Text S2: Variance estimation for mixture model detection functions

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Variances of both $\hat{N}$ and $\hat{P}_a$ can be estimated for both non-covariate models and covariate models using standard methods [1–3].

In the most general case, the variance of $\hat{N}$ is estimated by:

$$\hat{\mathrm{Var}}\left(\hat{N}\right) = \left(\frac{\partial \hat{N}}{\partial \hat{\boldsymbol{\Theta}}}\right)^{\mathrm{T}} \hat{\mathbf{I}}(\hat{\boldsymbol{\Theta}})^{-1} \frac{\partial \hat{N}}{\partial \hat{\boldsymbol{\Theta}}} + \sum_{i=1}^{n} \frac{(1-p_i)}{p_i^2}$$

where $\hat{\mathbf{I}}(\hat{\boldsymbol{\Theta}})^{-1}$ is the inverse of the Fisher information matrix, $\hat{\boldsymbol{\Theta}}$ is a vector of all of the maximum likelihood estimates of the parameters of the detection function ($\hat{\boldsymbol{\Theta}} = (\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\phi}})$) and all other notation is as in previous sections.

Then for the average detectability:

$$\hat{\mathrm{Var}}\left(\hat{P}_a\right) = \hat{P}_a^2 \left\{ \frac{\hat{\mathrm{Var}}(\hat{N})}{\hat{N}^2} + \frac{\left(\frac{\partial \hat{P}_a}{\partial \hat{\boldsymbol{\Theta}}}\right)^{\mathrm{T}} \hat{\mathbf{I}}(\hat{\boldsymbol{\Theta}})^{-1} \frac{\partial \hat{P}_a}{\partial \hat{\boldsymbol{\Theta}}} + \sum_{i=1}^{n}(1-p_i)}{n^2} - \frac{2\left(\left(\frac{\partial \hat{N}}{\partial \hat{\boldsymbol{\Theta}}}\right)^{\mathrm{T}} \hat{\mathbf{I}}(\hat{\boldsymbol{\Theta}})^{-1} \frac{\partial \hat{P}_a}{\partial \hat{\boldsymbol{\Theta}}} + \sum_{i=1}^{n} \frac{(1-p_i)}{p_i^2}\right)}{n\hat{N}} \right\}.$$

References

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3. Borchers DL, Buckland ST, Goedhart P, Clarke ED, Hedley SL (1998) Horvitz-Thompson estimators for double-platform line transect surveys. Biometrics : 1221–1237.