Branch 2

$$X_1 = 2\pi \times 50 \times 0.08 = 25.12\ \Omega$$

$$R_2 = 1\ \Omega$$

$$\therefore \quad Z_2 = \sqrt{(X_2)^2 + (R_2)^2} = \sqrt{(25.12)^2 + 1^2} = 25.14\ \Omega$$

$$I_2 = \frac{V}{Z_2} = \frac{100}{25.14} = 4\ \text{A}$$

Now, $\cos\phi_2 = \dfrac{1}{25.14} = 0.0397$

$$\sin\phi = \frac{25.12}{25.14} = 0.999$$

Now, resolving the components of currents I_1 and I_2,

X-components $= I_1\cos\phi_1 + I_2\cos\phi_2 = (12.5 \times 0.625) + 4 \times 0.0397 = 7.97$ A

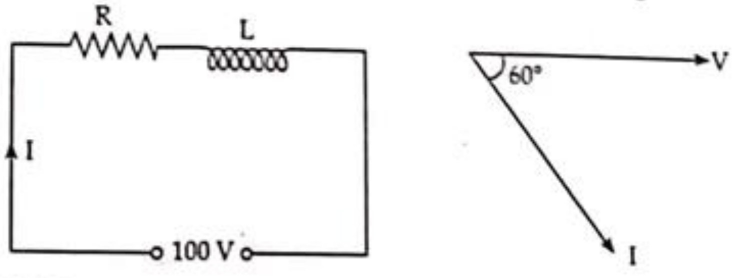
Y-components $= I_1\sin\phi_1 + I_2\sin\phi_2 = (12.5 \times 0.785) + 4 \times 0.999 = 13.8$ A

$$\therefore \quad I = \sqrt{(13.8)^2 + (7.97)^2} = 15.94\ \text{A}$$

$$\cos\phi = \left(\frac{7.97}{15.94}\right) = 0.5 \text{ lagg}$$

And, Power absorbed $= IV\cos\phi = 15.94 \times 100 \times 0.5 = 797$ watt

Again, the equivalent series circuits are shown in the figure below:



Here, $V = 100$ V

$I = 15.94$ A

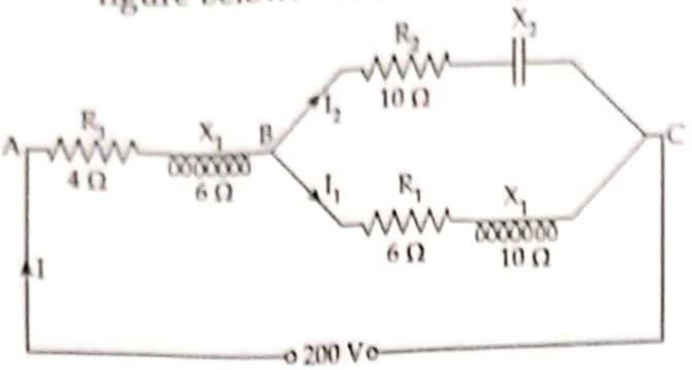
$\phi = 60°$

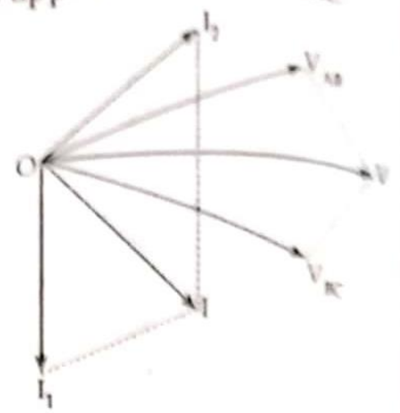
$$\therefore \quad \text{Impedance } (Z) = \frac{V}{I} = \frac{100}{15.94} = 6.27\ \Omega$$

Hence, $R = Z\cos\phi = 6.27\cos(60°) = 3.14\ \Omega$

and, $X = Z\sin\phi = 6.27\sin(60°) = 5.43\ \Omega$

8. Determine the current drawn by following circuit as shown in the figure below. When a voltage of 200 V is applied across a same





Solution:

From the given figure,

$$Z = 4 + j6 = 7.2 \angle 56.3^\circ$$
$$Z_1 = 6 + j10 = 11.7 \angle 58^\circ$$
$$Z_2 = 10 - j12 = 15.6 \angle -50.2^\circ$$

Now, equivalent resistance of $Z_1 \| Z_2 = \dfrac{(11.\angle 58)\,(15.6 \angle -50.2)}{(11.7 \angle 58) + (15.6 \angle -50.2)}$

$= 11.3 \angle 15.9^\circ$

Net equivalent impedance or circuit is;

$$Z' = Z + Z_1 \parallel Z_2 = (7.24 \angle 56.3) + 11.3 \angle 15.9 = 17.5 \angle 13.4^\circ$$

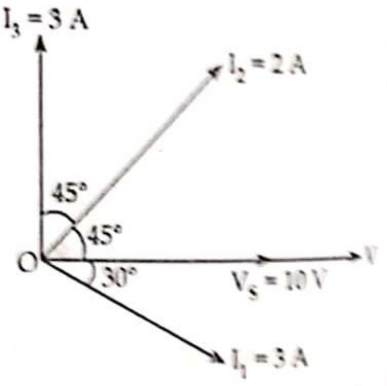
Now, Current $I = \dfrac{V}{Z} = \dfrac{200 \angle 0}{17.5 \angle 13.4} = 11.4 \angle 31.4$ Amp

The phasor diagram is shown in above figure. For drawing phasor diagram, we need

$$V_{AB} = I\,Z = 11.4 \angle -31.4 \times 7.2 \angle 56.3 = 82.2 \angle 24.9^\circ \text{ V}$$
$$V_{BC} = I\,Z_{BC} = 11.4 \angle -31.4 \times 11.3 \angle 15.9 = 128.8 \angle -15.5^\circ \text{ V}$$

Also $I_2 = \dfrac{V_{BC}}{Z_2} = \dfrac{128.8 \angle -15.5}{11.7 \angle 58^\circ} = 15.1 \angle -75.5^\circ$

9. In a particular AC circuits three impedances are connected in parallel, currents are shown in figure below are flowing through its parallel branches. Write the equations for the currents in terms of sinusoidal variations and drawn a waveforms. Also find the total current supplied by the source.



Take $V_S = 10$ V

Solution:

From given phasor

I_1 lags by 30° to the V

<u>For branch 1</u>

Current $(I_1) = 3A$

$\therefore \quad Z_1 = \frac{V_S}{I_1} = \frac{10}{3} = 3.333\ \Omega$

$\phi = 30^\circ$ (lagg)

$\therefore \quad R_1 = 3.333 \cos 30^\circ = 2.887\ \Omega$

$X_{L_1} = 3.333 \sin 30^\circ = 1.666\ \Omega$

For branch 2

I_2 leads V by 45°

and, $I_2 = 2A$

$\phi_2 = 45^\circ$

$\therefore \quad Z_2 = \frac{V}{I_2} = \frac{10}{2} = 5\ \Omega$

And, $R_2 = Z_2 \cos\phi = 5 \cos 45^\circ = 3.536\ \Omega$

And, $X_{12} = Z_2 \sin\phi = 5 \sin 45^\circ = 3.536\ \Omega$

For branch 3

I_3 leads V by 90°

$I_3 = 3A,\ \phi = 90^\circ$

$\therefore \quad Z_3 = \frac{10}{3} = 3.333\ \Omega$

Also, $R_3 = Z_3 \cos 90^\circ = 0$

$X_{C_3} = Z_3 \sin 90^\circ = 3.333\ \Omega$

Now, total current supplied by the source,

$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ &= 3\,[\cos 30^\circ - j \sin 30^\circ] + 2[\cos 45 + j \sin 45] + 3\,[0 + j1] \\ &= 4.0123 + j\,2.9142 = 4.96 \text{ Amp leading } V_S \text{ by } 36^\circ \end{aligned}$$

Again, expression for currents

Let supply frequency be 50 Hz

Then $V_{rms} = 10\sqrt{2} \sin(314\,t)$

$I_1 = 3\sqrt{2}\ \sin(314\,t - 30^\circ)$

$I_2 = 2\sqrt{2}\ \sin(314\,t + 45^\circ)$

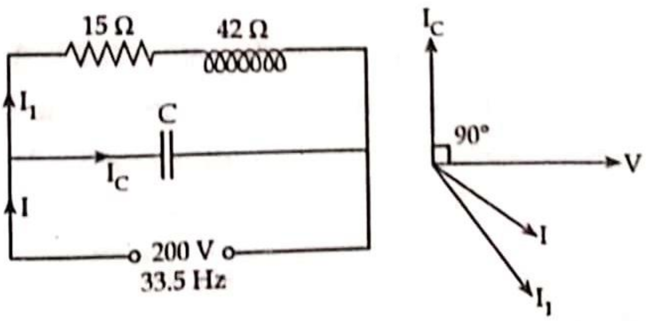
$I_3 = 3\sqrt{2}\ \sin(314\,t + 90^\circ)$

And, total current $i_{(t)} = 4.96\sqrt{2}\ \sin(314\,t + 36^\circ)$

Again, total power consumed $= IV \cos\phi = V.\,(I\cos\phi) = 10 \times 4.0123$

$= 40.123$ watt

10. An inductive coil of resistance 15 Ω and inductive resistance 42 Ω is connected in parallel with a capacitor of capacitive reactance of 47.6 Ω.



The combination is energized from a 200 V, 33.5 Hz ac. supply. Find the total current drawn by the circuit and its power factor. Draw to the scale. The phasor diagram of the circuit.

Solution:

Given that;

$$Z_1 = 15 + j42 = 44.6 \angle 70.36° \text{ (lagging)}$$

$$I_C = \frac{V}{X_C} = \frac{200}{47.6} = 4.2 \text{ Amp}$$

and, $$I_1 = \frac{V}{Z_1} = \frac{200}{44.6} = 4.484 \text{ Amp}$$

Now, current drawn in the circuit is,

$$I = 4.484\,(0.3355 - j.0.942) + j\,4.2 = 1.50 - j0.025 = 1.50 \angle -1°$$

The circuit diagram and phasor is shown in above figure.

For power calculation

<u>For branch A</u>

The current conjugate of (10 + j0) is → $I^* = (10 - j0)$

$\therefore \quad V_{I_A} = (120 + j160)\,(10 - j0) = 1200 + j1600$

Therefore, active power = 1200 watt

Reactive power = 1600 VAR

and, Volt ampere $= \sqrt{(1200)^2 + (1600)^2} = 2000$

$\therefore$ KVA = 2

<u>For branch B</u>

The conjugate current of (–4.0 + j8) is → $I^* = (-4.0 - j8)$

$\therefore \quad C_{I_B} = (120 + j160)\,(-4.0 - j8) = 800 - j\,1600$

Now,

Active power = 800 watt

Reactive power = 1600 VAR

and, Volt ampere $= \sqrt{(800)^2 + (1600)^2} = 1788$ VA = 1.788 KVA

Now, in the form of admittance,

$$Y = Y_A + Y_B = (0.03 - j\,0.04) + (0.02 + j\,0.04) = 0.05 + j0$$

$$I = VY = (120 + j160)\,(0.05 + j0)$$

$$= 6 + 8\,j$$

$$= 10 \angle 53.8°$$

and, $$I = I_A + I_B = (10 + j0) + (-4 + j8) = (6 + j8)$$

Since, both currents are same, hence, there is no phase difference.

Hence, power factor of circuit = cos 0 = 1

POWER IN AC CIRCUITS

7.1 POWER IN RESISTIVE CIRCUIT

Consider a purely resistive AC circuit. For this, we have voltage and current lies in the same phase. So,

$$V = V_m \sin \omega t$$

$$i = I_m \sin \omega t$$

Now, instantaneous power

$$P = V.i$$

or, $$P = V_m \sin \omega t \times I_m \sin \omega t$$

$$= V_m I_m \sin^2 \omega t$$

$$= \frac{V_m I_m}{2}(1 - \cos 2\,\omega t)$$

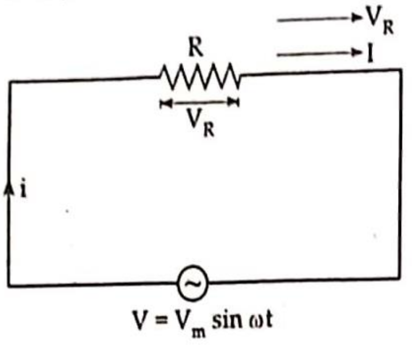
$$P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\,\omega t \qquad (1)$$

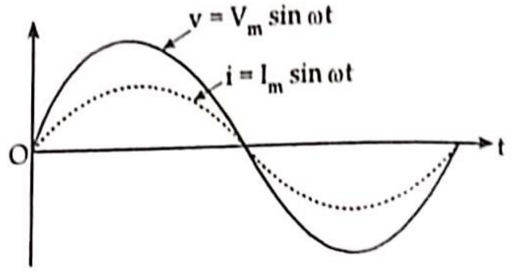
Here, we can see that power consists of a constant part $\frac{V_m I_m}{2}$ and a fluctuating part $\frac{V_m I_m}{2}$ cos 2 ωt of frequency double than that of voltage and current waves. For a complete cycle, the average value of $\frac{V_m I_m}{2}$ cos 2 ωt is zero. Hence, power for whole cycle is;

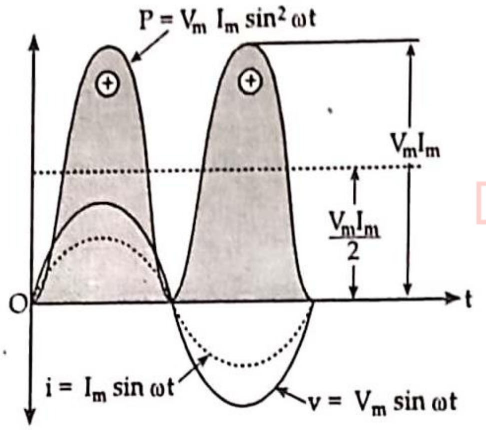
$$\therefore \quad P_{avg} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

or, $\boxed{P_{avg} = I \times V}$ Watt

where, V = rms value of the applied voltage, I = rms value of the current.







From above figure, we can see that no part of the power cycle becomes negative at any time. Hence, for purely resistive circuit, power is never zero because the instantaneous value of voltage and current are always either both positive or negative and hence, the product is always positive.

7.2 POWER IN INDUCTIVE AND CAPACITIVE CIRCUIT

7.2.1 Power in Inductive Circuit

Consider a purely inductive circuit containing an inductance L. Now, let, $V = V_m \sin \omega t$ and $I = I_m \sin(\omega t - 90°)$ is value of voltage and current in the circuit instantaneously. Here, voltage leads current by 90°. Then, instantaneous power is given as;

$$P = Vi = V_m \sin \omega t \times I_m \sin(\omega t - 90°)$$

or, $$P = V_m I_m \sin \omega t \times \sin(\omega t - 90°)$$

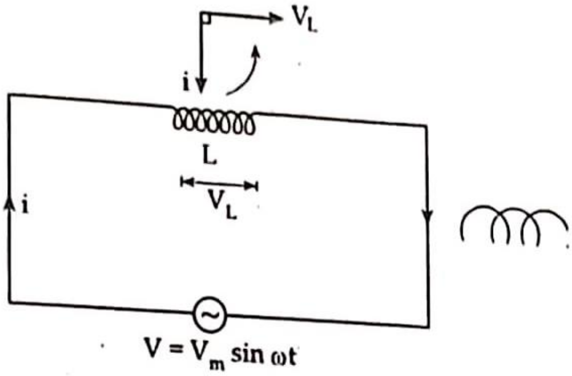
or, $$P = -V_m I_m \sin \omega t \cos \omega t \qquad [\because \sin(\omega t - 90°) = -\cos \omega t]$$

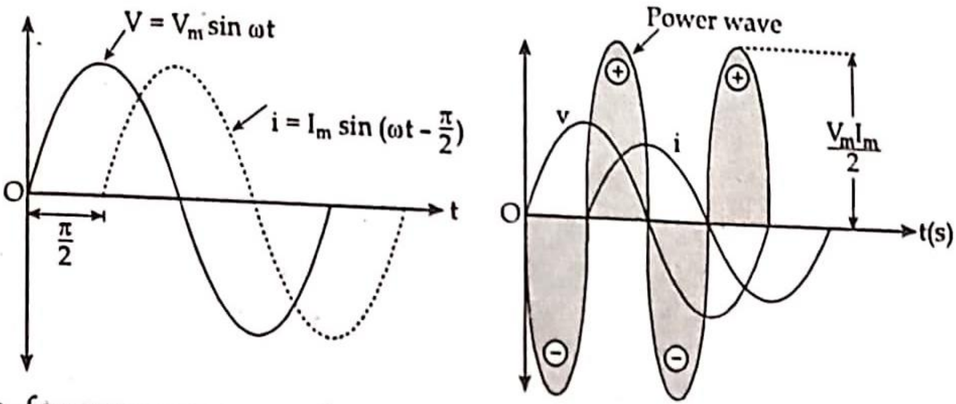
or, $P = \frac{-2 \times V_m I_m}{2} \sin \omega t \cos \omega t$

or, $P = -\frac{V_m I_m}{2} \sin 2 \omega t$

$[\because 2 \sin \omega t \cos \omega t = \sin 2 \omega t]$ (2)

The wave form of I, V and instantaneous power is shown in figure below.





Again, for average power for complete cycle, we have

$$P_{avg} = \frac{1}{\pi} \int_0^{\pi} -\frac{V_m I_m}{2} \sin 2 \omega t \, dt$$

or, $\boxed{P_{avg} = 0}$

Hence, in purely inductive circuit, average power for complete cycle is always zero. From above figure, we can see that the power wave is sine wave of frequency double than that the voltage and current waves. The maximum value of the instantaneous power is $\frac{V_m I_m}{2}$.

Also, when the current through the inductor is increasing, energy is transferred from the circuit to the magnetic field but this energy is returned when the current is decreasing and hence, the average power is zero.

7.2.2 Power in Capacitive Circuit

Consider a pure capacitive circuit for which voltage $V = V_m \sin \omega t$ is applied. For purely capacitive circuit, current flows leads the voltage by 90°. Thus, current is given as $I = I_m \sin (\omega t + 90°)$. Now, instantaneous power is given as;

$$P = Vi = V_m \sin \omega t \times I_m \sin (\omega t + 90°)$$

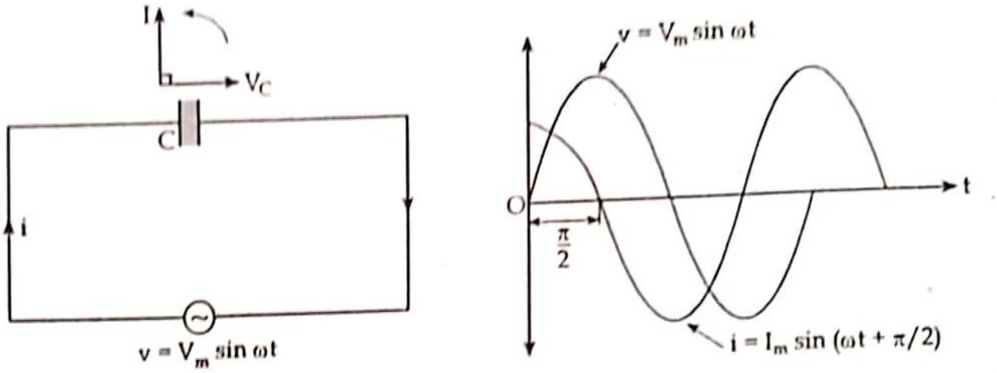
or, $\quad P = V_m I_m \sin \omega t \cos \omega t$ $\quad [\because \sin(\omega t + 90°) = \cos \omega t]$

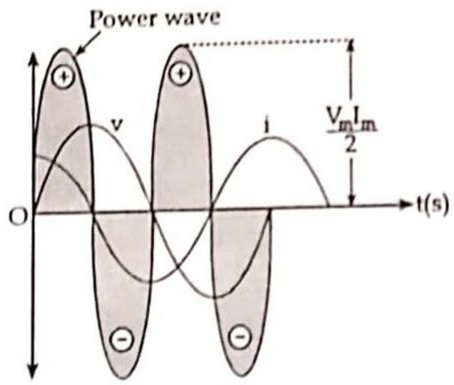
or, $\quad \boxed{P = \frac{1}{2} V_m I_m \sin 2\omega t}$

Again, average power for whole cycle is,

$$P_{avg} = \frac{1}{2} V_m I_m \frac{1}{\pi} \int_0^{\pi} \sin 2\omega t \, dt$$

or, $\quad \boxed{P_{avg} = 0}$





From above figure, we find that the average demand of power from supply is zero (same as purely inductive circuit). Again it is seen that power wave is sine wave of frequency double than that the voltage and current source waves. The maximum value of instantaneous power is $\frac{V_m I_m}{2}$.

7.3 POWER IN CIRCUIT WITH RESISTANCE AND REACTANCE

Consider an electric circuit containing resistance and either inductance or capacitance. Let, $V = V_m \sin \omega t$ be the voltage applied, then current should be $i = I_m \sin(\omega t \pm \theta)$ where, positive phase for capacitive circuit and negative phase for inductive circuit.

Now, instantaneous power is given by

$$P = V.i = V_m \sin \omega t \times I_m \sin(\omega t - \theta), \text{ (taking only negative phase)}$$

or, $\quad P = V_m I_m \sin \omega t \sin(\omega t - \theta)$

or, $\quad P = V_m I_m \times \frac{1}{2} [2 \sin \omega t \sin(\omega t - \theta)]$

or, $P = \frac{V_m I_m}{2} [-\cos(\omega t + \omega t - \theta) + \cos(\omega t - (\omega t - \theta))]$

or, $P = \frac{V_m I_m}{2} [\cos\theta - \cos(2\omega t - \theta)]$ (4)

Hence, the instantaneous power consists of two parts.

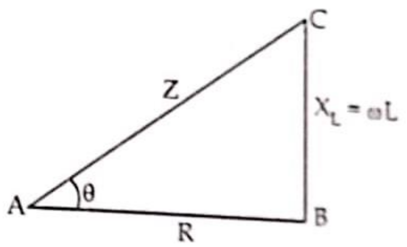
(i) A constant part $\frac{V_m I_m}{2} \cos\theta$. Which contributes to the real power.

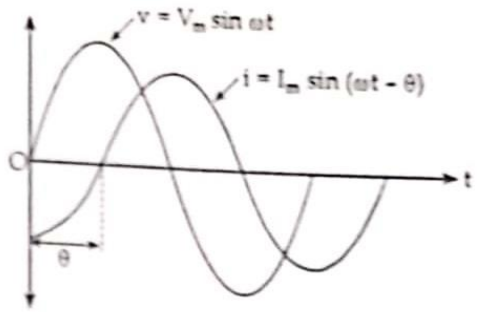
(ii) A Pulsating part $\frac{1}{2} V_m I_m \cos(2\omega t - \theta)$ which has a frequency twice than that of the voltage and current. It does not contribute to the actual power. Since its actual value over a complete cycle is zero. Hence, the average power consumed per cycle is,

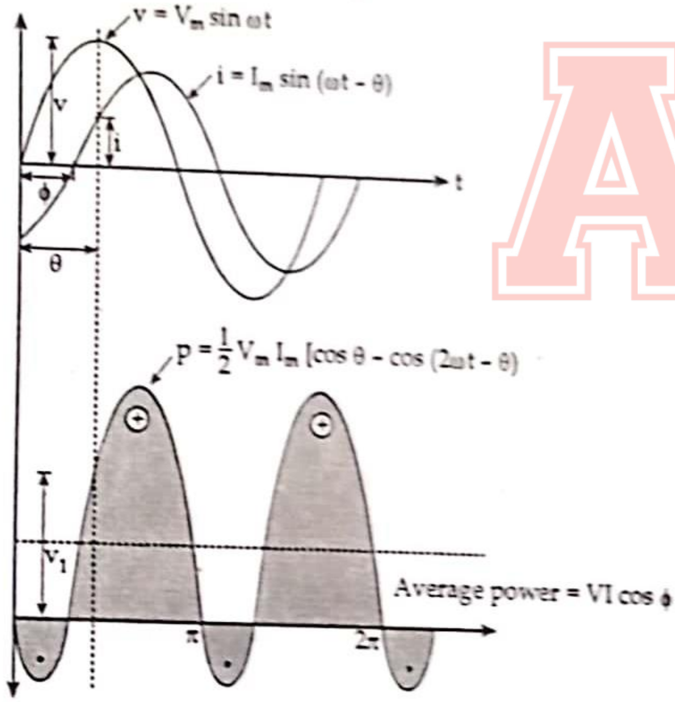
$$P_{avg} = \frac{1}{2} V_m I_m \cos\theta = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\theta$$

$$\boxed{P_{avg} = V_{rms} \times I_{rms} \times \cos\theta} \qquad (5)$$

Here, the term $\cos\theta$ is called power factor of the circuit. The graphical representation of the power consumed is shown in the figure below.







The above figure is for R_L circuit. Similar power curve can be obtained for RC circuit. The difference between the areas above and below the horizontal axis represent the heat loss due to circuit resistance.

Example 7.1

A coil takes a current of 2 A lagging 60° behind the applied voltage of 200 V at 50 Hz. Calculate the inductance, resistance and impedance of the coil. Also determine the power consumed when it is connected across 100 V, 25 Hz supply.

Solution:

Here, Supply voltage (V) = 200 V

Supply current (I) = 2 A

Also, reactance of coil,

$$Z_C = \frac{V}{I} = \frac{200}{2} = 100\ \Omega$$

Also given that, $\phi = 60°$

$R = Z \cos \phi$

or, $R = 100 \cos 60° = 50\ \Omega$

and, $X_L = Z \sin \phi$

or, $X_L = 100 \sin 60°$

or, $X_L = 86.6\ \Omega$

$\therefore$ $X_L = 86.6\ \Omega$

and, $L = \frac{X_L}{2\pi f}$

or, $L = \frac{86.6}{2\pi \times 50}$

or, $L = 0.275H$

Again, when the coil is connected with 100 V, 25 H, then its resistance remains same but inductance changed, since, frequency is changed.

$\therefore$ $X_L = 2\pi fL = 2\pi \times 25 \times 0.275$

or, $X_L = 43.3\ \Omega$

and, $Z = \sqrt{(50)^2 + (43.3)^2} = 66.1\ \Omega$

and, Current through coil

$$I = \frac{100}{66.1} = 1.513\ A$$

and, Power factor $(\cos \theta) = \frac{R}{Z} = \frac{50}{66.1} = 0.76$

and, Power consumed by the coil is

$$P = VI \cos \theta = 100 \times 1.513 \times 0.76 = 114.998 = 115 \text{ watt}$$

Example 7.2

In a series circuit containing pure resistance and pure inductance. The current and voltage is given as $i(t) = 5 \sin\left(314t + \frac{2\pi}{3}\right)$ and $V(t) = 15 \sin\left(314t + \frac{5\pi}{6}\right)$.

(a) What is the impedance of the circuit.

(b) What is the value of the resistance.

(c) What is the value of inductance.

(d) What is the average power consumed by the circuit.

(e) What is the power factor?

Solution:

Given that,

$$i(t) = 5 \sin\left(314t + \frac{2\pi}{3}\right)$$

$$V(t) = 15 \sin\left(314t + \frac{5\pi}{6}\right)$$

so, $I_m = 5A$

$V_m = 15\ V$

$\therefore \quad Z = \dfrac{V_m}{I_m} = \dfrac{15}{5} = 3\ \Omega$

and, $\omega = 314$ and phase angle of current is $\theta_I = \dfrac{2\pi}{3} = \dfrac{2 \times 180}{3} = 120°$

Phase angle for voltage

$$\theta_V = \frac{5\pi}{6} = \frac{5 \times 180°}{6} = 150°$$

Hence, $\theta = \theta_V - \theta_I = 30$; voltage leads current by 30°. So, it is R-L circuit.

Now, $\omega = 2\pi f$

or, $314 = 2\pi f$

or, $f = 50\ Hz$

and, Power factor

$\dfrac{R}{Z} = \cos\theta$

or, $\dfrac{R}{3} = \cos 30°$

or, $R = 3 \times \cos 30°$

or, $R = 2.6\ \Omega$

Also, $\sin 30° = \dfrac{X_L}{Z}$

or, $X_L = 3 \times \dfrac{1}{2} = 1.5\ \Omega$

or, $2\pi fL = 1.5$

or, $L = \dfrac{1.5}{2 \times \pi \times 50} = 4.78\ mH$

Now,

(a) Impedance = 3 Ω

(b) Resistance = 2.6 Ω

(c) Inductance = 4.78 mH

(d) Power = $I_{rms} \times V_{rms} \times \cos\theta$

$$= \frac{V_m I_m}{2} \cos\theta$$

$$= \frac{5 \times 15}{2} \times \cos 30^\circ$$

$$= 32.476 \text{ Watt}$$

(e) Power factor = $\cos 30^\circ = 0.866$ (lag)

7.4 ACTIVE, REACTIVE AND APPARENT POWER

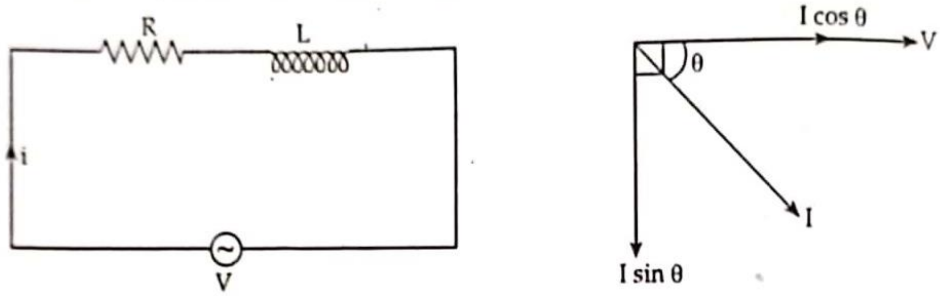


Figure: Series R-L circuit and phasor diagram

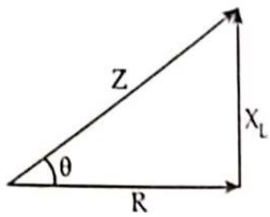
Let us consider a series R-L circuit, draw current I when an alternating voltage of rms value V is applied to it. Suppose, the current lags behind the voltage by angle θ. From phasor diagram, component of I cos θ is in phase with V and I sin θ is quadrature component of current. Then, we have three types of power, that are;

(i) Active power

The power due to the active component of current is called active power. It is the product of voltage V and in phase component of current I cos θ.

From figure;

$$\cos\theta = \frac{R}{Z}$$



Mathematically,

Active power (P) = $V \times I \cos\theta$

Unit = Watt (W)

Here;

$$P = VI\cos\theta$$

or, $$P = (IZ)\,.\,I \times \frac{R}{Z}$$

or, $$P = I^2\,.\,R$$

Active power is also called real power. It is power that is consumed in circuit and produce heat. So it is also called consumed power.

(ii) Reactive power

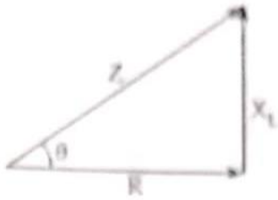
The power due to reactive component of the current is called reactive power. It is product of voltage and quadrature component of current I sin θ.

Mathematically,

Reactive power (Q) = VI sin θ

Its unit is volt ampere reactive (VAR)

Here;



$$\sin\theta = \frac{X_L}{Z}$$

so, $$Q = VI\left(\frac{X_L}{Z}\right)$$

or, $$Q = (IZ) \times I \times \left(\frac{X_L}{Z}\right)$$

or, $$\boxed{Q = I^2 X_L}$$

The reactive power Q, consume nothing to the total energy transfer. It is the term used in the power generating, distribution and utilization of electrical energy.

NOTE

Inductive reactive power is positive reactive power.

Capacitive reactive power is negative reactive power.

(iii) Apparent power

Apparent power is the total power in the circuit and it is defined as the product of voltage and current and its unit is volt-ampere.

Mathematically,

Apparent power (S) = P + jQ

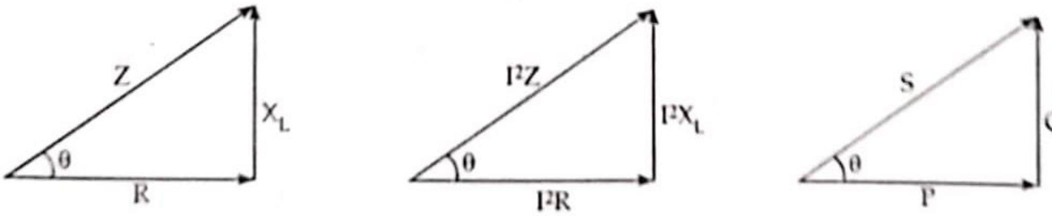
or, $$S = \sqrt{P^2 + Q^2}$$

Also $$\boxed{S = VI = I^2 \times Z}$$

It is an important quantity because almost all AC products are rated as volt-ampere and not Watt.

Power triangle

From the impedance triangle, called power triangle, the power triangle is a right angled triangle, with P and Q as two sides and S as hypotenous. The angle between the base and hypotenous is θ.



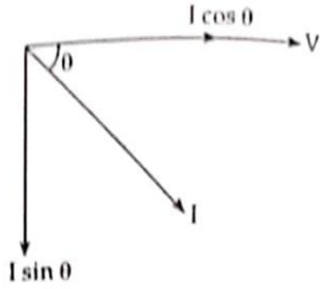
Therefore, from power triangle

$$S = \sqrt{P^2 + Q^2}$$

$$\text{Power factor } (\cos\phi) = \left(\frac{R}{Z}\right) = \frac{P}{S}$$

7.5 POWER FACTOR AND ITS PRACTICAL IMPORTANCE

The cosine of angle between voltage and current in an AC circuit is known as power factor.



In an AC circuit, there is generally a phase difference θ between voltage and current. The term cos θ is called the power factor of the circuit. If the circuit is inductive the current lags behind the voltage and the power factor is referred to as lagging. However in a capacitive circuit, current leads the voltage and power factor is said to be leading.

From figure, I cos θ is active component and I sin θ is reactive component. The reactive component is measure of the power factor. If the reactive component is small, the phase angle θ is small and hence, power factor cos θ will be high. Therefore a circuit having small reactive current (I sin θ) will have high power factor and vice versa. It may be noted that, value of power factor never be more than unity.

- We have to write lagging or leading with power factor while solving numerical problems. So, if the circuit has a power factor of 0.5 and it lags the voltage, we generally write power factor as 0.5 lags.
- Some time it may be expressed in percentage *i.e.*, 50%

Cause of low power factor

Low power factor is undesirable from economics point of view. Normally, the power factor of the whole load on the supply system is lower than 0.8. The following are causes of, low power factor;

i) Most of the motors are inductive type, which have low power factor lagging. These motors works at a power factor which is extremely small on light load (0.2 to 0.3) and rises to 0.8 or 0.9 at full load.

ii) Arc lamps, electric discharge lamps and industrial heating furnace operates at low lagging power factor.

iii) The load in the power system is varying, being high during morning and evening and low at other times. During low load period, supply voltage increased which increases the magnetization current. This results in the decreased power factor.

Disadvantages of low power factor

The power factor plays an important role in AC circuits. Since, power consumed depends upon this factor.

$$P = V_L I_L \cos\theta$$

or, $$\boxed{I_L = \frac{P}{V_L \cos\theta}}$$

It is clear from above that, for fixed power and voltage, the load current is inversely proportional to the power factor. Lower the power factor higher is the load currents and vice versa. A power factor less than unity of equipments cause following disadvantages;

i) Large KVA rating of equipments.
ii) Large conductor size required.
iii) Large copper losses.
iv) Poor voltage regulation.
v) Reduced handling capacity of system.

7.6 IMPROVEMENT OF POWER FACTOR

The low power factor is due to the loads, which take lagging currents. So, to improve power factor, some device taking leading power factor should be connected in parallel with the load such as capacitor. The capacitor draws a leading current and partly or completely neutralises the lagging reactive component of load current. This rises the power factor of load.

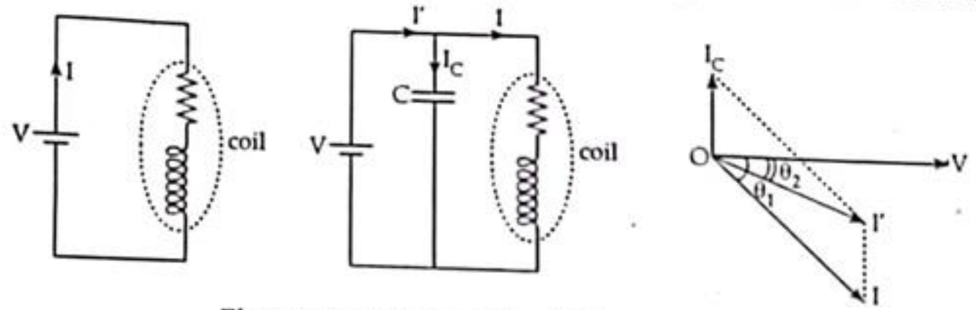


Figure: Arrangement for PF improvement

Consider a single phase load, taking lagging current I at power factor cos θ as shown in above figure. The capacitor C is connected in parallel with the load and draw current I_C, which leads the voltage by 90°. The resulting line current I' is the phasor sum of I and I_C and its lag angle is θ_2 as shown in figure (c). It is clear that θ_2 is less than θ_1, so that cos θ_2 is greater than cos θ_1 and hence the power factor of the load is improved.

- Current I' after power factor correction is less than I.
- The active components remain same before and after power factor correction because only the lagging reactive component is reduced by the capacitor.

  i.e., $I \cos \theta_1 = I' \cos \theta_2$

  and $VI \cos \theta_1 = VI' \cos \theta_2$

  So active power also remains unchanged.
- The lagging reactive component is reduced by I_C.

  $\therefore \quad I' \sin \theta_2 = I \sin \theta_1 - I_C$

  also, $VI' \sin \theta_2 = IV \sin \theta_1 - VI_C$

  Hence, reactive power reduced after power factor correction by amount of power consumed at capacitor. *i.e.,* VI_C.

Power factor improvement equipments

(a) Static capacitor

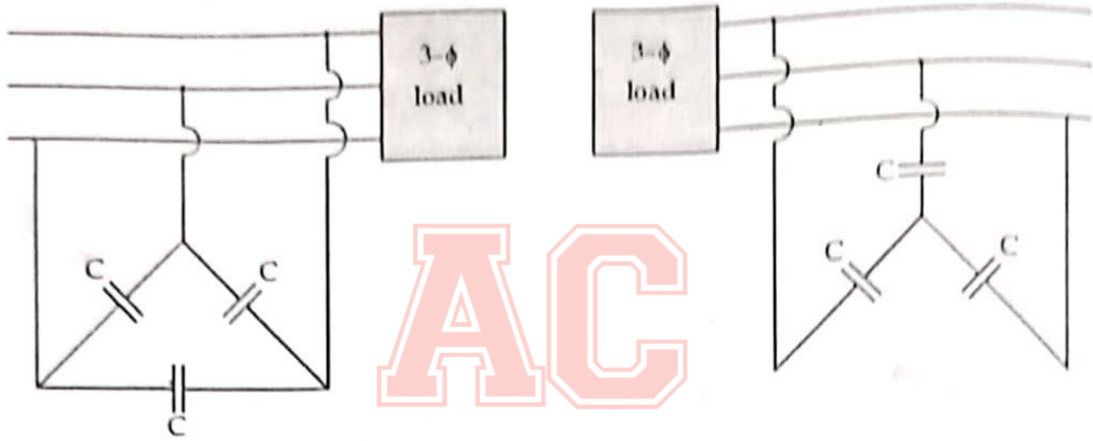


Figure: Static Capacitor

The power factor can be improved by connecting capacitors in parallel with the equipment operating at lagging power factor. The capacitor draws leading current and neautralise the lagging reactive component of load current. This rises the power factor of the load. For three phase loads, the capacitors can be connected in either star or delta as above figure.

(b) Synchronous condenser

A synchronous motor takes the leading current when over excited and therefore, behaves as a capacitor. An over excited synchronous motor running on no load is called synchronous condenser. When such a machine is connected in parallel with the supply, it takes leading current which partly nutralises the lagging reactive component of the load. Thus power factor is improved.

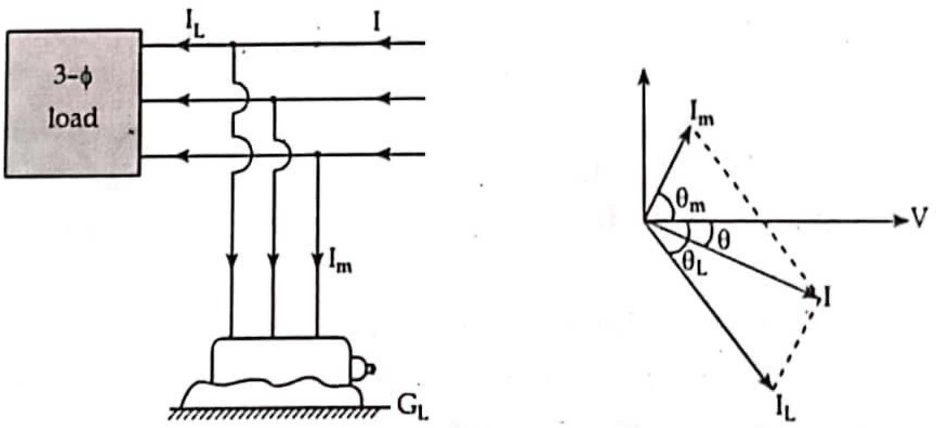
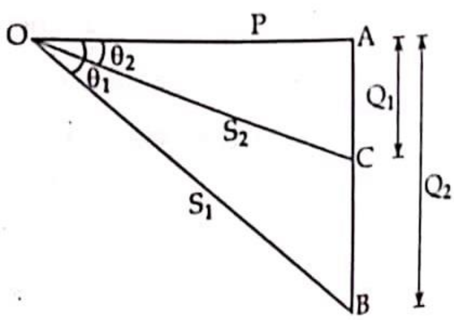


Figure: 3-ϕ synchronous motor

Power triangle

The power factor correction can also be illustrated from power triangle. In the figure below, the power triangle OAB is for power factor cos θ_1 where as power triangle OAC is for the improved power factor cos θ_2. It may be seen that active power (OA) does not change with power



factor improvement. However the lagging reactive power of the load is reduced by the power factor correction equipment. Thus improving the power factor to $\cos\theta_2$.

Now, leading K-VAR supplied by power factor correction equipment is;

$= BC$

$= AB - AC$

$= Q_2 - Q_1$

$= OA \tan\theta_1 - OA \tan\theta_2$

$= OA (\tan\theta_1 - \tan\theta_2)$

$= P(\tan\theta_1 - \tan\theta_2)$

Now, by knowing the leading kVAR supplied by the power correction equipment, the desired result can be obtained.

Example 7.3

An alternator is supplying a load of 300 kW at a p.f. of 0.6 lagging. If the power factor is raised to unity, how many more kilowatts can alternator supply for the same kVA loading?

Solution:

$$kVA = \frac{kW}{\cos\varphi} = \frac{300}{0.6} = 500 \text{ kVA}$$

kW at 0.6 p.f. = 300 kW

kW at 1 p.f. = 500 × 1 = 500 kW

∴ Increased power supplied by the alternator is;

= 500 − 300 = 200 kW.

NOTE

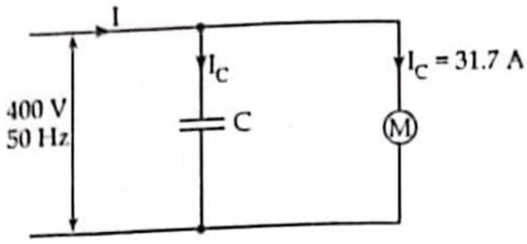
When the p.f. of the alternator is unity, the 500 kVA are also 500 kW and the engine driving the alternator has to be capable of developing this power together with the losses in the alternator. But when the power factor of the load is 0.6, the power is only 300 kW. Therefore, the engine is developing only 300 kW, though the alternator is supplying its rated output of 500 kVA.

Example 7.4

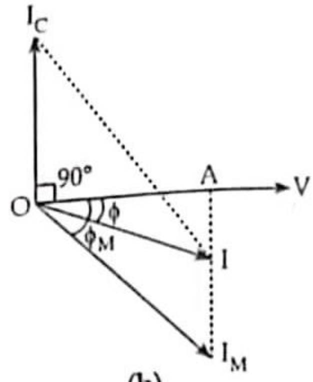
A single phase motor connected to 400 V, 50 Hz supply takes 31.7A at a power factor of 0.7 lagging. Calculate the capacitance required in parallel with the motor to raise the power factor to 0.9 lagging.

Solution:

The circuit and phasor diagrams are shown in Figures (a) and (b) respectively. Here motor M is taking a current I_M of 31.7A. The current I_C taken by the capacitor must be such that when combined with I_M, the resultant current I lags the voltage by an angle ϕ where $\cos\phi = 0.9$



(a)



(b)

Referring to the phasor diagram in figure (b).

Active component of $I_M = I_M \cos \phi_M = 31.7 \times 0.7 = 22.19$ A

Active component of $I = I \cos \phi = I \times 0.9$

These components are represented by OA in Fig. 6.9.

$$\therefore \quad I = \frac{22.19}{0.9} = 24.65 \text{ A}$$

Reactive component of $I_M = I_M \sin \phi_M = 31.7 \times 0.714 = 22.6$ A

Reactive component of $I = I \sin \phi \; = 24.65 \sqrt{(1 - 09)^2}$

$$= 24.65 \times 0.436 = 10.75 \text{ A}$$

It is clear from figure (b) that :

$$I_C = \text{Reactive component of } I_M - \text{Reactive component of I}$$

$$= 22.6 - 10.75 = 11.85\text{A}$$

But $\quad I_C = \dfrac{V}{X_C} = V \times 2\pi f C$

or, $\quad 11.85 = 400 \times 2\pi \times 50 \times C$

$\therefore \quad C = 94.3 \times 10^{-6} \text{ F} = 94.3\ \mu\text{F}$

NOTE

The effect of connecting a 94.3 μF capacitor in parallel with the motor. The current taken from the supply is reduced from 31.7 A to 24.65 A without altering the current or power taken by the motor. This enables an economy to be affected in the size of generating plant and in the cross-sectional area of the conductors.

Example 7.5

A single phase a.c. generator supplies the following loads:

(i) Lighting load of 20 kW at unity power factor.

(ii) Induction motor load of 100 kW at p.f. 0.707 lagging.

(iii) Synchronous motor load of 50 kW at p.f. 0.9 leading.

Calculate the total kW and kVA delivered by the generator and the power factor at which it works.

Solution:

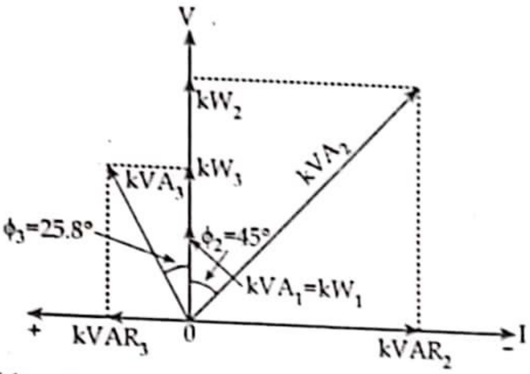
Using the suffixes 1, 2 and 3 to indicate the different loads, we have,

$$kVA_1 = \frac{kW_1}{\cos\phi_1} = \frac{20}{1} = 20 \text{ kVA}$$

$$kVA_2 = \frac{kW_2}{\cos\phi_2} = \frac{100}{0.707} = 141.4 \text{ kVA}$$

$$kVA_3 = \frac{kW_3}{\cos\phi_3} = \frac{50}{0.9} = 55.6 \text{ kVA}$$

These loads are represented in the given figure. The tree kVAs' are not in phase. In other to find the total kVA, we resolve each kVA into rectangular components - kW and kVAR as shown in figure 6.10. The total kW and kVAR may then be combined to obtain total kVA.



$$kAVR_1 = kVA_1 \sin\phi_1 = 20 \times 0 = 0$$

$$kAVR_2 = kVA_2 \sin\phi_2 = -141.4 \times 0.707 = -100 \text{ kVAR}$$

$$kAVR_3 = kVA_3 \sin\phi_3 = +55.6 \times 0.436 = +24.3 \text{ kVAR}$$

Note that $kVAR_2$ and $kVAR_3$ are in opposite direction; $kVAR_2$ being a lagging while $kVAR_3$ being a leading kVAR

$$\text{Total kW} = 20 + 100 + 50$$

$$= 170 \text{ kW}$$

$$\text{Total kVAR} = 0 - 100 + 24.3$$

$$= -75.7 \text{ kVAR}$$

$$\text{Total kVA} = \sqrt{(kW)^2 (kVAR)^2}$$

$$= \sqrt{(170)^2 + (75.7)^2}$$

$$= 186 \text{ kVA}$$

$$\text{Power factor} = \frac{\text{Total kW}}{\text{Total kVA}}$$

$$= \frac{170}{186} = 0.914 \text{ lagging}$$

The power factor must be lagging since the resultant kVAR is lagging.

Example 7.6

A 3-phase, 50 Hz, 400 V motor develops 100 H.P. (74.6 kW), the power factor being 0.75 lagging and efficiency 93%. A bank of capacitors is connected in delta across the supply terminals and power factor raised to

0.95 lagging. Each of the capacitance units of 4 similar 100 V capacitors. Determine the capacitance of each capacitor.

Solution:

Original p.f. $\cos\phi_1 = 0.75$ lag

Final p.f. $\cos\phi_2 = 0.85$ lag

Motor input $\quad P = \dfrac{\text{Output}}{h} = \dfrac{74.6}{0.93} = 80 \text{ kW}$

$$\phi_1 = \cos^{-1}(0.75) = 41.41°$$

$$\tan\phi_1 = \tan 41.41° = 0.8819$$

$$\phi_2 = \cos{-1}\,(0.95) = 18.19°$$

$$\tan\phi_2 = \tan 18.19° = 0.3288$$

Leading kVAR taken by the condenser bank

$$= P(\tan\phi_1 - \tan\phi_2)$$

$$= 80(0.8819 - 0.3288)$$

$$= 44.25 \text{ kVAR}$$

Leading kVAR taken by each of three sets

$$= \frac{44.25}{3} = 14.75 \text{ kVAR} \quad (1)$$

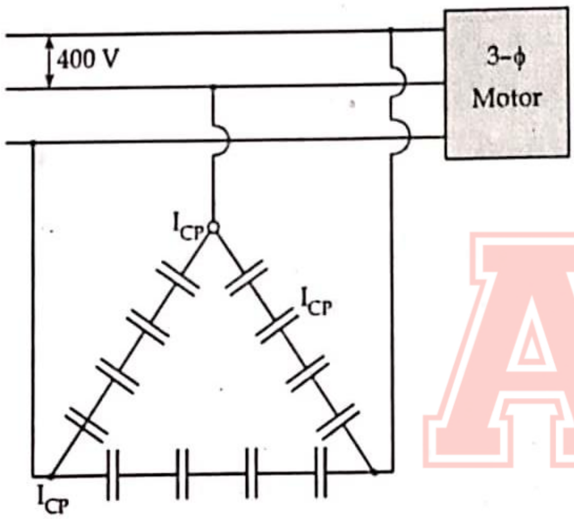


Figure shows the delta connected condenser bank. Let C farad be the capacitance of 4 capacitors in each phase.

Phase current of capacitor is

$$L_{CP} = \frac{V_{ph}}{X_C} = 2\pi f C V_{ph}$$

$$= 20 \times 50 \times C \times 400$$

$$= 1{,}25{,}600 \text{ C amperes}$$

$$\text{kVAR/phase} = \frac{V_{ph} I_{CP}}{1000}$$

$$= \frac{400 \times 1{,}25{,}600 \text{ C}}{1000}$$

$$= 50240 \text{ C} \quad (2)$$

Equating express (1) and (2), we get;

$$50240 = 14.75$$

$$C = \frac{14.75}{50,240} = 293.4 \times 10^{-6}\ F = 293.4\ \mu F$$

Since it is the combine capacitance of four equal capacitors joined in series.

Therefore, capacitance of each capacitor = $4 \times 293.4 = 1173.6\ \mu F$

Example 7.7

The load on an installation is 800 kW, 0.8 lagging p.f. which works for 3000 hours per annum. The tariff is Rs. 100 per kVA plus 20 paise per kWh. If the power factor is improved to 0.9 lagging by means of loss-free capacitors costing Rs. 60 per kVAR, calculate the annual saving effected. Allow 10% annum for interest and depreciation on capacitors.

Solution:

Load, P = 800 kW

$$\cos\phi_1 = 0.8 : \tan\phi_1 = \tan(\cos^{-1})\ 0.8) = 0.75$$

$$\cos\phi_2 = 0.9 : \tan\phi_2 = \tan(\cos^{-1} 0.9) = 0.4843$$

Leading kVAR taken by the capacitors

$$= P(\tan\phi_1 - \tan\phi_2) = 800\ (0.75 - 0.4843 = 212.56$$

Annual cost before p.f. correction

$$\text{Max. kVA demand} = \frac{800}{0.8} = 1000$$

kVA demand charges = Rs. 100 × 1000 = Rs. 100000

Units consumed/year = 800 × 3000 = 2400000 kWh

Energy charges/year = Rs. 0.2 × 2400000 = Rs. 480000

Total annual cost = Rs. (100000 + 180000) = Rs. 580000

Annual cost after p.f. correction

$$\text{Max. kVA demand} = \frac{800}{0.9} = 888.89$$

kVA demand charges = Rs. 100 × 888.89 = Rs. 88889

Energy charges = Same as before *i.e.*, Rs. 480000

Capital cost of capacitors = Rs. 60 × 212.56 = Rs. 12750

Total annual cost = Rs. (88889 + 480000 + 1275) = Rs. 570164

∴ Annual saving = Rs (5800000 – 570164) = Rs. 9836 n-type majority ... holes minority

DIODES

INTRODUCTION

The diode is one of the oldest, simplest, and most important semiconductor devices used in all sorts of electrical and electronic systems. In this chapter, we will study about diodes, its types, and application areas. But before exploring characteristics of diodes, deep **insights** on semiconductors is mandatory.

SEMICONDUCTORS

A semiconductor is a material that has a resistivity somewhere between that of a good conductor and that of an insulator. But the resistivity (or conductivity) of semiconductor changes considerably when even minute amounts of certain other substances, called the impurities, are added to them. Most semiconductor materials used in electronics industry have negative temperature coefficients. Examples of semiconductor materials include silicon (Si), germanium (Ge), gallium arsenide (GaAs), and indium arsenide (InAs).

Pure semiconductor material is known as intrinsic material. Before intrinsic material can be used in the manufacture of a device, impurity atoms must be added to improve its conductivity. The process of adding the atoms is termed doping. Two different types of doping are possible: donor doping and acceptor doping. Donor doping generates free electrons in the conduction band. Acceptor doping produces valence-band holes, or a shortage of valence electrons in the material. After doping, the semiconductor material is known as extrinsic material.

Donor-doped semiconductor is known as n-type semiconductor and acceptor-doped semiconductor is known as p-type semiconductor. Typical donor atoms (also known as pentavalent atoms) are antimony, phosphorus, and arsenic. Typical acceptor atoms (also known as trivalent atoms) are boron, aluminum, and gallium.

In n-type semiconductor, electrons are the majority charge carriers and holes are the minority charge carriers. Whereas in p-type semiconductor, holes are the majority charge carriers and electrons are the minority charge carriers.

A Silicon Atom

If we look at an isolated silicon atom, it contains four electrons in its valence shell. When atoms combine to form a solid crystal, each atom positions itself between four other silicon atoms in such a way that the valence shells overlap from one atom to another. This causes each individual valence electron to be shared by two atoms as shown in **Figure 2.1**. By sharing the electrons between four adjacent atoms, each individual silicon atom appears to have eight electrons in its valence shell. This sharing of valence electrons is called covalent bonding.

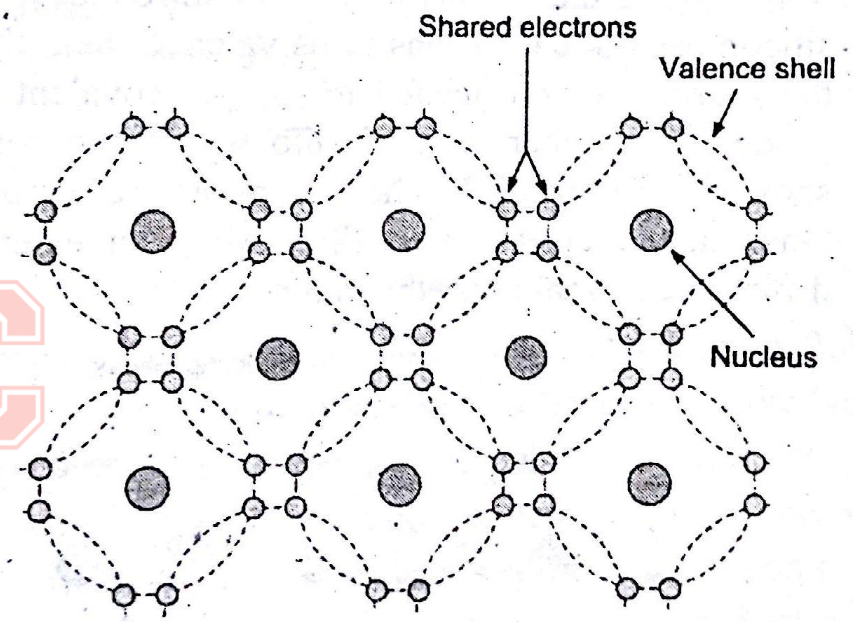


Figure 2.1 ***Lattice showing covalent bonding***

In its pure state, silicon is an insulator because the covalent bonding rigidly holds all of the electrons leaving no free (easily loosened) electrons to conduct current. If, however, an atom of a different element (i.e., an impurity) is introduced that has five electrons in its valence shell, a surplus electron will be present, as shown in **Figure 2.2**. These free electrons become available for use as charge carriers and they can be made to move through the lattice by applying an external potential difference to the materials.

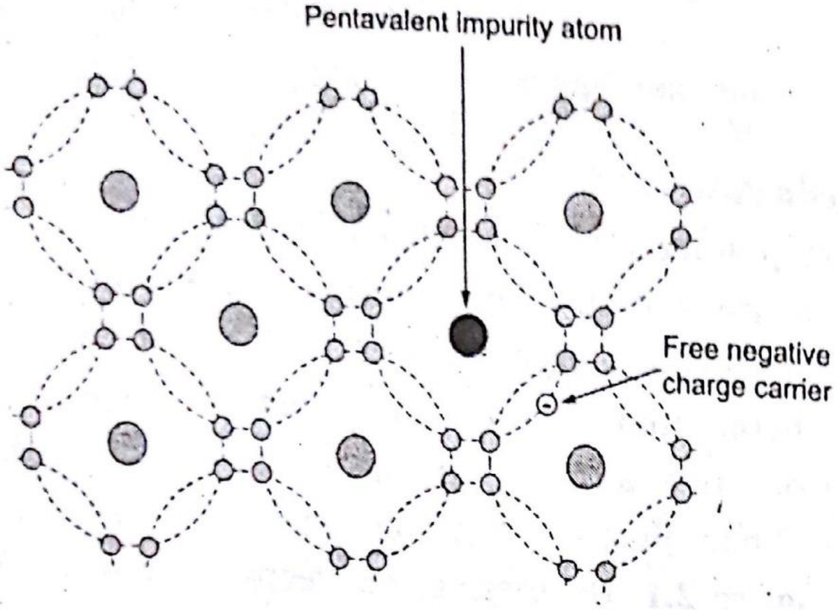


Figure 2.2 ***Free negative charge carriers (electrons) produced by introducing a pentavalent impurity***

Similarly, if the impurity element introduced into the pure silicon has three electrons in its valence shell, the absence of the fourth electron needed for proper covalent bonding will produce a number of gaps into which electrons can fit as shown in **Figure 2.3**. These gaps are referred to as holes. Once again, current will flow when an external potential difference is applied to the material.

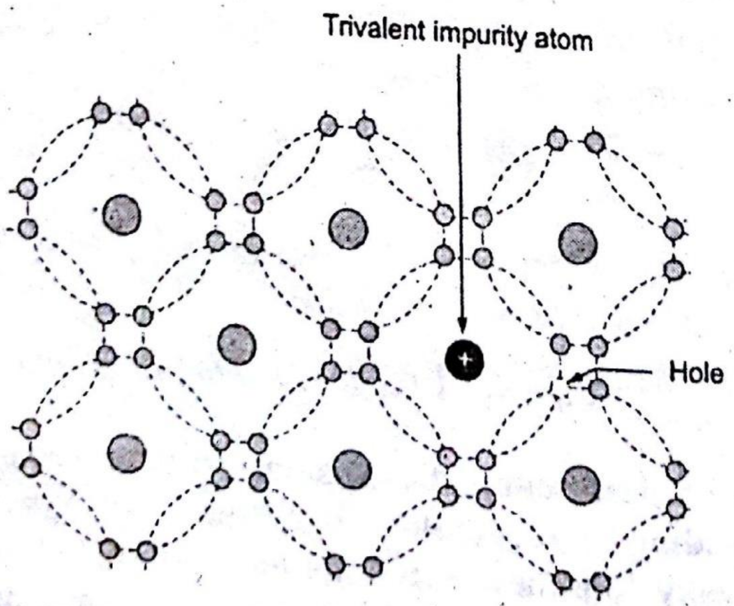


Figure 2.3 ***Holes produced by introducing a trivalent impurity***

Why Semiconductors?

The devices (such as diodes, transistors, etc.) manufactured from semiconductors have many advantages such as compact size, low cost, light weight, rugged construction, more resistive to shocks and vibrations, instantaneous operation (no heating required), low operating voltage, high operating efficiency (no heat loss), and long life with essentially no ageing effect if operated with permissible limits of temperature and frequency. Detailed description of these devices are given in the subsequent topics.

DIODES

The term "diode" refers to a two-electrode, or two-terminal device. Put simply, a diode is a one-way device, offering a low resistance when forward-biased, and behaving almost as an open switch when reverse biased.

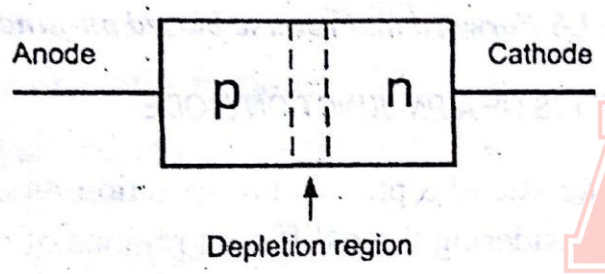


Figure 2.4 ***A pn-junction diode***

A semiconductor junction diode is shown in **Figure 2.4.** The connection to the p-type material is referred to as the "anode" while that to the n-type material is called the "cathode". With no externally applied potential, electrons from the n-type material will cross into the p-type region and fill some of the vacant holes. This action will result in the production of a region either side of the junction in which there are no free charge carriers. This zone is known as the "depletion region".

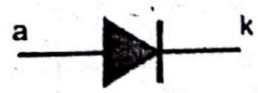


Figure 2.5 ***Diode circuit symbol***

Figure 2.6 shows a junction diode in which the anode is made positive with respect to the cathode. This is forward-biased condition in which the diode freely passes current. It also shows a diode with the cathode made positive with respect to the anode. This is reverse-biased condition in which the diode passes a negligible amount of current.

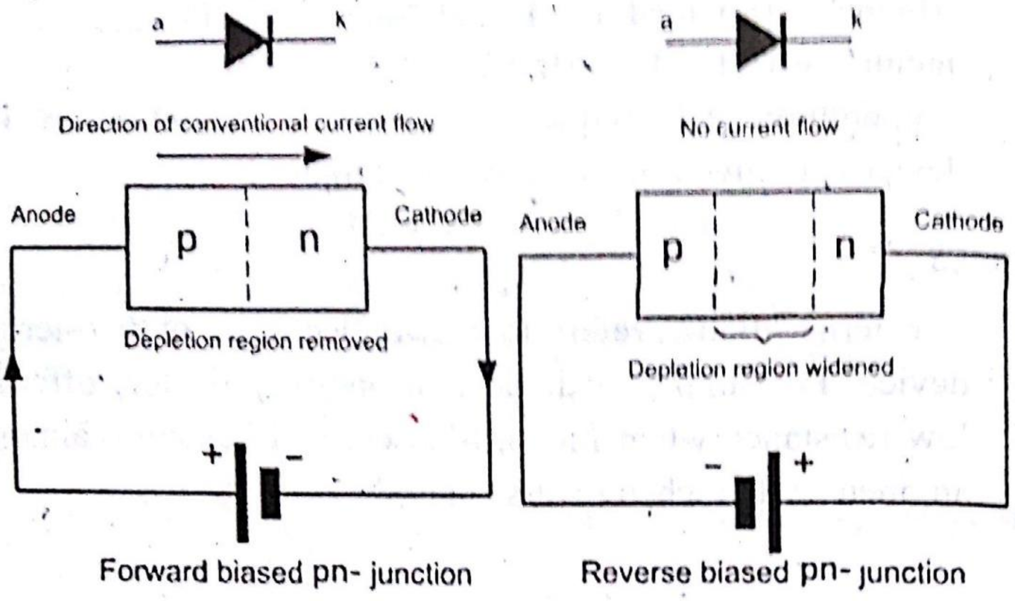


Figure 2.6 *Forward and reverse biased pn-junction*

I-V CHARACTERISTICS OF A PN-JUNCTION DIODE

The i-v characteristic of a practical pn-junction diode can be best explained by considering three different regions of operation.

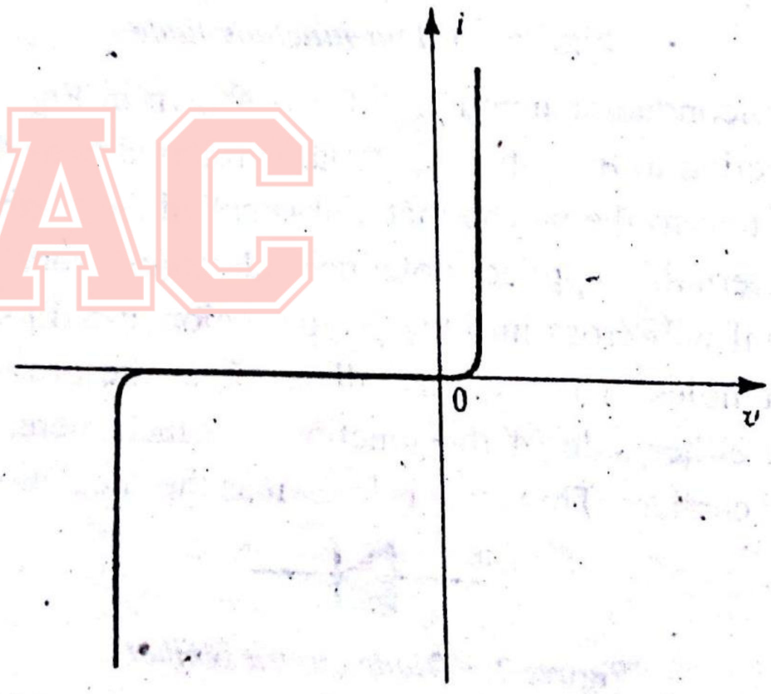


Figure 2.7 The i-v characteristics of a practical silicon junction diode

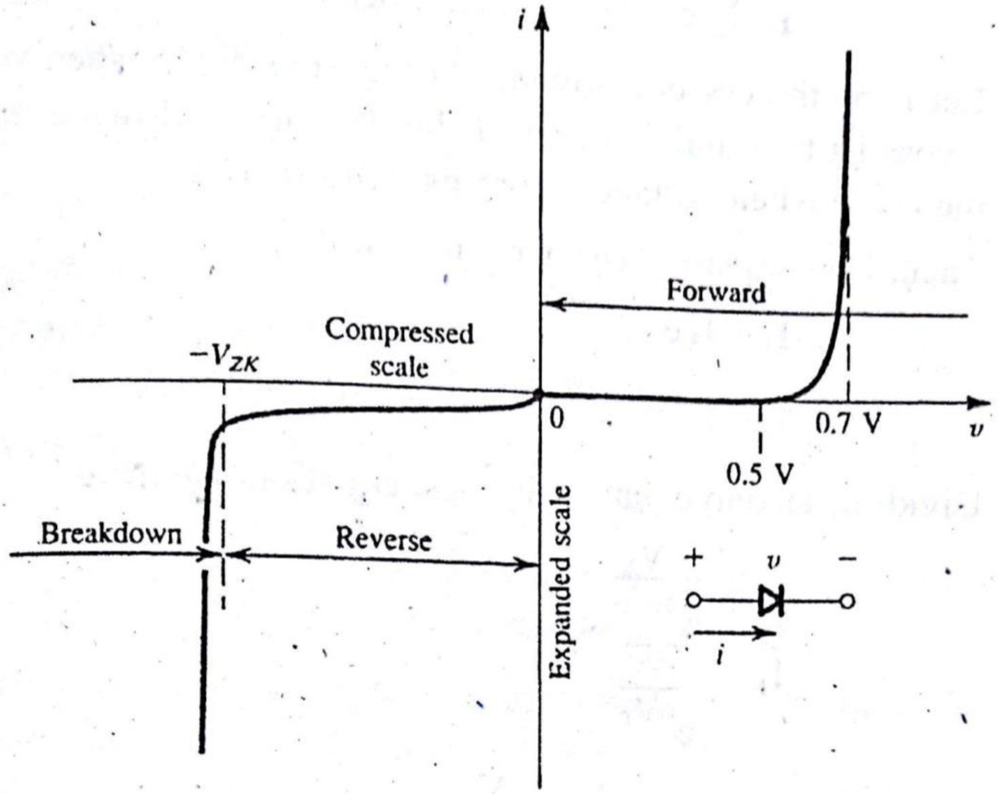


Figure 2.8 ***The i-v relationship of a practical silicon junction diode with some scale expanded and others compressed in order to reveal details***

i. The forward-bias region:

The forward region of operation is entered when the terminal voltage v is positive. In the forward region, the i-v relationship is closely approximated by

$$i = I_S \left(e^{v/nV_T} - 1\right) \text{——— (i)}$$

where I_S = A constant for a given diode at a given temperature known as saturation current or scale current.

$$V_T = \frac{KT}{q} = \text{thermal voltage}$$

where K = Boltzmann's constant

T = the absolute temperature in kelvins

q = the magnitude of electronic charge

n = a constant depends on the material and the physical structure of the diode.

For appreciable current i in the forward direction, specially for $i >> I_S$, equation (i) can be approximated as

$$i \approx I_S e^{v/nV_T} \ .... (ii)$$

Let I_1 be the current flowing through the diode when voltage across its terminal is V_1 and I_2 be the current flowing through the diode when voltage across its terminal is V_2.

Then, from equation (ii), we can write

$$I_1 = I_S e^{V_1/nV_T}$$

$$I_2 = I_S e^{V_2/nV_T}$$

Dividing second equation by first equation, we have

$$\frac{I_2}{I_1} = \frac{e^{\frac{V_2}{nV_T}}}{e^{\frac{V_1}{nV_T}}}$$

$$\text{or, } \frac{I_2}{I_1} = e^{\frac{V_2}{nV_T} - \frac{V_1}{nV_T}}$$

$$\text{or, } \frac{I_2}{I_1} = e^{\frac{V_2 - V_1}{nV_T}}$$

Taking natural logarithm on both sides,

$$ln \frac{I_2}{I_1} = \frac{V_2 - V_1}{nV_T}$$

$$\text{or, } V_2 = V_1 + nV_T \, ln \frac{I_2}{I_1}$$

$$\therefore V_2 = V_1 + 2.3 \, nV_T \log \frac{I_2}{I_1}$$

A glance at the i–v characteristics in the forward region reveals that the current is negligibly small for v smaller than about 0.5 V. This value is usually referred to as the cut-in voltage. But for a "fully-conducting" diode, the voltage drop lies in a narrow range, approximately 0.6 V to 0.8 V.

ii. The reverse-bias region:

The reverse-bias region of operation is entered when the diode voltage v is made negative. Equation (i) predicts that if v is negative and a few times larger than V_T (25 mV) in magnitude, the exponential term becomes negligibly small compared to unity, and the diode current becomes

$$i \approx -I_S$$

iii. **The breakdown region:**

The third distinct region of diode operation is the breakdown region, which can be easily identified on the diode i–v characteristic (see **Figure 2.8**). The breakdown region is entered when the magnitude of the reverse voltage exceeds a threshold value that is specific to the particular diode, called the "breakdown voltage". This is the voltage at the "knee" of the i–v curve in and is denoted V_{ZK}, where the subscript Z stands for Zener and K denotes knee.

An ideal diode on the other hand would exhibit the nature as demonstrated by the graph that follows.

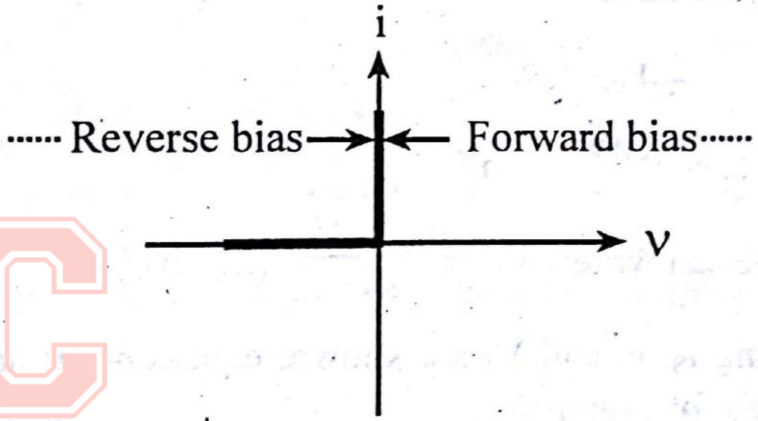


Figure 2.9 ***i-v characteristics of an ideal diode***

MODELING THE SEMICONDUCTOR DIODE FORWARD CHARACTERISTIC

The representation of any device with equivalent electric elements such as resistors, capacitors, inductors, voltage/current sources, etc. is called modeling and the circuit representation of any device with equivalent electric elements without the loss of its exact functional behavior is called "model of the device".

i. **The Exponential Model**

The most accurate description of the diode operation in the forward region is provided by the exponential model.

However, its severely nonlinear nature makes this model the most difficult to use.

Graphical analysis of diode circuit using the exponential model:

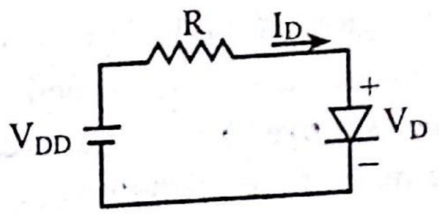


Figure 2.10 *A simple circuit used to illustrate the analysis of circuits in which the diode is forward conducting*

The circuit consists of a DC source V_{DD}, a resistor R, the diode voltage V_D, and current I_D.

Representing the diode i–v characteristic by the exponential relation, we have

$$i = I_S\,(e^{v/nV_T} - 1) \simeq I_S\,e^{v/nV_T}$$

$$\text{or, } I_D = I_S\,e^{V_D/nV_T} \quad \text{....(i)}$$

Also, we can write: $I_D = \dfrac{V_{DD} - V_D}{R}$ ..... (ii)

Assuming I_S, n, and V_T are known, equations (i) and (ii) can be solved for I_D and V_D.

Graphical method is one of the methods to determine I_D and V_D.

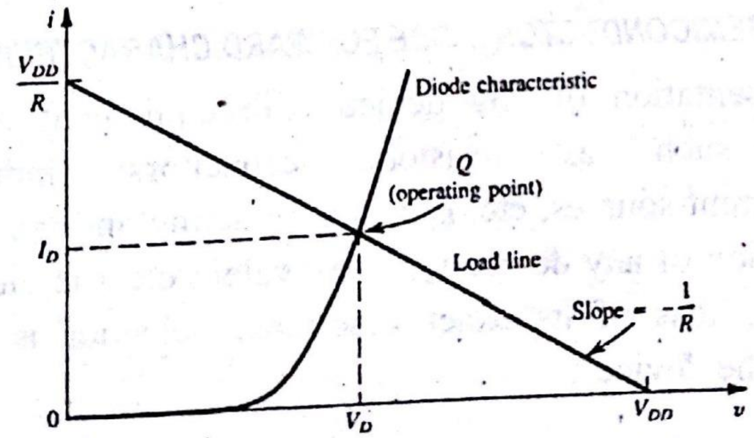


Figure 2.11 *Graphical analysis of the circuit using the exponential diode model*

Graphical analysis is performed by plotting the relationships of equations (i) and (ii). The solution is the coordinates of the point of intersection of the two graphs. The curve represents equation (i), and the straight line represents equation (ii). Such a straight line is known as the "load line". The load line intersects the diode curve at point Q, which represents the "operating point" of the circuit. Its coordinates give the values of I_D and V_D. Graphical analysis aids in the visualization of circuit operation.

Iterative analysis using the exponential model:

The two equations, namely

$$I_D = I_S e^{V_D/nV_T}, \qquad I_D = \frac{V_{DD} - V_D}{R}$$

can be solved using a simple iterative procedure, as illustrated below.

Problem 2.1

Determine the current I_D and the diode voltage V_D for the circuit with $V_{DD} = 5V$ and $R = 1k\Omega$. Assume that the diode has a current of 1mA at a voltage of 0.7V and that its voltage drop changes by 0.1V for every decade change in current.

Solution:

$$I_D = \frac{V_{DD} - V_D}{R} = \frac{5-0.7}{1\times10^3} = 4.3 \text{ mA}$$

For better estimation of V_D, we use

$$V_2 - V_1 = 2.3\, n\, V_T \log\frac{I_2}{I_1} \quad \text{......(i)}$$

Given, $V_2 - V_1 = 0.1$ V for $\frac{I_2}{I_1} = 10$

$\therefore 2.3\, n\, V_T = 0.1$

Using equation (i),

$$V_2 = V_1 + 2.3\, nV_T \log\frac{I_2}{I_1}$$

Taking $V_1 = 0.7$ V, $I_1 = 1$ mA, and $I_2 = 4.3$ m A, we get

$$V_2 = 0.7 + 0.1 \log\frac{4.3}{1} = 0.763 \text{ V}$$

Thus, the results of the first iteration are

$$I_D = 4.3 \text{ mA and } V_D = 0.763 \text{ V}.$$

The second iteration yields

$$I_D = \frac{V_{DD} - V_D}{R} = \frac{5 - 0.763}{1 \times 10^3} = 4.237 \text{ mA}.$$

$$V_2 = V_1 + 2.3\, n\, V_T \log \frac{I_2}{I_1} = 0.763 + 0.1 \log \frac{4.237}{4.3} = 0.762\text{V}$$

Thus, the second iteration yields I_D = 4.23 m A and V_D = 0.762V. Since these values are not much different from the values obtained after the first iteration, no further iterations are necessary, and the solution is I_D = 4.237 mA and V_D = 0.762V.

ii. The Piecewise -Linear Model

The straight-lines (or piecewise-linear) model can be described by

$$i_D = 0,\ v_D \le V_{DO} \ldots\ldots\ (i)$$

$$i_D = (v_D - V_{DO})/r_D,\ v_D \ge V_{DO} \ldots\ldots\ (ii)$$

where V_{DO} is the intercept of line B on the voltage axis and r_D is the inverse of the slope of line B. The piecewise-linear model is depicted in the graph shown below.

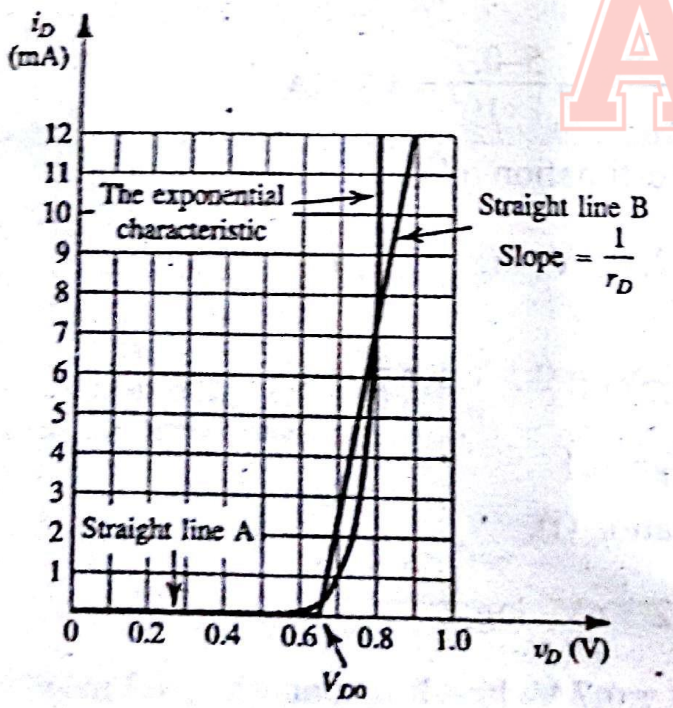


Figure 2.12 *Approximating the diode forward characteristic with two straight lines: the piecewise-linear model*

The piecewise-linear model described by equations (i) and (ii) can be represented by the equivalent circuit shown in **Figure 2.13** below. Note that an ideal diode is included in this model to constrain i_D to flow in the forward direction only. This model is also known as the battery-plus-resistance model.

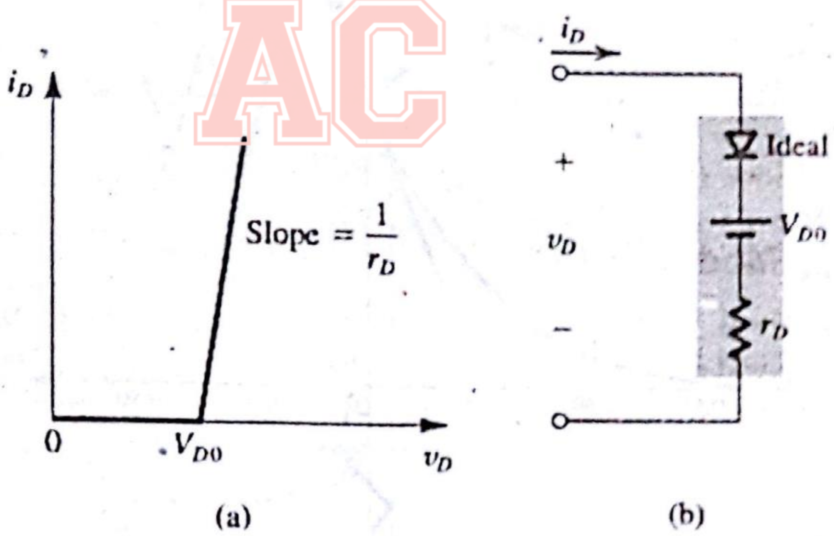


Figure 2.13 Piecewise-linear model of the diode forward characteristic and its equivalent circuit representation

iii. The Small-Signal Model

There are applications in which a diode is biased to operate at a point on the forward i–v characteristic and a small AC signal is superimposed on the DC quantities. For this situation, we first have to determine the DC operating point (V_D and I_D) of the diode. Then, for small-signal operation around the DC bias point, the diode is best modeled by a resistance equal to the inverse of the slope of the tangent to the exponential i–v characteristic at the bias point.

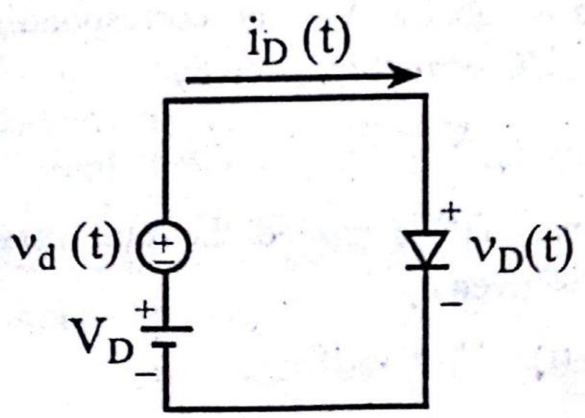


Figure 2.14 ***Circuit for developing the small-signal model of a diode***

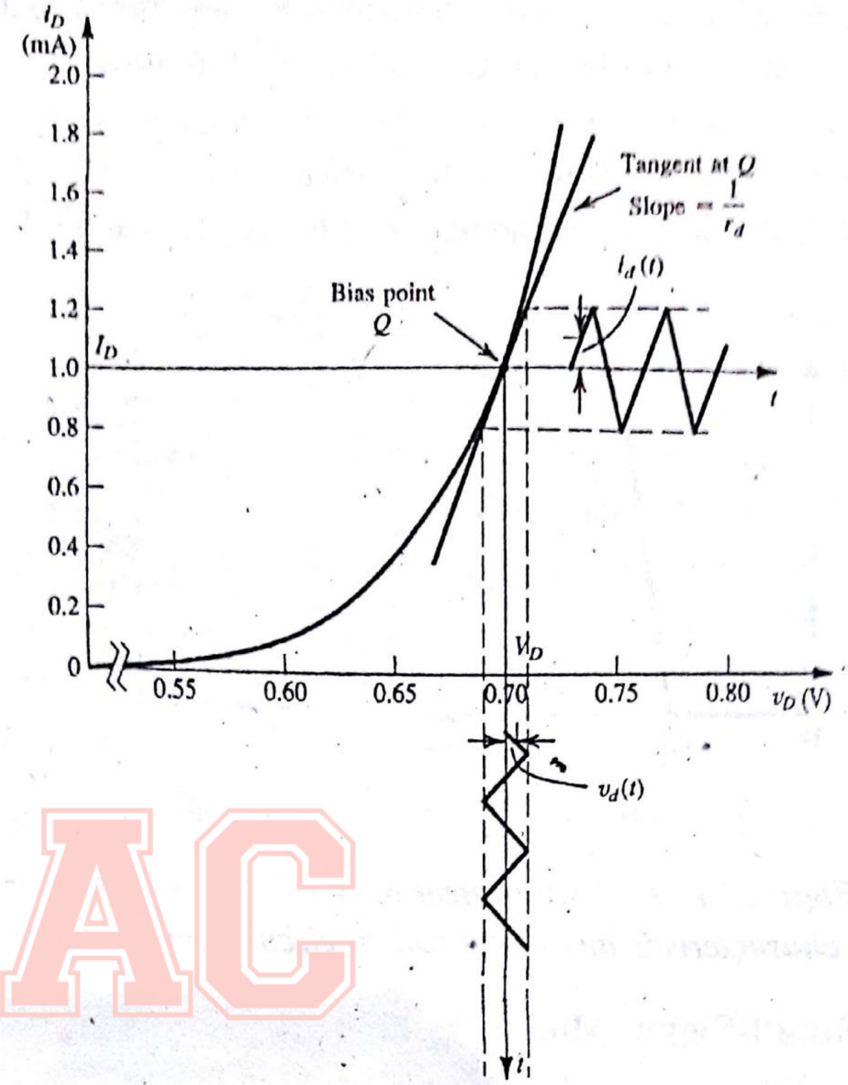


Figure 2.15 ***i_D–v_D characteristics curve***

Consider the conceptual circuit as shown in **Figure 2.14** and the corresponding graphical representation as shown in **Figure 2.15**. A DC voltage V_D, represented by a battery, is applied to the diode, and a time-varying signal v_d (t), assumed (arbitrarily) to have a triangular waveform, is superimposed on the DC voltage V_D. In the absence of the signal v_d (t), the diode voltage is equal to V_D, and correspondingly, the diode will conduct a DC current I_D given by

$$I_D = I_S e^{V_D/nV_T} \quad \text{.......}(i)$$

When the signal v_d (t) is applied, the total instantaneous diode voltage $v_D(t)$ is given by

$$v_D(t) = V_D + v_d(t)$$

Correspondingly, the total instantaneous diode current i_D (t) will be

$$i_D(t) = I_S e^{v_D/nV_T}$$

$$\text{or, } i_D(t) = I_S e^{\frac{V_D + v_d}{nV_T}}$$

$$\text{or, } i_D(t) = I_S e^{V_D/nV_T} e^{v_d/nV_T}$$

Using equation (i), we get

$$i_D(t) = I_D e^{v_d/nV_T} \text{ ...... (ii)}$$

Now, if the amplitude of the signal v_d (t) is kept sufficiently small such that

$$\frac{v_d}{nV_T} << 1$$

then, we may expand equation (ii) in a series and truncate the series after the first two terms to get

$$i_D(t) \simeq I_D \left(1 + \frac{v_d}{nV_T}\right).$$

This is the "small-signal approximation." It is valid for signals whose amplitudes are smaller than about 10 mV for n =2 and 5 mV for n =1.

$$\text{or, } I_D + i_d = I_D + \frac{I_D}{nV_T} v_d$$

Comparing like terms, we get

$$i_d = \frac{I_D}{nV_T} v_d$$

The quantity relating the signal current i_d to the signal voltage v_d has the dimensions of conductance, mhos (℧), and is called the "diode small-signal conductance". The inverse of this parameter is the "diode small-signal resistance, " or "incremental resistance", or "AC resistance", or "dynamic resistance" denoted by r_d.

$$r_d = \frac{nV_T}{I_D}$$

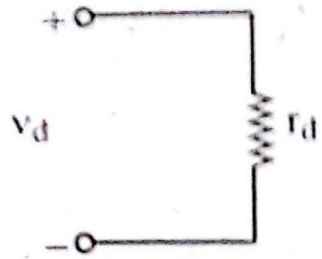


Figure 2.16 *Small signal model of a diode*

DIODE CIRCUITS

Clampers

Clampers are diode networks that will "clamp" a signal to a different dc level. The network must have a capacitor, a diode, and a resistive element, but it can also employ an independent dc supply to introduce an additional shift. The magnitude of R and C must be chosen such that the time constant τ = RC is large enough to ensure that the voltage across the capacitor does not discharge significantly during the interval the diode is non conducting. There are two basic types of clampers:

i. **Positive clamper**

A positive clamper shifts its input waveform in a positive direction, so that it lies above a dc reference voltage.

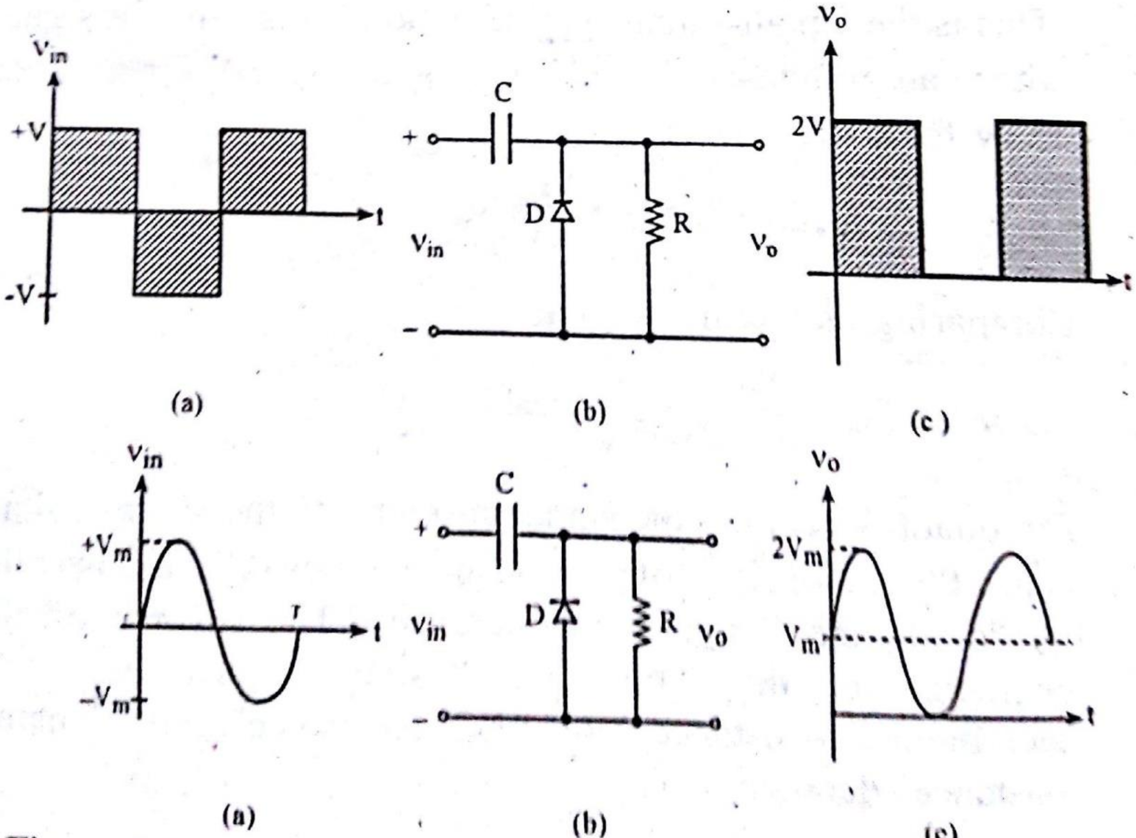
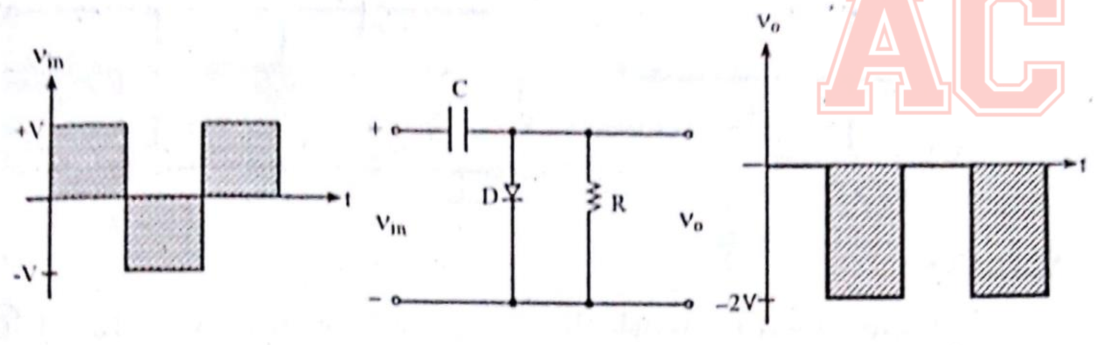


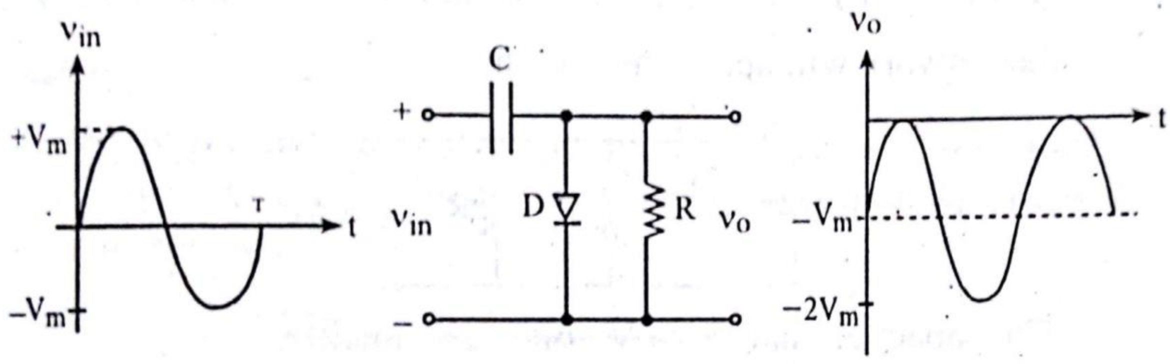
Figure 2.17 ***(a) Input waveform (b) A positive clamper (c) Output waveform***

ii. Negative clamper

A negative clamper shifts its input waveform in a negative direction, so that it lies below a dc reference voltage.



(a) (b) (c)



(a) (b) (c)

Figure 2.18 ***(a) Input waveform (b) A negative clamper (c) Output waveform***

In general, the following steps may be helpful when analyzing clamping networks:

i. Start the analysis by examining the response of the portion of the input signal that will forward bias the diode.

ii. During the period that the diode is in the "on" state, assume that the capacitor will charge up instantaneously to a voltage level determined by the surrounding network.

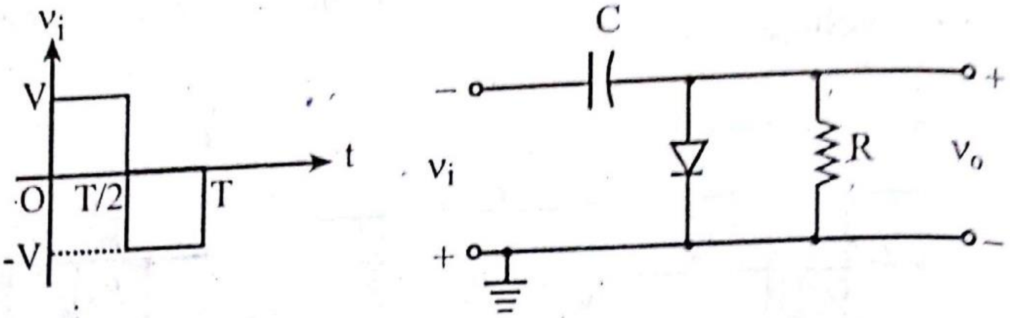
iii. Assume that during the period when the diode is in the "off" state the capacitor holds on to its established voltage level.

iv. Throughout the analysis, maintain a continual awareness of the location and defined polarity for v_0 to ensure that the proper levels are obtained.

v. Check that the total swing of the output matches that of the input.

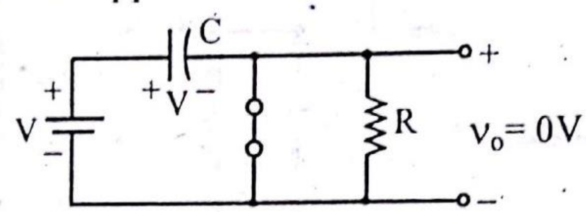
Problem 2.2

Find v_o for the network given below for the input indicated.



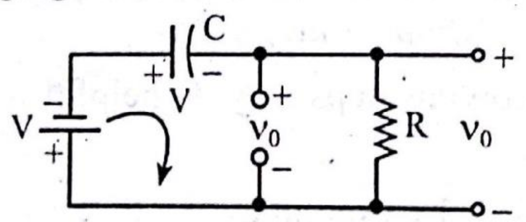
Solution:

For the given network the diode will be forward biased for the positive portion of the applied signal. For the interval $0 \to \frac{T}{2}$ the network will appear as shown.



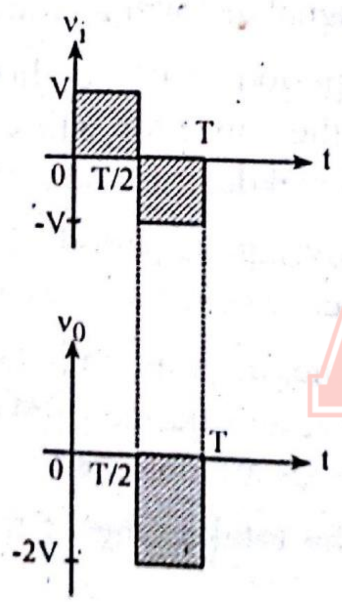
The capacitor charges to V volts very quickly.

During the interval $\frac{T}{2} \to$ T the network will appear as shown below. The capacitor retains its voltage (V volts) since the discharging time constant τ = RC chosen is very large.



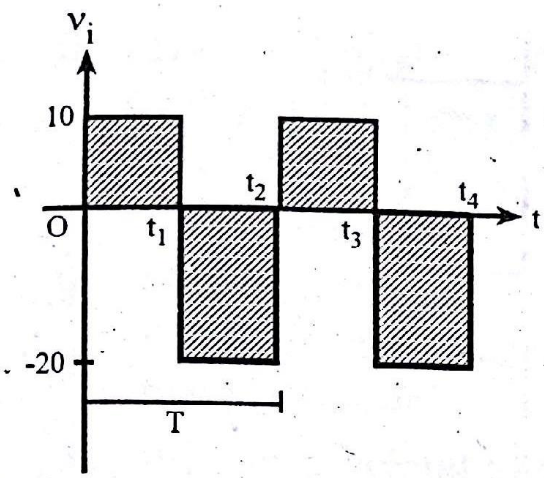
$-V-V-v_0=0$

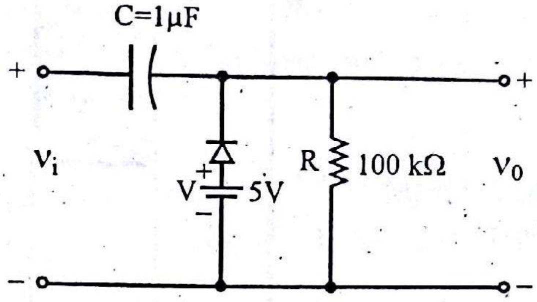
$\therefore v_0 = -2V$



Problem 2.3

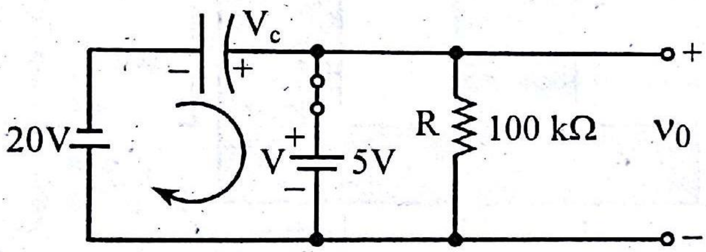
Determine v_o for the network given below for the input indicated.





Solution:

We begin the analysis considering the interval t_1 to t_2 because in this interval, the diode will be forward biased. The network will appear as shown.



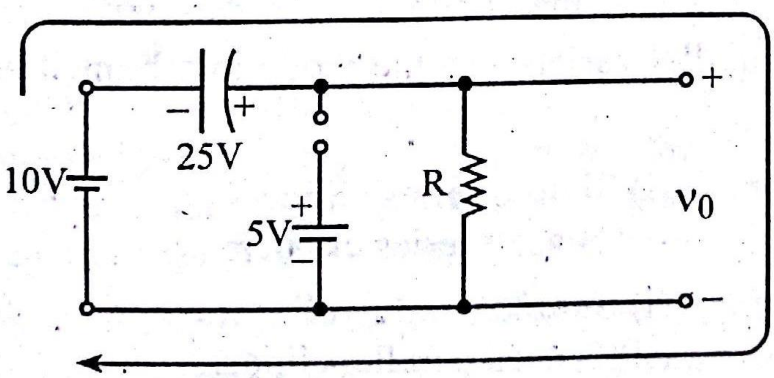
$$-20 + V_C - 5 = 0$$

$$\therefore V_C = 25\text{ V}$$

The capacitor will therefore charge up to 25 V.

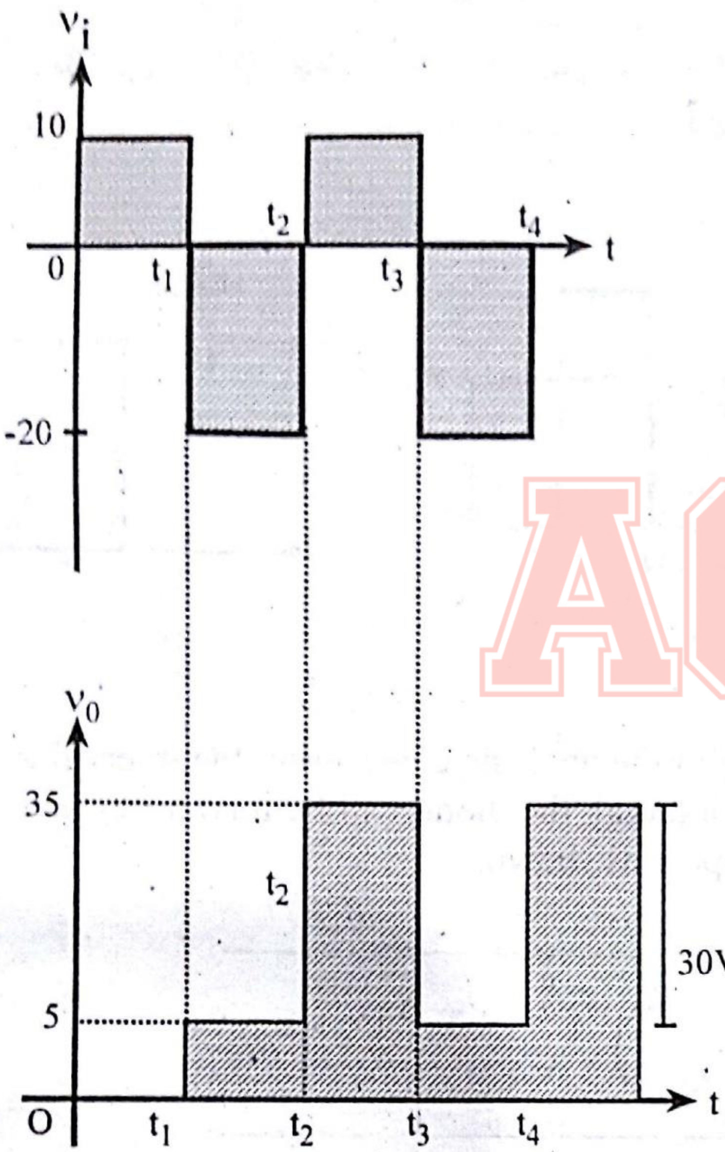
$$v_0 = 5\text{V}$$

For the period $t_2 \rightarrow t_3$ the network will appear as shown.



$$+10 + 25 - v_0 = 0$$

$$\therefore v_0 = 35\text{ V}$$



Clippers (sometimes called limiters)

Clippers are networks that employ diodes to "clip" away a portion of an input signal without distorting the remaining part of the applied waveform. There are two general categories of clippers: series and parallel. The series configuration is defined as one where the diode is in series with the load, whereas the parallel variety has the diode in a branch parallel to the load.

Series clippers - (i) Simple series clippers
(ii) Biased series clippers

Parallel clippers - (i) Simple parallel clippers
(ii) Biased parallel clippers

Clippers are useful in signal shaping, circuit protection, and communication.

Analysis of series clippers

Important steps:

i. Take careful note of where the output voltage is defined.

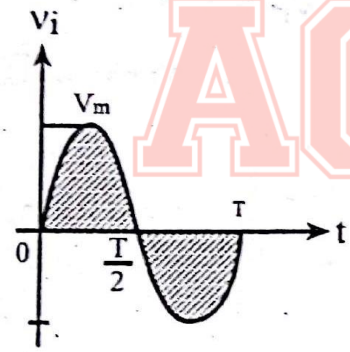
ii. Try to develop an overall sense of the response by simply noting the "pressure" established by each supply and the effect it will have on the conventional current direction through the diode.

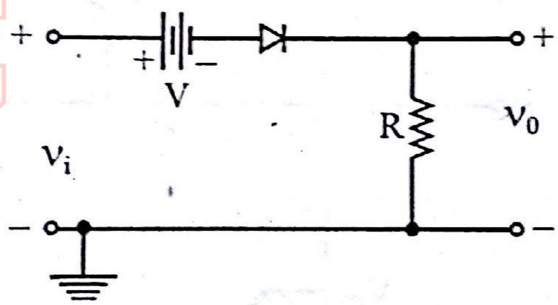
iii. Determine the applied voltage (transition voltage) that will result in a change of state for the diode from the "off" to the "on" state.

iv. It is often helpful to draw the output waveform directly below the applied voltage using the same scales for the horizontal axis and the vertical axis.

Problem 2.4

Determine the output waveform for the input shown below.





Solution:

For the network in the above example, the direction of the diode suggests that the signal v_i must be positive to turn it on. The dc supply further requires that the voltage v_i be greater than V volts to turn the diode on. The negative region of the input signal is "pressurizing" the diode into the "off" state, supported further by the dc supply. In general, therefore, we can be quite sure that the diode is an open circuit ("off" state) for the negative region of the input signal.

Now, we determine the applied voltage (transition voltage) that will cause a change in state for the diode. For the ideal diode the transition between states will occur at the point on the characteristics where $v_d = 0$ and $i_d = 0$.

The level of v_i that will cause a transition in state is $v_i = V$.

For $v_i = V$, $v_o = 0V$.

For an input voltage greater than V volts the diode is in the short-circuit state, while for input voltages less than V volts it is in the open-circuit or "off" state.

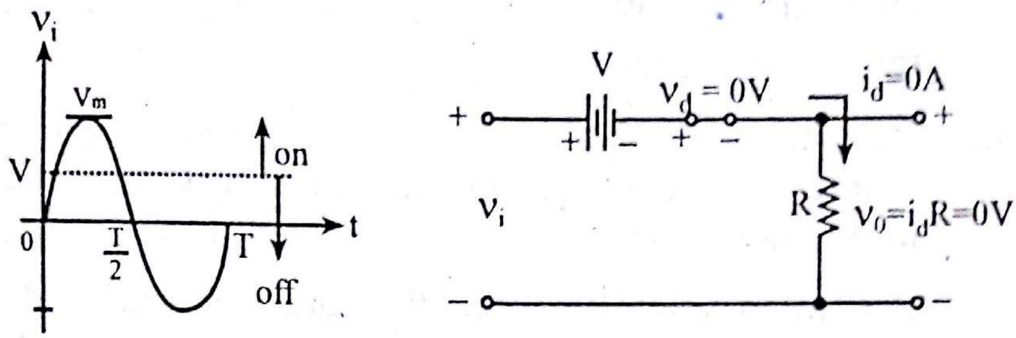
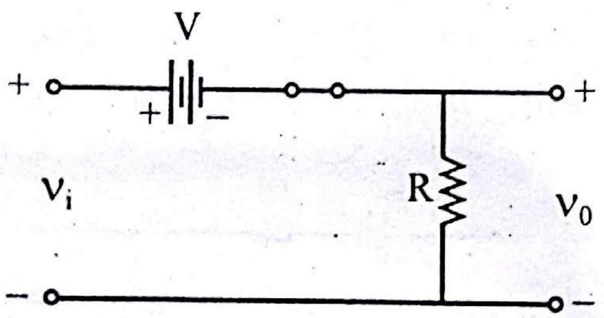
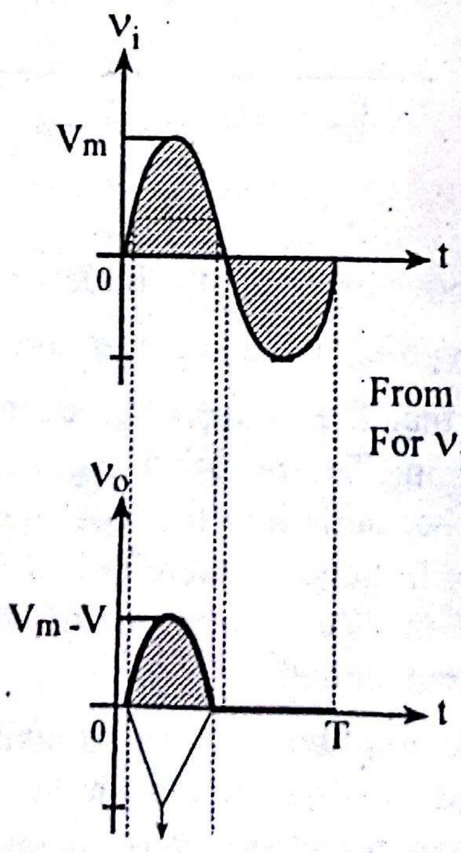


Fig.: Determining the transition level for the circuit



$v_i - V - v_o = 0$

$\therefore v_0 = v_i - V$ ....(i)

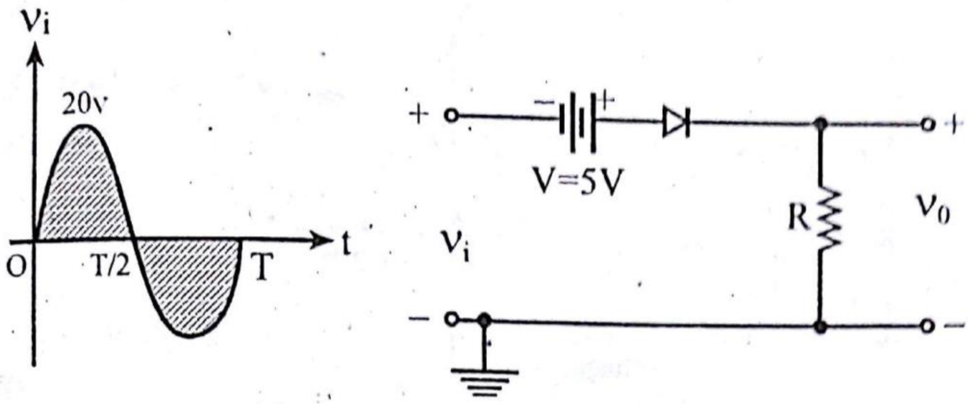


From equation (i),
For v_i= Vm, v_o = Vm-V

v_i=V(diode changes state)

Problem 2.5

Determine the output waveform for the sinusoidal input of Fig. shown below.

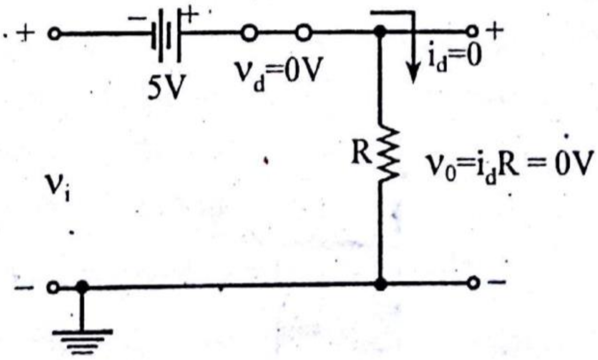


Solution:

Step 1: The output is again directly across the resistor R

Step 2: The positive region of v_i and the dc supply are both applying "pressure" to turn the diode on. The result is that we can safely assume the diode is in the "on" state for the entire range of positive voltages for v_i. Once the supply goes negative, it would have to exceed the dc supply voltage of 5V before it could turn the diode off.

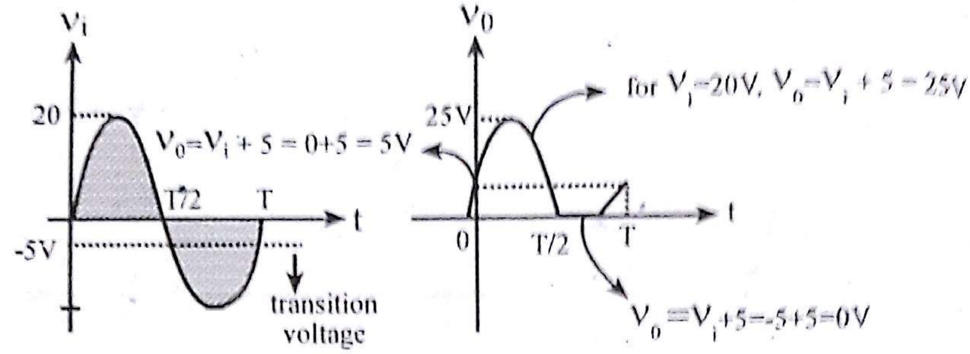
Step 3: The transition from one state to the other will occur when



$$v_i + 5 = 0$$
$$\therefore v_i = -5V$$

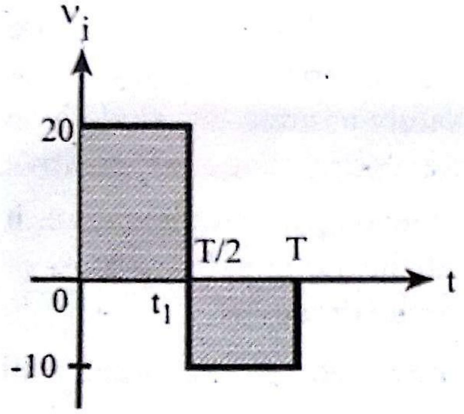
Fig.: Determining the transition level for the circuit

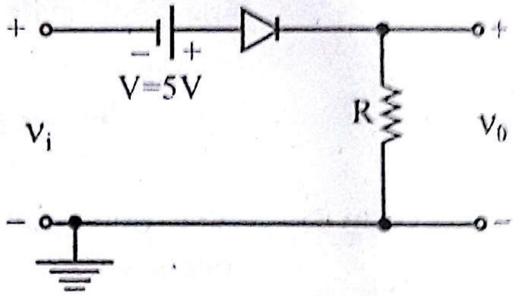
Step 4: In Fig. shown below, a horizontal line is drawn through the applied voltage at the transition level. For voltages less than –5V the diode is in the open-circuit state and the output is 0V, as shown in the sketch of v_o. Using Fig. below, we find that for conditions when the diode is on and the diode current is established the output voltage will be the following, as determined using KVL:



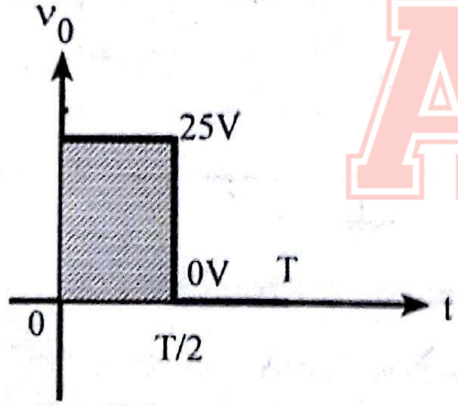
Problem 2.6

Find the output voltage for the network shown.





Solution:

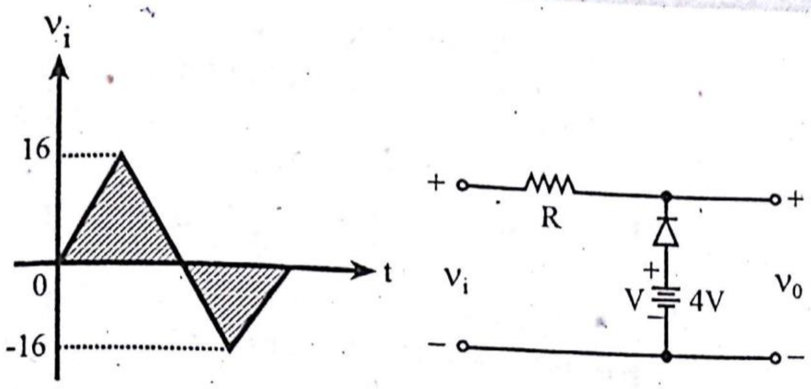


Analysis of parallel clippers

The analysis of parallel clippers is very similar to that applied to series configurations, as demonstrated in the coming examples:

Problem 2.7

Determine v_o for the network of Fig. shown below.



Solution:

Step 1: The output is defined across the series combination of the 4V supply and the diode, not across the resistor R.

Step 2: The polarity of the dc supply and the direction of the diode strongly suggest that the diode will be in the "on" state for a good portion of the negative region of the input signal. In fact, it is interesting to note that since the output is directly across the series combination, when the diode is in its short-circuit state the output voltage will be directly across the 4V dc supply, requiring that the output be fixed at 4V. In other words, when the diode is on the output will be 4V. Other than that, when the diode is an open circuit, the current through the series network will be 0 mA and the voltage drop across the resistor will be 0 V. That will result in $v_o = v_i$ whenever the diode is off.

Step 3: The transition level of the input voltage is

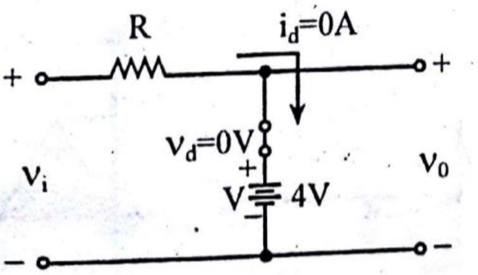
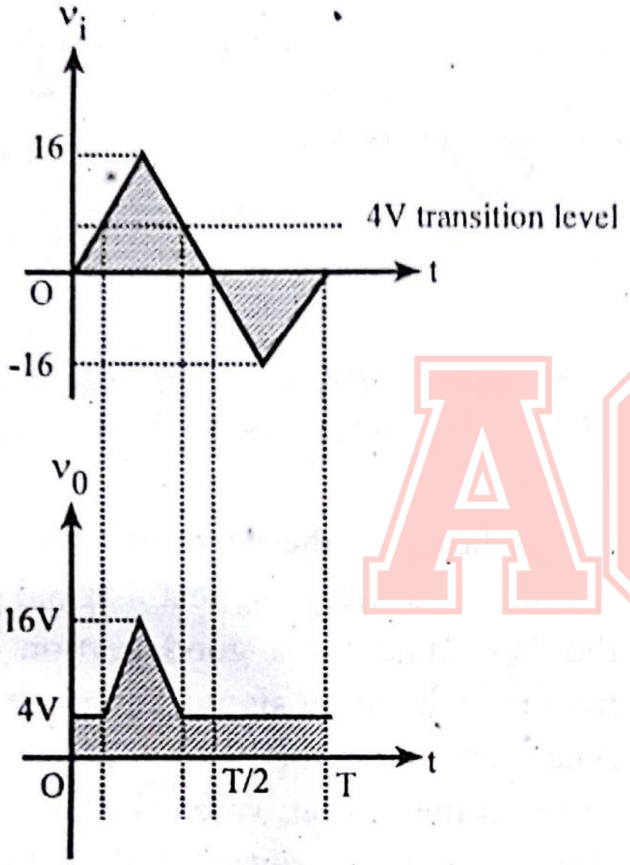


Fig.: Determining the transition level of the circuit.

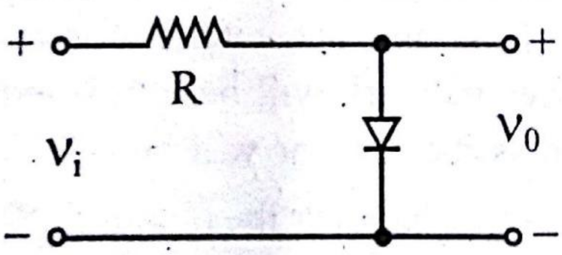
Change in state is at $v_i = 4V$

Step 4: The transition level is drawn along with $v_o = 4V$ when the diode is on. For $v_i \geq 4V$, $v_o = 4V$, and the waveform is simply repeated on the output plot.

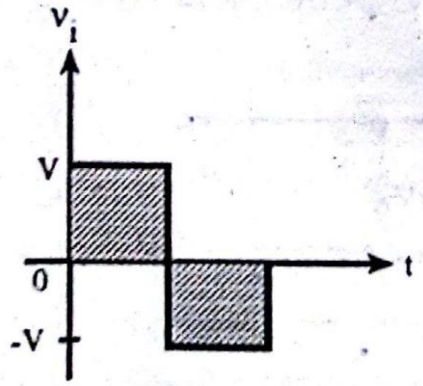


Problem 2.8

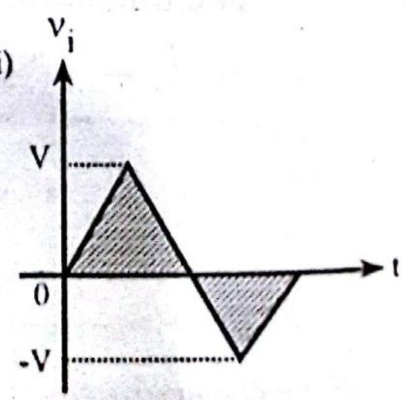
For the network given below, find the output waveform if the input waveform is



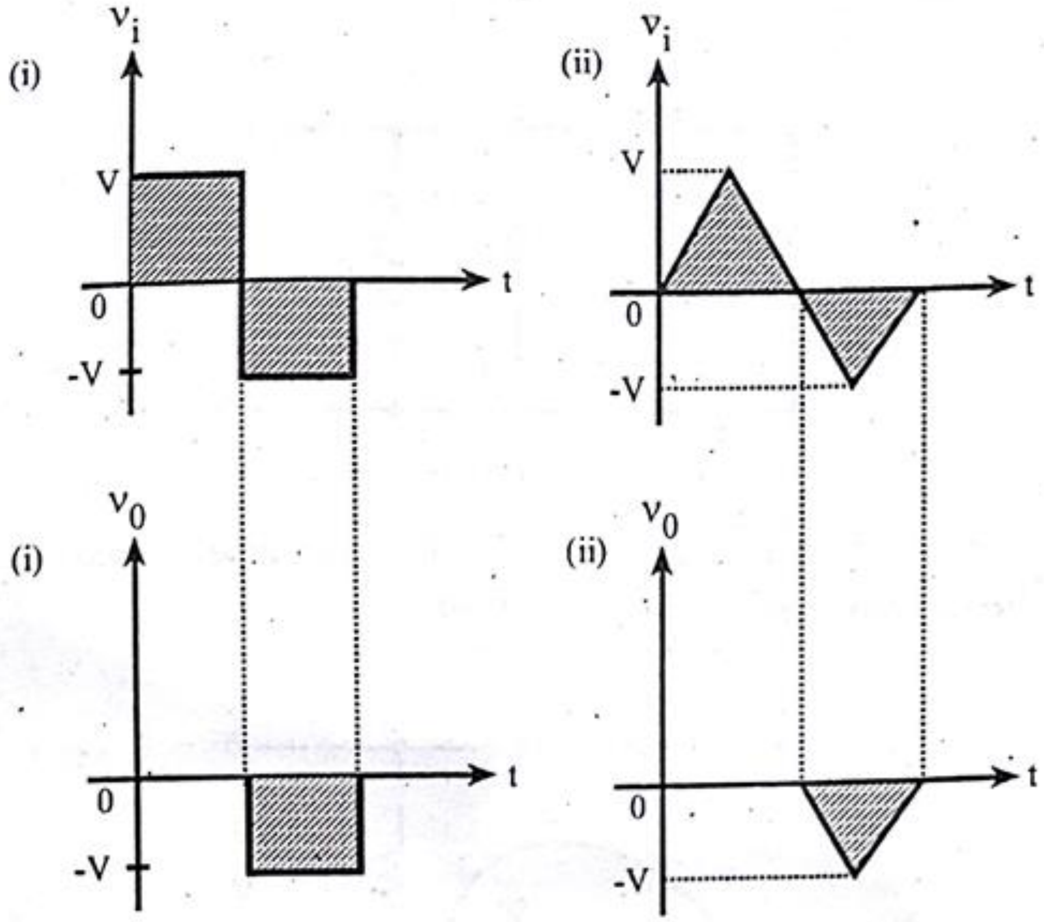
(i)



(ii)

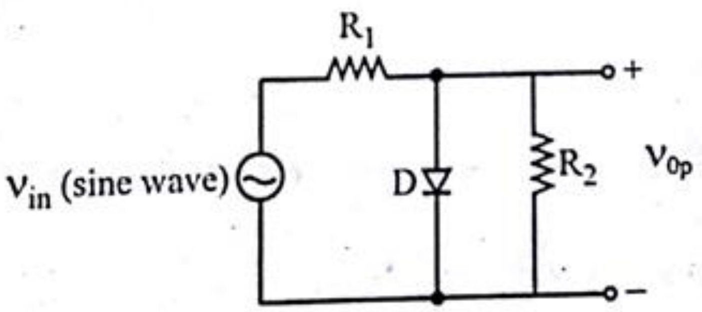


Solution:



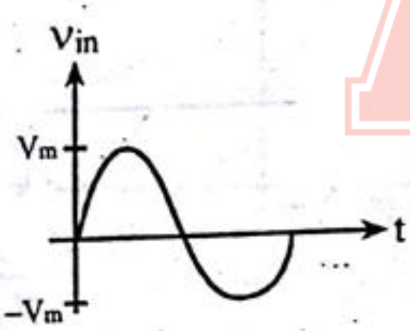
Problem 2.9

Draw the output waveform of the circuit and indicate the peak output voltage. Assume diode is ideal.

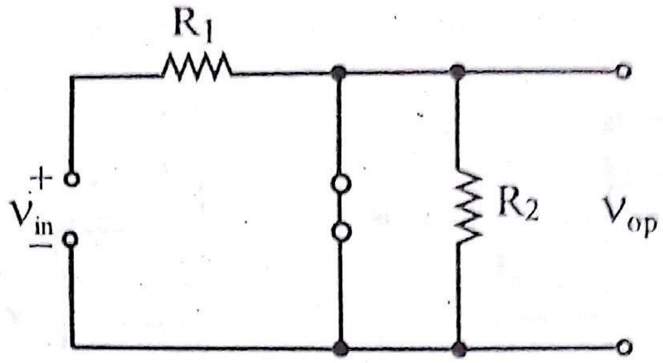


Solution:

Input waveform for $v_{in} = V_m \sin \omega t$ is

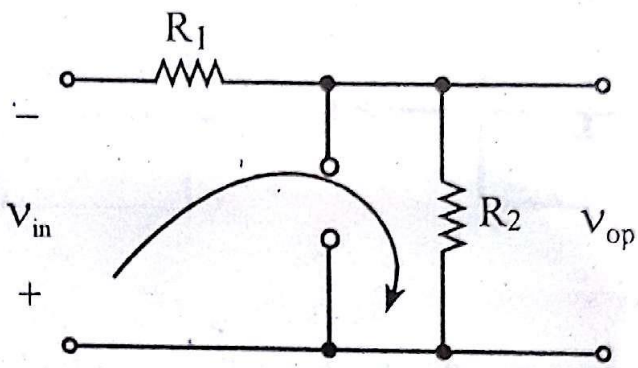


For positive half cycle, diode D is forward biased and is hence, replaced by a short circuit.



$$\therefore \ v_{op} = 0 \text{ V}.$$

For negative half cycle, diode D is reverse biased and is hence, replaced by an open circuit.

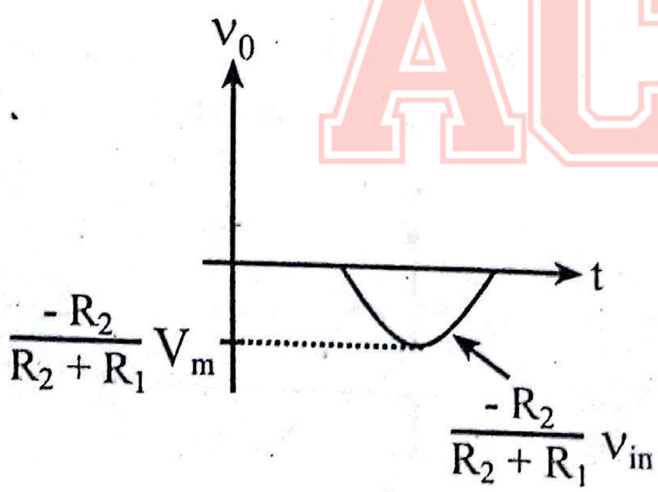


$$v_{op} = \frac{R_2}{R_1 + R_2}(-v_{in})$$

For $v_{in} = V_m$,

$$v_{op} = \frac{R_2}{R_1 + R_2}(-V_m) = \frac{-R_2}{R_1 + R_2}(V_m)$$

The output waveform is



ZENER DIODE

A Zener diode is a special type of diode that is designed to operate in the reverse breakdown region. An ordinary diode operated in this region will usually be destroyed due to excessive current. This is not the case for the Zener diode.

The circuit symbol for Zener diode and its conduction direction is shown below.

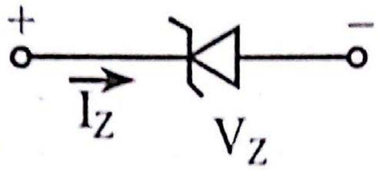


Figure 2.18 ***Zener diode***

The i-v characteristics of a Zener diode is shown below.

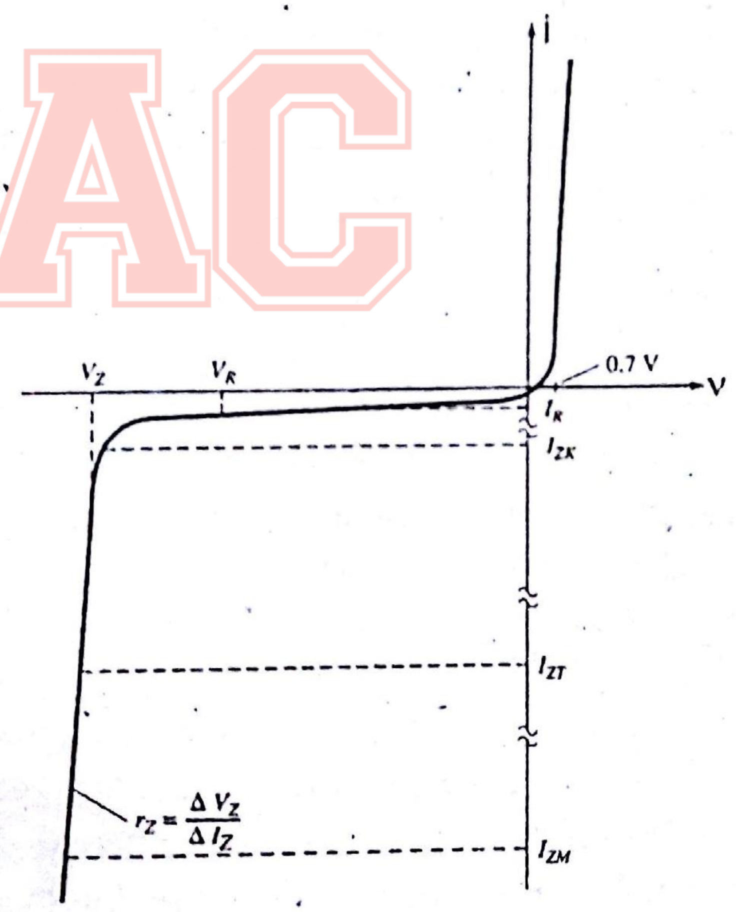


Figure 2.19 ***Zener diode characteristics with the equivalent model for each region***

V_Z: Zener breakdown voltage

I_{ZT}: Test current for measuring V_Z

I_{ZK}: Reverse current near the knee of the characteristic, the minimum reverse current to sustain breakdown

I_{ZM}: Maximum Zener current, limited by the maximum power dissipation

ZENER EFFECT

In a heavily doped pn-junction diode with a very narrow depletion region, the electric field strength (volts/width) produced by a reverse bias voltage can be very high. The high intensity electric field causes electrons to break away from their atoms, thus converting the depletion region from an insulating material into a conductor. This effect is called Zener effect. This is ionization by electric field and it usually occurs with reverse bias voltage less than 5V i.e., $V_Z < 5V$.

AVALANCHE EFFECT

In a highly doped pn-junction diode with depletion region too wide for Zener effect, increase in voltage across the diode increases the velocity of the minority carriers responsible for reverse saturation current I_S. Eventually, their velocity and associated kinetic energy will be sufficient to release additional carriers through collisions with other stable atomic structure. This is ionization by collision. These additional carriers can then aid the ionization process to the point where a high avalanche current (I_Z) is established. This effect is called Avalanche effect. It usually occurs with reverse bias voltage levels above 5V i.e., $V_Z > 5V$.

LIGHT EMITTING DIODES (LEDS)

LEDs are optoelectronic devices which emit a fairly narrow bandwidth of visible (usually red, orange, yellow or green) or invisible (infrared) light when its internal diode junction is stimulated by a forward electric current/voltage (power).

The operation of LED is based on the phenomenon of electroluminance. Electroluminance is the emission of light from a semiconductor under the influence of an electric field.

LEDs are broadly divided into two categories:

(i) Surface - emitting LEDs

(ii) Edge - emitting LEDs

Light emitting diodes are available in various formats with the round types being most popular. The symbol for a LED is shown in the figure below.

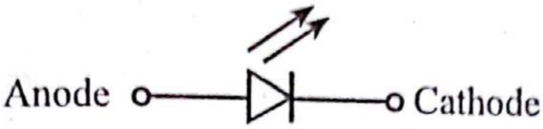


Figure 2.20 ***Symbol of a LED***

Application of LEDs

i. LEDs are used in burglar-alarm systems.

ii. They are used in solid-state video displays.

iii. They are used in the field of optical fibre communication system.

iv. They are used for numeric displays in hand-held or pocket calculators.

PHOTODIODE

Photodiode is a two terminal semiconductor pn-junction device having a small transparent window to allow light to strike the pn-junction and is designed to operate with reverse bias.

If a reverse-biased pn-junction is illuminated- that is, exposed to incident light- the photons impacting the junction cause covalent bonds to break, and thus, electron-hole pairs are generated in the depletion layer. The electric field in the depletion region then sweeps the liberated electrons to the n-side and the holes to the p-side, giving rise to a reverse current across the junction. This current, known as photocurrent, is proportional to the intensity of the incident light. Such diodes are called photodiodes which can be used to convert light signals into electrical signals. The definition of photodiode is now clear.

The basic biasing arrangement, construction, and symbol for the device appears below.

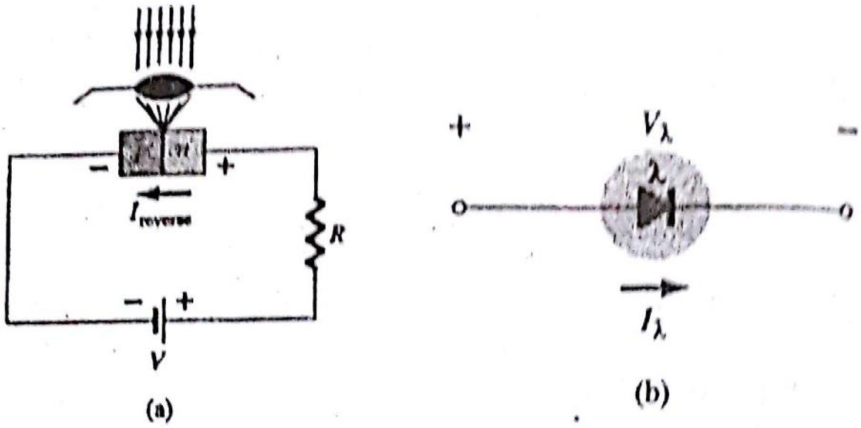


Figure 2.21 *Photoiode: (a) Basic biasing arrangement and construction (b) symbol*

Relation between reverse current (I_λ) and luminous flux (f_c)

The almost equal spacing between the curves for the same increment in luminous flux reveals that the reverse current and the luminous flux are almost linearly related. In other words, an increase in the light intensify will result in a similar increase in reverse current. A plot of the two to show this linear relationship appears below for a fixed voltage V_A of 20 V.

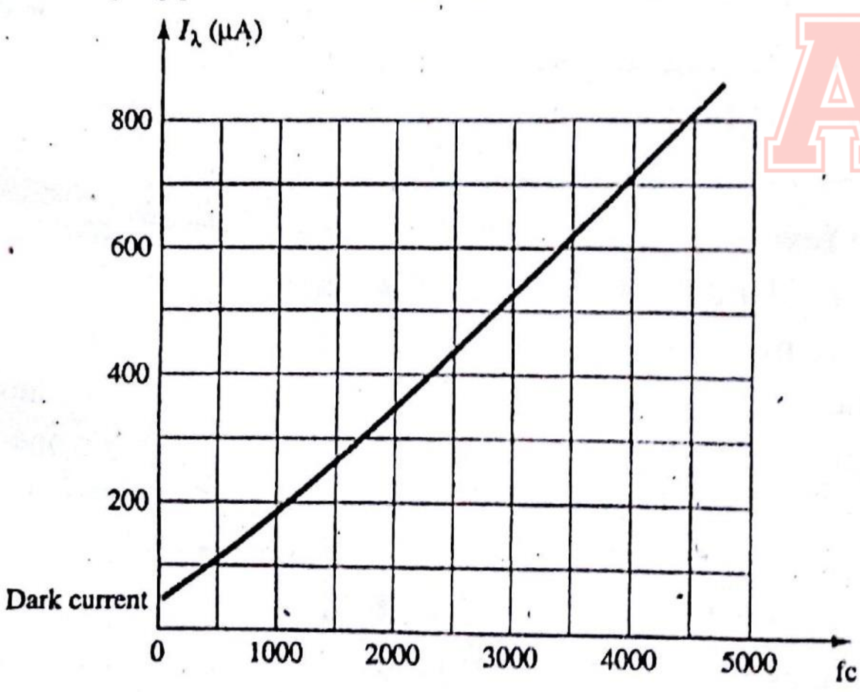


Figure 2.22 *I_λ (µA) versus f_c (at V_A = 20V) for the photodiode*

Application of photodiode

i. Photodiode is used to count items on a conveyor belt.
ii. Photodiode is used in an alarm system.
iii. Photodiodes are also used in logic circuits that require stability and high speed.

Special diodes that are fabricated to be used as voltage-variable capacitors are known as varactor diodes or simply, varactors. Varactor diode is a reverse-biased diode and its mode of operation depends on the capacitance that exist at the pn-junction. It is also called varicap or VVC (voltage variable capacitor) or tuning diode.

The pn-junction capacitance (transition capacitance) is given by

$$C_T = \varepsilon \frac{A}{w_d}$$

where ε = permittivity of the semiconductor
A = the pn–junction area
w_d = depletion width

The symbol for a varactor diode is shown below.

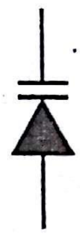


Figure 2.23 *Symbol of a varicap diode*

Working principle

As the reverse-bias potential increases the width of the depletion region increases, which in turn reduces the transition capacitance. The characteristics of a typical commercially available varicap are shown below.

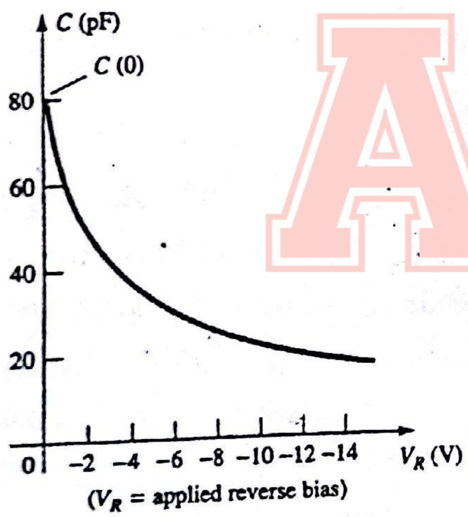


Figure 2.24 *Varicap characteristics: C(pF) versus V_R*

Particularly, the junction capacitance varies inversely with the square root of the reverse bias voltage (V_R).

i.e., $C_T \alpha \frac{1}{\sqrt{V_R}}$

Applications of varactor diode

i. FM modulators
ii. Automatic frequency control (AFC)
iii. Automatic frequency tuning (AFT)
iv. Band pass filter.

TUNNEL DIODE

A tunnel diode is a high conductivity two terminal pn-junction diode doped heavily-about 1000 times higher than a conventional junction diode. Due to heavy doping, the width of depletion layer is extremely reduced to a small value of the order of 10^{-5} mm. This reduced depletion layer can result in carriers punching through the junction even when they do not possess enough energy to overcome the potential barrier. The result is that large forward current is produced at relatively low forward voltage. Such a mechanism of conduction in which charge carriers (possessing very little energy) punch through a barrier directly instead of climbing over it is called tunneling. That is why, such diodes are called tunnel diodes.

Figure 2.25 ***Symbol of a tunnel diode***

Working principle

The depletion region is an insulator because it locks charge carriers and usually charge carriers can cross it only when the external bias is large enough to overcome the barrier potential. However, because the depletion region in a tunnel diode is so narrow, it does not constitute a large barrier to electron flow. Consequently, a small forward or reverse bias can give charge carriers sufficient energy to cross the depletion region. When this occurs, the charge carriers are said to be tunneling through the barrier.

Characteristics of tunnel diode

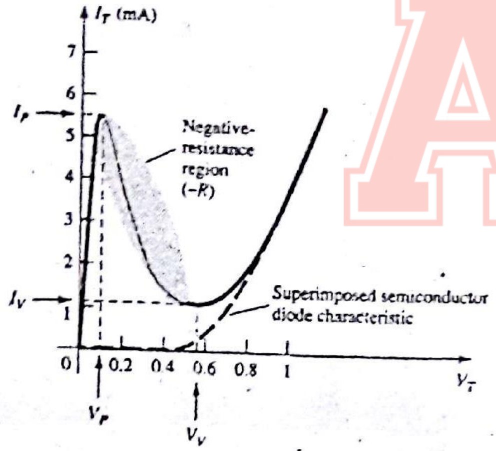


Figure 2.26 *Tunnel diode characteristics*

A forward biased tunnel diode initially increases forward current with the increase of forward voltage (V_T). Eventually, a peak level of tunneling is reached, and then further increase in forward voltage causes forward current to decrease. The decrease in forward current continues until the normal process of current flows across a forward-biased junction.

Application

Used in high-speed applications such as in computers where switching times in the order of nanoseconds or picoseconds are desirable.

RECTIFIER CIRCUITS

Rectifier is a device used for converting alternating current (AC) to direct current (DC).

1. Half-Wave Rectifier

The process of removing one-half the input signal to establish a dc level is called half-wave rectification. The half-wave rectifier utilizes alternate half-cycles of the input sinusoid. For analysis, the circuit of a half-wave rectifier, assuming an ideal diode is shown in the **Fig.** below.

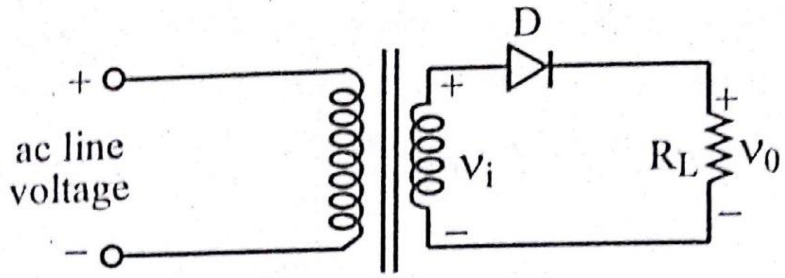


Figure 2.27 *Half-wave rectifier*

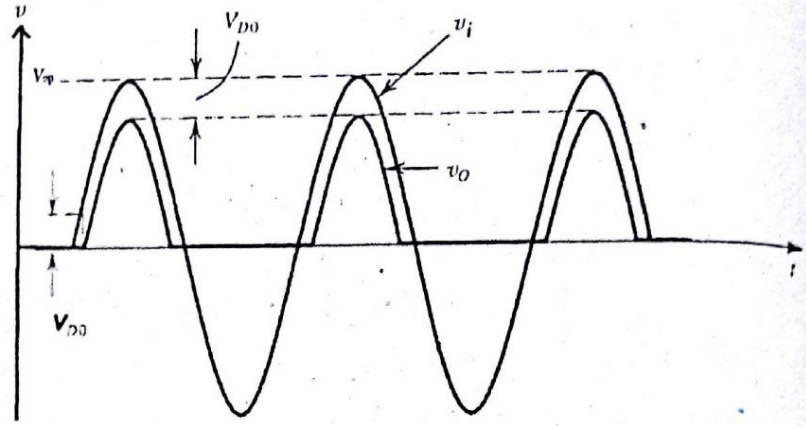


Figure 2.28 *Input and output waveforms*

During the positive half-cycles of the input voltage, v_i is positive and thus, current is conducted through diode D, and R_L. During the negative half-cycles of the input voltage, v_i will be negative, which makes diode D in reverse-biased condition.

The characteristics of half-wave rectifier can be summarized as follows:

i. DC output current: $I_{dc} = \frac{I_{op}}{\pi}$

ii. DC output voltage: $V_{dc} \simeq \frac{V_{op}}{\pi}$

iii. RMS value of current: $I_{rms} = \frac{I_{op}}{2}$

iv. RMS value of output voltage: $V_{rms} = \frac{V_{op}}{2}$

v. Form factor: $K_f = \frac{\text{RMS value}}{\text{Average value}} = 1.57$

vi. Peak factor: $K_p = \frac{\text{Peak value}}{\text{RMS value}} = 2$

vii. Ripple factor: $\gamma = 1.21$

viii. Rectification efficiency:

$$\eta = \frac{\text{DC power delivered to the load}}{\text{AC input power from the transformer}} = \frac{P_{dc}}{P_{ac}} = 40.6\%$$

The advantages and disadvantages of a half-wave rectifier are given below:

Advantages

i. Simple circuit and low cost.

Disadvantages

i. Ripple factor is high, and an elaborate filtering is, therefore, required to give steady dc output.

ii. The power output, and therefore, rectification efficiency is quite low.

iii. DC saturation of transformer core resulting in magnetizing current, hysteresis losses and generation of harmonics.

2. Full-Wave Rectifier

The full-wave rectifier utilizes both halves of the input sinusoid. To provide a unipolar output, it inverts the negative halves of the sine wave.

i. Center-tapped transformer full-wave rectifier

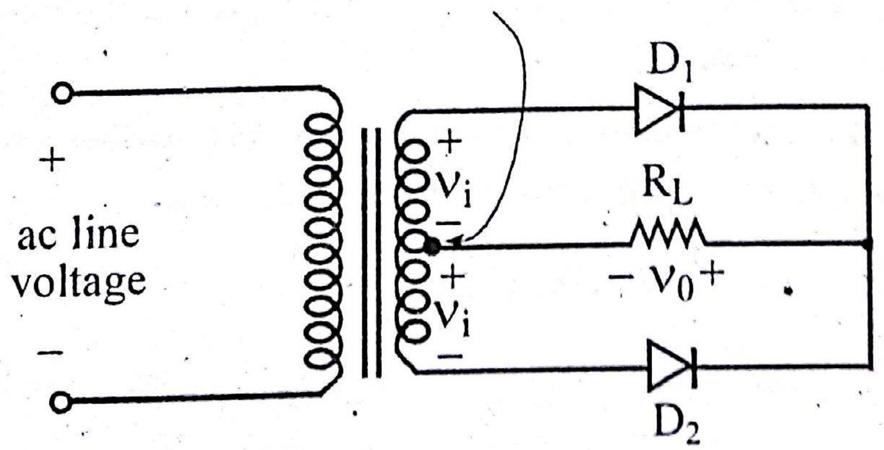


Figure 2.29 *Center-tapped transformer full-wave rectifier*

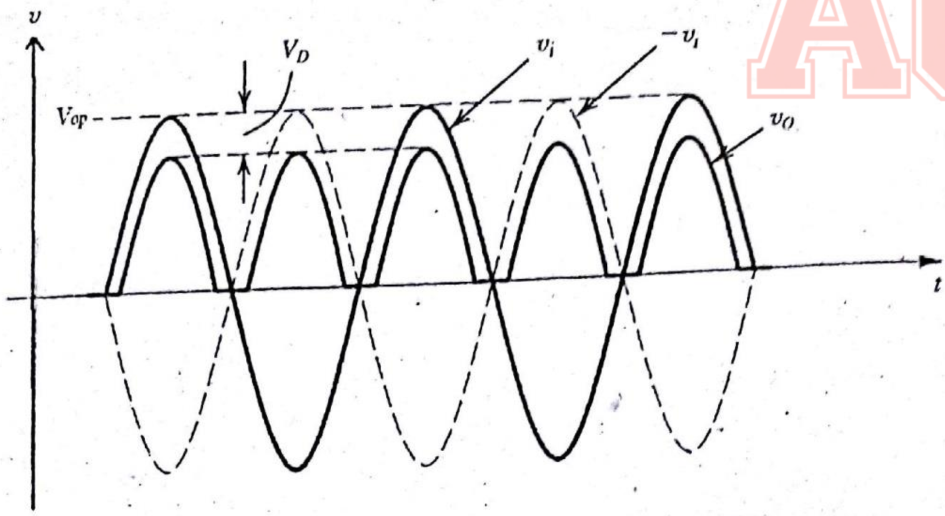


Figure 2.30 ***Input and output waveforms***

As shown in the circuit, the transformer winding is center-tapped to provide two equal voltages v_i across the two halves of the secondary winding with the polarities indicated. During the positive half-cycle of the ac line voltage (primary side), both of the signals labeled v_i will be positive. In this case, D_1 will conduct and D_2 will be reverse biased. The current through D_1 will flow through R_L and back to the center tap of the secondary. During the negative half-cycle of the ac line voltage (primary side), both of the voltages labeled v_i will be negative. Thus, D_1 will be cut off while D_2 will conduct. The current conducted by D_2 will flow through R_L and back to the center tap.

The characteristics of center-tapped transformer full-wave rectifier can be summarized as follows:

i. DC output current: $I_{dc} = \frac{2I_{op}}{\pi}$

ii. DC output voltage: $V_{dc} = \frac{2V_{op}}{\pi} = \frac{2I_{op}}{\pi} R_L$

iii. RMS value of current: $I_{rms} = \frac{I_{op}}{\sqrt{2}}$

iv. RMS value of output voltage: $V_{rms} = \frac{V_{op}}{\sqrt{2}} = \frac{I_{op}}{\sqrt{2}} R_L$

v. Form factor: $k_f = 1.11$

vi. Peak factor: $k_p = \sqrt{2}$

vii. Ripple factor: $\gamma = 0.482$

viii. Rectification efficiency: $\eta = 81.2\%$

ii. Full-wave bridge rectifier

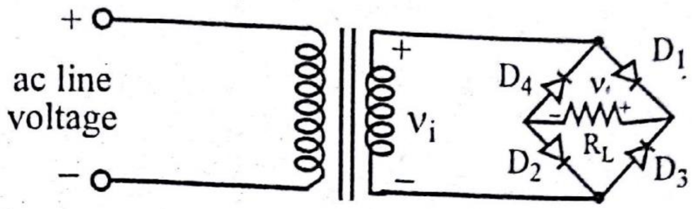


Figure 2.31 *Full-wave bridge rectifier*

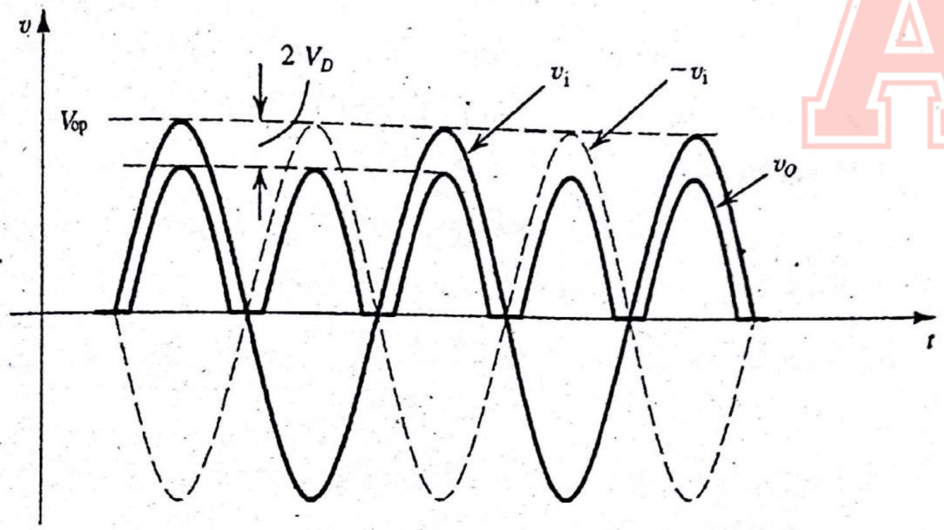


Figure 2.32 *Input and output waveform*

The bridge rectifier circuit operates as follows: During the positive half-cycles of the input voltage, v_i is positive, and thus, current is conducted through diode D_1, resistor R, and diode D_2. Meanwhile, diodes D_3 and D_4 will be reverse biased. During the negative half-cycles of the input voltage, v_i will be negative, and thus $-v_i$ will be positive, forcing current through D_3, R_L, and D_4. Meanwhile, diodes D_1 and D_2 will be reverse biased. It should be noted that, during both half-cycles of the input voltage, current flows through R_L in the direction (from right to left).

The characteristics of full-wave bridge rectifiers can be summarized as follows:

i. DC output current: $I_{dc} = \dfrac{2I_{op}}{\pi}$

ii. DC output voltage: $V_{dc} = \frac{2V_{op}}{\pi} = \frac{2I_{op}}{\pi} R_L$

iii. RMS value of current: $I_{rms} = \frac{I_{op}}{\sqrt{2}}$

iv. RMS value of output voltage: $V_{rms} = \frac{V_{op}}{\sqrt{2}} = \frac{I_{op}}{\sqrt{2}} R_L$

v. Form factor: $k_f = 1.11$

vi. Peak factor: $k_p = \sqrt{2}$

vii. Ripple factor: $\gamma = 0.482$

viii. Rectification efficiency: $\eta = 81.2\%$

Now, we will see the advantages and disadvantages of full-wave rectifiers over half-wave rectifiers.

Advantages

i. The rectification efficiency of a full-wave rectifier is double of that of a half-wave rectifier.

ii. Ripple factor is low, and so simple filtering circuit is required in a full-wave rectifier.

iii. Higher output voltage and higher output power in case of a full-wave rectifier.

Disadvantages

i. Full-wave rectifier needs more circuit elements and is costlier.

RIPPLE VOLTAGE OF CAPACITOR FILTER WHEN CONNECTED ACROSS A RECTIFIER

i. For half-wave rectifier

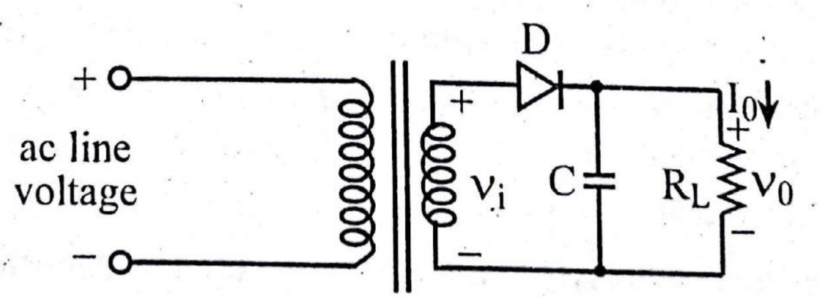


Figure 2.33 ***Half-wave rectifier with a capacitor filter***

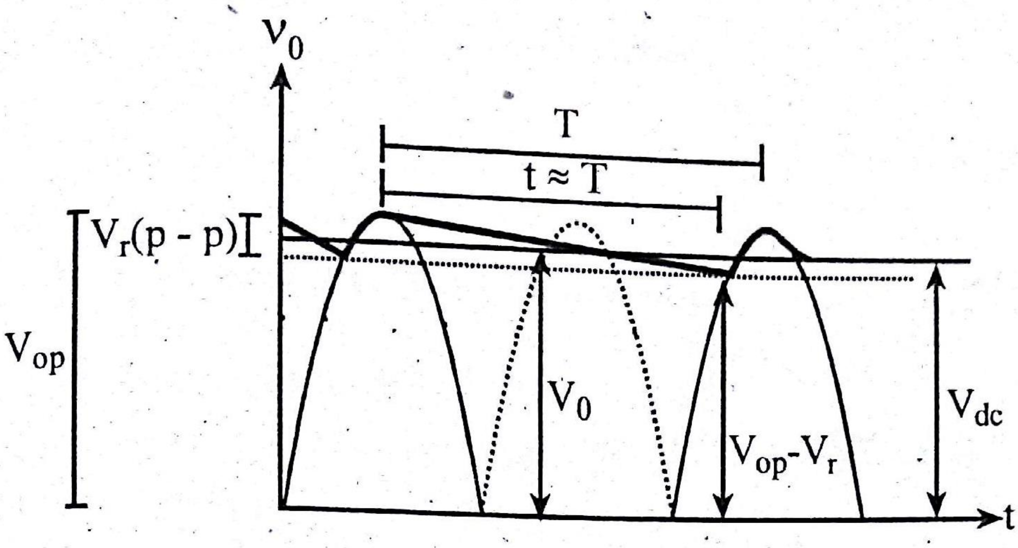


Figure 2.34 *Approximate triangular ripple voltage for capacitor filter (in case of half-wave rectifier)*

$$V_o = \{V_{op} + (V_{op} - V_r)\}/2 = V_{op} - \frac{V_r}{2}$$

If $V_r << V_{op}$, then $V_0 \simeq V_{op}$

The amount of voltage discharge (V_r) in a period of time $t = T$ is given by

$$Q = CV_r = I_0 T; \; I_0 = \frac{V_0}{R_L}$$

$$\text{or, } CV_r = \frac{V_0}{R_L}\frac{1}{f}$$

$$\therefore V_r = \frac{V_0}{fCR_L} \simeq \frac{V_{op}}{fCR_L}$$

where V_r = ripple voltage in peak to peak value.

ii. For full-wave rectifier

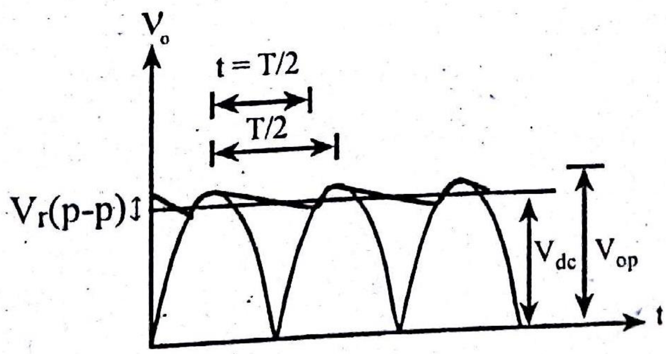


Figure 2.35 *Approximate triangular ripple voltage for capacitor filter (in case of full wave rectifier)*

$$V_o = \{V_{op} + (V_{op} - V_r)\}/2$$

$$= V_{op} - \frac{V_r}{2}$$

If $V_r << V_{op}$, then $V_o \simeq V_{op}$

The amount of voltage discharge (V_r) in a period of time $t = T$ is given by

$$Q = CV_r = I_0 \frac{T}{2};\ I_0 = \frac{V_0}{R_L}$$

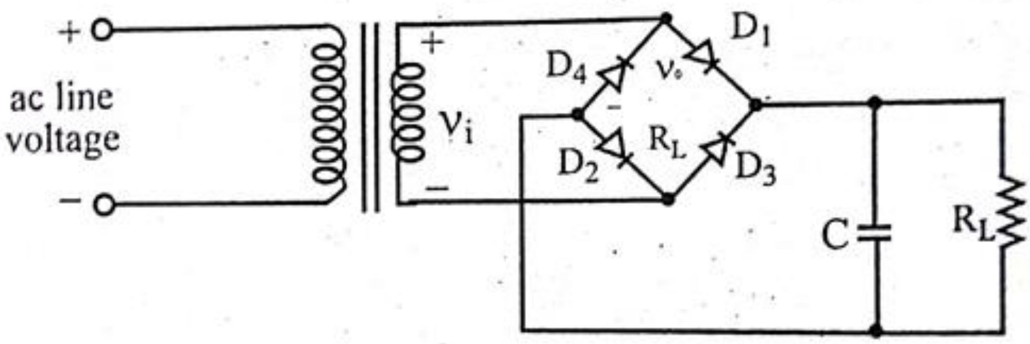
$$\text{or, } CV_r = \frac{V_0}{R_L} \frac{1}{2f}$$

$$\therefore V_r = \frac{V_0}{2fCR_L} \simeq \frac{V_{op}}{2fCR_L}$$

Problem 2.18

Express the ripple factor if smoothing capacitor, C is connected to the bridge rectifier circuit.

Solution:



The ripple factor of a voltage is defined by

$$r = \frac{\text{rms value of ac component of signal}}{\text{average value of signal}}$$

$$= \frac{V_r(\text{rms})}{V_{dc}}$$

$$= \frac{\frac{V_r(\text{p-p})}{2\sqrt{3}}}{V_{dc}}$$

$$= \frac{\frac{V_{op}}{2fCR_L}}{2\sqrt{3}V_{dc}} = \frac{1}{4\sqrt{3}fCR_L} \cdot \frac{V_{op}}{V_{dc}}$$

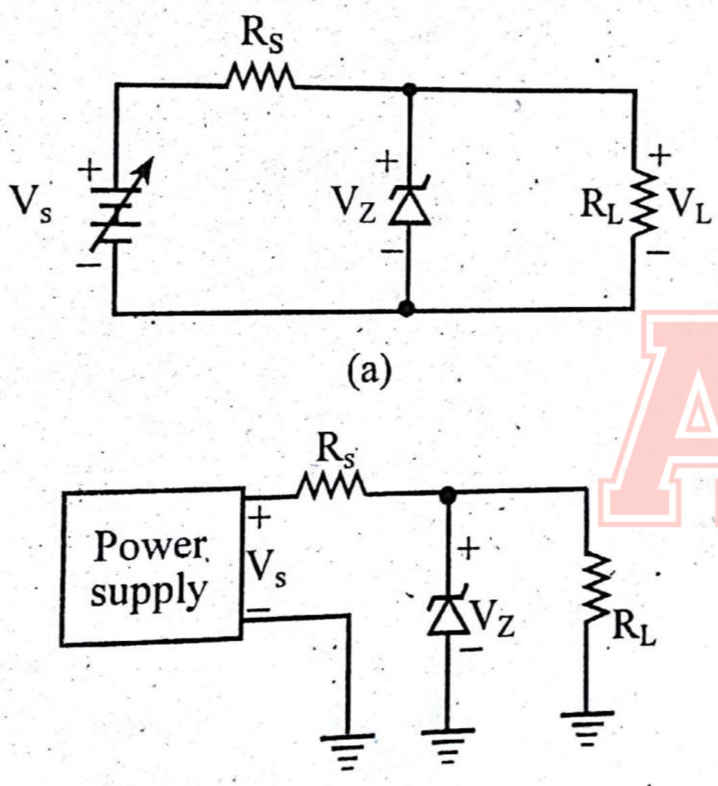
If ripple content is very small, $V_{op} \approx V_{dc}$

$$\boxed{\therefore r = \frac{1}{4\sqrt{3} f C R_L}}$$

Note: For half-wave rectifier, $r = \frac{1}{2\sqrt{3} f C R_L}$

ZENER VOLTAGE REGULATOR

Figure 2.36 (a) shows a loaded Zener regulator and **Figure 2.36 (b)** shows the same circuit with grounds. The Zener diode operates in the breakdown region and holds the load voltage constant. Even if the source voltage changes or the load resistance varies, the load voltage will remain fixed and equal to the Zener voltage.



(b)

Figure 2.36 ***Loaded Zener regulator: (a) Basic circuit (b) practical circuit***

Explanation:

Series current - Assuming that the Zener diode is operating in the breakdown region, the current through the series resistor is given by:

$$I_S = \frac{V_S - V_Z}{R_S}$$

This is Ohm's law applied to the current-limiting resistor. It is the same whether or not there is a load resistor. In other words, if you disconnect the load resistor, the current through the series resistor still equals the voltage across the resistor divided by the resistance.

Load current - Ideally, the load voltage equals the Zener voltage because the load resistor is in parallel with the Zener diode.

As an equation:

$$V_L = V_Z$$

This allows us to use Ohm's law to calculate the load current:

$$I_L = \frac{V_L}{R_L}$$

Zener current - With Kirchhoff's current law:

$$I_S = I_Z + I_L$$

The Zener diode and the load resistor are in parallel. The sum of their currents has to equal the total current, which is the same as the current through the series resistor.

We can rearrange the foregoing equation to get this important formula:

$$I_Z = I_S - I_L$$

This tells you that the Zener current no longer equals the series current, as it does in an unloaded Zener regulator. Because of the load resistor, the Zener current now equals the series current minus the load current.

Working principle

Case I: Regulation when the input source voltage varies

To analyze the working principle, the load resistance (R_L) is kept fixed and the input source voltage (V_S) is varied.

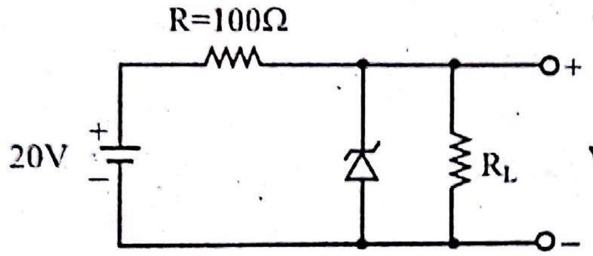
For instance, let's suppose that the V_S increases. This increases I_S, as a result, the current through Zener diode (I_Z) will increase without affecting load current (I_L). As I_S has increased, voltage drop across R_S is increased keeping load voltage (V_L) unchanged. Same principle applies when the source voltage decreases.

Case II: Regulation when the load resistance varies

The input source voltage (V_S) is kept fixed and the load resistance (R_L) is varied for analysis. For instance, suppose that R_L decreases. This will increase I_L but decrease I_Z which makes $I_S = I_Z + I_L$ constant. So, voltage drop across R_S is unchanged and hence, the load voltage (V_L) is held constant. Same principle applies when R_L increases.

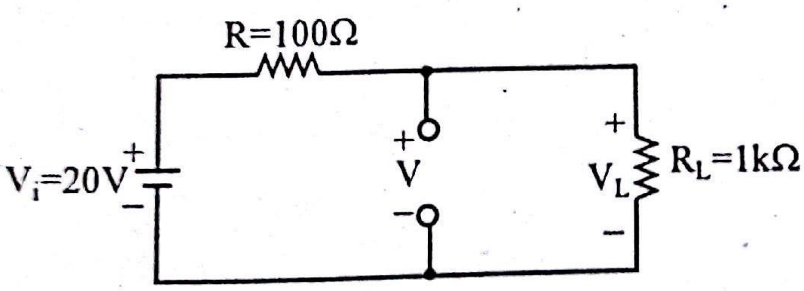
Problem 2.19

Find the Zener current in the following circuit when $R_L = 1\ k\Omega$ and $R_L = 200\ \Omega$ Assume $V_Z = 12\ V$.



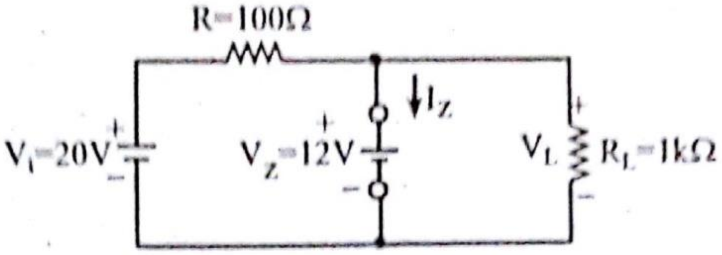
Solution:

a. R_L has a fixed value of 1kΩ. Also $V_i = 20$ V is fixed. We proceed by determining the state of the Zener diode by removing it from the network and calculating the voltage across the resulting open circuit.



$$V = V_L = \left(\frac{R_L}{R + R_L}\right) V_i = \left(\frac{1000}{100 + 1000}\right) \times 20 = 18.18\ V$$

Since V = 18.18 V is greater than $V_Z = 12$ V, the diode is in the "on" state and the following network will result.



But $V_L = V_Z = 12$ V

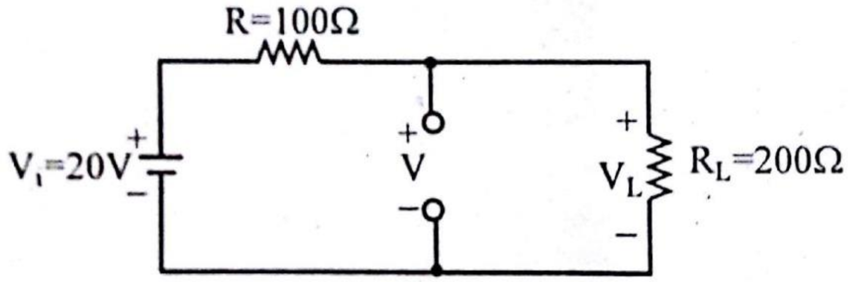
$V_R = V_i - V_L = 20-12 = 8$ V

$$I_L = \frac{V_L}{R_L} = \frac{12}{1000} = 0.012 \text{ A}$$

$$I_R = \frac{V_R}{R} = \frac{8}{100} = 0.08\text{A}$$

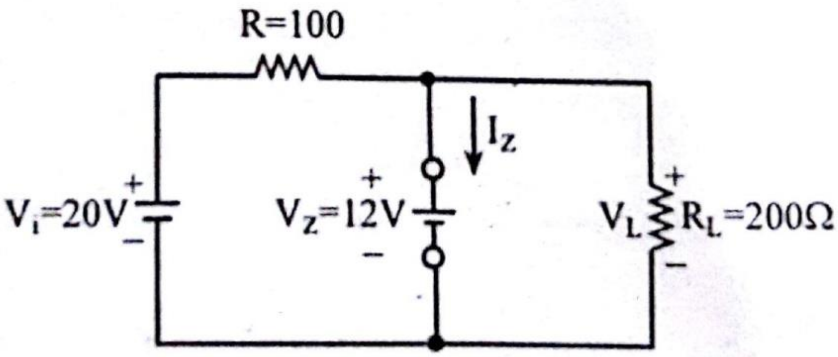
$\therefore I_Z = I_R - I_L = 0.068$ A which is the required Zener current.

b. R_L has a fixed value of 200 Ω. Also $V_i = 20$ V is fixed. We proceed by determining the state of the Zener diode by removing it from the network and calculating the voltage across the resulting open circuit.



$$V = V_L \left(\frac{R_L}{R + R_L}\right) V_i = \left(\frac{200}{100+200}\right) \times 20 = 13.333 \text{ V}$$

Since V = 13.333 V is greater than $V_Z = 12$ V, the diode is in the "on" state and the following network will result.



But $V_L = V_Z = 12$ V

$V_R = V_i - V_L = 20-12 = 8$ V

$$I_L = \frac{V_L}{R_L} = \frac{12}{200} = 0.06 \text{ A}$$

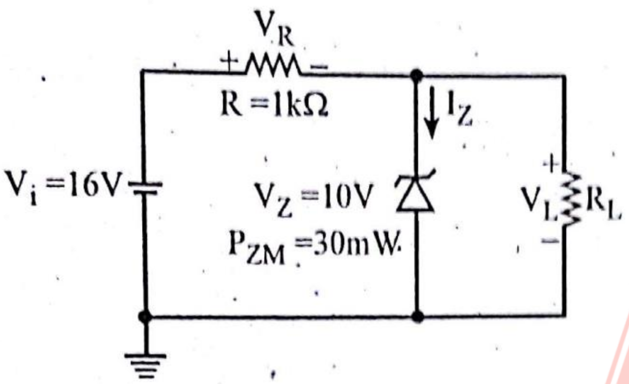
$$I_R = \frac{V_R}{R} = \frac{8}{100} = 0.08 \text{ A}$$

$$\therefore I_Z = I_R - I_L = 0.08 - 0.06$$

$$= 0.02 \text{ A which is the required Zener current.}$$

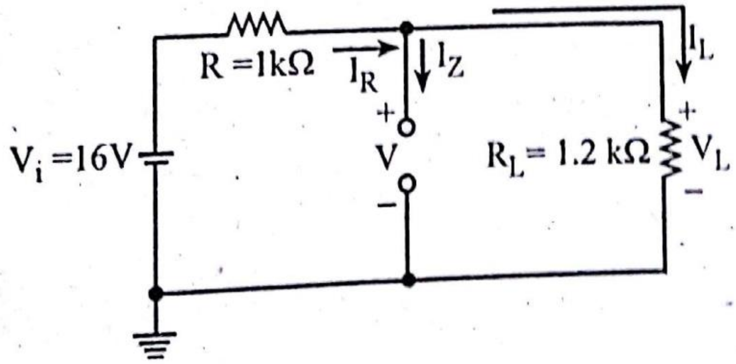
Problem 2.20

For the Zener diode network shown below, determine V_L, V_R, I_Z and P_Z for $R_L = 1.2\ k\Omega$ and $R_L = 3\ k\Omega$.



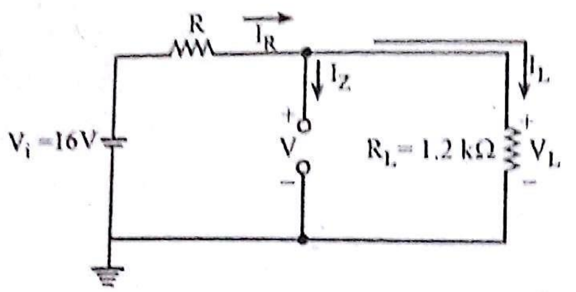
Solution:

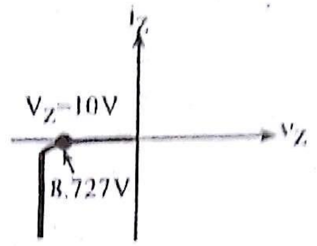
a. R_L has a fixed value of 1.2 kΩ. Also V_i = 16V is fixed. We proceed by determining the state of the Zener diode by removing it from the network and calculating the voltage across the resulting open circuit.



$$V = V_L = \left(\frac{R_L}{R + R_L}\right) V_i = \left(\frac{1.2}{1 + 1.2}\right) \times 16 = 8.727$$

Since V = 8.727 V is less than V_Z = 10V, the diode is in the "off" state and the following network will result.





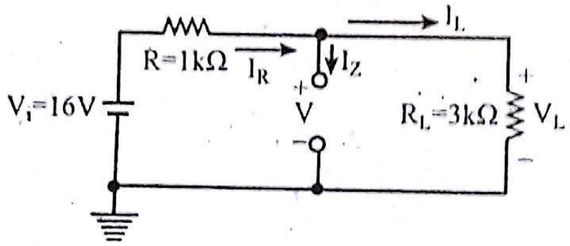
$V_L = V = 8.727$ V

$V_R = V_i - V_L = 16 - 8.727 = 7.27$ V

$I_Z = 0A$

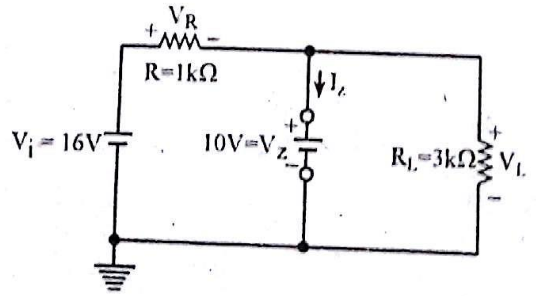
$P_Z = V_Z I_Z = 0W$

b. R_L has a fixed value of 3kΩ. Also $V_i = 16$ V is fixed. We proceed by determining the state of the Zener diode by removing it from the network and calculating the voltage across the resulting open circuit.



$$V = V_L = \left(\frac{R_L}{R + R_L}\right) V_i = \left(\frac{3}{1+3}\right) \times 16 = 12 \text{ V}$$

Since V = 12 V is greater than $V_Z = 10V$, the diode is in the "on" state and the following network will result.



$V_L = V_Z = 10$ V, $V_R = V_i - V_L = 16 - 10 = 6V$

$$I_L = \frac{V_L}{R_L} = \frac{10}{3\times10^3} = 3.33 \times 10^{-3} \text{ A}$$

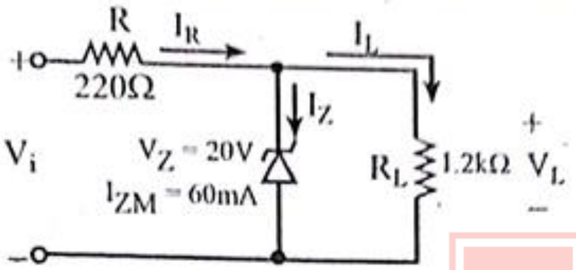
$$I_R = \frac{V_R}{R} = \frac{6}{1\times10^3} = 6 \times 10^{-3} \text{ A}$$

$\therefore I_Z = I_R - I_L = 6\times10^{-3} - 3.33\times10^{-3} = 2.67 \times 10^{-3}$ A

$P_Z = V_Z I_Z = 10\times2.67 \times10^{-3} = 26.7$ m W which is less than the specified $P_{ZM} = 30$ mW.

Problem 2.21

Determine the range of values of V_i that will maintain the Zener diode (see Fig.) in the 'on' state.



Solution:

I_{ZM} = maximum Zener current

V_{imin} = minimum turn-on voltage

V_{imax} = maximum turn-on voltage

$$V_{imin} = \frac{(R_L + R)V_Z}{R_L} = \frac{(1200 + 220)(20)}{1200} = 23.67V$$

$$I_L = \frac{V_L}{R_L} = \frac{V_Z}{R_L} = \frac{20}{1.2} = 16.67 \text{ mA}$$

$$I_{Rmax} = I_{ZM} + I_L = 60 + 16.67 = 76.67 \text{ mA}$$

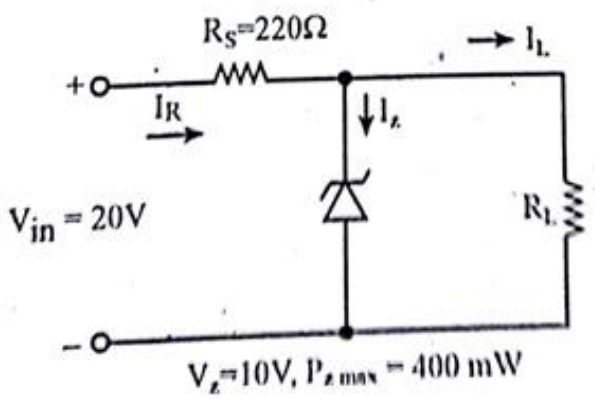
$$V_{imax} = I_{Rmax} R + V_Z = (76.67)(0.22) + 20 = 36.87 \text{ V}$$

Hence, the range of V_i is 23.67 - 36.87 V

Problem 2.22

Determine V_L, I_L, I_Z and I_R for the network shown in figure below for following conditions.

a. If $R_L = 180\ \Omega$ b. If $R_L = 470\ \Omega$



Solution:

[Do it yourself]

TRANSISTORS & MOSFET

INTRODUCTION

The bipolar junction transistor (BJT) is a three-layer semiconductor device which is able to amplify a signal and consists of either two n- and one p- type layers of material or two p- and one n- type layers of material. The former is called an npn transistor, and the latter is called a pnp transistor. It is called "bipolar" because the conduction takes place due to both electrons as well as holes. There are three terminals in the BJT namely the base, the collector, and the emitter.

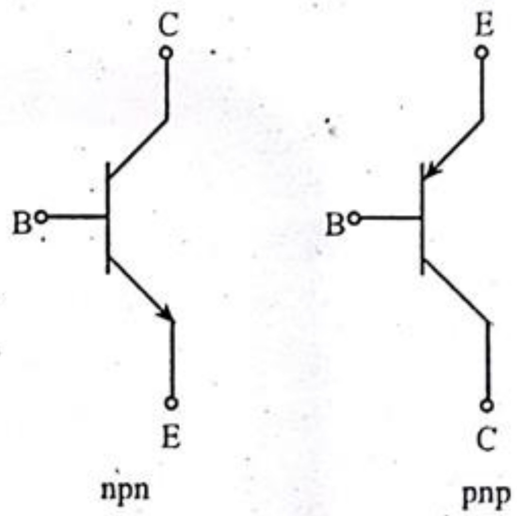
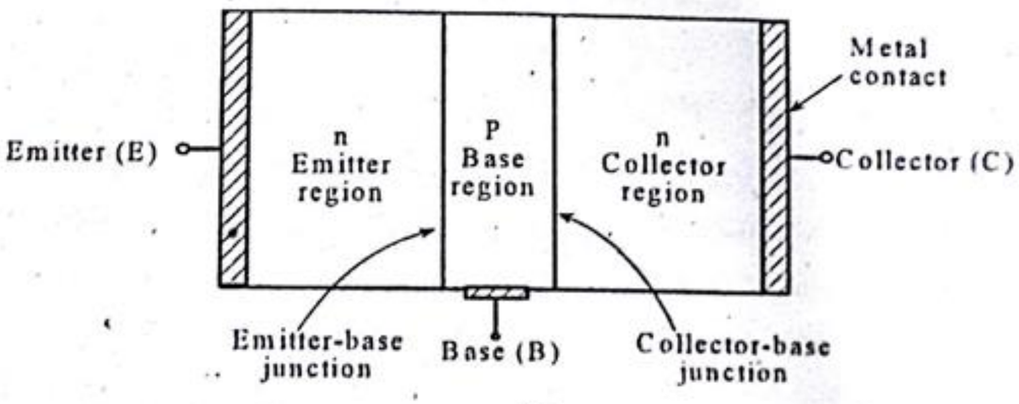


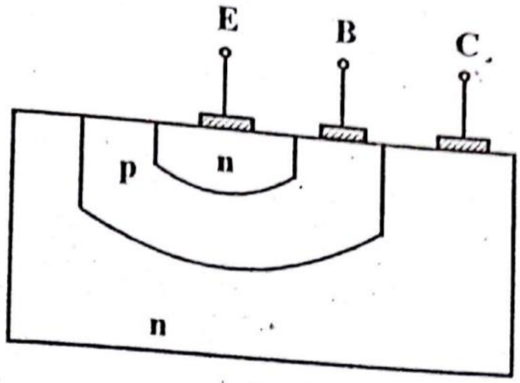
Figure 3.1 ***Circuit symbols for BJTs***

The basic amplifying action of transistor was produced by transferring a current I from a low-to high-resistance circuit. The combination of the two terms in italics results in the label transistor; that is, transfer + resistor → transistor

CONSTRUCTION AND OPERATION OF NPN BJT



(a)



(b)

Figure 3.2 *(a) Simplified sketch of an npn transistor (b) Cross-section of an npn transistor*

As can be seen from **Figure 3.2 (a)**, the BJT is formed from two back-to-back pn junctions: (i) base-emitter junction (ii) base-collector junction. In practice, most of the BJTs are asymmetrically constructed as shown in **Figure 3.2 (b)**. The figure shows that base-collector junction and base-emitter junction have very differently sized surface contact areas. That is, area of base-collector junction is greater than that of base-emitter junction. Emitter region is doped heaviest, collector region is doped moderately, and base region is doped very lightly.

For normal operation (as an amplifier), base-emitter junction is forward biased and base-collector junction is reverse biased. This mode of operation is called active mode of operation.

The forward bias at base-emitter junction reduces its barrier potential (depletion layer is diminished). As a result, electrons from E-region flow into B- region (The electrons are said to be emitted into the base region; hence, the name "emitter"). Holes also flow from p-type B- region into n- type E-region. But because the base is much lightly doped than the emitter, large number of electrons that couldn't recombine with the holes of the base region tend to accumulate in the base region. These electrons behave like minority charge carriers for reverse-biased base-collector junction. Hence, they flow into collector region as minority carrier current. The reverse-biased condition at the base-collector junction causes the base-collector depletion layer (region) to penetrate deeper into the base region. Thus, the thin base region becomes more thinner. As a result, the electrons emitted from E-region into B- region immediately approach the base-collector junction where large positive collector reverse biased voltage is present, causing almost all these electrons to cross the base-collector junction and flow into collector region as collector current. Due to both of these phenomena, more than 99% of emitter current becomes collector current. Very few electrons only succeed recombining with the holes of the base forming base current. it is only 1% or less of the emitter current.

BETA (β) AND ALPHA (α)

The beta (β) of transistor is defined as the ratio of the dc collector current to the dc base current.

$$\beta = I_C/I_B$$

The beta is also known as the "current gain" because a small base current produces a much larger collector current. For low-power transistor (under 1 W), the current gain is typically 100 to 300. High-power transistors (over 1 W) usually have current gains of 20 to 100.

The alpha (α) of a transistor is defined as the ratio of the dc collector current to the dc emitter current.

$$\alpha = \frac{I_C}{I_E}$$

Since the collector current almost equals the emitter current, the alpha is slightly less than 1. For instance, in a low-power transistor, the alpha is typically greater than 0.99. Even in a high-power transistor, the alpha is typically greater than 0.95.

Relationship between α and β

For any transistor,

$$I_E = I_C + I_B$$

$$\text{or, } \frac{I_E}{I_C} = 1 + \frac{I_B}{I_C}$$

Since $\alpha = \frac{I_C}{I_E}$ and $\beta = \frac{I_C}{I_B}$, we have

$$\frac{1}{\alpha} = 1 + \frac{1}{\beta}$$

Simplification will yield

$$\boxed{\alpha = \frac{\beta}{\beta + 1}, \quad \beta = \frac{\alpha}{1 - \alpha}}$$

BJT BIASING

For a transistor to work properly, it should be biased adequately. The purpose of biasing is to establish a proper working point (quiescent or Q-point) of the transistor circuit. Biasing provides a proper voltage across emitter-base junction and base-collector junction so that a constant and required dc collector (emitter) current is established.

i. Emitter-feedback bias

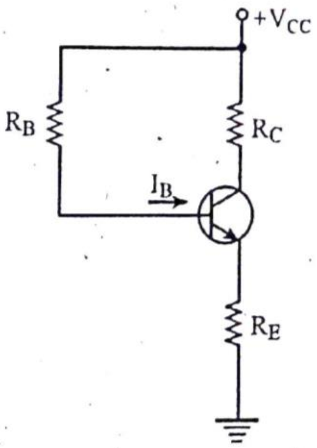


Figure 3.3 ***Emitter-feedback bias***

Historically, the first attempt at stabilizing the Q point was emitter-feedback bias, shown in **Figure 3.3**.

Notice that an emitter resistor has been added to the circuit. The basic idea is this: If I_c increases, V_E increases, causing V_B to increase. More V_B means less voltage across R_B. This results in less I_B, which opposes the original increase in I_c. It's called feedback because the change in emitter voltage is being fed back to the base circuit. Also, the feedback is called negative because it opposes the original change in collector current. Emitter-feedback bias never became popular. The movement of the Q point is still too large for most applications that have to be mass-produced. Here are the equations for analyzing the emitter-feedback bias:

$$I_E = \frac{V_{CC} - V_{BE}}{R_E + R_B/\beta}$$

$$V_E = I_E R_E$$

$$V_B = V_E + 0.7V$$

$$V_C = V_{CC} - I_C R_C$$

ii. **Collector-feedback bias**

Figure 3.4 shows collector-feedback bias (also called self-bias). Historically, this was another attempt at stabilizing the Q point. Again, the basic idea is to feed back a voltage to the base in an attempt to neutralize any change in collector current.

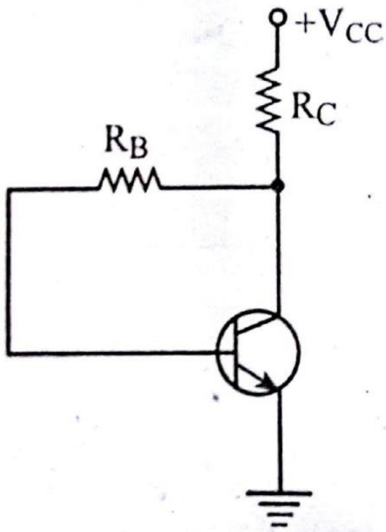


Figure 3.4 ***Collector- feedback bias***

For instance, suppose the collector current increases. This decreases the collector voltage, which decreases the voltage across the base resistor. In turn, this decreases the base current, which opposes the original increase in collector current. Like emitter-feedback bias, collector-feedback uses negative feedback in an attempt to reduce the original change in collector current. Here are the equations for analyzing collector-feedback bias:

$$I_E = \frac{V_{CC} - V_{BE}}{R_c + R_B/\beta}$$

$$V_B = 0.7V$$

$$V_C = V_{CC} - I_C R_C$$

The Q point is usually set near the middle of the load line by using a base resistance of:

$$R_B = \beta R_C$$

iii. **Collector- and emitter-feedback bias**

Emitter-feedback bias and collector-feedback bias were the first steps toward a more stable bias for transistor circuits. Even though the idea of negative feedback is sound, these circuits fall short because there is not enough negative feedback to do the job. This is why the next step in biasing was the circuit shown in **Figure 3.5.**

The basic idea is to use both emitter and collector feedback to try to improve the operation.

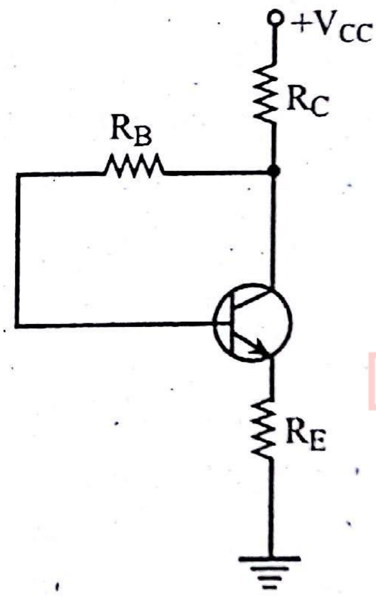


Figure 3.5 *Collector-emitter feedback bias*

As it turns out, more is not always better. Combining both types of feedback in one circuit helps but still falls short of the performance needed for mass production. Here are the equations for analyzing it.

$$I_E = \frac{V_{CC} - V_{BE}}{R_c + R_E + R_B/\beta}$$

$V_E = I_E R_E$

$V_B = V_E + 0.7V$

$V_C = V_{CC} - I_C R_C$

iv. Voltage-divider bias

Figure 3.6 shows the most widely used biasing circuit. Notice that the base circuit contains a voltage divider (R_1 and R_2). Because of this, this circuit is called voltage-divider bias (VDB).

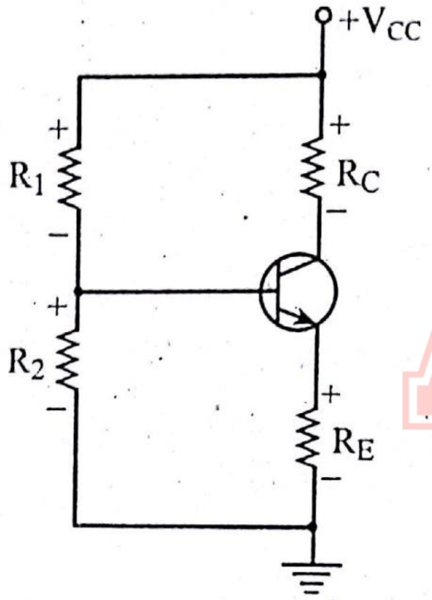
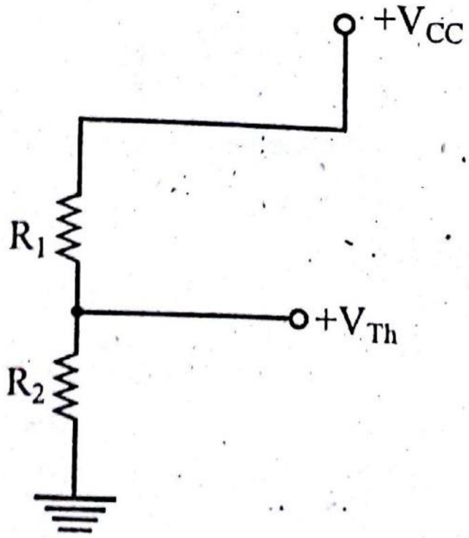


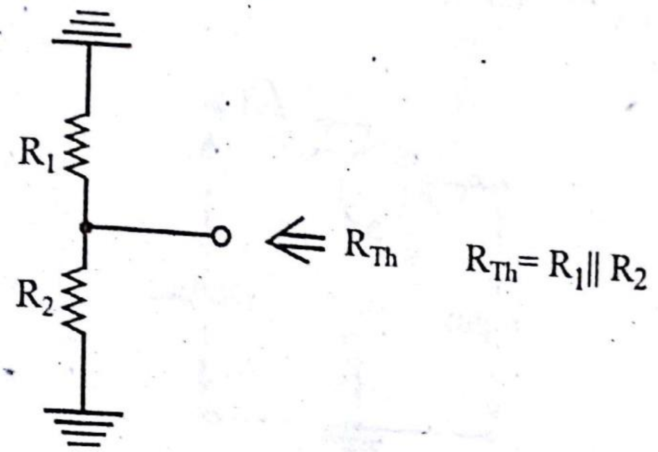
Figure 3.6 ***Voltage-divider bias***

In any well-designed VDB circuit, the base current is much smaller than the current through the voltage divider. Since the base current has a negligible effect on the voltage divider, we can mentally open the connection between the voltage divider and the base to get the equivalent circuit as shown.

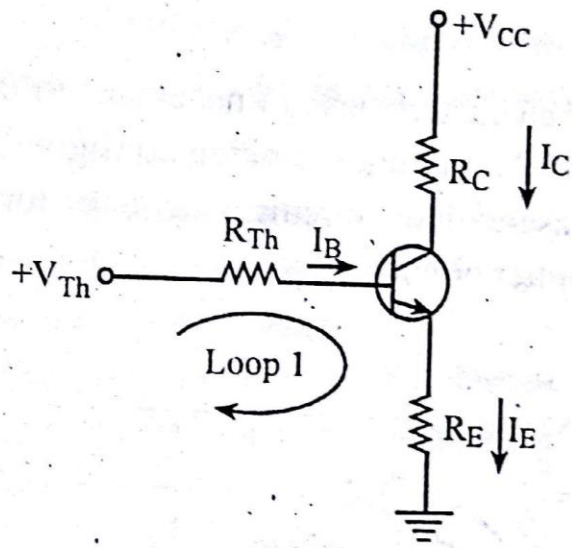


$$V_{Th} = \left(\frac{R_2}{R_1 + R_2}\right) \times V_{CC}$$

Looking back into the voltage divider with V_{CC} grounded, we see R_1 in parallel with R_2.



Finally, the circuit with its Thevenin's equivalent is



Applying KVL in loop 1,

$$V_{Th} - I_B R_{Th} - V_{BE} - I_E R_E = 0$$

$$\text{or, } V_{Th} - \frac{I_E}{(\beta + 1)} R_{Th} - V_{BE} - I_E R_E = 0$$

$$\therefore I_E = \frac{V_{Th} - V_{BE}}{R_E + R_{Th} / (\beta + 1)}$$

BJT CONFIGURATION

(1) Common-emitter configuration

It is the most frequently encountered transistor configuration. It is used in about 90 to 95 percent of all transistor applications. Its application areas include preamplifier and power amplifier circuits. It is called the "common-emitter configuration" because the emitter is common or reference to both the input and output terminals (in this case, common to both the base and collector terminals).

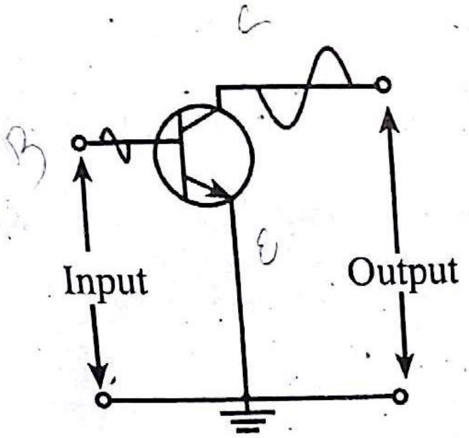
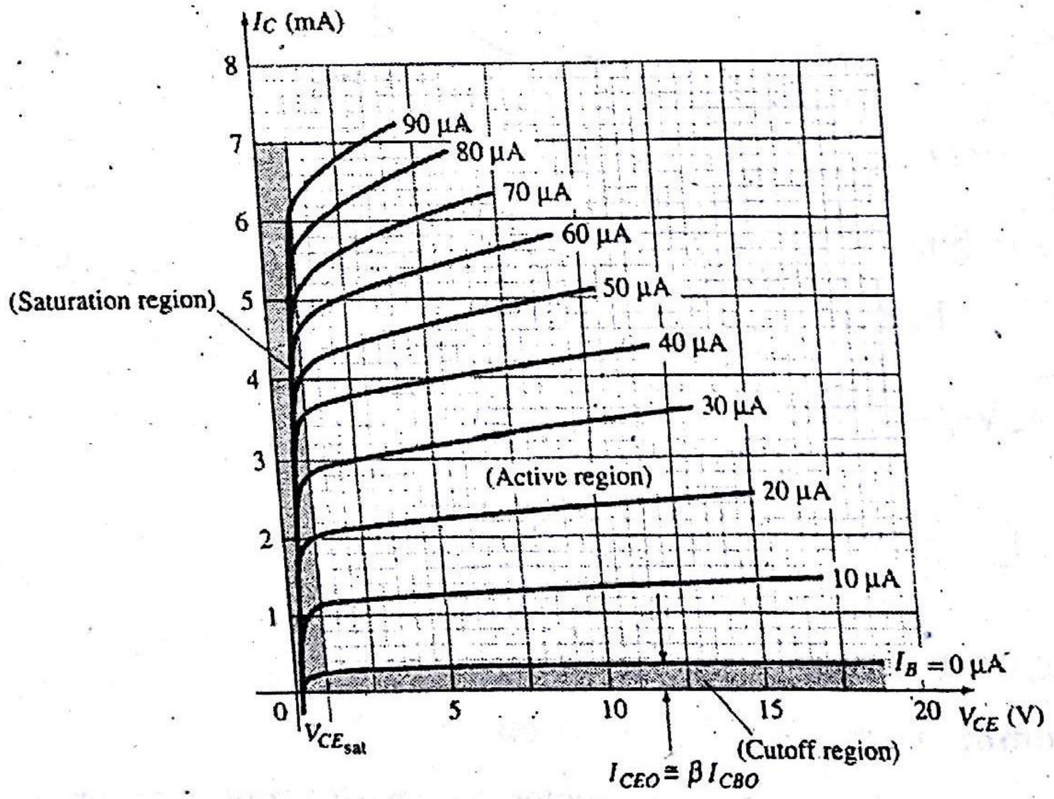
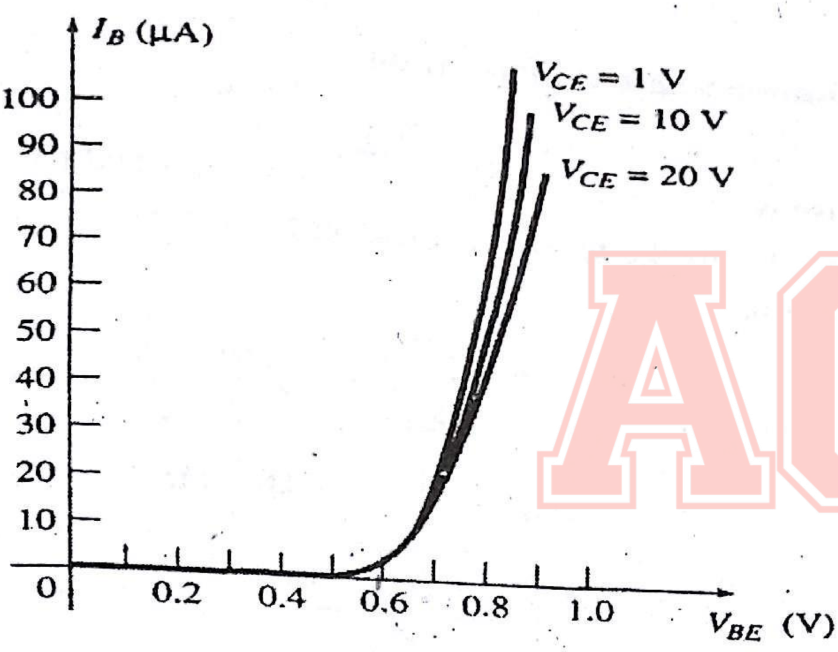


Figure 3.7 ***Common- emitter configuration (npn transistor)***

Two sets of characteristics are necessary to describe fully the behaviour of the common-emitter configuration: one for the input or base-emitter circuit and one for the output or collector-emitter circuit.



(a)



(b)

Figure 3.8 *Characteristics of a silicon transistor in the common-emitter configuration: (a) output characteristics (b) input characteristics*

For the common-emitter configuration, the output characteristics are a plot of the output current (I_C) versus output voltage (V_{CE}) for a range of values of input current (I_B). The input characteristics are a plot of the input current (I_B) versus the input voltage (V_{BE}) for a range of values of output voltage (V_{CE}).

The active region for the common-emitter configuration is that portion of the upper-right quadrant that has the greatest linearity, that is, that region in which the curves for I_B are nearly straight and equally spaced. In **Figure 3.8 (a)**, this region exists to the right of the vertical dashed line at $V_{CE}(sat)$ and above the curve for I_B equal to zero. The region to the left of $V_{CE}(sat)$ is called the saturation region.

The active region of the common-emitter configuration can be employed for voltage, current, or power amplification. The cutoff region is the region between I_B = 0 μA and the V_{CE}-axis. This region (below I_B = 0 μA) is to be avoided if an undistorted output signal is required.

(2) Common-base configuration

In this configuration, input is applied between emitter and base while output is taken across collector and base. Thus, the base forms the terminal common to both the input and output circuits.

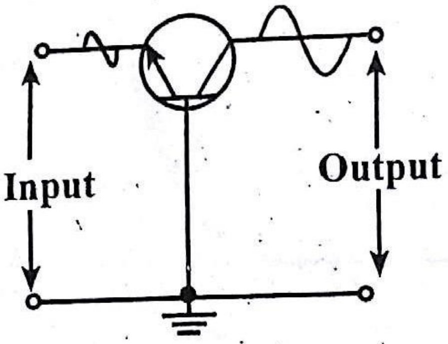
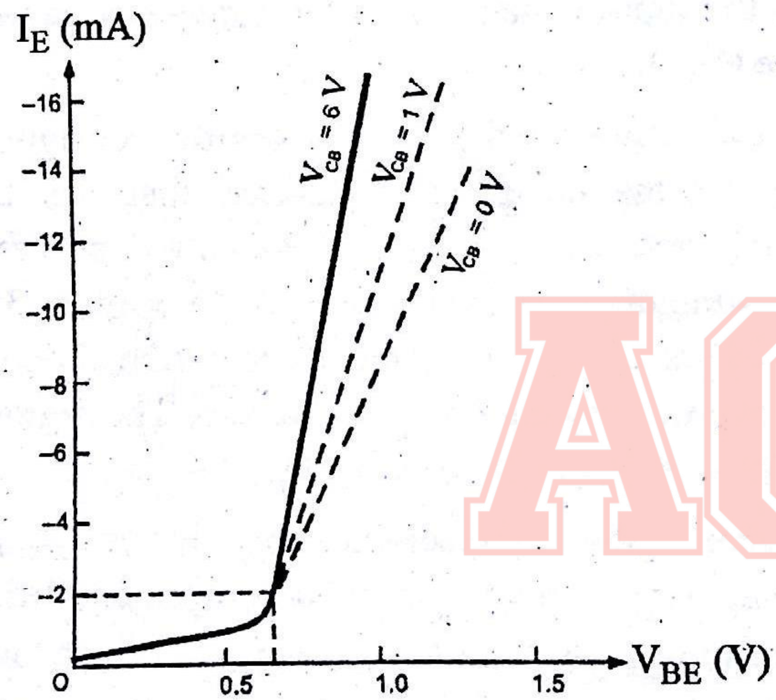
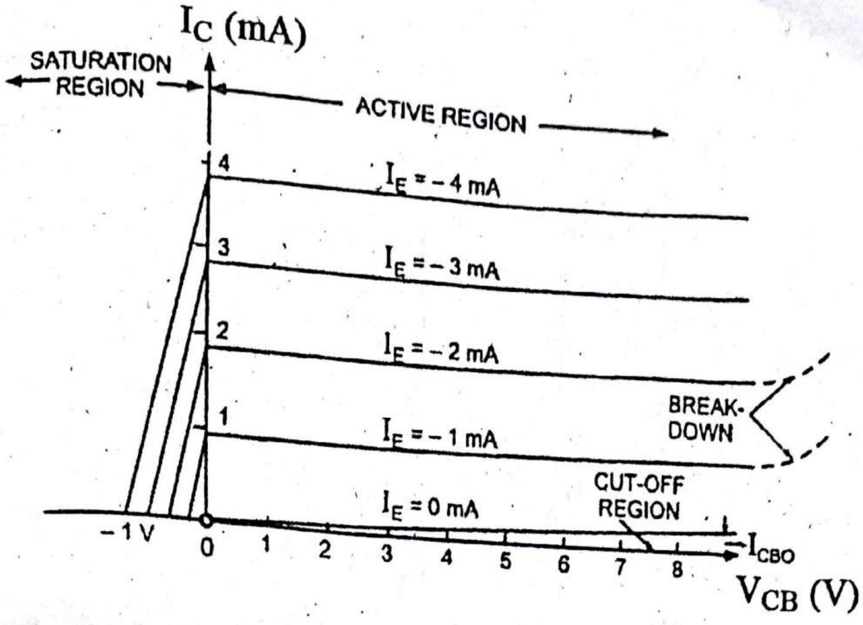


Figure 3.9 *Common-base configuration (npn transistor)*

Two sets of characteristic are required to explain the nature of common-base configuration: input characteristic curve and output characteristic curve. The curve drawn between emitter current I_E and base-emitter voltage V_{BE} for a given value of collector-base voltage V_{CB} is known as input characteristic. While the curve drawn between collector current I_C and collector-base voltage V_{CB} for a given value of emitter current I_E is known as output characteristic.



(a)



(b)

Figure 3.10 *Characteristic curves: (a) Input characteristics (b) Output characteristics*

(3) Common-collector configuration

In this circuit arrangement, input is applied between base and collector while the output is taken across emitter and collector. Thus, the collector forms the terminal common to both the input and output circuits.

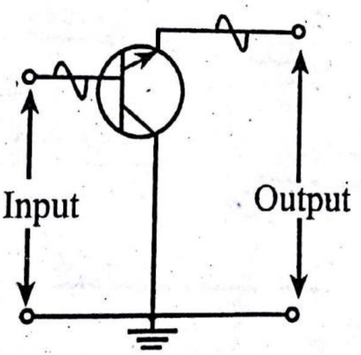
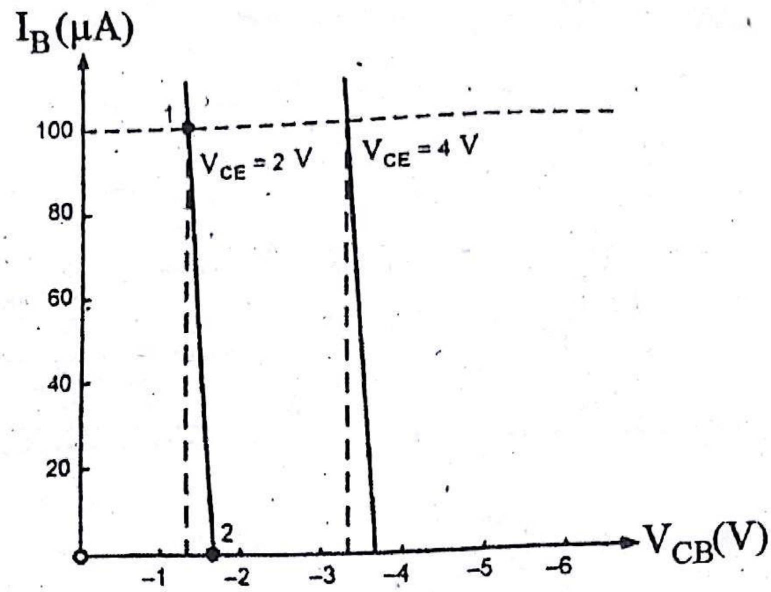
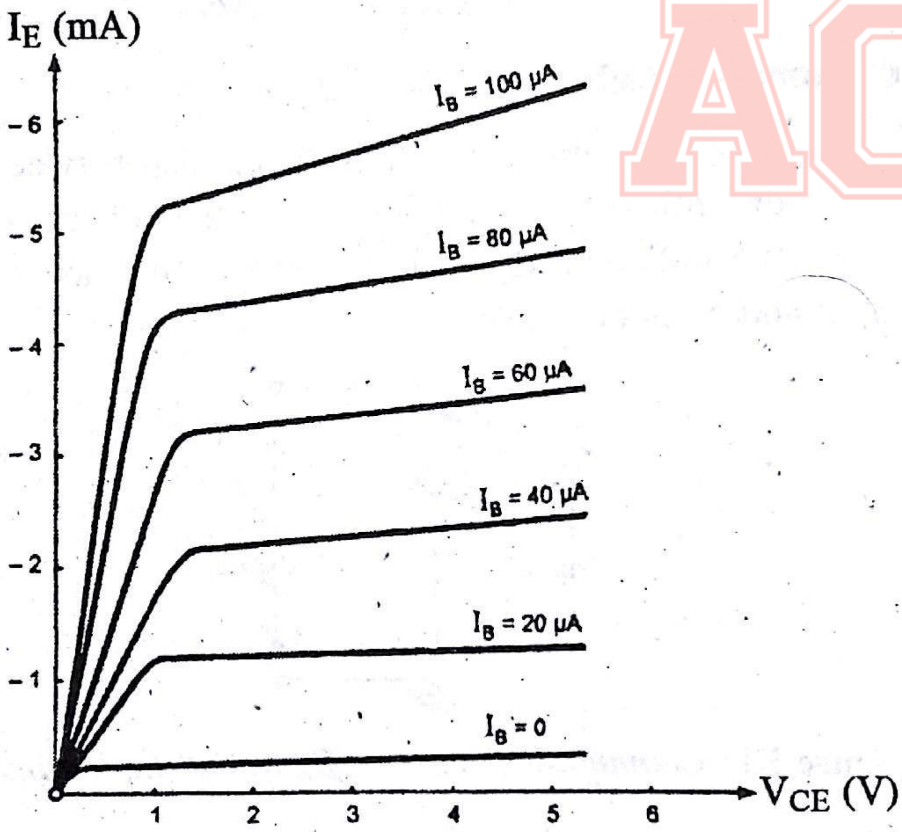


Figure 3.11 *Common-collector configuration (npn transistor)*

The performance of transistor can be determined from two characteristic curves: input characteristic curve and output characteristic curve. The curve drawn between base current I_B and collector-base voltage V_{CB} for a given value of collector-emitter voltage V_{CE} is known as input characteristic. While the curve drawn between emitter current I_E and collector-emitter voltage V_{CE} for a given value of base current I_B is known as output characteristic.



(a)



(b)

Figure 3.12 *Characteristic curves: (a) Input characteristics (b) Output characteristics*

DIFFERENT MODES OF OPERATION OF BJT

The following table illustrates the different BJT modes of operation. EBJ stands for emitter-base junction and CBJ stands for collector-base junction.

Mode	EBJ	CBJ
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Reverse active	Reverse	Forward
Saturation	Forward	Forward

Table 3.1 *BJT modes of operation*

BJT SWITCH AND LOGIC CIRCUITS

i. The BJT Inverter (Transistor Switch)

NOT gate

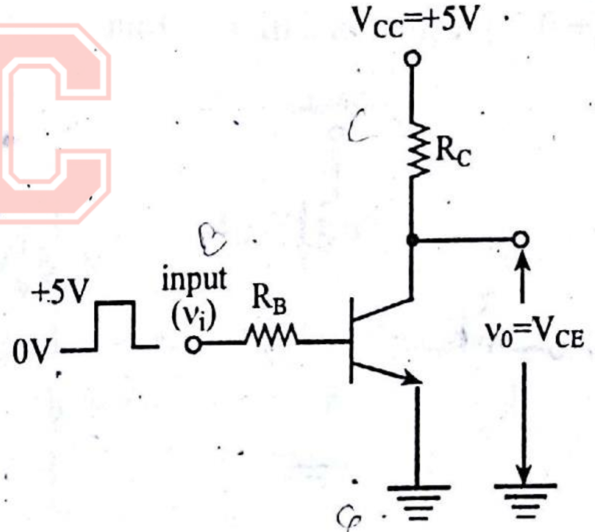


Figure 3.13 *An npn transistor inverter or switch*

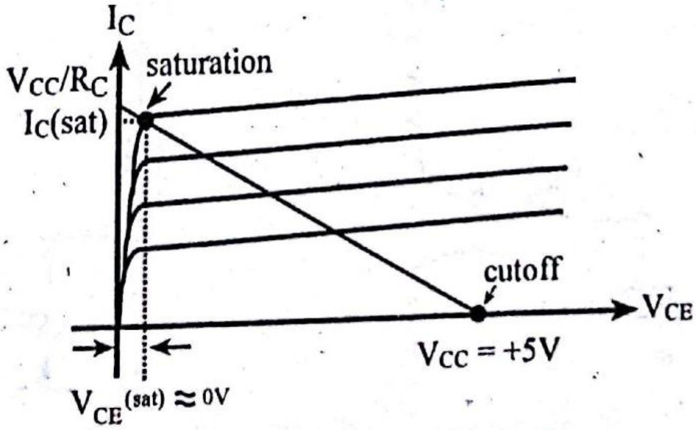


Figure 3.14 *A load line plotted on I_C versus V_{CE} characteristics*

Consider an npn transistor inverter as shown in the **Figure 3.13** above. When the input to the transistor is high (+ 5 V), the base-emitter junction is forward biased and current flows through R_B into the base. The values of R_B and R_C are chosen (designed) so that the amount of base current flowing is enough to saturate the transistor. Note that the value of V_{CE} corresponding to a point in the saturation region, called V_{CE} (sat) is very nearly 0 (typically about 0.1V). When the transistor is saturated, it is said to be ON. The conclusion reached is: A high input to the inverter (+5 V) results in a low output (≈0 V).

When the input to the transistor is low (0 V), the base-emitter junction has no forward bias applied to it, so no base current, and hence, no collector current flows. There is therefore, no voltage drop across R_C, and it follows that V_{CE} must be the same as V_{CC}: +5 V. In this situation, the transistor is in the cutoff region. The conclusion reached is: A low input to the inverter (+ 0 V) results in a high output (+ 5 V).

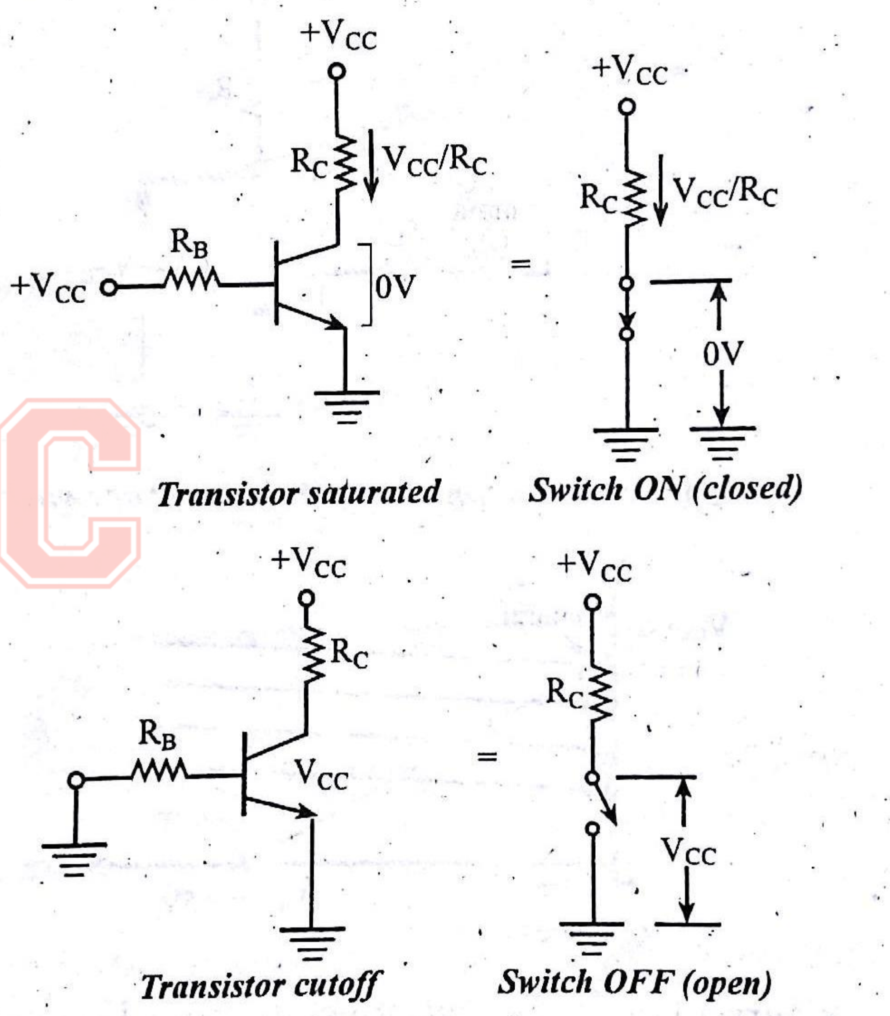


Figure 3.15 *The transistor as a voltage-controlled switch. A high input closes the switch and a low input opens it.*

ii. OR gate

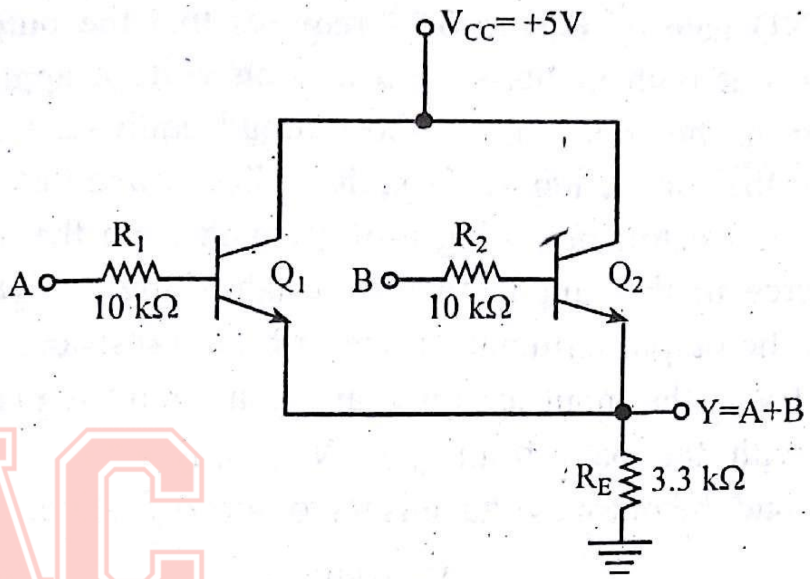


Figure 3.16 *OR gate*

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Table 3.2 *Truth table*

If both A and B of the OR gate of **Figure 3.16** have a low or 0V input, both transistors are off (cutoff), and the impedance between the collector and the emitter of each transistor can be approximated by an open circuit. Mentally replacing both transistors by open circuits between the collector and the emitter will remove any connection between the applied bias of 5V and the output. The result is zero current through each transistor and through the 3.3 kΩ resistor. The output voltage is therefore 0V or "low". On the other hand, if transistor Q_1 is on and Q_2 is off due to a positive voltage at the base of Q_1 and 0V at the base of Q_2, then the short-circuit equivalent between the collector and emitter for transistor Q_1 can be applied, and the voltage at the output is 5V or "high". Finally, if both transistors are turned on by a positive voltage applied to the base of each, they will both ensure that the output voltage is 5V or "high".

iii. AND gate

The AND gate of **Figure 3.17** requires that the output be high only if both inputs have a turn-on voltage applied. If both are in the "on" state, a short-circuit equivalent can be used for the connection between the collector and the emitter of each transistor, providing a direct path from the applied 5V source to the output, thereby establishing a high or 1 state at the output terminal. If one or both transistors are off due to 0V at the input terminal, an open circuit is placed in series with the path from the 5V supply voltage to the output, and the output voltage is 0V or an "off" state.

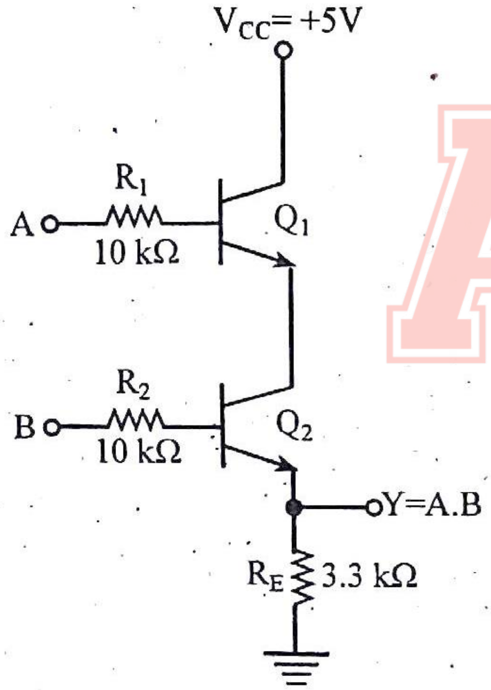


Figure 3.17 ***AND gate***

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Table 3.3 ***Truth table***

iv. **NOR gate**

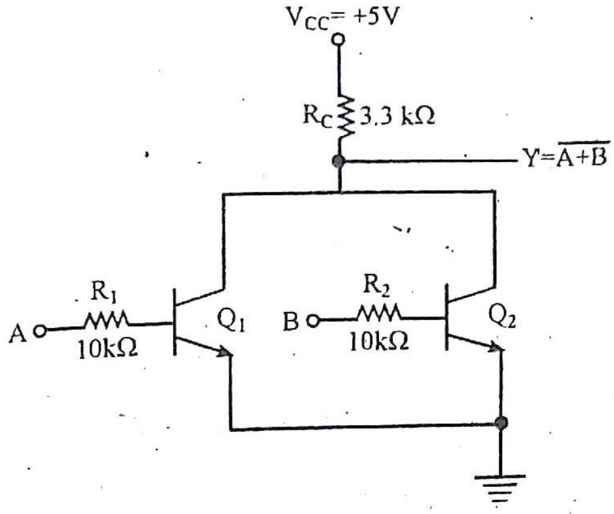


Figure 3.18 ***NOR gate***

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

Table 3.4 ***Truth table***

v. **NAND gate**

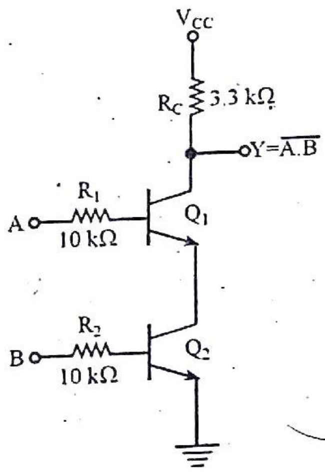


Figure 3.19 ***NAND gate***

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Table 3.5 ***Truth table***

vi. NOT gate

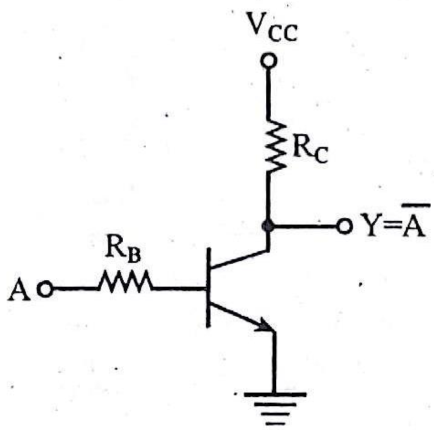


Figure 3.20 *NOT gate*

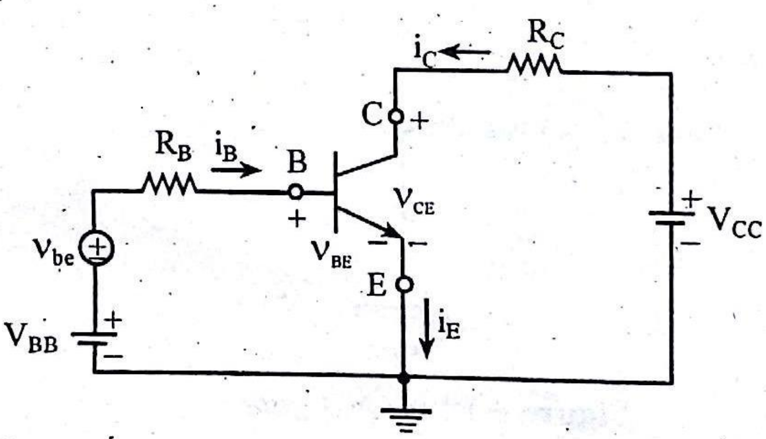
A	Y
0	1
1	0

Table 3.6 *Truth table*

TRANSISTOR MODELING

A model is a combination of circuit elements, properly chosen, that best approximates the actual behavior of a semiconductor device under specific operating conditions.

Consider a circuit in common-emitter configuration as shown in the **Figure 3.21** below.



$i_C = I_C + i_c$

$v_{BE} = V_{BE} + v_{be}$

$v_{CE} = V_{CE} + v_{ce}$

Figure 3.21 ***Conceptual circuit to illustrate the operation of the transistor as an amplifier***

i. DC analysis (Large signal model)

For DC analysis, all the AC components are removed (here, v_{in} is replaced by a short circuit).

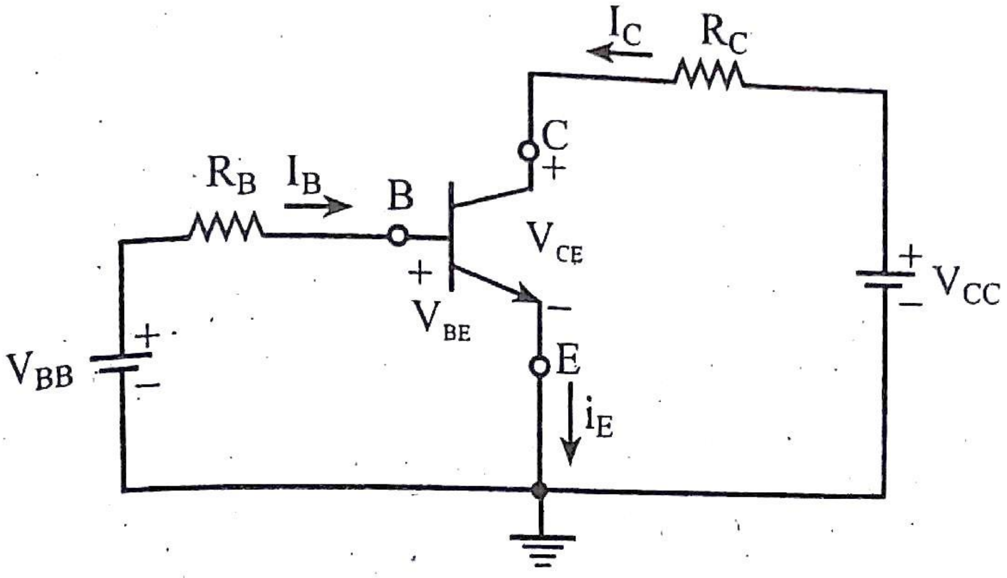


Figure 3.22 *Circuit for DC analysis*

Looking through the base terminal (B), we notice a forward biased diode between base (B) and emitter (E) and the current I_B flows into the terminal. Looking at the collector terminal, we see that current I_C flows into the terminal.

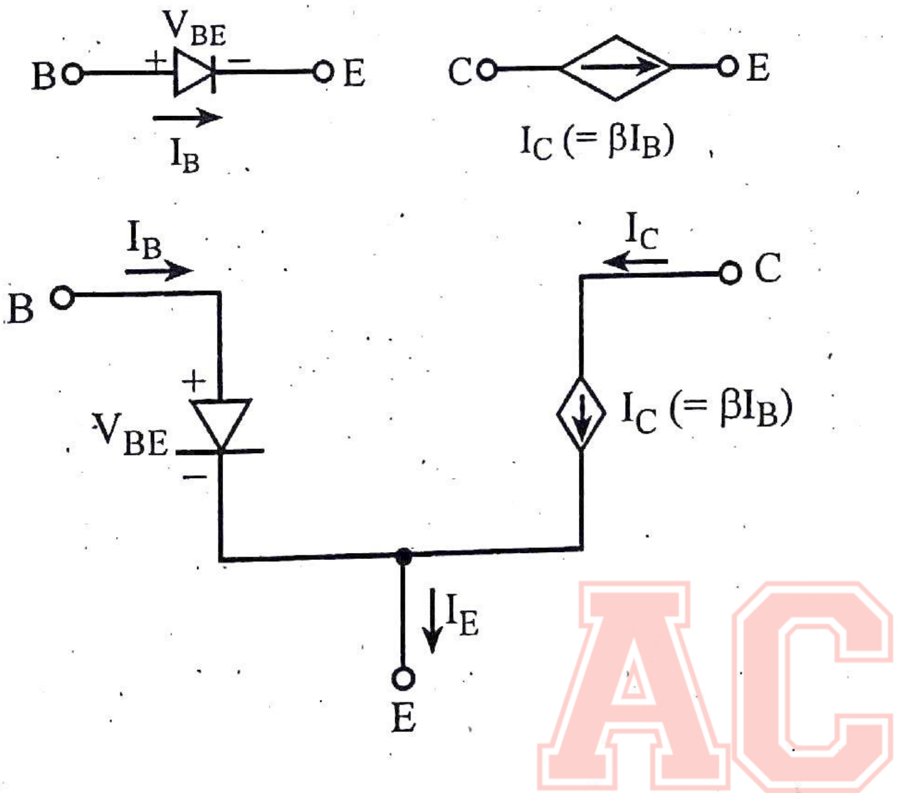


Figure 3.23 *Large signal model [DC (bias) equivalent circuit]*

ii. AC analysis (small signal model)

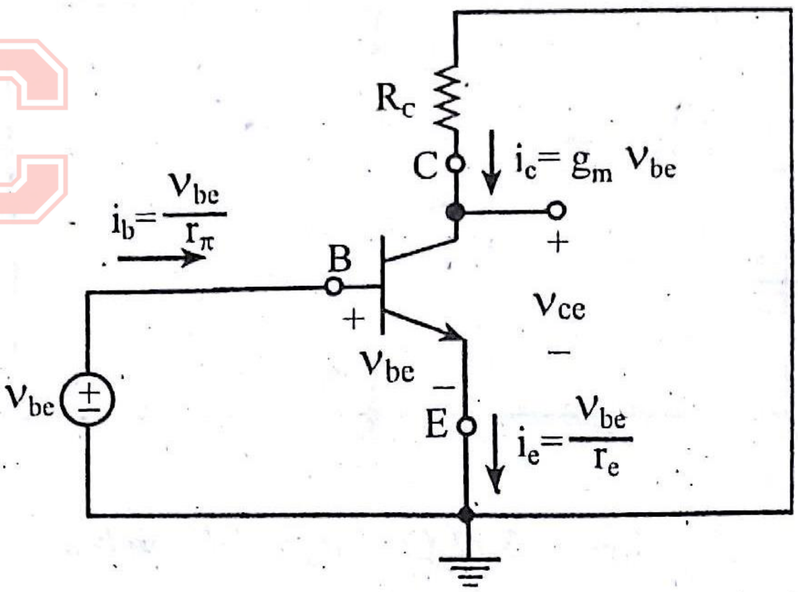


Figure 3.24 *Small signal model (AC equivalent circuit)*

For AC analysis, we eliminate (short circuit) the DC sources and thus, only the signal components are present. Note that, this is a representation of the signal operation of the BJT and not an actual amplifier circuit.

THE HYBRID-π MODEL

This is the most widely used model for the BJT. The model (**Figure 3.25 (a)**) represents the BJT as a voltage-controlled current source and explicitly includes the input resistance looking into the base, r_π. The model obviously yields $i_c = g_m v_{be}$ and $i_b = \frac{v_{be}}{r_\pi}$. This model also yields the correct expression for i_e as follows:

$$i_e = \frac{v_{be}}{r_\pi} + g_m\, v_{be}$$

$$= \frac{v_{be}}{r_\pi}(1 + g_m\, r_\pi) = \frac{v_{be}}{r_\pi}(1 + \beta) = \frac{v_{be}}{\left(\frac{r_\pi}{1+\beta}\right)} = \frac{v_{be}}{r_e}$$

A slightly different equivalent circuit model (**Figure 3.25 (b)**) can be obtained by expressing the current of the controlled source ($g_m v_{be}$) in terms of the base current i_b as follows:

$$g_m v_{be} = g_m(i_b r_\pi) = (g_m r_\pi)\, i_b = \beta i_b$$

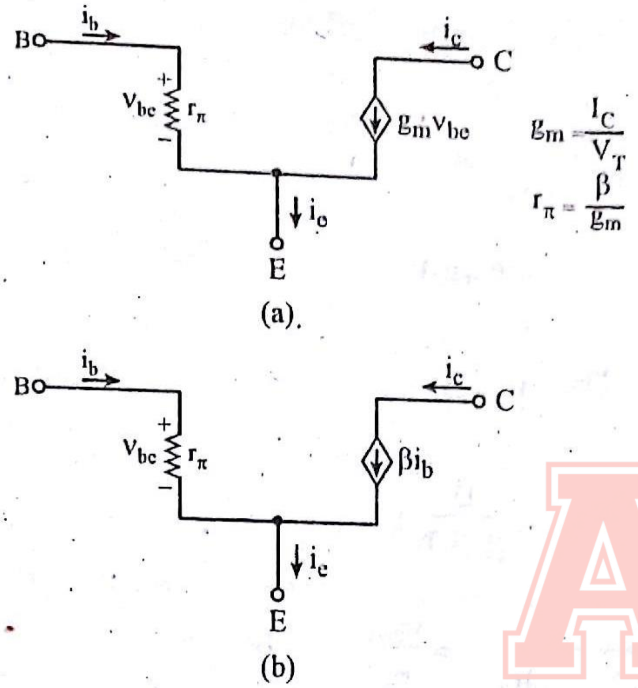


Figure 3.25 *Two slightly different versions of the simplified hybrid-π model for the small-signal operation of the BJT. The equivalent circuit in (a) represents the BJT as a voltage-controlled current source (a transconductance amplifier), and in (b) represents the BJT as a current-controlled current source (a current amplifier).*

THE T MODEL

Although the hybrid-π model can be used to carry out small-signal analysis of all transistor circuits, there are situations in which an alternative model, called the T model, is much more convenient.

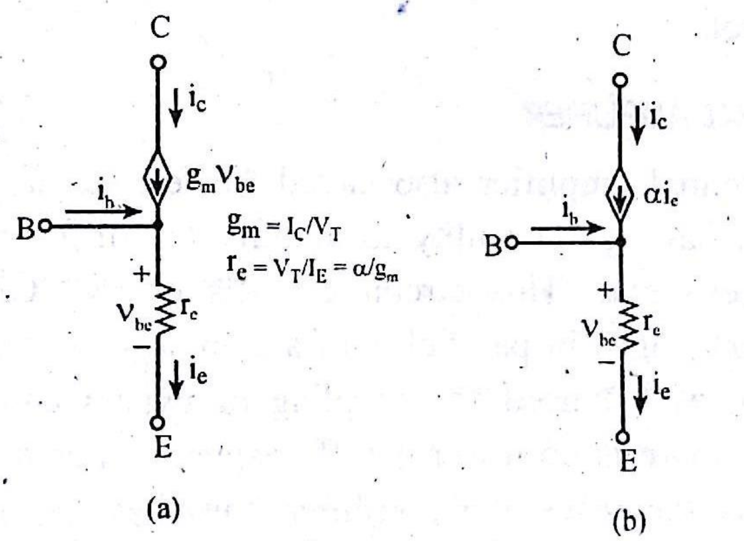


Figure 3.26 *Two slightly different versions of the T model of the BJT. The circuit in (a) is a voltage-controlled current source representation, and in (b) is a current-controlled current source representation.*

The value of i_b is expressed as

$$i_b = \frac{v_{be}}{r_e} - g_m v_{be}$$

$$= \frac{v_{be}}{r_e}(1 - g_m r_e)$$

$$= \frac{v_{be}}{r_e}(1 - \alpha)$$

$$= \frac{v_{be}}{r_e}\left(1 - \frac{\beta}{\beta + 1}\right)$$

$$= \frac{v_{be}}{(\beta + 1) r_e} = \frac{v_{be}}{r_\pi}$$

If in the model of **Figure 3.26 (a)**, the current of the controlled source is expressed in terms of the emitter current as follows:

$$g_m v_{be} = g_m (i_e r_e)$$

$$= (g_m r_e)\, i_e = \alpha\, i_e$$

we obtain the alternative T model (see **Figure 3.26 (b)**).

Both of these models explicitly show the emitter resistance r_e rather than the base resistance r_π featured in the hybrid-π model.

DIFFERENTIAL AMPLIFIER

Differential amplifier also called "difference amplifier" is a circuit having the ability to amplify the difference of input voltage signal. This circuit consists of two CE (common emitter) stages in parallel with a common emitter resistor. It eliminates the need for coupling or bypass capacitors, and hence, there is no lower cutoff frequency. For this and other reasons, the differential amplifier is used as the input stage of almost every IC op-amp.

Ideally, the circuit (see **Figure 3.27**) has identical transistors that remain biased in forward active region, and equal collector resistors. The two base terminals are the two signal inputs v_{i1} and v_{i2}.The two collector terminals are the two outputs v_{o1} and v_{o2}. The basic relationship between the input and output in differential amplifier is

$$v_{o1} - v_{o2} = -A\,(v_{i1} - v_{i2})$$

where A is the differential voltage gain of the amplifier.

The ac output voltage is defined as the voltage between the collectors with the polarity shown i.e.,

$$v_{out} = v_{o2} - v_{o1} = A\,(v_{i1} - v_{i2})$$

Input v_{i1} is referred to as a "non-inverting input" as a positive voltage v_{i1} acting alone provides a positive output voltage. Similarly, input v_{i2} acting alone provides a negative output voltage, and is therefore, called "inverting input". Consequently, the base terminal to which v_{i1} is impressed is known as "non-inverting terminal" and the base terminal to which v_{i2} is applied is known as "inverting terminal".

Note: Here, ac voltages also includes 0 Hz as a special case.

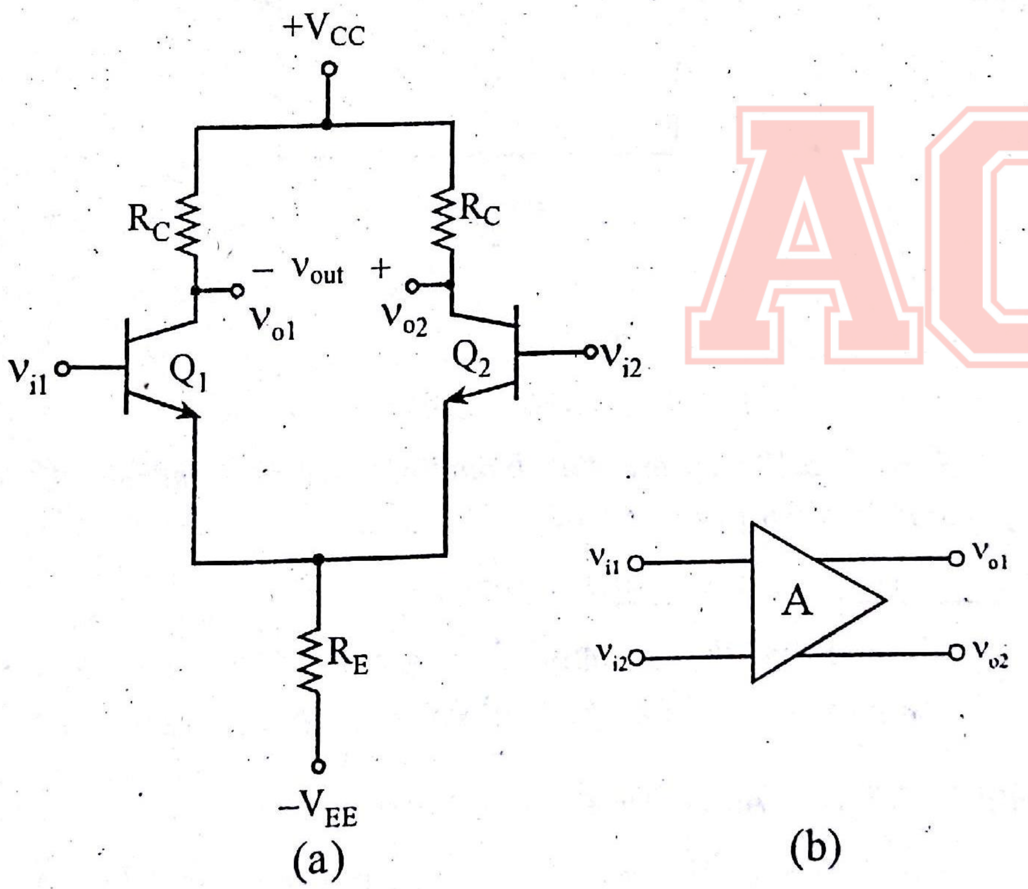


Figure 3.27 ***(a) Basic differential amplifier circuit (b) schematic symbol***

There are three configurations of a differential amplifier.

Case-I: Single-ended input

If a signal is applied to either input with other input connected to ground, the operation is referred to as single-ended input.

Case-II: Double-ended (differential) input

If opposite-polarity signals are applied to inputs, the operation is referred to as double-ended or differential mode input. The waveform at the collector terminals for sinusoidal differential inputs is depicted below.

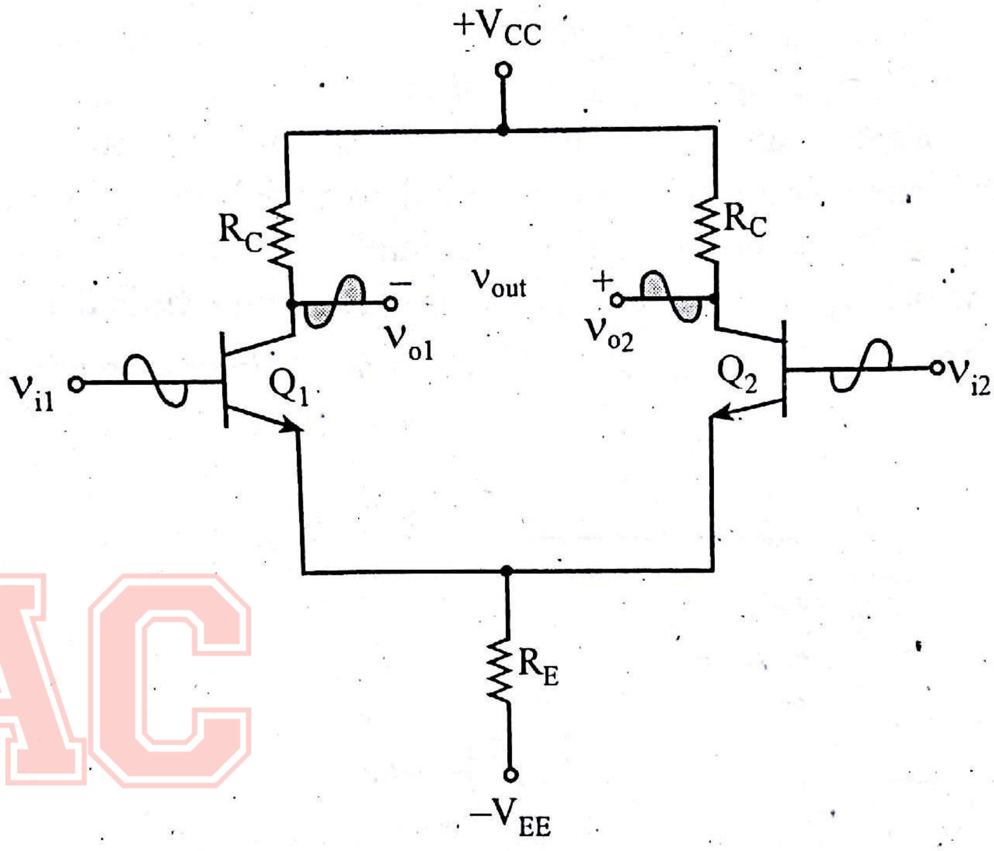


Figure 3.28 ***Differential mode of operation of a differential amplifier with waveforms.***

Case-III: Common-mode input

If same signals are applied to inputs, the operation is called common-mode input. The output voltage v_{out} is zero in this case.

FIELD-EFFECT TRANSISTORS

The field-effect transistor is a semiconductor device which depends for its operation on the control of current by an electric field. There are two types of field-effect transistor junction field-effect transistor (JFET) and metal-oxide-semiconductor field-effect transistor (MOSFET).

The FET enjoys several advantages over the conventional transistor:

i. Its operation depends upon the flow of majority carriers only. It is therefore a "unipolar" (one type of carrier) device. The vacuum tube is another example of a unipolar device. The conventional transistor is a "bipolar" device.

ii. It is relatively immune to radiation.

iii. It exhibits a high input resistance, typically many megaohms.

iv. It is less noisy than a tube or a bipolar transistor.

v. It exhibits no offset voltage at zero drain current, and hence, makes an excellent signal chopper.

vi. It has thermal stability.

The main disadvantage of the FET is its relatively small gain-bandwidth product in comparison with that which can be obtained with a conventional transistor.

MOSFET

The Enhancement- Type MOSFET

Physical structure

The physical structure of the n-channel enhancement-type is shown below. The transistor is fabricated on a p-type substrate, which is a single-crystal silicon wafer that provides physical support for the device. Two heavily doped n-type regions, as indicated by n^+ are created in the substrate as source and drain. A thin layer of silicon dioxide (SiO_2) of thickness, t_{ox} (typically 2-50 nm), which is an excellent electrical insulator, is grown on the face of substrate as shown in **Figure 3.29** that follows. Metal is deposited on top of the oxide layer to form the gate electrode of the device. Metal contacts are also made to the source region, the drain region, and the substrate (body). Thus, four terminals are brought out: the gate terminal (G), the source terminal (S), the drain terminal (D), and the substrate or body terminal (B).

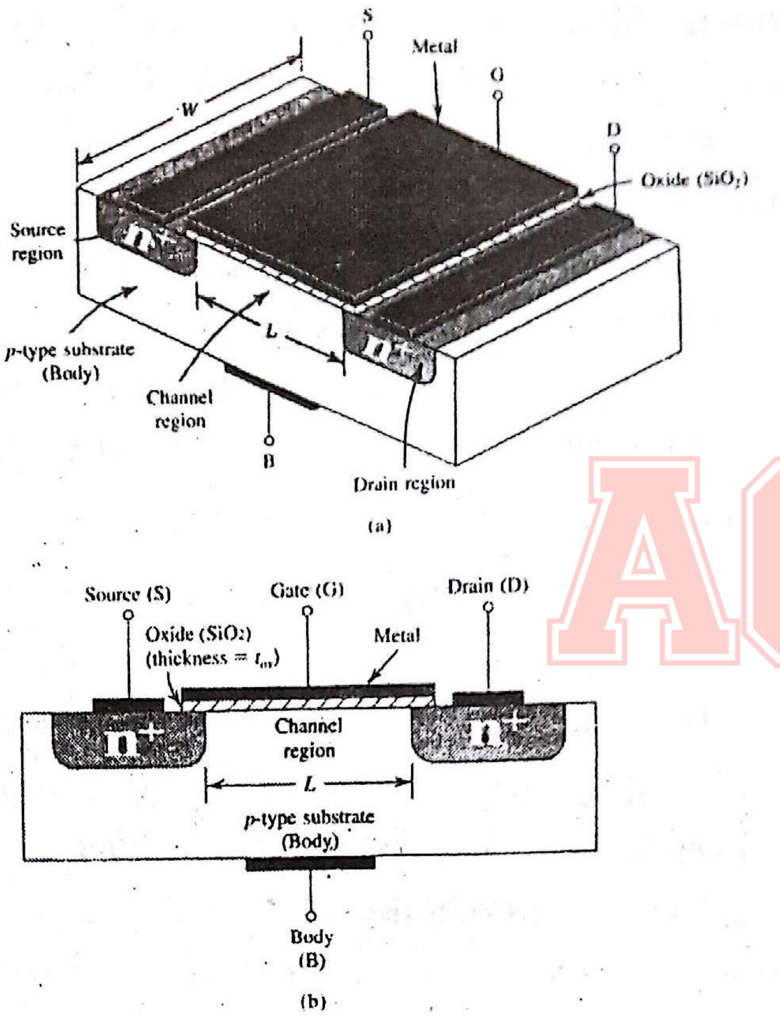


Figure 3.29 *Physical structure of the enhancement-type NMOS transistor: (a) perspective view (b) cross section*

Operation

i. Operation with no gate voltage

With no bias voltage applied to the gate, two back-to-back diodes exist in series between drain and source. One diode is formed by the pn-junction between the n^+ drain region and the p-type substrate, and the other diode is formed by the pn-junction between the p-type substrate and the n^+ source region. These back-to-back diodes prevent current conduction from drain to source when a voltage v_{DS} is applied.

ii. Creating a channel for current flow

The source and the drain are grounded and a positive voltage is applied to the gate (since the source is grounded, the gate voltage appears in effect between gate and source and thus is denoted v_{GS}). This v_{GS} will induce n-type channel as shown in the **Figure 3.30** that follows. The value of v_{GS} at which a sufficient number of mobile electrons accumulate in the channel region to form a conducting channel (that is, a channel is just induced) is called the threshold voltage and is denoted by V_t.

iii. Applying a small v_{DS}

Having induced a channel, we now apply a positive voltage v_{DS} between drain and source. A small v_{DS} causes a current i_D to flow through the induced n-channel. Note the graph shown below which depicts that the MOSFET is operating as a linear resistance whose value is controlled by v_{GS}.

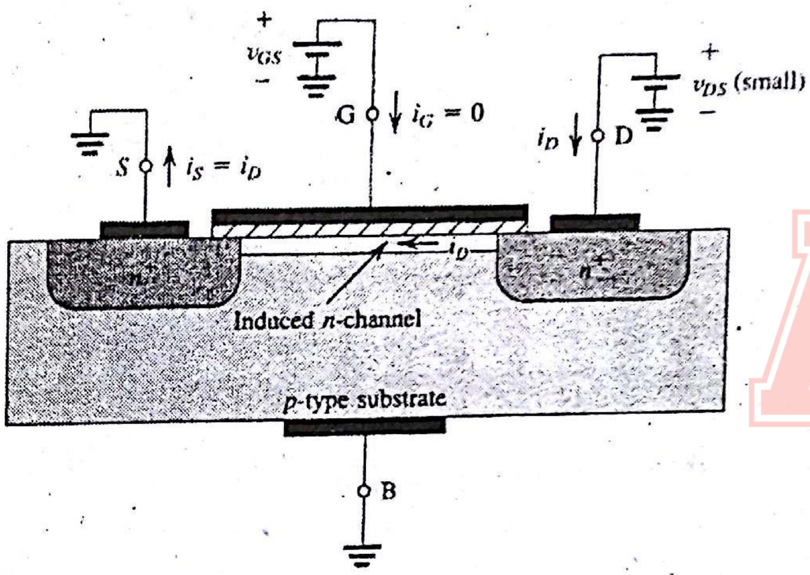


Figure 3.30 *An NMOS transistor with $v_{GS} > V_t$ and with small v_{DS} applied*

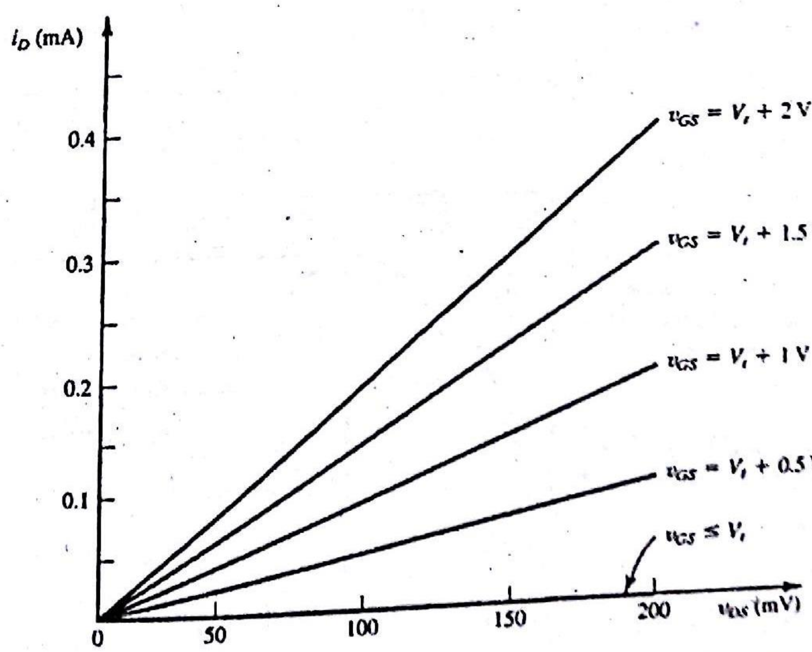


Figure 3.31 *The i_D-v_{DS} characteristics of the MOSFET when the voltage applied between drain and source, v_{DS} is kept small*

Hence, for the MOSFET to conduct, a channel has to be induced. Then, increasing v_{GS} above V_t enhances the channel, hence, the names enhancement-mode operation and enhancement-type MOSFET.

iv. Operation as v_{DS} is increased

First we held v_{GS} constant at a value greater than V_t and increase v_{DS}. The voltage v_{DS} appears as a voltage drop across the length of the channel. Now, the channel becomes more tapered. Thus, the i_D–v_{DS} curve does not continue as a straight line but bends as shown in the **Figure 3.32.**

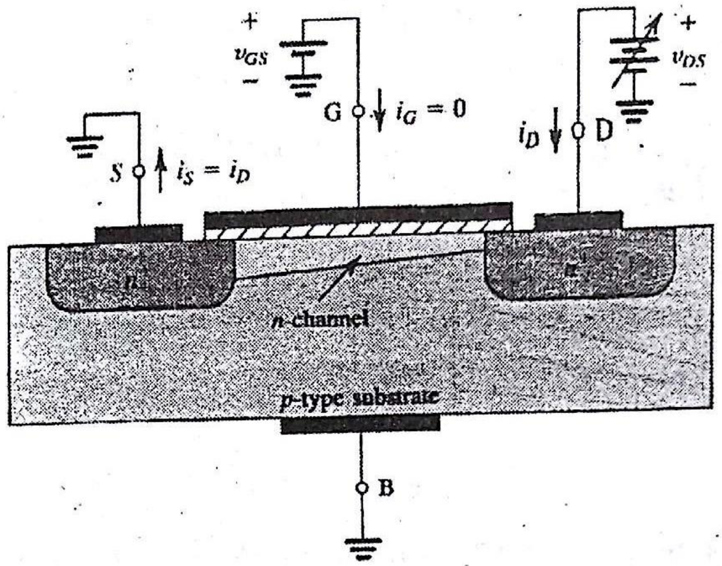


Figure 3.32 *Operation of the enhancement NMOS transistor as v_{DS} is increased*

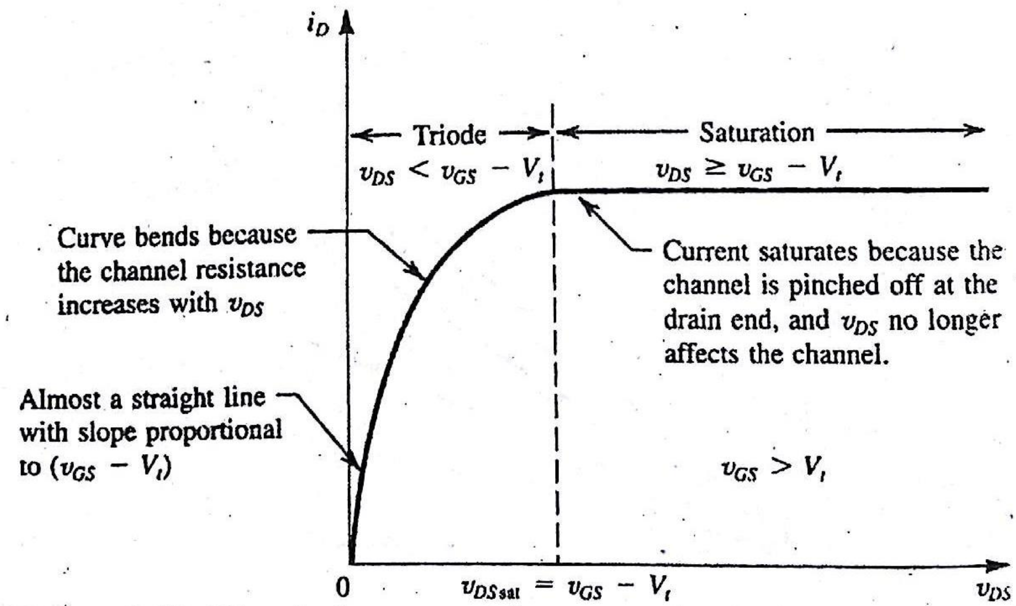


Figure 3.33 *The drain current i_D versus the drain-to-source voltage v_{DS} for an enhancement-type NMOS transistor operated with $v_{GS} > V_t$*

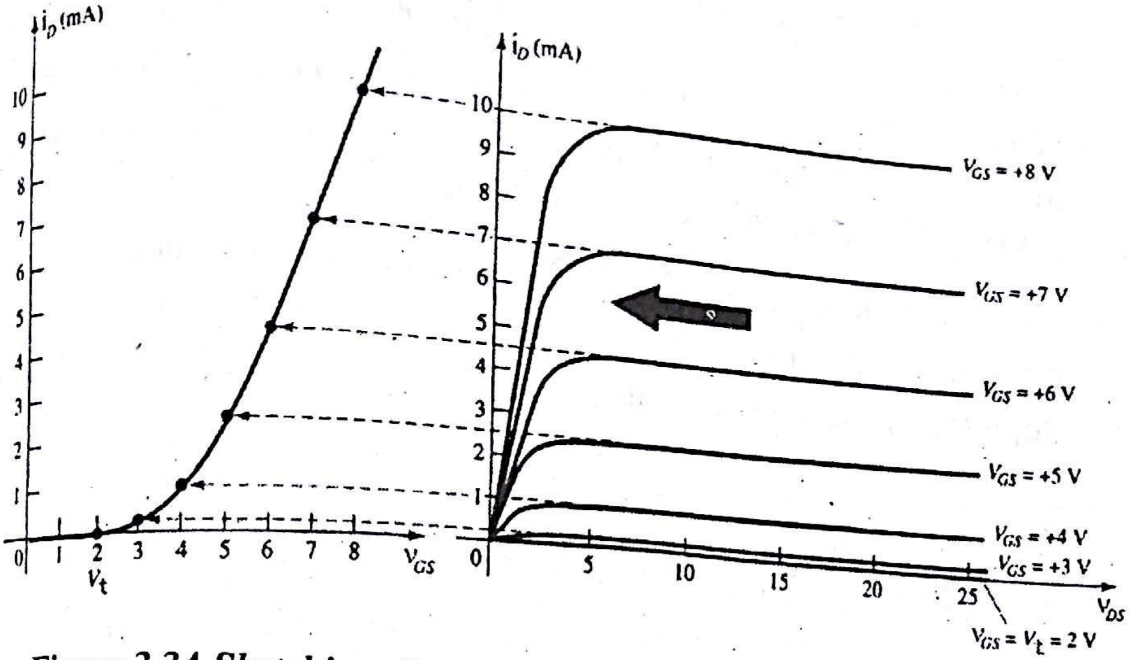


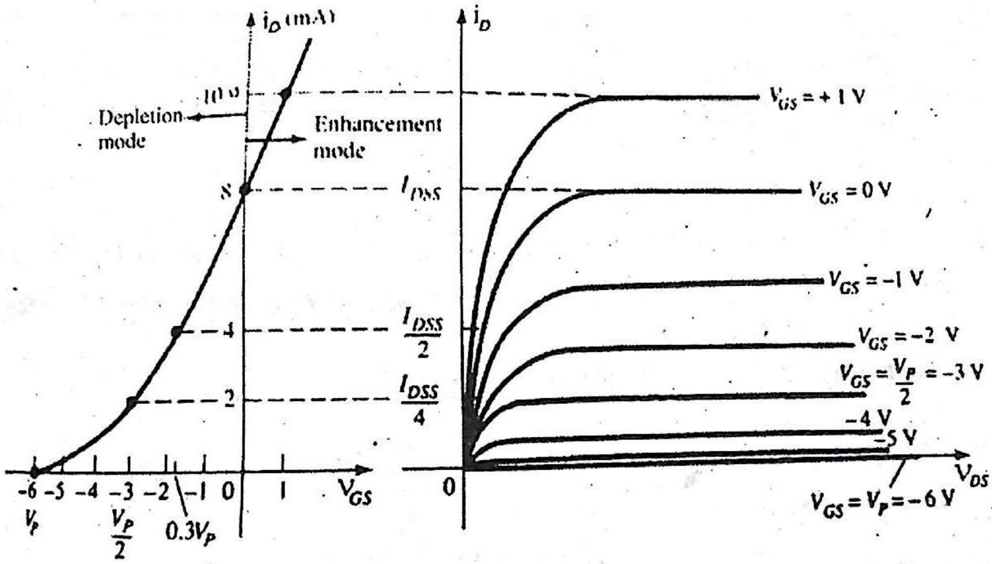
Figure 3.34 *Sketching the transfer characteristics for an n-channel enhancement-type MOSFET from the drain characteristics*

The Depletion-Type MOSFET

The structure of the depletion-type MOSFET is similar to that of the enhancement-type MOSFET with one important difference. The depletion MOSFET has a physically implanted channel. Thus, an n-channel depletion type MOSFET has an n-type silicon region connecting the n^+ source and the n^+ drain regions at the top of the p-type substrate. Thus, if a voltage v_{DS} is applied between drain and source, a current i_D flows for $v_{GS} = 0$. In other words, there is no need to induce a channel, unlike the case of the enhancement MOSFET.

The channel depth and hence, its conductivity can be controlled by v_{GS} in exactly the same manner as in the enhancement -type device. Applying a positive v_{GS} enhances the channel by attracting more electrons into it. Here, however, we also can apply a negative v_{GS}, which causes electrons to be repelled from the channel, and thus, the channel becomes shallower and its conductivity decreases. The negative v_{GS} is said to deplete the channel of its charge carriers, and this mode of operation (negative v_{GS}) is called depletion mode. As the magnitude of v_{GS} is increased in the negative direction, a value is reached at which the channel is completely depleted of charge carriers and i_D is reduced to zero even though v_{DS} may be still applied. This negative value of v_{GS} is the threshold voltage of the n-channel depletion-type MOSFET and is called pinch-off voltage, V_P.

Hence, a depletion-type MOSFET can be operated in the enhancement mode by applying a positive v_{GS} and in the depletion mode by applying a negative v_{GS}.

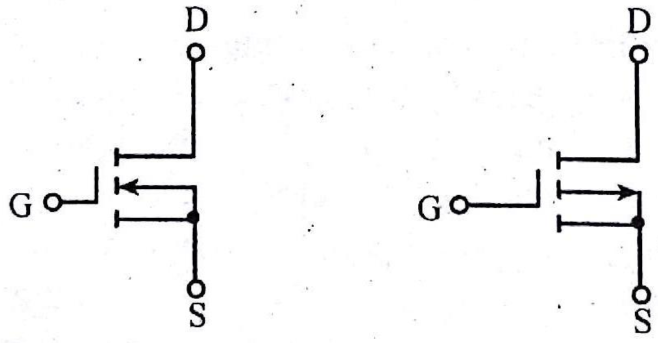


I_{DSS} is the saturation drain current defined by $v_{GS} = 0V$ and $v_{DS} > |V_p|$.

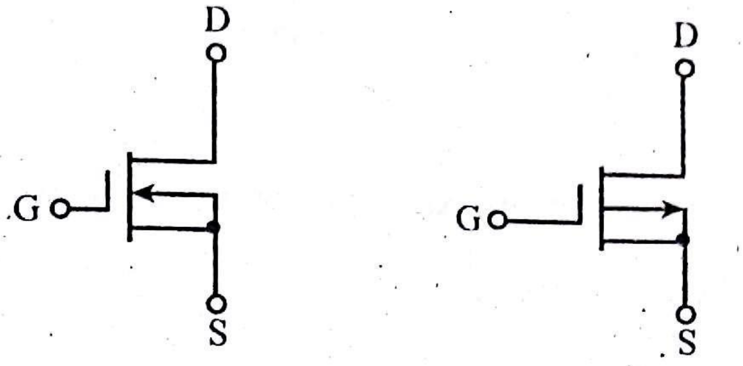
Figure 3.35 ***Drain and transfer characteristics for an n-channel depletion-type MOSFET***

The application of a positive gate-to-source voltage enhances the level of free carriers in the channel compared to that encountered with v_{GS} = 0V. For this reason, the region of positive gate voltages on the drain or transfer characteristics is often referred to as the "enhancement region", with the region between cutoff and the saturation level of I_{DSS} referred to as the "depletion region".

The symbols of MOSFETs are shown below.

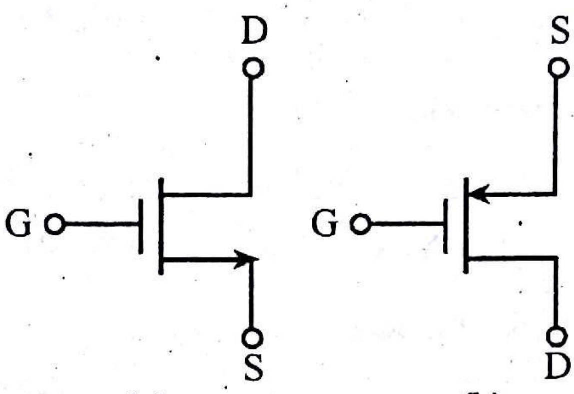


(a) (b)



(c) (d)

Figure 3.36 *(a) n-channel enhancement-type MOSFET (b) p-channel enhancement-type MOSFET (c) n-channel depletion-type MOSFET (d) p-channel depletion-type MOSFET*



(a) (b)

Figure 3.37 *(a) Simplified circuit symbol of NMOS to be used when the source is connected to the body or when the effect of the body on device operation is unimportant. (b) Simplified circuit symbol of PMOS to be used when the source is connected to the body or when the effect of the body on device operation is unimportant.*

NMOS as a Switch (Logic NOT Gate)

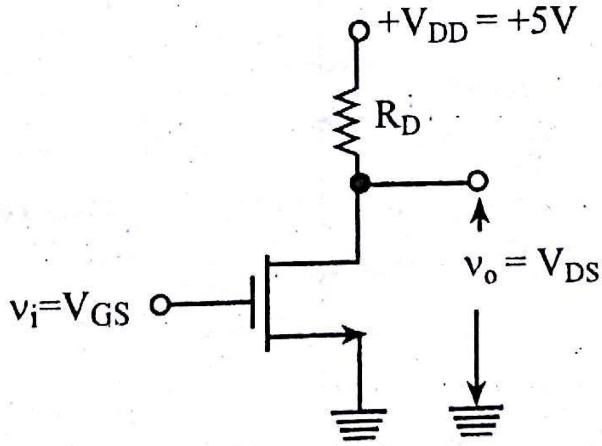


Figure 3.38 *NMOS NOT gate*

Case I: When $v_i = V_{GS} = 0V$ = logic 0

$V_{GS} < V_t$.

$\therefore$ No channel exists so that $R_{DS} \approx 10\ G\Omega$.

Let $R_D = 5\ k\Omega$.

We have,

$$V_{DS} = \frac{R_{DS}}{R_{DS} + R_D} V_{DD} \quad = \frac{10G\Omega}{10G\Omega + 5\ k\Omega} \approx 5\ V$$

$\therefore v_o = V_{DS} = 5\ V$ = logic 1.

Case II: When $v_i = V_{GS} = +5\ V$ = logic 1

$V_{GS} >> V_t$.

$\therefore$ Large channel is created so that $R_{DS} \approx 100\ \Omega$ (large channel = small resistance).

Let $R_D = 5\ k\Omega$. We have,

$$V_{DS} = \frac{R_{DS}}{R_{DS} + R_D} V_{DD} = \frac{100\Omega}{100\Omega + 5000\ \Omega} \approx 0.1\ V$$

$\therefore v_o = V_{DS} = 0.1\ V$ = logic 0.

NMOS NAND gate

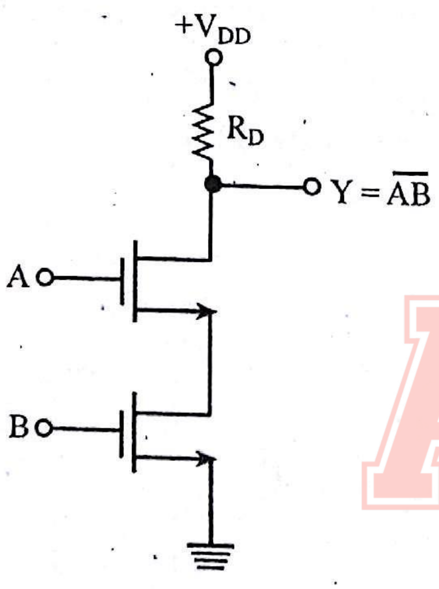


Figure 3.39 *NAND gate*

NMOS NOR gate

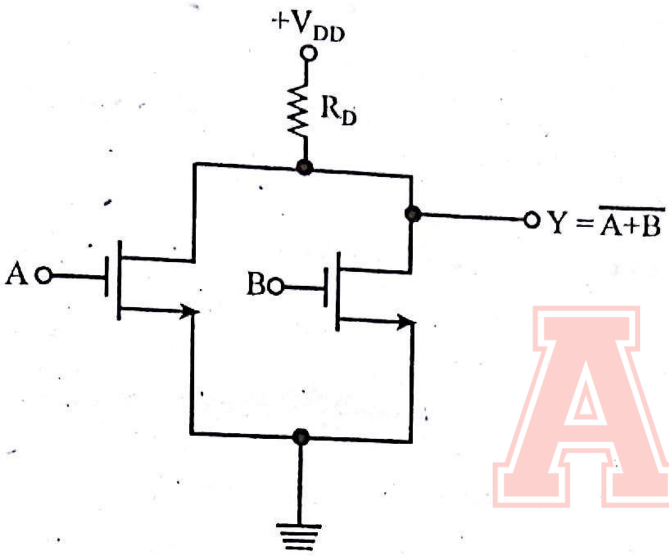


Figure 3.40 *NOR gate*

COMPLEMENTARY MOS OR CMOS

As the name implies, complementary MOS technology employs MOS transistors of both polarities, and is currently the dominant MOS technology. Although CMOS circuits are somewhat more difficult to fabricate than NMOS, the availability of complementary devices makes possible many powerful circuit-design possibilities.

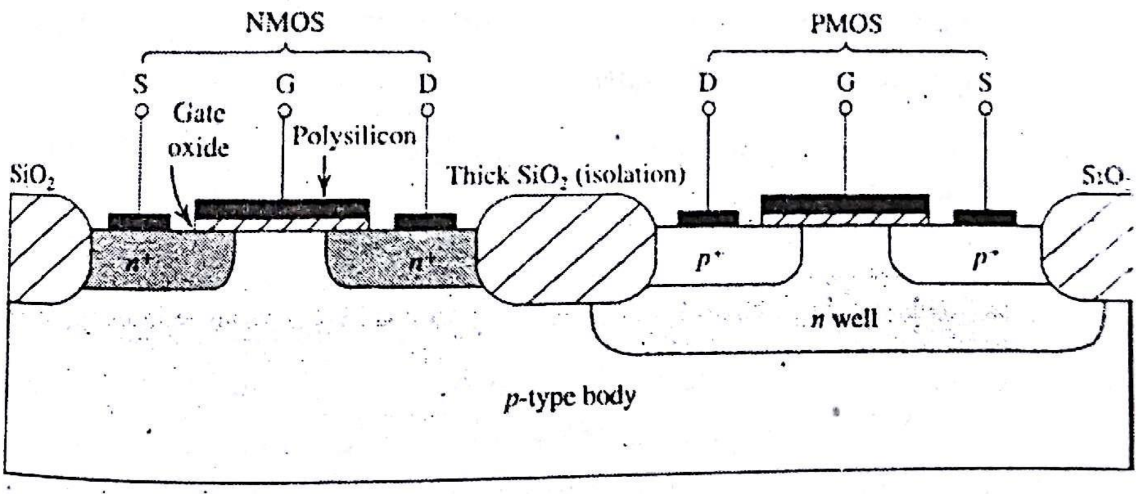


Figure 3.41 *Cross-section of a CMOS integrated circuit*

Figure 3.41 above shows a cross-section of a CMOS chip illustrating how the PMOS and NMOS transistors are fabricated. Note that, the NMOS transistor is implemented directly in the p-type substrate while the PMOS transistor is fabricated in a specially created n region, known as "n well".

The two devices are isolated from each other by a thick region of oxide that functions as an insulator. Not shown on the diagram are the connections made to the p-type body and to the n well. The latter connection serves as the body terminal for the PMOS transistor.

CMOS Inverter (Logic NOT Gate)

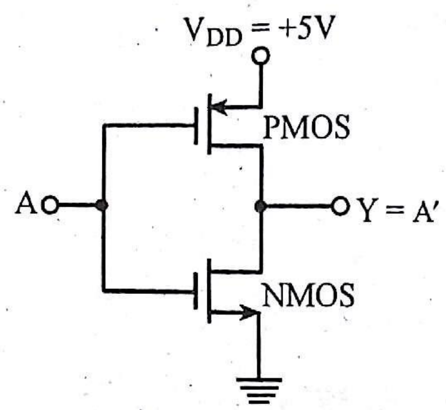


Figure 3.42 ***CMOS inverter***

To understand the operation of the CMOS inverter, we must know that

1. The n-channel MOSFET conducts when its gate-to-source voltage is positive.
2. The p-channel MOSFET conducts when its gate-to-source voltage is negative.
3. Either type of device is turned off if its gate-to-source voltage is zero.

When the input is low, i.e., A = 0V, both gates are at zero potential. The input is at $-V_{DD}$ relative to the source of the p-channel MOSFET and at 0V relative to the source of the n-channel MOSFET. The result is that the p-channel MOSFET is turned on and the n-channel MOSFET is turned off. Under these conditions, there is a low-impedance path from V_{DD} to the output and a very-high-impedance path from output to ground. Therefore, the output voltage approaches the high level V_{DD}, i.e., $Y \approx 5V$ under normal loading conditions. When the input is high, i.e., A = 5V, both gates are at V_{DD} and the situation is reversed. The p-channel MOSFET is off and the n-channel MOSFET is on. The result is that the output approaches the low level of 0V, i.e., $Y \approx 0V$.

CMOS NAND Gate

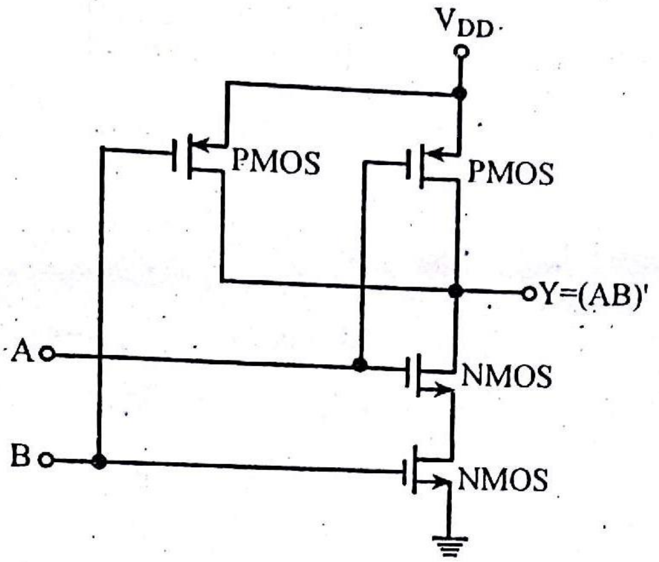


Figure 3.43 *CMOS NAND gate*

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Table 3.7 *Truth table*

CMOS NOR Gate

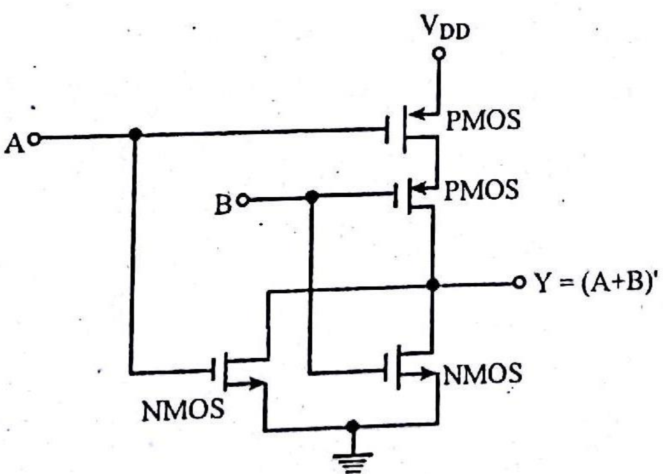


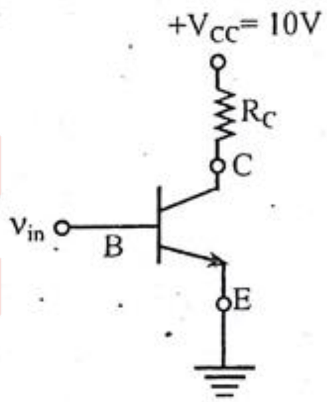
Figure 3.44 *CMOS NOR gate*

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

Table 3.8 ***Truth table***

Problem 3.1

In BJT circuit, if $V_{CC} = 10$ V, and $R_C = 8$ kΩ, draw the dc load line. Determine the Q-point (operating point) for zero input signal if $I_B = 15$ μA and $\beta = 40$.



Solution:

$$I_C = \beta I_B = 40 \times 15 \times 10^{-6} = 0.6 \times 10^{-3} \text{ A}$$

$$V_{CC} - I_C R_C - V_{CE} = 0$$

$$\text{or, } V_{CC} - \beta I_B R_C - V_{CE} = 0$$

$$\text{or, } +10 - 40 \times 15 \times 10^{-6} \times 8 \times 10^3 - V_{CE} = 0$$

$$\therefore V_{CE} = 5.2 \text{ V} = V_{CEQ}$$

$$\therefore \text{Q-point} = (I_{CQ}, V_{CEQ}) = (6 \times 10^{-4} \text{ A}, 5.2 \text{ V})$$

To draw the dc load line, we use the equation

$$V_{CC} - I_C R_C - V_{CE} = 0$$

For $I_C = 0$,

$$V_{CE} = V_{CC} = 10 \text{ V}$$

For $V_{CE} = 0$,

$$I_C = \frac{V_{CC}}{R_C} = \frac{10}{8 \times 10^3} = 1.25 \times 10^{-3} \text{ A.}$$

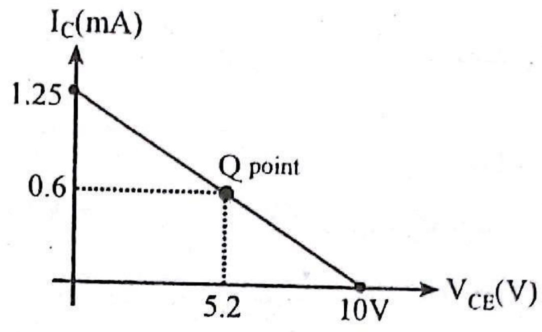
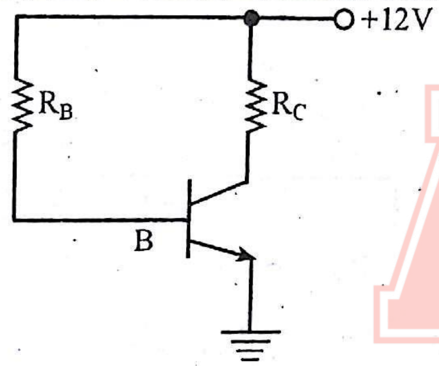


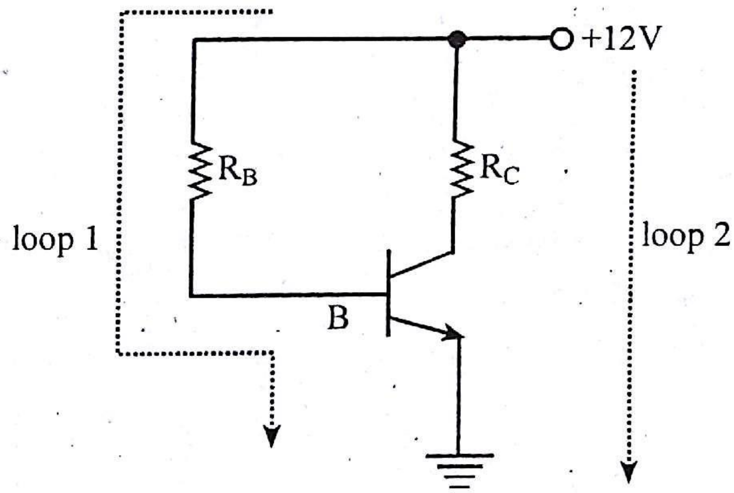
Fig.: Load line

Problem 3.2

Find R_B and R_C in the given circuit. Given data are: $I_C = 1.2$ mA, $V_{CE} = 6V$, and $\beta = 100$.



Solution:



Applying KVL in loop 2,

$+12 - I_C R_C - V_{CE} = 0$

or, $+12 - 1.2 \times 10^{-3} R_C - 6 = 0 \Rightarrow R_C = 5000\ \Omega = 5\ k\Omega$

Applying KVL in loop 1,

$+12 - I_B R_B - V_{BE} = 0$

Here, $V_{BE} = 0.7$ V

$I_B = \frac{I_C}{\beta}$ ($\because I_C = \beta I_B$)

$\therefore \quad I_B = \frac{1.2 \times 10^{-3}}{100} = 1.2 \times 10^{-5}$ A

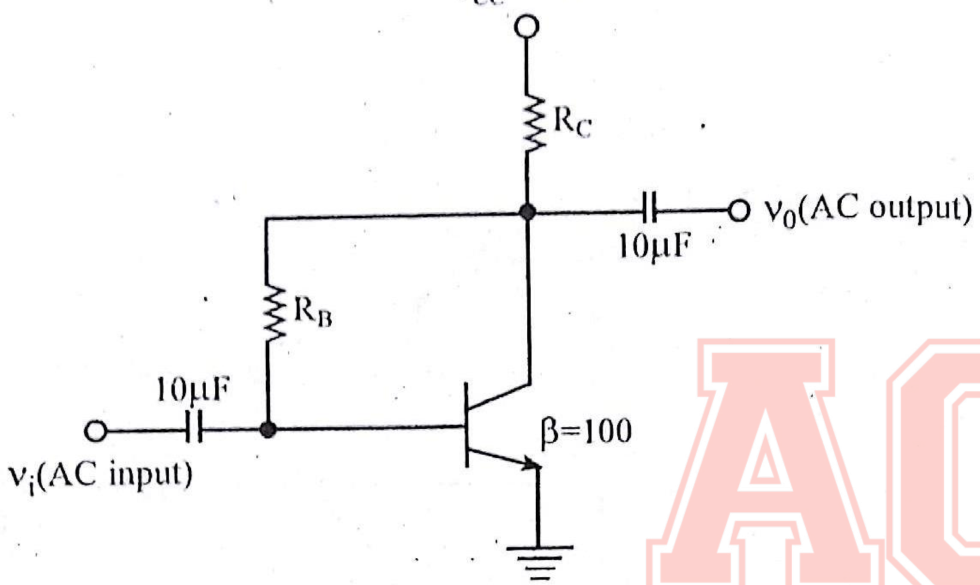
So, $+12 - 1.2 \times 10^{-5} R_B - 0.7 = 0$

$\therefore \quad R_B = 941666.666\ \Omega = 941.666\ k\Omega$

Hence, $R_B = 941.666\ k\Omega$, $R_C = 5\ k\Omega$

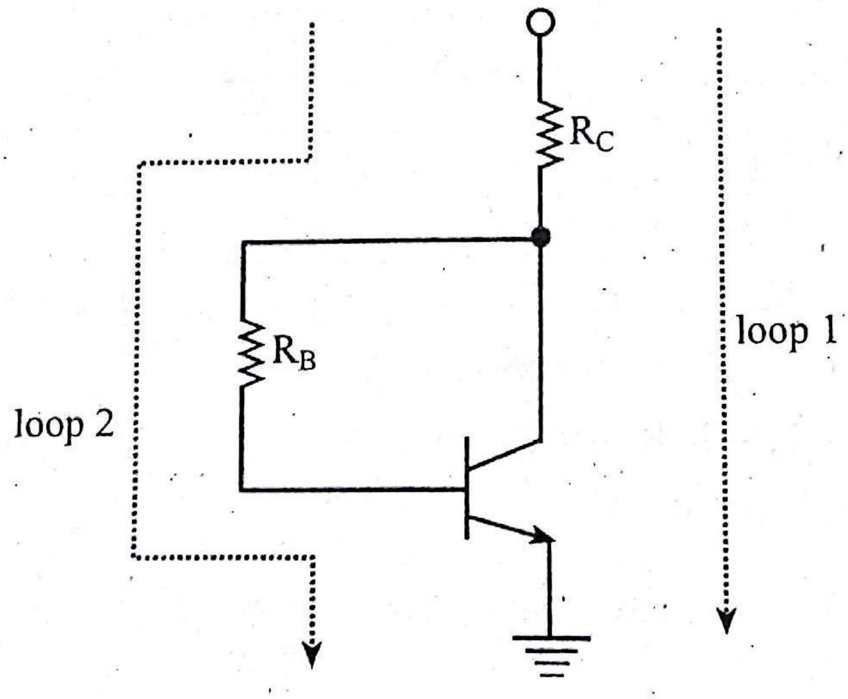
Problem 3.3

Find R_C and R_B in the given circuit. Given data are: $I_C = 1.2$ mA, $V_{CE} = 6V$, and $\beta = 100$.



Solution:

Since the capacitor acts as an open circuit for DC, the circuit can be redrawn as



Applying KVL in loop 1,

$V_{CC} - (I_C + I_B) R_C - V_{CE} = 0$

$I_C = 1.2\times 10^{-3}$ A, $\quad I_B = \frac{I_C}{\beta} = \frac{1.2 \times 10^{-3}}{100} = 1.2 \times 10^{-5}$ A

or, $+12 - (1.2 \times 10^{-3} + 1.2 \times 10^{-5}) R_C - 6 = 0$

$\therefore R_C = 5000\Omega \quad = 5$ kΩ

Applying KVL in loop 2,

$V_{CC} - (I_C+I_B) R_C - I_B R_B - V_{BE} = 0$

$V_{BE} = 0.7$ V, $I_B = 1.2 \times 10^{-5}$ A

So, $+12-(1.2\times10^{-3}+1.2\times10^{-5}) \times 5000 - 1.2\times10^{-5}R_B - 0.7 = 0$

$\therefore \quad R_B = 441666.666\Omega = 441.666$ kΩ

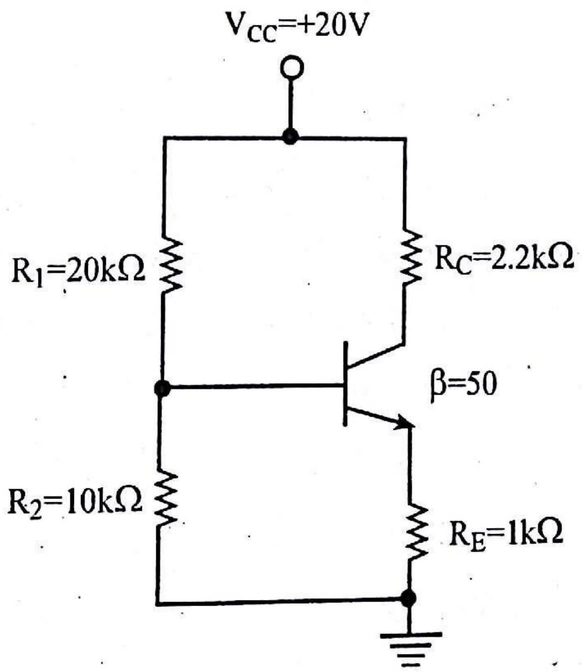
Hence,

$R_C = 5$ kΩ

$R_B = 441.666$ kΩ

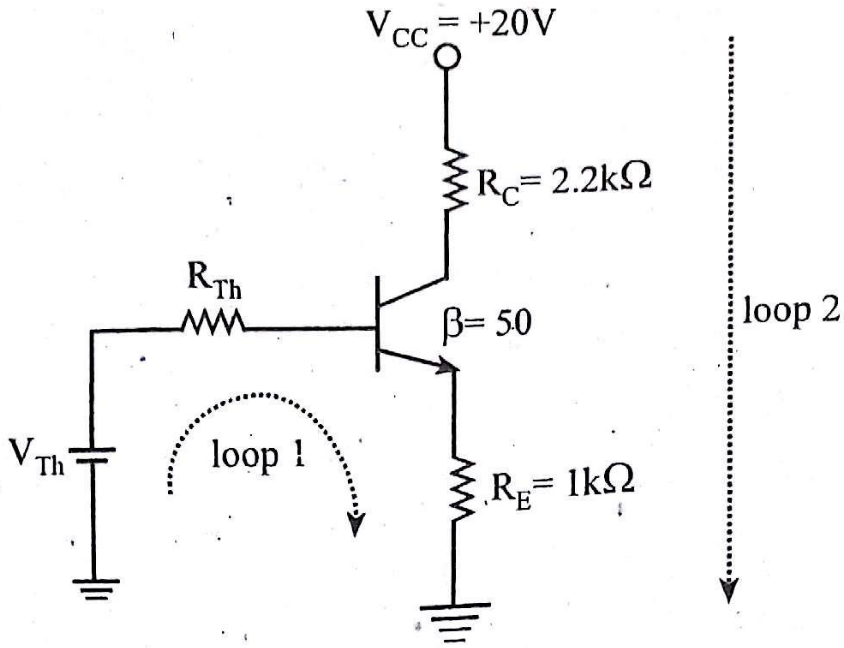
Problem 3.4

Find the value of I_C and V_{CE} for the given circuit.



Solution:

The DC circuit to the left of base terminal, B can be replaced by a Thevenin's equivalent circuit as shown in the figure below.



V_{Th} is determined from the original figure looking to the left from the base terminal, B.

$$V_{Th} = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \left(\frac{10}{20 + 10}\right) \times 20 = 6.67 \text{ V}$$

Looking back into the voltage divider with V_{CC} grounded in the original figure, we see R_1 in parallel with R_2. So,

$$R_{Th} = R_1 \parallel R_2 = \frac{20 \times 10}{20 + 10} = 6.67 \text{ k}\Omega$$

Applying KVL in loop 1,

$$V_{Th} - I_B R_{Th} - V_{BE} - I_E R_E = 0$$

or, $V_{Th} - I_B R_{Th} - V_{BE} - (\beta + 1) I_B R_E = 0$

or, $+6.67 - I_B \times 6.67 \times 10^3 - 0.7 - 51 \times 1 \times 10^3 \times I_B = 0$

$\therefore$ $I_B = 1.0352 \times 10^{-4}$ A

$I_C = \beta I_B = 50 \times 1.0352 \times 10^{-4} = 5.176 \times 10^{-3}$ A

$I_E = (\beta + 1) I_B = 51 \times 1.0352 \times 10^{-4} = 5.27952 \times 10^{-3}$ A

Applying KVL in loop 2,

$V_{CC} - I_C R_C - V_{CE} - I_E R_E = 0$

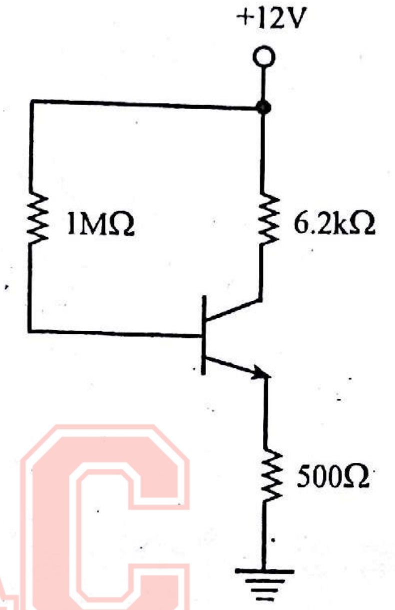
or, $+20 - 5.176 \times 10^{-3} \times 2.2 \times 10^3 - V_{CE} - 5.2795 \times 10^{-3} \times 1 \times 10^3 = 0$

$\therefore$ $V_{CE} = 3.33$ V

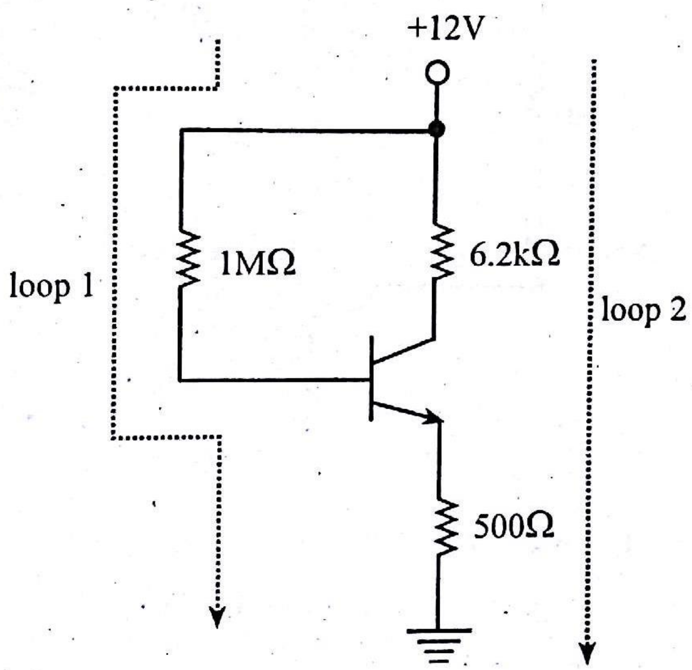
Hence, $I_C = 5.176 \times 10^{-3}$ A, $V_{CE} = 3.33$ V

Problem 3.5

Find I_C and V_{CE} for the given circuit.



Solution:



Applying KVL in loop 1,

$$+12 - I_B \times 1 \times 10^6 - 0.7 - I_E \times 500 = 0$$

or, $+12 - I_B \times 10^6 - 0.7 - (\beta + 1)\, I_B \times 500 = 0$

Take $\beta = 100$

Solve and obtain the value of I_B.

Applying KVL in loop 2,

$$+12 - I_C \times 6.2 \times 10^3 - V_{CE} - I_E \times 500 = 0$$

Here, $I_C = \beta I_B$

$I_E = (\beta + 1)\, I_B$

Solve and obtain the value of V_{CE}.

Problem 3.6

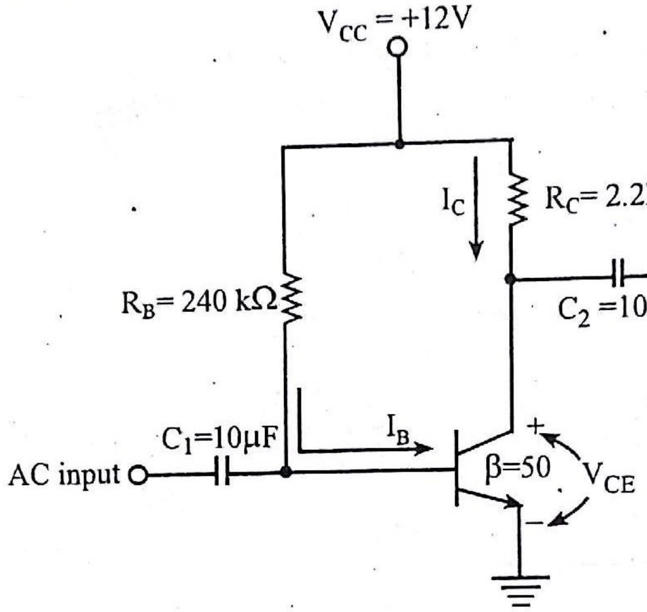
Determine the following for the fixed–bias configuration of figure shown below.

a. I_{BQ} *and* I_{CQ}

b. V_{CEQ}

c. V_B *and* V_C

d. V_{BC}



Solution:

a. $V_{CC} - I_{BQ} R_B - V_{BE} = 0$

or, $12 - I_{BQ} \times 240 \times 10^3 - 0.7 = 0 \Rightarrow I_{BQ} = 47.08 \times 10^{-6}$ A

b. $V_{CC} - I_{CQ} R_C - V_{CEQ} = 0$

Here, $I_{CQ} = \beta\, I_{BQ} = 50 \times 47.08 \times 10^{-6} = 2.35 \times 10^{-3}$ A

$\therefore V_{CEQ} = V_{CC} - I_{CQ} R_C = 12 - (2.35 \times 10^{-3})(2.2 \times 10^3)$

$= 6.83$ V

c. $V_B = V_{BE} = 0.7$ V

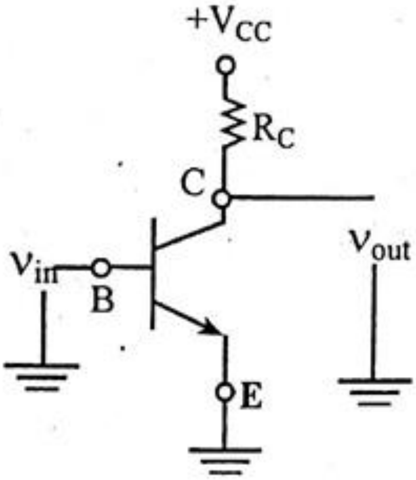
$V_C = V_{CEQ} = 6.83$ V

d. $V_{BC} = V_B - V_C = 0.7\text{ V} - 6.83\text{ V} = -6.13$ V with the negative sign revealing that the junction is reversed – biased, as it should be for linear amplification.

Problem 3.7

Find the volume of collector current, Q-point, dc load line for common-emitter circuit having $V_{CC} = 15V$, $R_C = 10K\Omega$, $I_B = 10\mu A$, and $\beta = 40$.

Solution:



$I_C = \beta I_B$

$= 50 \times 10 \times 10^{-6} = 5 \times 10^{-4} A = 0.5 \times 10^{-3} A$

Using,

$V_{CC} - I_C R_C - V_{CE} = 0$

or, $+ 15 - 5\times10^{-4} \times 10\times 10^{3} - V_{CE} = 0$

$\therefore V_{CE} = 10V = V_{CEQ}$

Q-point $= (I_{CQ}, V_{CEQ}) = (0.5\times 10^{-3}A, 10V)$

To draw the dc load line, we use the equation

$V_{CC} - I_C R_C - V_{CE} = 0$

For $I_C = 0$,

$V_{CE} = V_{CC} = 15V$

For $V_{CE} = 0$,

$$I_C = \frac{V_{CC}}{R_C}$$

$$= \frac{15}{10\times 10^{3}} = 1.5 \times 10^{-4} A$$

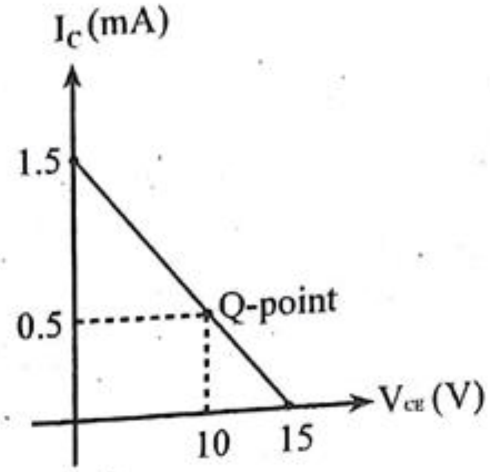


Fig.: Load line

Problem 3.8

Draw collector-feedback type dc biasing circuit. If V_{CC} = 10V, R_B = 950 KΩ, R_C = 2.2 KΩ, and β = 150, calculate the dc operating collector current (I_{CQ}).

Solution:

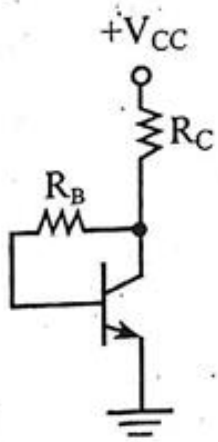
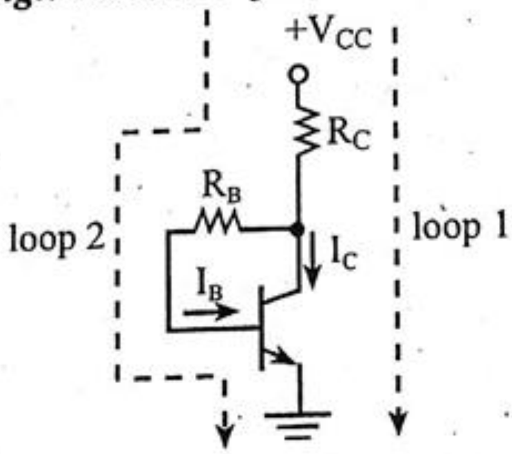


Fig.: Collector- feedback bias circuit



Using KVL in loop 2,

$$+V_{CC} - (I_C + I_B)R_C - I_BR_B - V_{BE} = 0$$

$$\text{or, } + V_{CC} - \left(I_C + \frac{I_C}{\beta}\right)R_C - \frac{I_C}{\beta}R_B - V_{BE} = 0$$

or, $10 - \frac{\beta+1}{\beta} I_C R_C - \frac{I_C}{\beta} R_B - V_{BE} = 0$

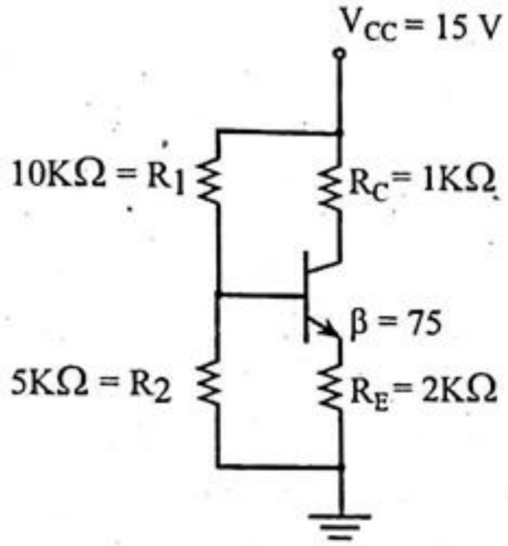
or, $10 - \frac{151}{150} \times 2.2 \times 10^3 I_C - \frac{950\times10^3}{150} I_C - 0.7 = 0$

$\therefore I_C = 1.0879 \times 10^{-3} A = I_{CQ}$

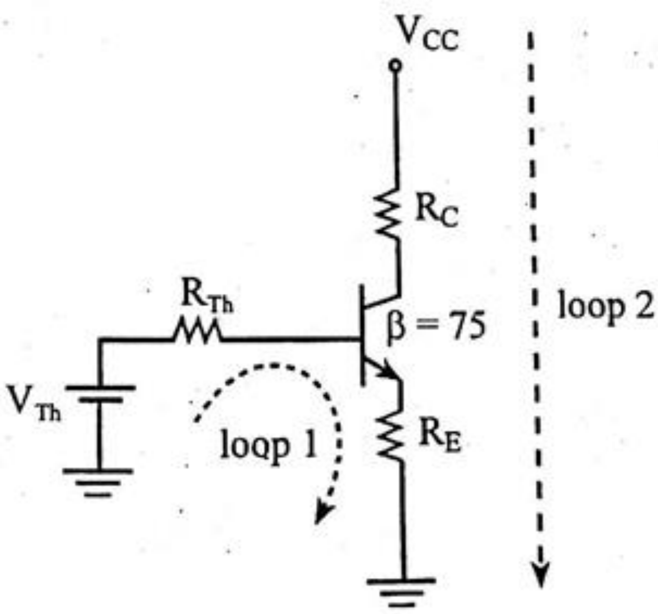
Problem 3.9

Draw the dc load line and determine the Q-point of the voltage divider biased transistor circuit having V_{CC}=15V, R_C=1 KΩ, R_1=10 KΩ, R_2=5 KΩ, R_E=2 KΩ, and β=75.

Solution:



The DC circuit to the left of base terminal can be replaced by a Thevenin's equivalent circuit as shown in the figure below.



where $V_{Th} = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \frac{5}{15} \times 15 = 5V$

$$R_{Th} = R_1 || R_2 = 10 || 5 = \frac{10 \times 5}{10 + 5} = 3.33\ K\Omega$$

Applying KVL in loop 1,

$V_{Th} - I_B R_{Th} - V_{BE} - I_E R_E = 0$

or, $V_{Th} - I_B R_{Th} - V_{BE} - (\beta + 1) I_B R_E = 0$

or, $5 - I_B \times 3.33 \times 10^3 - 0.7 - 76 \times I_B \times 2 \times 10^3 = 0$

or, $4.3 - (3.33 \times 10^3 + 152 \times 10^3) I_B = 0$

$\therefore I_B = 2.768 \times 10^{-5}$ A

$I_C = \beta I_B = 75 \times 2.768 \times 10^{-5} = 2.076 \times 10^{-3}$ A

$I_E = (\beta+1) I_B = 76 \times 2.768 \times 10^{-5} = 2.1036 \times 10^{-3}$ A.

Applying KVL in loop 2,

$V_{CC} - I_C R_C - V_{CE} - I_E R_E = 0$

or, $15 - 2.076 \times 10^{-3} \times 1 \times 10^3 - V_{CE} - 2.1036 \times 10^{-3} \times 2 \times 10^3 = 0$

or, $15 - 2.076 - V_{CE} - 4.2072 = 0$

or, $V_{CE} = 8.716V$

Hence, Q-point = $(I_C, V_{CE}) = (2.076 \times 10^{-3} A, 8.716V) = (I_{CQ}, V_{CEQ})$

To draw the load line, we use the equation

$V_{CC} - I_C R_C - V_{CE} - I_E R_E = 0$

or, $V_{CC} - I_C R_C - V_{CE} - \frac{\beta+1}{\beta} I_C R_E = 0$

For $I_C = 0$,

$V_{CE} = V_{CC} = 15$ V

For $V_{CE} = 0$,

$$15 - I_C \times 1 \times 10^3 - 0 - \frac{76}{75} \times I_C \times 2 \times 10^3 = 0$$

$\therefore I_C = 4.9559 \times 10^{-3}$A

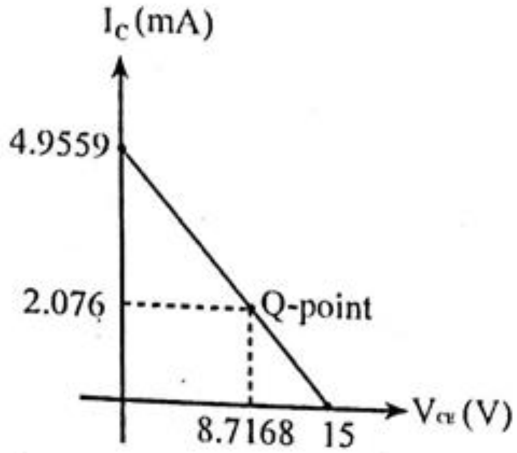


Fig.: Load line

Problem 3.10

Draw emitter-feedback bias circuit of BJT by labeling all the circuit components. Find I_C and V_{CE} in the circuit if V_{CC} = +12V, R_B = 430 KΩ, R_C = 2 KΩ, R_E = 1 KΩ, and β = 50.

Solution:

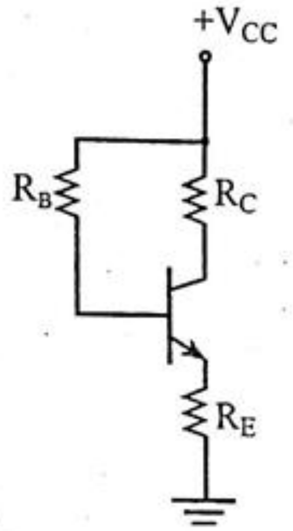
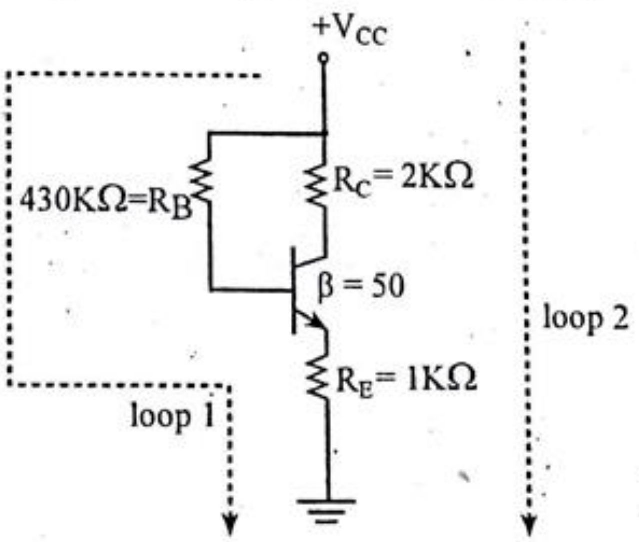


Fig.: Emitter-feedback bias circuit



Using KVL in loop 1,

$+V_{CC} - I_B R_B - V_{BE} - I_E R_E = 0$

or, $V_{CC} - I_B R_B - V_{BE} - (\beta+1) I_B R_E = 0$

or, $12 - 430 \times 10^3 \times I_B - 0.7 - 51 \times 1 \times 10^3 I_B = 0$

$\therefore I_B = 23.4 \times 10^{-6}$ A

$I_C = \beta I_B = 50 \times 23.4 \times 10^{-6} = 1.17 \times 10^{-3}$ A

$I_E = (\beta+1) I_B = 51 \times 23.4 \times 10^{-6} = 1.193 \times 10^{-3}$ A

Using KVL in loop 2,

$+ V_{CC} - I_C R_C - V_{CE} - I_E R_E = 0$

or, $+12 - 1.17 \times 10^{-3} \times 2 \times 10^3 - V_{CE} - 1.193 \times 10^{-3} \times 1 \times 10^3 = 0$

or, $V_{CE} = 8.467$ V

Hence, $I_C = 1.17 \times 10^{-3}$ A, $V_{CE} = 8.467$ V

The Operational Amplifier

INTRODUCTION

The operational amplifier is a versatile device that can be used to amplify dc as well as ac input signals and was originally designed for performing mathematical operations such as addition, subtraction, multiplication, and integration. Thus, the name "operational amplifier" stems from its original use for these mathematical operations and is abbreviated to op-amp. With the addition of suitable external feedback components, the modern day op-amp can be used for a variety of applications such as ac and dc signal amplification, active filters, oscillators, comparators, regulators, displays, testing and measuring systems, and others.

INTERNAL BLOCK DIAGRAM OF AN OP – AMP

A typical op-amp is made up of three types of amplifier circuit: a differential amplifier, a voltage amplifier, and a push-pull amplifier as shown in **Figure 4.1**.

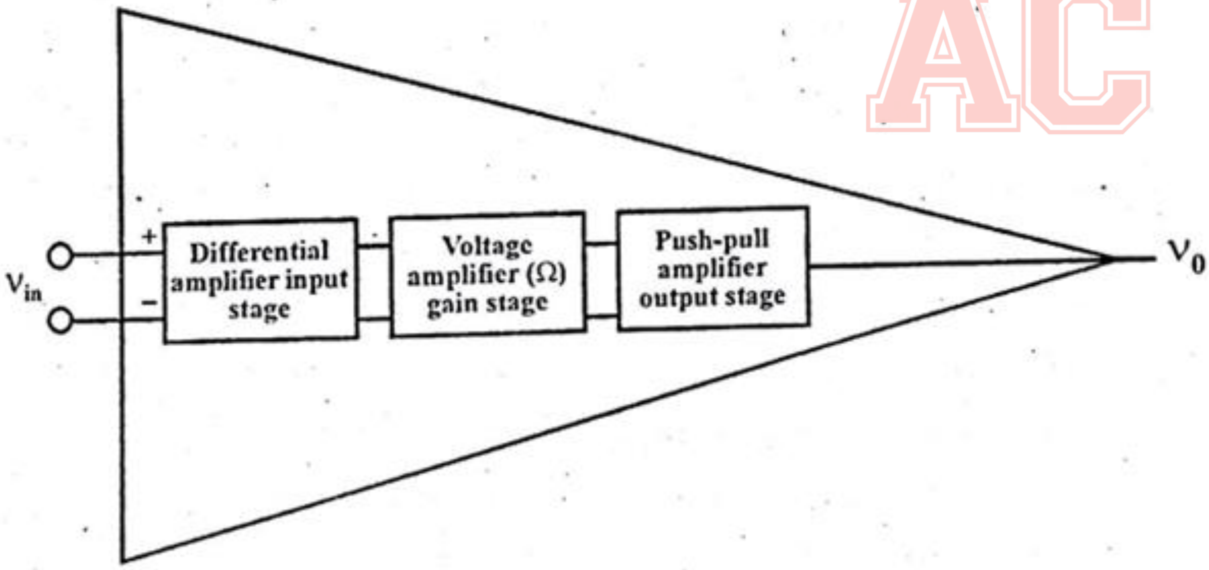


Figure 4.1 *Internal block diagram of an op-amp*

THE IDEAL OP- AMP

An ideal op-amp would exhibit the following electrical characteristics:

1. **Infinite voltage gain, A_0.**
2. **Infinite input resistance, R_i** so that almost any signal source can drive it and there is no loading of the preceding stage.
3. **Zero output resistance, R_0** so that the output can drive an infinite number of other devices.
4. **Zero output voltage when input voltage is zero.**
5. **Infinite bandwidth** so that any frequency signal from 0 to ∞ Hz can be amplified without attenuation.
6. **Infinite common-mode rejection ratio** so that the output common-mode noise voltage is zero.
7. **Infinite slew rate** so that output voltage changes occur simultaneously with input voltage changes.

SOME BASIC TERMS

1. **Open-loop voltage gain and closed-loop voltage gain**

   The maximum possible voltage gain from a given op-amp is called *open-loop voltage gain* and is denoted by A_0. The term "open-loop" indicates a circuit condition when there is no feedback path from the output to the input of op-amp. The typical value of A_0 for a 741 op-amp is 200000.

   When a feedback path is present, the resulting circuit gain is referred to as *closed-loop voltage gain* and is denoted by the symbol A.

2. **Bandwidth**

   All electronic devices work only over a limited range of frequencies. This range of frequencies is called *bandwidth*.

3. **Gain-bandwidth product.**

   The *gain-bandwidth product* (GBW), or unity - gain bandwidth, of an operational amplifier is the open loop gain at a given frequency multiplied by the frequency.

4. **Common-mode rejection ratio**

   The *common-mode rejection ratio* (CMRR) (expressed in dB) defines how good the amplifier is at attenuating common mode input voltages.

5. **Slew rate**

The *slew rate* of an op-amp is a measure of how fast the output voltage can change and is measured in volts per microsecond (V/μs).

6. **Input bias current**

The *input bias current* (I_{IB}) is the base current for the op-amp input stage transistors. The typical value of I_{IB} for a 741 op-amp 80 nA.

7. **Input offset voltage and input offset current**

Ideally, the input stage transistors should be perfectly matched, so that zero voltage difference between the two input (base) terminals should produce a zero output voltage. In practice, there is always some difference between the base-emitter voltages of the transistors and this result in an *input offset voltage (V_{Io})* for a 741 op- amp is 1.0 mV. Similarly, because of mismatch of input transistors, there is also an *input offset current* (I_{Io}). The typical value of I_{Io} for a 741 op-amp is 20 nA.

8. **Input and output resistance**

The *input resistance* (R_i) and the *output resistance* (R_o) are the resistances at the op-amp terminals when no feedback is involved. The typical values of R_i and R_o for a 741 op-amp are 2MΩ and 75 Ω respectively.

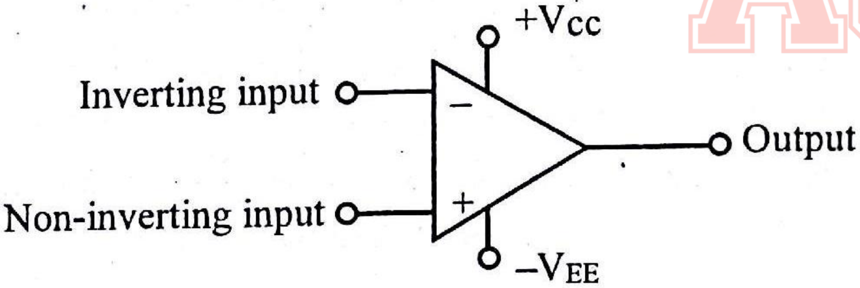


Figure 4.2 ***Op-amp circuit symbol***

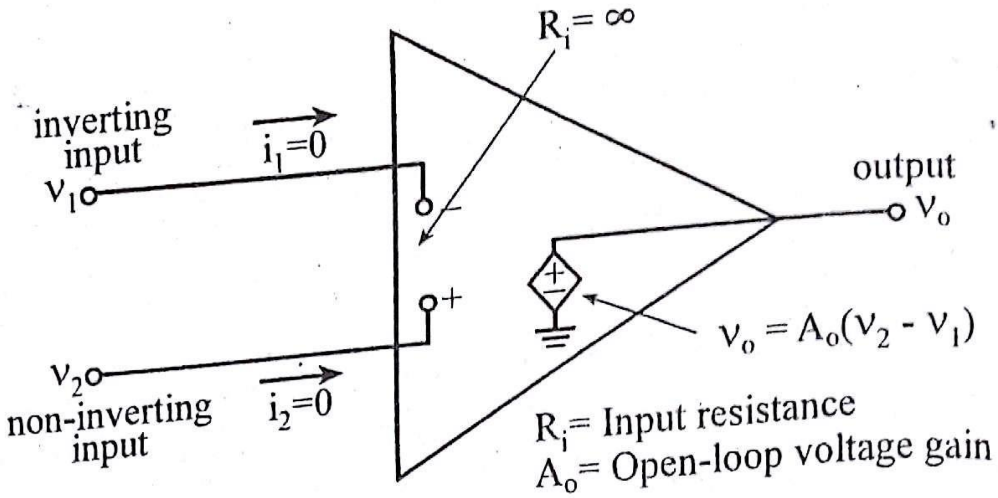


Figure 4.3 *Model of an ideal op-amp*

VIRTUAL GROUND

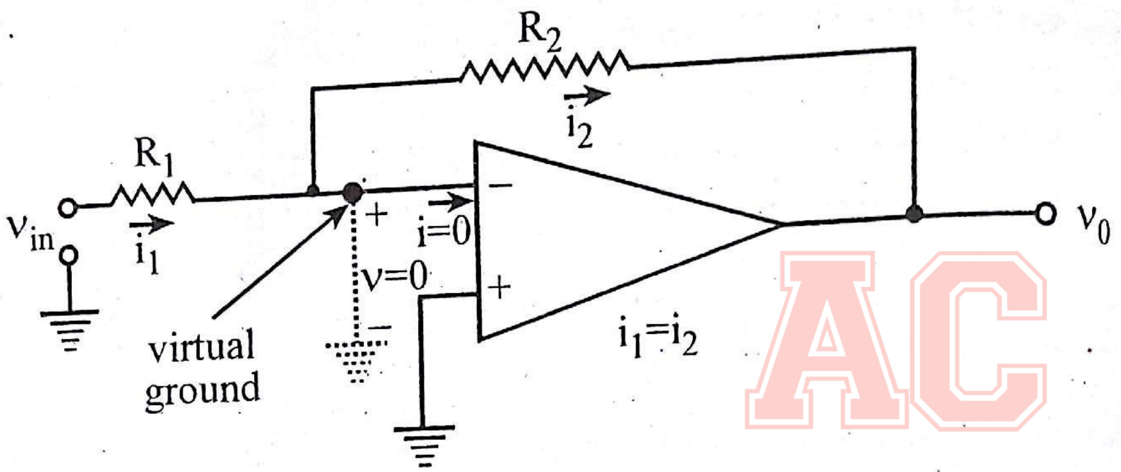


Figure 4.4 *The concept of virtual ground*

When we connect a piece of wire between some point in a circuit and ground, the voltage of the point becomes zero. Furthermore, the wire provides a path for current to flow to ground. A "mechanical ground" (a wire between a point and ground) is ground to both voltage and current.

A "virtual ground" is different. This type of ground is a widely used shortcut for analyzing an inverting amplifier. With a virtual ground, the analysis of an inverting amplifier and related circuits becomes incredibly easy.

The concept of a virtual ground is based on an ideal op-amp. When an op-amp is ideal, it has infinite open-loop voltage gain and infinite input resistance. Because of this, we can deduce the following ideal properties for the inverting amplifier of **Figure 4.4**.

1. Since R_i is infinite, $i = 0$.

2. Since A_o is infinite, $v = 0$.

Since i is 0, the current through R_2 must equal the input current through R_1, as shown. Furthermore, since $v = 0$, the virtual ground shown means that the inverting input acts like a ground for voltage but an open for current.

Virtual ground is very unusual. It is like half of a ground because it is a short for voltage but an open for current. To remind us of this half-ground quality, **Figure 4.4** uses a dashed line between the inverting input and ground. The dashed line means that no current can flow to ground. Although virtual ground is an ideal approximation, it gives very accurate answers when used with heavy negative feedback.

VIRTUAL SHORT

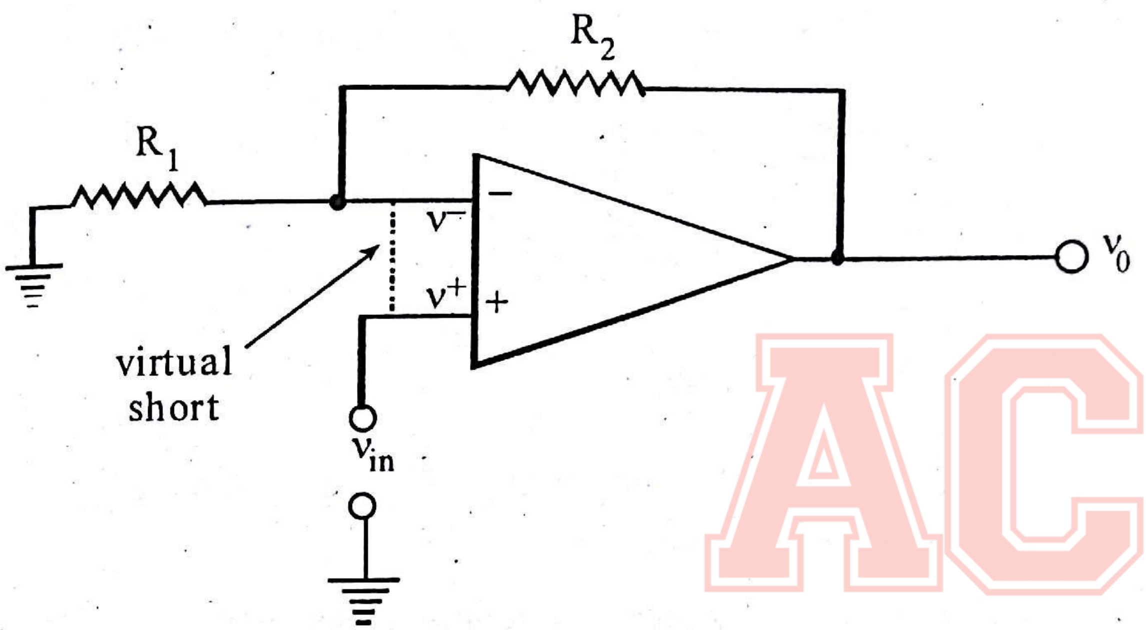


Figure 4.5 *The concept of virtual short*

When we connect a piece of wire between two points in a circuit, the voltage of both points with respect to ground is equal. Furthermore, the wire provides a path for current to flow between the two points. A "mechanical short" (a wire between two points) is a short for both voltage and current.

A "virtual short" is different. This type of short can be used for analyzing non-inverting amplifiers. With a virtual short, we can quickly and easily analyze non-inverting amplifiers and related circuits.

The virtual short uses these two properties of an ideal op-amp:

1. Since R_i is infinite, both input currents are zero.
2. Since A_o is infinite, $v^+ - v^- = 0$.

Figure 4.5 shows a virtual short between the input terminals of the op-amp. The virtual short is a short for voltage but an open for current. As a reminder, the dashed line means that no current can flow through it. Although the virtual short is an ideal approximation, it gives very accurate answers when used with heavy negative feedback.

APPLICATIONS OF OP-AMPS

1. Inverting amplifier

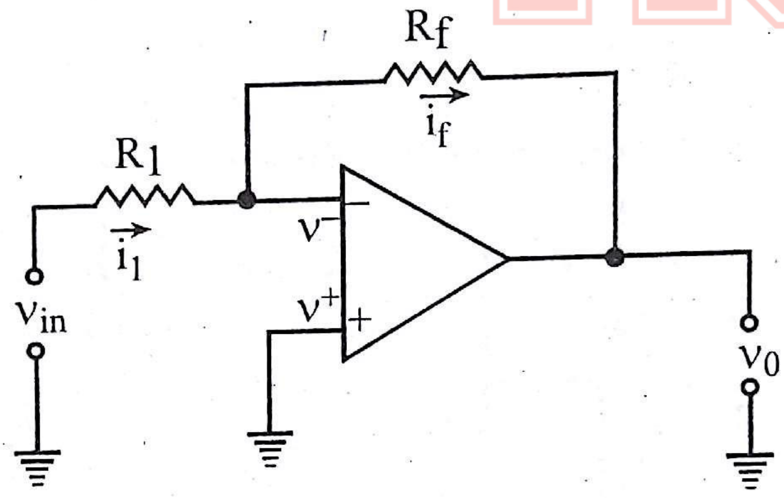


Figure 4.6 ***Circuit diagram of an inverting amplifier***

Our assumptions are:

i. The input resistance (i.e., the resistance that appears between the inverting and non-inverting input terminals, R_i) is infinite.

ii. The open loop voltage gain (i.e., the ratio of v_o to v_{in} with no feedback applied) is infinite.

As a consequence,

i. The voltage appearing between the inverting and non-inverting terminals will be zero. This will make v^- equals to v^+.

ii. The current flowing into the op-amp chip will be zero. This will make i_1 equals to i_f.

Using, $i_1 = i_f$

or, $\frac{v_{in} - v^-}{R_1} = \frac{v^- - v_o}{R_f}$

or, $\frac{v_{in} - v^+}{R_1} = \frac{v^+ - v_o}{R_f}$

or, $\frac{v_{in} - 0}{R_1} = \frac{0 - v_o}{R_f}$

or, $\frac{v_o}{v_{in}} = \frac{-R_f}{R_1}$

$\therefore$ Closed - loop voltage gain (A) $= \frac{v_o}{v_{in}} = \frac{-R_f}{R_1}$

Multiplier and divider

$$v_o = \frac{-R_f}{R_1} v_{in}$$

If $R_f > R_1$, the circuit acts as a multiplier.

If $R_f < R_1$, the circuit acts as a divider.

2. Non- inverting amplifier

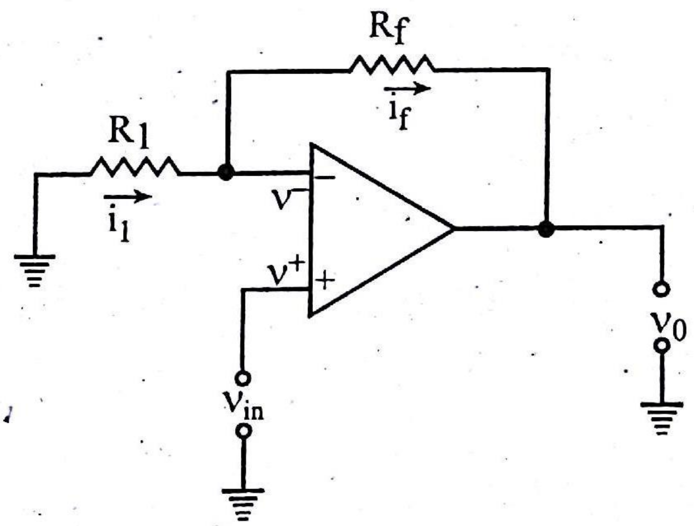


Figure 4.7 *Circuit diagram of a non-inverting amplifier*

Using, $i_1 = i_f$

or, $\frac{0 - v^-}{R_1} = \frac{v^- - v_o}{R_f}$

$$\text{or, } \frac{-v^+}{R_1} = \frac{v^+ - v_o}{R_f}$$

$$\text{or, } \frac{-v_{in}}{R_1} = \frac{v_{in} - v_o}{R_f}$$

$$\text{or, } \frac{v_o}{R_f} = \frac{v_{in}}{R_f} + \frac{v_{in}}{R_1}$$

$$\text{or, } \frac{v_o}{v_{in}} = R_f\left(\frac{1}{R_f} + \frac{1}{R_1}\right)$$

$$\text{or, } \frac{v_o}{v_{in}} = 1 + \frac{R_f}{R_1}$$

$$\boxed{\therefore A = \frac{v_o}{v_{in}} = 1 + \frac{R_f}{R_1}}$$

3. Adder

This circuit performs the addition of signals with amplification (if desired).

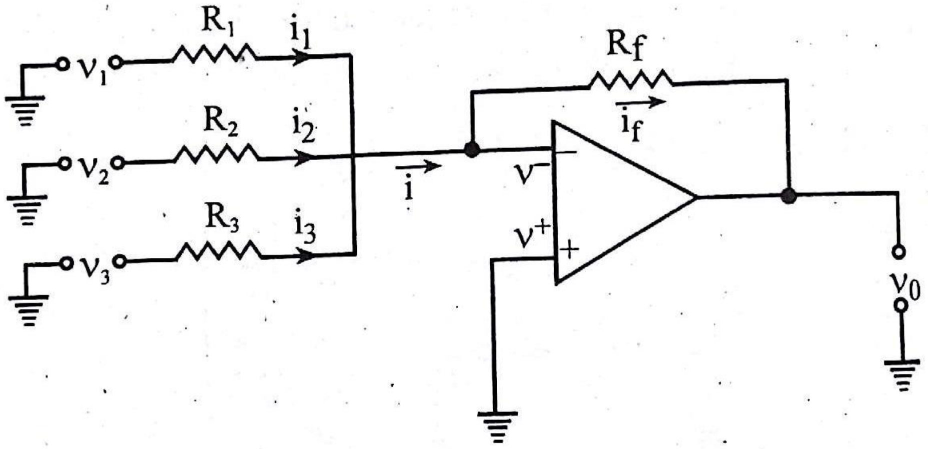


Figure 4.8 ***Circuit diagram of an adder***

Using, $i = i_f$

or, $i_1 + i_2 + i_3 = i_f$

$$\text{or, } \left(\frac{v_1 - v^-}{R_1}\right) + \left(\frac{v_2 - v^-}{R_2}\right) + \left(\frac{v_3 - v^-}{R_3}\right) = \frac{v^- - v_o}{R_f}$$

$$\text{or, } \left(\frac{v_1 - v^+}{R_1}\right) + \left(\frac{v_2 - v^+}{R_2}\right) + \left(\frac{v_3 - v^+}{R_3}\right) = \frac{v^+ - v_o}{R_f}$$

$$\text{or, } \left(\frac{v_1 - 0}{R_1}\right) + \left(\frac{v_2 - 0}{R_2}\right) + \left(\frac{v_3 - 0}{R_3}\right) = \frac{0 - v_o}{R_f}$$

$$\text{or, } \frac{v_1}{R_1}+\frac{v_2}{R_2}+\frac{v_3}{R_3}=\frac{-v_0}{R_f}$$

$$\text{or, } v_0=-\left[\frac{R_f}{R_1}v_1+\frac{R_f}{R_2}v_2+\frac{R_f}{R_3}v_3\right]$$

For $R_1 = R_2 = R_3 = R_f$, we have

$$\boxed{v_0=-(v_1+v_2+v_3)}$$

4. Subtractor

This circuit is used for subtraction of two input signals.

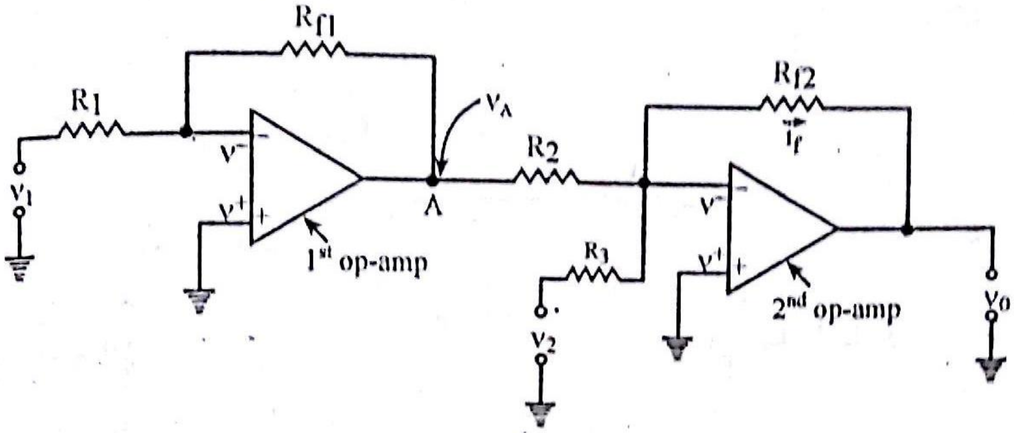


Figure 4.9 *Circuit diagram of a subtractor*

The output voltage due to first op-amp is

$$v_A=\frac{-R_{f1}}{R_1}v_1$$

For $R_{f1} = R_1$, $v_A = -v_1$

The second op-amp has two input signals $-v_1$ and v_2 fed to the inverting terminal.

For input signal $-v_1$, output voltage due to second op- amp is

$$v_o'=\frac{-R_{f2}}{R_2}(-v_1)$$

For $R_{f2} = R_2$, $v_o' = v_1$

For input signal v_2, output voltage due to second op-amp is

$$v_o''=\frac{-R_{f2}}{R_3}(v_2)$$

For $R_{f2} = R_3$, $v_o'' = -v_2$

Using superposition principle, the output voltage due to $-v_1$ and v_2 is given as

$$v_o = v_o' + v_o''$$

$$\boxed{\therefore\ v_o = v_1 - v_2}$$

5. Integrator

A circuit that performs the mathematical integration of input signal is called an integrator. For example, if the input to the integrator is a square wave, the output will be a triangular wave.

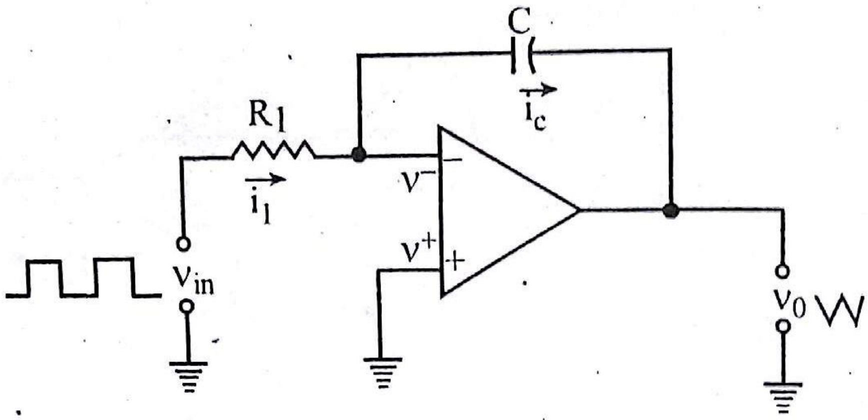


Figure 4.10 *Circuit diagram of an integrator*

Using, $i_1 = i_c$

or, $\dfrac{v_{in} - v^-}{R_1} = C\dfrac{dv_c}{dt}$

or, $\dfrac{v_{in} - v^-}{R_1} = C\dfrac{d(v^- - v_o)}{dt}$

or, $\dfrac{v_{in} - v^+}{R_1} = C\dfrac{d}{dt}(v^+ - v_o)$

or, $\dfrac{v_{in} - 0}{R_1} = C\dfrac{d}{dt}(0 - v_o)$

or, $\dfrac{v_{in}}{R_1} = -C\dfrac{dv_o}{dt}$

or, $dv_o = \dfrac{-1}{R_1C} v_{in}\, dt$

On integration, $\boxed{v_0 = \dfrac{-1}{R_1C}\int v_{in}\, dt}$

6. **Differentiator**

A circuit that performs the mathematical differentiation of input signal is called a differentiator. For example, if the input to the differentiator is a triangular wave, the output will be a square wave.

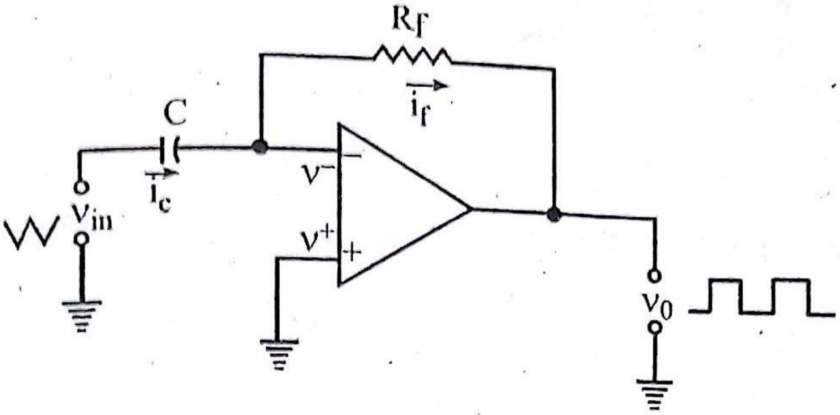


Figure 4.11 *Circuit diagram of a differentiator*

Using, $i_c = i_f$

$$\text{or, } C\frac{dv_c}{dt} = \frac{v^- - v_o}{R_f}$$

$$\text{or, } C\frac{d}{dt}(v_{in} - v^-) = \frac{v^- - v_o}{R_f}$$

$$\text{or, } C\frac{d}{dt}(v_{in} - v^+) = \frac{v^+ - v_o}{R_f}$$

$$\text{or, } C\frac{d}{dt}(v_{in} - 0) = \frac{0 - v_o}{R_f}$$

$$\text{or, } C\frac{dv_{in}}{dt} = \frac{-v_o}{R_f}$$

$$\boxed{\therefore\ v_o = -R_f C\frac{dv_{in}}{dt}}$$

Problem 4.1

Realize a circuit to obtain $v_0 = -2v_1 + 3v_2 + 4v_3$ using an operational amplifier. Use minimum value of resistance as 10kΩ

Solution:

For adder circuit,

$$v_o = -\left[\frac{R_f}{R_1}(v_1) + \frac{R_f}{R_2}(v_2) + \frac{R_f}{R_3}(v_3)\right] \ldots\ldots \text{(i)}$$

Given,

$v_o = -2v_1 + 3v_2 + 4v_3 = -[2v_1 - 3v_2 - 4v_3]$ ...... (ii)

From equation (i) & (ii),

$$\frac{R_f}{R_1} = 2, \frac{R_f}{R_2} = 3, \frac{R_f}{R_3} = 4$$

$\therefore R_f = 4R_3$

Let $R_3 = 10\ K\Omega$

$\therefore R_f = 4 \times 10 = 40k\Omega$

$$R_f = 3R_2 \Rightarrow R_2 = \frac{R_f}{3} = \frac{40}{3} = 13.33\ k\Omega$$

$$R_f = 2R_1 \Rightarrow R_1 = \frac{R_f}{2} = \frac{40}{2} = 20\ k\Omega$$

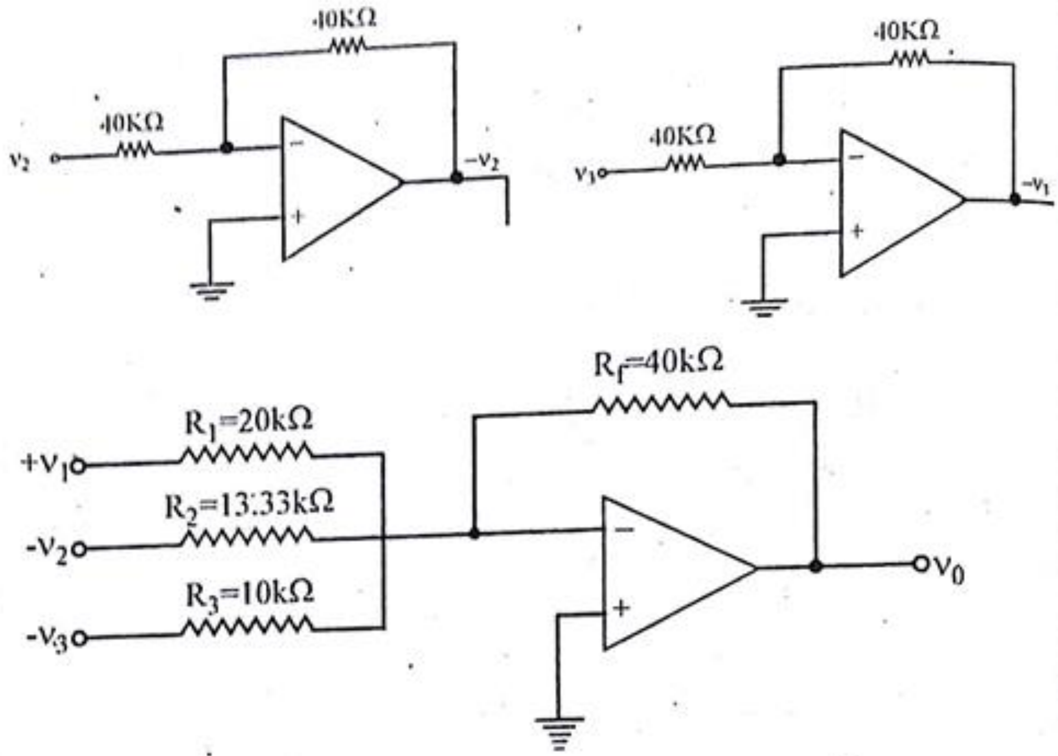


Fig.: Circuit realization of $v_o = -2v_1 + 3v_2 + 4v_3$

Problem 4.2

Design a summer circuit using op-amp to get the output voltage as: $v_o = -(v_1 + 10v_2 + 25v_3)$.

Solution:

For a summer circuit,

$$v_o = -\left[\frac{R_f}{R_1}(v_1) + \frac{R_f}{R2}(v_2) + \frac{R_f}{R3}(v_3)\right] \text{..............................(i)}$$

Given,

$v_o = -(v_1 + 10v_2 + 25v_3)$ ....(ii)

From equations (i) and (ii), we get

$\frac{R_f}{R_1} = 1, \frac{R_f}{R_2} = 10, \frac{R_f}{R_3} = 25$

$\therefore R_f = 25R_3$

Let $R_3 = 10\ k\Omega$

$\therefore R_f = 25 \times 10 = 250\ k\Omega$

$R_f = 10R_2 \Rightarrow R_2 = \frac{R_f}{10} = \frac{250}{10} = 25\ k\Omega$

$R_f = R_1 \Rightarrow R_1 = R_f = 250\ k\Omega$

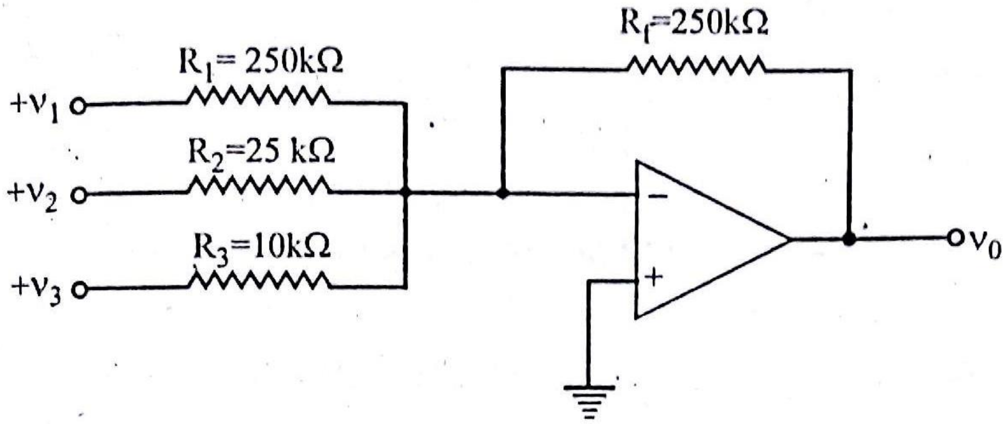


Fig.: Circuit realization of $v_o = -(v_1 + 10v_2 + 25v_3)$

FEEDBACK

The process of injecting a fraction of output energy of some device back to the input is known as feedback.

Positive Feedback

When the feedback energy (voltage or current) is in phase with the input signal and thus aids it, it is called positive feedback.

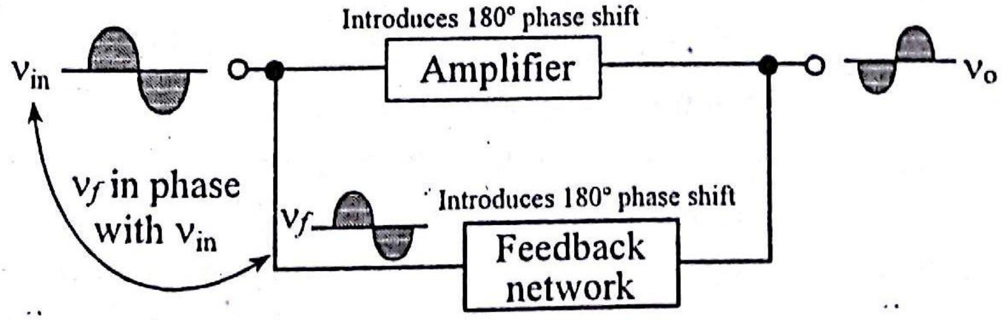


Figure 4.12 ***Illustration of positive feedback***

One important use of positive feedback is in oscillators about which we will be discussing shortly.

Disadvantages of positive feedback

i. Increased distortion

ii. Instability

Negative Feedback

When the feedback energy (voltage or current) is out of phase with the input signal and thus opposes it, it is called negative feedback.

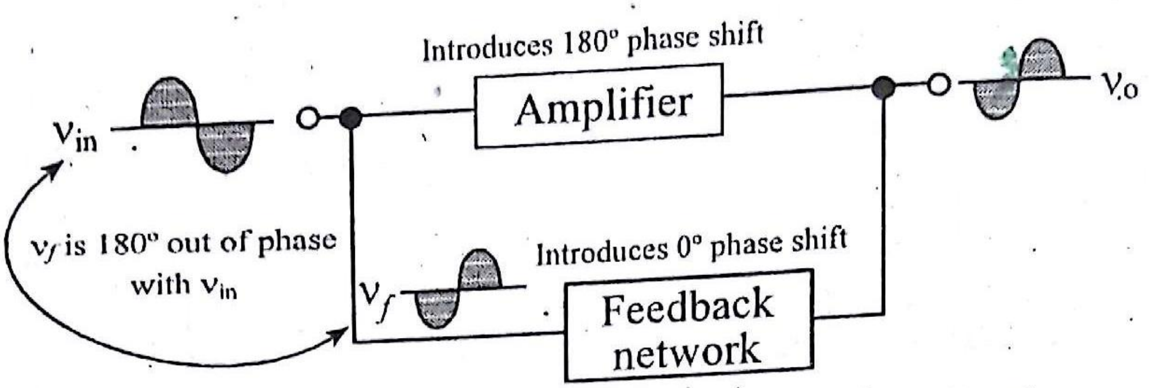


Figure 4.13 ***Illustration of negative feedback***

The concept of negative feedback is frequently employed in amplifiers.

Advantages of negative feedback

i. Gain stability of the amplifier can be achieved.

ii. Reduces non-linear distortion in large signal amplifiers.

iii. Improves frequency response of the amplifier.

iv. Increases circuit stability of the amplifier.

v. Increases input impedance and decreases output impedance of the amplifier.

• • •

The End