Multiple detection functions in density surface models

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Abstract

We often want to combine data from multiple surveys into one spatial model. In this case we usually want to include multiple detection functions and use them with a single GAM. Here I show how to do that.

1 Introduction

Generally for a DSM we have:

$$\mathbb{E}\left[n_i|\boldsymbol{\beta},\boldsymbol{\lambda},p(\hat{\boldsymbol{\theta}};\mathbf{z}_i)\right] = a_i p(\hat{\boldsymbol{\theta}};\mathbf{z}_i)\exp\left(\beta_0 + \sum_m f_m(x_{im})\right), \tag{1}$$

where the number of individuals per segment (of area a_i), n_i, follows some count distribution such as quasi-Poisson, Tweedie or negative binomial and we assume a log link. The f_m are smooth functions of environmental covariates, x_{im}, represented by a basis expansion (i.e., $f_m(x) = \sum_j \beta_j b_j(x)$ for some basis functions b_j) penalized by a (sum of) quadratic penalty (or penalties); β_0 is an intercept term, included in parameter vector $\boldsymbol{\beta}$; $\boldsymbol{\lambda}$ is a vector of smoothing parameters which control the wiggliness of the smooth components of the model.

When we have multiple detection functions, we write $p(\hat{\boldsymbol{\theta}};\mathbf{z}_i)$ as a piecewise function:

$$p(\hat{\boldsymbol{\theta}};\mathbf{z}_i) = \begin{cases} p(\hat{\boldsymbol{\theta}}_1;\mathbf{z}_i) & \text{observation } i \text{ in detection function 1} \\ \vdots & \\ p(\hat{\boldsymbol{\theta}}_2;\mathbf{z}_i) & \text{observation } i \text{ in detection function 2} \\ \vdots & \\ p(\hat{\boldsymbol{\theta}}_K;\mathbf{z}_i) & \text{observation } i \text{ in detection function } K \end{cases},$$

so that a given observation is related to one detection function only (indexed by $k = 1, \ldots, K$). No assumption is made about the specific form of the detection function. We can concatenate all of the detection function parameters $\hat{\boldsymbol{\theta}} = (\hat{\boldsymbol{\theta}}_1, \hat{\boldsymbol{\theta}}_2, \ldots, \hat{\boldsymbol{\theta}}_K)$. We can then fit model (1) as usual.

1.1 Variance estimation

The above only addresses mean effects, what about variance?

1.2 Delta method

Calculate $\mathrm{CV}(\hat{N}) = \sqrt{\mathrm{CV}_{\mathrm{GAM}}(\hat{N})^2 + \sum_k \mathrm{CV}(\hat{p}_k)^2}$. There, the $\mathrm{Var}_{\mathrm{GAM}}(\hat{N})$ (hence $\mathrm{CV}_{\mathrm{GAM}}(\hat{N})$) is calculated using the usual GAM estimator (see the varprop paper for details).

1.3 Variance via variance propagation

Thinking about the variance propagation method of Bravington, Miller and Hedley (2018)[1], we need not only the $p(\hat{\boldsymbol{\theta}}; \mathbf{z}_i)$s but also their derivatives wrt $\hat{\boldsymbol{\theta}}$ and the Hessian corresponding to the detection functions. Following from the varprop paper, we fit the following model to estimate variance:

$$\log \mathbb{E}\left[n_i | \boldsymbol{\beta}, \boldsymbol{\lambda}, \hat{p}_i\right] = \log a_i \hat{p}_i + X_i \boldsymbol{\beta} + \kappa_i \boldsymbol{\delta},$$

defining the vectors $\boldsymbol{\delta} \triangleq \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0$ and $\kappa_i \triangleq \left.\frac{d \log p(\boldsymbol{\theta}, z_i)}{d\boldsymbol{\theta}}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}$. Extending this to our case of multiple detection functions, the definition of $\boldsymbol{\delta}$ follows simply and $\kappa_i \triangleq \left.\frac{d \log p(\boldsymbol{\theta}, z_i)}{d\boldsymbol{\theta}}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}$, where entries where i does not belong to detection function k are 0.

We then also need to form the covariance matrix for the detection functions. We assume no covariance between the detection functions[2], so we have:

$$\mathbf{V}_{\boldsymbol{\theta}} = \begin{pmatrix} \mathbf{V}_{\boldsymbol{\theta}_1} & 0 & \ldots & 0 \\ 0 & \mathbf{V}_{\boldsymbol{\theta}_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & \mathbf{V}_{\boldsymbol{\theta}_K} \end{pmatrix}.$$

This can then be used as the covariance matrix for the random effect $\boldsymbol{\delta}$.

Do we need to do something special about scale parameter stuff? Do we need to think about different smoopars for the detection functions?!?!?!?!

[1] `https://arxiv.org/abs/1807.07996`

[2] There might be cases where we have some information about covariance between the detection functions, but we ignore that for now.

2 Implementation

To implement this in `dsm` you need to be able to identify each observation as being from one detection function. You also need to be able to know which segments relate to a given detection function. This leads to some implementation differences in dsm.

Detection function: One detection function for each data subset (e.g., per cruise etc).

Observation table: The observation table is unchanged (it seems easiest to concatenate the tables used to fit the detection function) *but* you must ensure that the object IDs (column `object`) are unique and match those used to fit the detection functions.

Segment table: Additional column `ddfobj`, which refers to which detection function is used for each set of segments.

The call to `dsm` now lets you use a list of detection functions, the order of the detection functions in the list relates to the numbering in the `ddfobj` column in the segment table.

3 Some special cases and changes to `dsm`

This approach allows us to do some other stuff that we couldn't do before in `dsm`.

3.1 Strip transects

A new function (`dummy_ddf`) is used to construct dummy detection functions for use with `dsm`. This takes the object IDs, group sizes, truncation and transect type and constructs a model object that can be used with `dsm`. This replaces the `strip.width` way of specifying strip transects. This also works with whatever one might call the point transect equivalent ("circle transects"?).

3.2 Changes

- No longer need to supply the `transect=` argument, this is determined from the (dummy) detection functions. This means you can mix points and line and strips and circles.
- No `strip.width` argument, as this is handled by dummy detection functions.
- No `transect` argument as this can always be determined from the detection functions (dummy or not).