Response to reviewers

We thank the reviewers for their constructive comments and have corrected the minor (spelling/grammar) mistakes. The more major points have also been corrected and some explanation/addition information is given below.

Reviewer 1

- *You mention that the larger class of functions considered in Duchon (1977) has been ignored in the statistical literature. Can you elaborate on why this might be? Fundamental to the argument for using your methods is that the penalty associated with the larger class of functions is effective for smoothing in high dimensions. It is not clear why this would not have been considered in the statistical literature.*

We think that there are several reasons why Duchon's wider class of smoothers have not been taken up, none of them really statistical.

1. Duchon (1977) is not written in an accessible manner. Duchon commented to SW that he believes that the paper is very little read, despite being highly cited. We were only able to understand it with help from Duchon himself (with whom SW was in touch about something else).
2. Although Duchon `got there first', other authors were working on the problem and also developed the TPS version of Duchon splines. So TPS became the focus, while Duchon did other things, rather than promoting his invention. A good example of this effect is in smoothing on the sphere. Duchon has a suggestion for doing this based directly on Duchon Splines, but he never published it. In simulations it out performs conventional thin plate spline based TPS like splines on the sphere (Wahba's pseudosplines on the sphere and the Wahba and Wendelberger exact version).
3. There was no software for Duchon's wider class of smooths, and you had to recognize that Wood (2003) could be used for this class in order to make a really practical smoother.

- *You choose $s = D / 2 - 1$. It would be helpful to discuss appropriate choices for s in the Duchon penalty. It is not clear why the choice of $D / 2 - 1$ is made other than the fact that it is the smallest value that satisfies the continuity constraint. Since this penalty will be new to most readers, some discussion of the choice of s would be helpful.*
- *I agree that your method is somewhat more apparent about what is being penalized when smoothing when compared to the method in Wang and Ranalli (2007); however, your penalty introduces a weighting function that depends on the value s. It is not clear how to connect the value of s to the nature of the smoother, whereas the order of the differential penalty can have a natural interpretation (e.g. minimizing the overall surface*

tension, as mentioned in the soap film method). Some elucidation about the role of s and some general recommendations in practice would strengthen the argument for the use of the proposed method.

We have added a comment to the end of section 3.2 to address both of these queries. Increasing *s* would increase the penalty weighting on the high frequency parts of the smooth, which we are not worried about here (just the nullspace). It is possible that other values of *s* may be appropriate in other situations but we would not like to speculate without further data.

- *Again, a point of clarification is needed regarding the null space basis and its relation to boundary consideration. It would be of great help to readers to have laid out (here or somewhere else in the paper) what the authors mean by the null space basis and how the two methods differ in their boundary considerations.*

We have added an informal definition of the nullspace at it's first mention in section 2.2 and an further information on the consequences on the definition in that section. We also now point out those functions which lie in the nullspace of the TPS penalty (i.e. when s=0) towards the end of section 3.2.

Reviewer 2

- The observation with the generalized distances are mapped to a Euclidean space. Since the configurations are not unique (eg rotations), some explanation is needed as to why this does not affect the smoothing.

A short explanation has been added to section 3.2 explaining that the spline basis used is rotation invariant and hence rotations of the MDS configuration will not have an adverse effect on the resulting smooth functions.