**MOL-42166 Rheology**

**1.Midterm exam**

14.10.2019

Ilari Jönkkäri

The formulary is enclosed, other literature is forbidden. Calculators are permitted.

Answer to all questions

**1. Short questions**

**a)** What is the melt flow index?

**b)** What causes shear thinning of polymer melts? Draw a viscosity curve for a polymer melt.

**c)** Does the temperature have a stronger impact on the viscosity of polycarbonate or polyethylene? Why?

**d)** What is thixotrophy and what can cause it? Name two applications where thixotrophy is important.

**2. Essay.** Name six things that effect on the viscosity of a suspension and explain how they effect.

**3. Essay.** Compare traditional rheometer and process equipment rheometer. Explain what they are and what are their strengths and weaknesses.

**4. Calculation.** A polymer melt was measured with a capillary rheometer using two capillaries: L/D = 40 mm / 1mm and L/D = 10mm / 1mm. The table below shows the measured pressure drops at different flow rates. Calculate the true viscosities and shear rates for the data points.

| Q [$mm^3/s$] | Δp [bar] L/D = 40/1 | Δp [bar] L/D =10/1 |
|---|---|---|
| 4.909 | 73.2 | 21.6 |
| 19.64 | 137.0 | 44.3 |
| 49.09 | 200.0 | 65.0 |
| 98.18 | 259.8 | 84.6 |

Vilma Turunen

**MOL-42166 Rheology** Midterm exam

22.10.2018

Ilari Jönkkäri

The formulary is enclosed, other literature is forbidden. Calculators are permitted.

Answer to all questions

**1. Short questions**

a) What is the melt flow index?

b) What is an edge fracture and where it may appear?

c) What is the wall slip and how it can be prevented?

**2. Essay.** Draw a typical **shear** viscosity curve for a polymer melt. Explain what happens to the polymer chains at different regions of the curve. Comparison of viscosity curves can give a lot of qualitative information about the polymeric material. Explain how the following properties effect on the shape and location of the viscosity curve:

Molecular weight

Molecular weight distribution

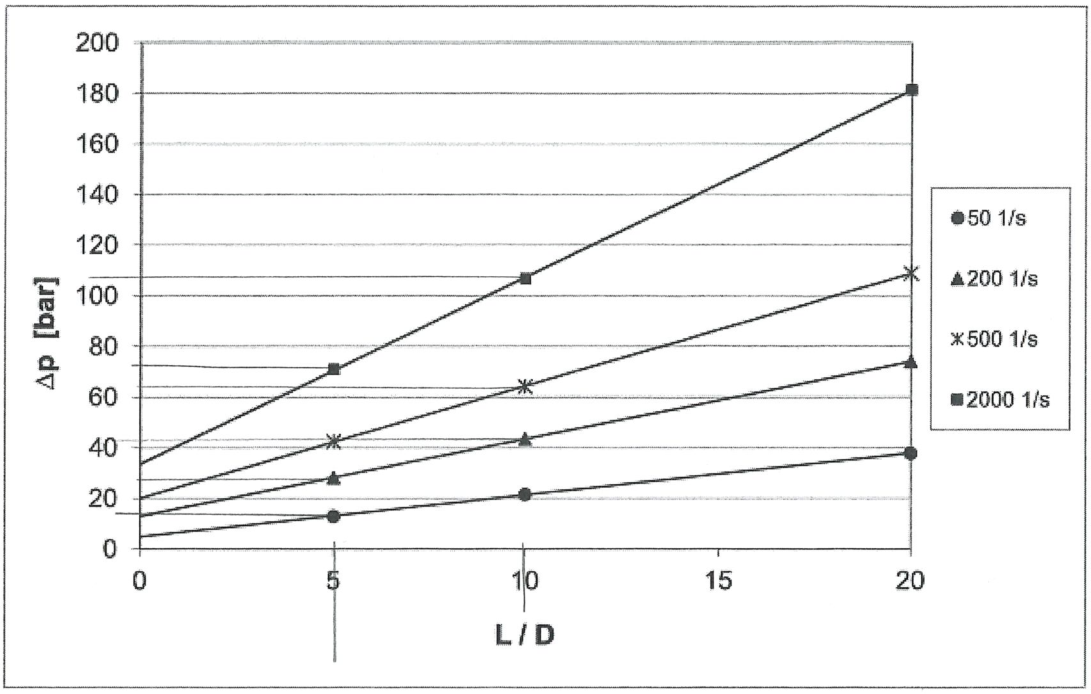
Branching

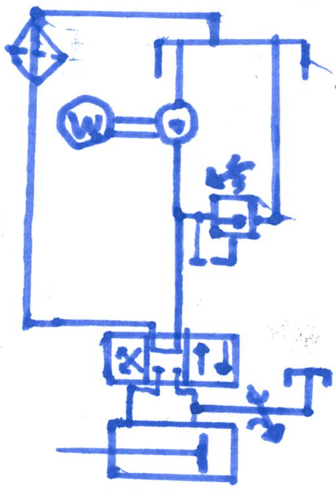
**3. Calculation** A polymer melt was measured with a rotational rheometer using a cone-plate geometry. The cone angle was $4^0$ and radius of the plate 12.5mm. Calculate shear rates, stresses and viscosities based on the data in table 1. Fit a power law model on the appropriate part of the viscosity curve. What are the values of the power law parameters?

**Table 1** Rotational rheometer data

| Torque [mNm] | Angular Velocity [rad/s] |
|---|---|
| 0.11 | 0.000701 |
| 1.06 | 0.00701 |
| 9.25 | 0.0701 |
| 47.00 | 0.701 |
| 66.30 | 7.02 |

**4. Calculation.** The figure below shows the pressure drop *Δp* as function of the *L/D*-ratio (L= length of the capillary, D=diameter) at apparent shear rates of 50, 200, 500 and 2000 1/s. This is the so-called Bagley plot. The diameter of the capillaries is 1mm. Determine the true shear rates and viscosities. Read the information you need from the figure.



Aleksi Virtanen



**MOL-42166 Rheology**

19.12.2017

Ilari Jönkkäri

The formulary is enclosed, other literature is forbidden. Calculators are permitted.

Answer to all questions

1. Answer briefly to the following questions:

   a) How the molecular mass and the molecular mass distribution effect on the polymers viscosity?
   b) What is an edge fracture and where it may appear?
   c) What phenomenon can be modelled with a Kelvin/Voigt model? – viscoelastic solid
   d) What conclusions may be made about the polymer structure based on the intersection of storage G' and loss moduli G''? – entanglement

- haaroittuneisuus
- dimensiot?
- sula näyte hankala
- eri tavat kiertyä

2. **Essay** Why the viscosity is sometimes measured under extensional deformation instead of shear deformation? What challenges extensional viscosity measurements have?

3. **Essay** Compare traditional rheometer and process equipment rheometer. How they work and what are their strengths and weaknesses.

4. **Calculation** A polymer melt was measured with a rotational rheometer using a plate-plate geometry. The gap between the plates was 1mm and radius of the plate 12.5mm. Calculate shear rates, stresses and viscosities based on the data in table 1. Fit a power law model on the appropriate part of the viscosity curve. What are the values of the power law parameters?

**Table 1** Rotational rheometer data

| Torque [mNm] | Angular Velocity [rad/s] |
|---|---|
| 0.10 | 0.001 |
| 0.96 | 0.010 |
| 8.38 | 0.100 |
| 39.50 | 1.001 |
| 70.60 | 10.147 |

Perinteinen
+ Pieni näytemäärä
+ luotettava
+ pienet shear
- suuret shear

Process
+ ei erillisiä laitteistoja
+ samat parametrit kuin tehtaassa
+ suuri shear
- suuri näytemäärä
- vaikea asentaa

5. **Calculation** The figure 1 shows results of a creep-recovery test (strain vs. time) for a polymer melt. The test was done in a linear region and the stress was 15 Pa. How are you able to determine if the creep test is done in a linear region? Determine from the figure: the zero viscosity $\eta_0$, the steady-state creep compliance $J_s^0$ and the steady-state recoverable compliance $R_\infty^0$. Evaluate what the strain would be at the end of the creep phase, if the stress was 5 Pa.

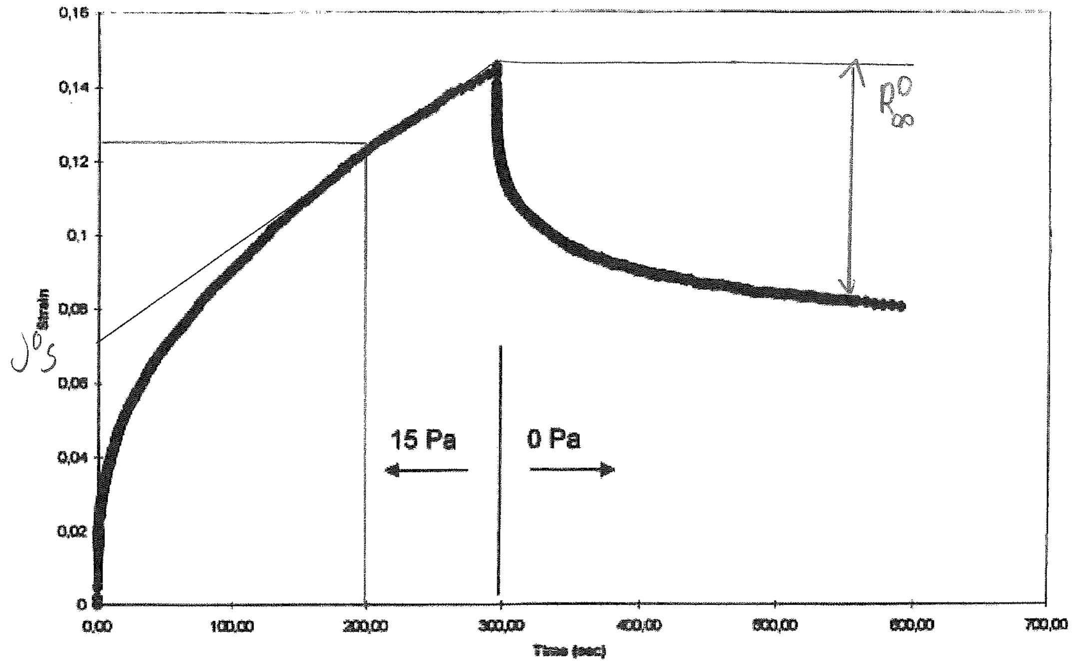



***Figure 1.*** *The creep recovery test*

**MSE.432 Rheology**

17.10.2023

Ilari Jönkkäri

The formulary is enclosed, other literature is forbidden. Calculators are permitted.

Answer to all questions

**1. Short questions (1.5 pts/each)**

a) How the molecular mass and the molecular mass distribution effect on the polymers viscosity?

b) What causes shear thinning of polymer melts? Draw a viscosity curve for a polymer melt.

c) Does the temperature have a stronger impact on the viscosity of polycarbonate or polyethylene? Why?

d) What corrections are needed when calculating the true viscosity and shear rate for a polymer melt when the measurements are done with a capillary rheometer? Explain also why they are needed.

**2. Essay.** What different things effect on the viscosity of a suspension and how? (6 pts)

**3. Essay.** You are not able to use capillary or rotational rheometers, but you need to study the viscosity of the polymer melt as function of the shear rate. Explain two different ways how you might do it. What weaknesses these approaches have compared to capillary or rotational rheometers? (6 pts)

**4. Calculation** A polymer melt was measured with a rotational rheometer using a cone-plate geometry. The cone angle was 4°, radius of the plate 12.5mm. Calculate shear rates, stresses and viscosities based on the data in table 1. Fit a power law model on the appropriate part of the viscosity curve. What are the values of the power law parameters? (6 pts)

**Table 1** Rotational rheometer data

| Torque [mNm] | Angular Velocity [rad/s] |
|---|---|
| 1,06 | 0,00701 |
| 9,25 | 0,0701 |
| 47,00 | 0,701 |
| 66,30 | 7,02 |

MSE.432 Rheology
Välikoe 1 2024

T1

a) Disadvantages of melt index device and Brookfield viscosimeter compared to capillary and rotational rheometer. How can you get moredata.

b) Slit-die capillary advantages and disadvantages compared to capillary rheometer.

c) What corrections are needed for capillary rheometry and why?

T2. What things affect viscosity the viscosity of ~~a~~ polymer melt (internal and external)

T3. What rotational rheometer geometries there are. Advantages disadvantages and what materials can be measured.

T4. Bagley-plot at start. Calculate true shear rates and viscosities.

**MSE.432 Rheology**

**2.midterm exam**

12.12.2023

Ilari Jönkkäri

The formulary is enclosed, other literature is forbidden. Calculators are permitted.

Answer to all questions

**1. Short questions**

a) What type of measurements are the frequency and amplitude sweeps? Which one should be done first and why?

b) What conclusions may be made about the polymer structure based on the intersection of storage G' and loss moduli G'' and how?

c) What is the linear region when talking about viscoelastic properties? How are you able to make sure that you are in the linear region when doing the measurements?

d) What is the time temperature superposition principle?

**2. Essay.** What are the possible error sources in rheological measurements? How they effect on the results?

**3. Calculation.** The figure 1 shows results of a creep-recovery test (strain vs. time) for a polymer melt. The test was done in a linear region and the stress was 15 Pa. How are you able to determine if the creep test is done in a linear region? Determine from the figure: the zero viscosity $\eta_0$, the steady-state creep compliance $J_s^0$ and the steady-state recoverable compliance $R_\infty^0$. Evaluate what the strain would be at the end of the creep phase, if the stress was 5 Pa.

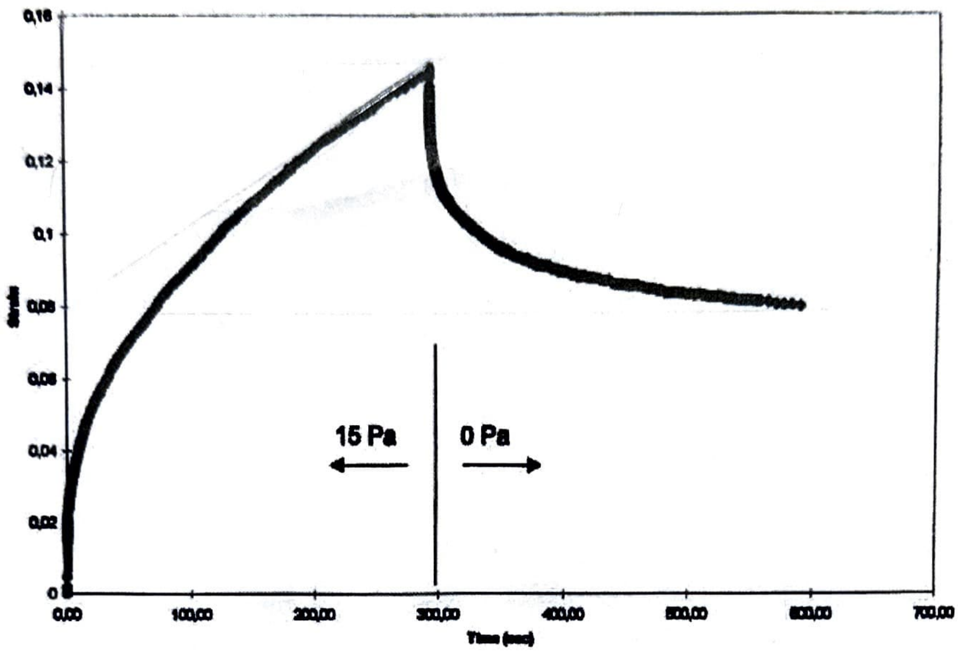



*__Figure 1.__ The creep recovery test*

**4. Calculation** A frequency sweep measurement was performed for a polymer melt with a strain amplitude of 10 % that is within the linear region. The results of the measurement are in table 1. Based on this data determine the shear viscosity and fit the power-law model on the appropriate shear rate range. The polymer melt follows the Cox-Merz rule.

**Table 1.** Results of an oscillatory measurement.

| $\omega$ (rad/s) | $\delta$ (-) | $\tau_0$ (Pa) |
|---|---|---|
| 0,01 | 1,56 | 3003 |
| 0,1 | 1,47 | 28964 |
| 1 | 1,03 | 209641 |
| 10 | 0,78 | 733747 |
| 100 | 0,53 | 2086236 |

**MSE.432 Rheology**

03.03.2023

Ilari Jönkkäri

The formulary is enclosed, other literature is forbidden. Calculators are permitted.

Answer to all questions

1. Answer briefly to the following questions:

   a) What property of polymer chains is more visible from extensional than shear viscosity data? How it effects on the shape of the curves?
   b) What is a process equipment rheometer and what benefits it has?
   c) What forces may act on the particles in a suspension?
   d) What is the time temperature superposition principle?

2. **Essay** What are the possible error sources in rheological measurements? How they effect on the results?

3. **Essay** What is the principle of oscillatory measurements (SAOS)? How the viscoelasticity of the sample effects on the measured data? What quantities are typically determined from the measured data?

4. **Calculation** A polymer melt was measured with a capillary rheometer using two capillaries: L/D = 40 mm / 1mm and L/D = 10mm / 1mm. The table 1 shows the measured pressure drops at different flow rates. Make Bagley and Rabinowitch corrections for the data and calculate the true viscosities and shear rates for the measured data points.

**Table 1.** Capillary rheometer data

| Q [$mm^3/s$] | Δp [bar] L/D = 40/1 | Δp [bar] L/D =10/1 |
|---|---|---|
| 4.909 | 73.2 | 21.6 |
| 19.64 | 137.0 | 44.3 |
| 49.09 | 200.0 | 65.0 |
| 98.18 | 259.8 | 84.6 |

5. **Calculation** The figure 2 shows the storage modulus G' (ω) for a polymer melt that follows time-temperature superposition principle at temperatures 160, 200 and 240 °C. Determine the temperature shift factor $\alpha_T$, at temperatures 160 and 240 °C, when the reference temperature is 200 °C. Does the shift factor follow the Arrhenius equation? The intersection of *G'* and *G"* at 200°C is at 3.5 rad/s. At which angular frequency the *G'* and *G"* are crossing at 240°C?

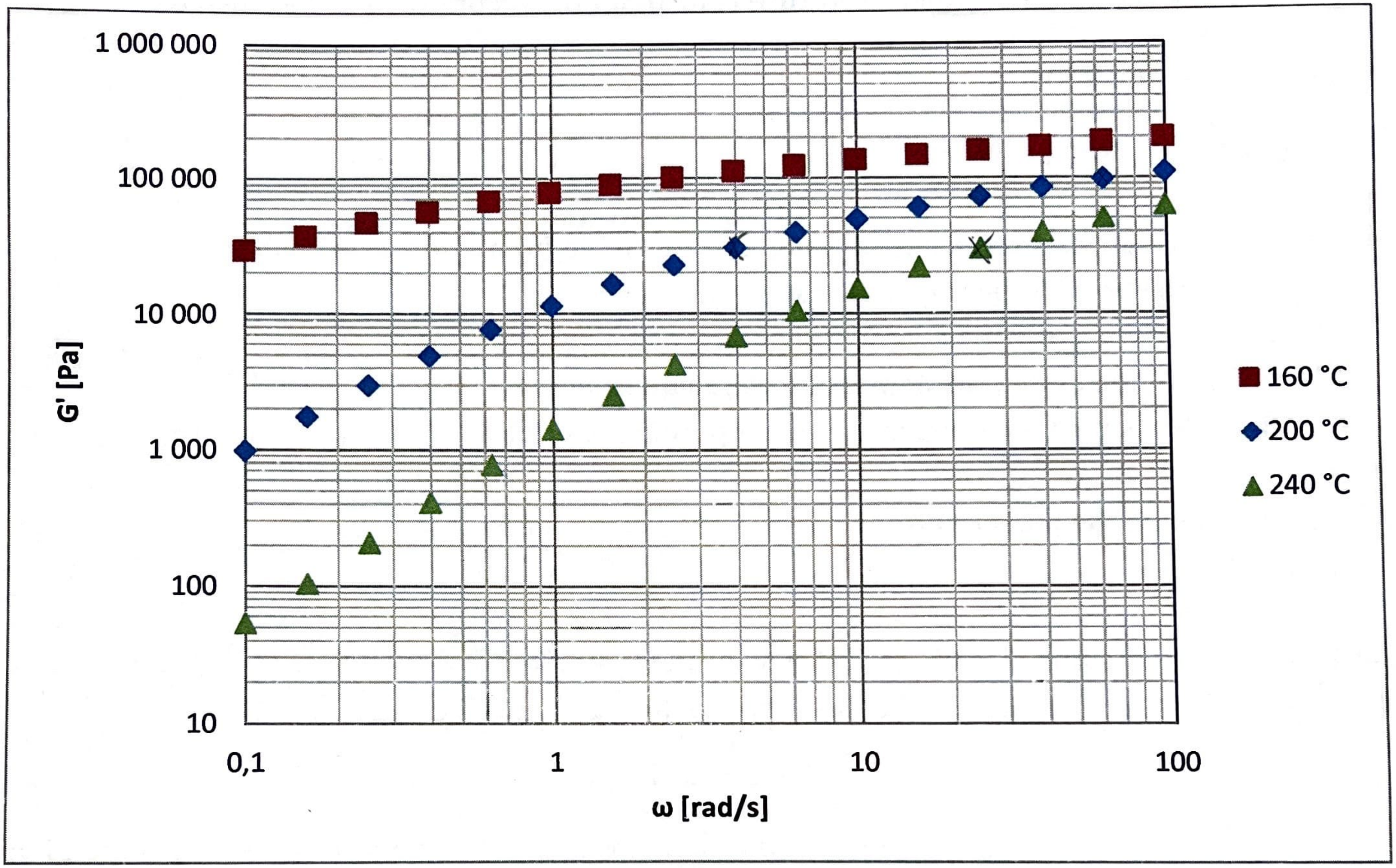



**Figure 2.** The storage modulus as function of the angular frequency measured at three different temperatures

**MSE.432 Rheology**

2.Midterm exam

14.12.2022

Ilari Jönkkäri

The formulary is enclosed, other literature is forbidden. Calculators are permitted.

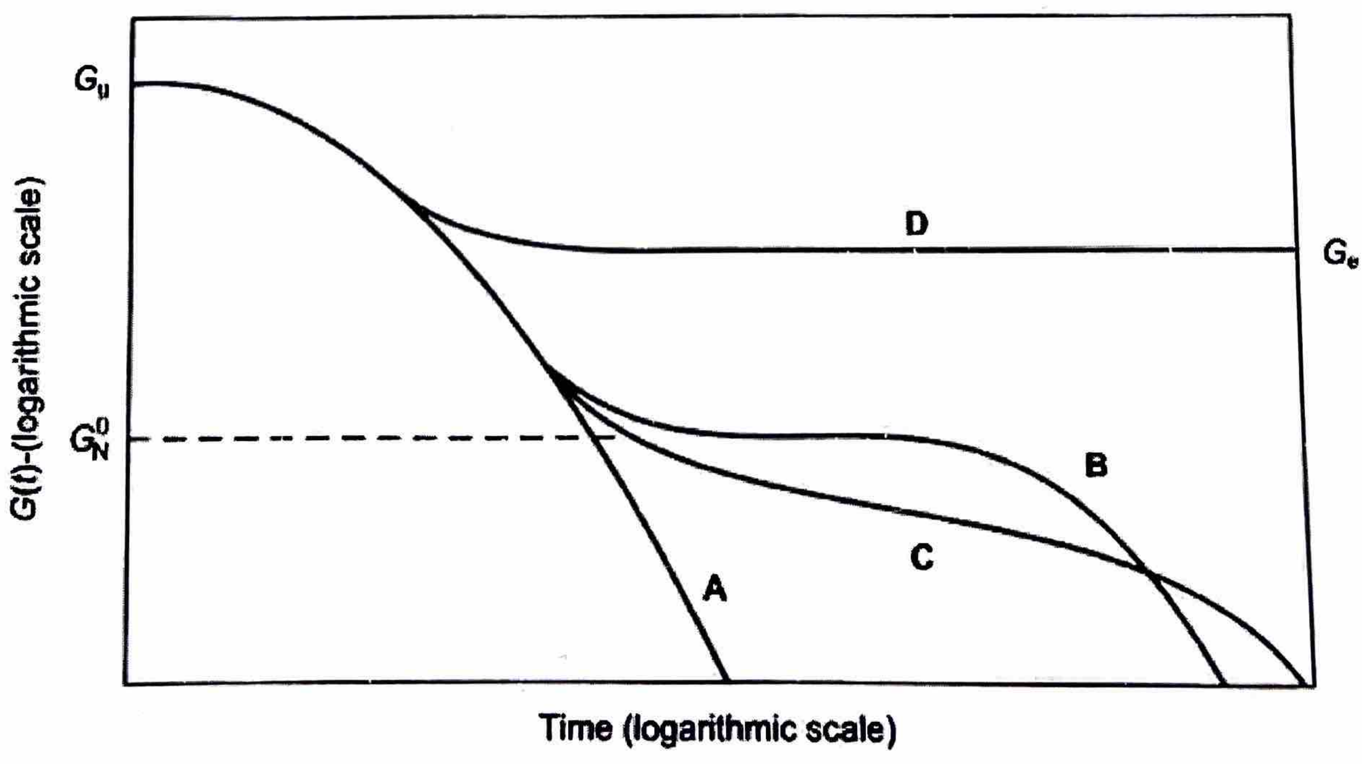
Answer to all questions

**1. Short questions**

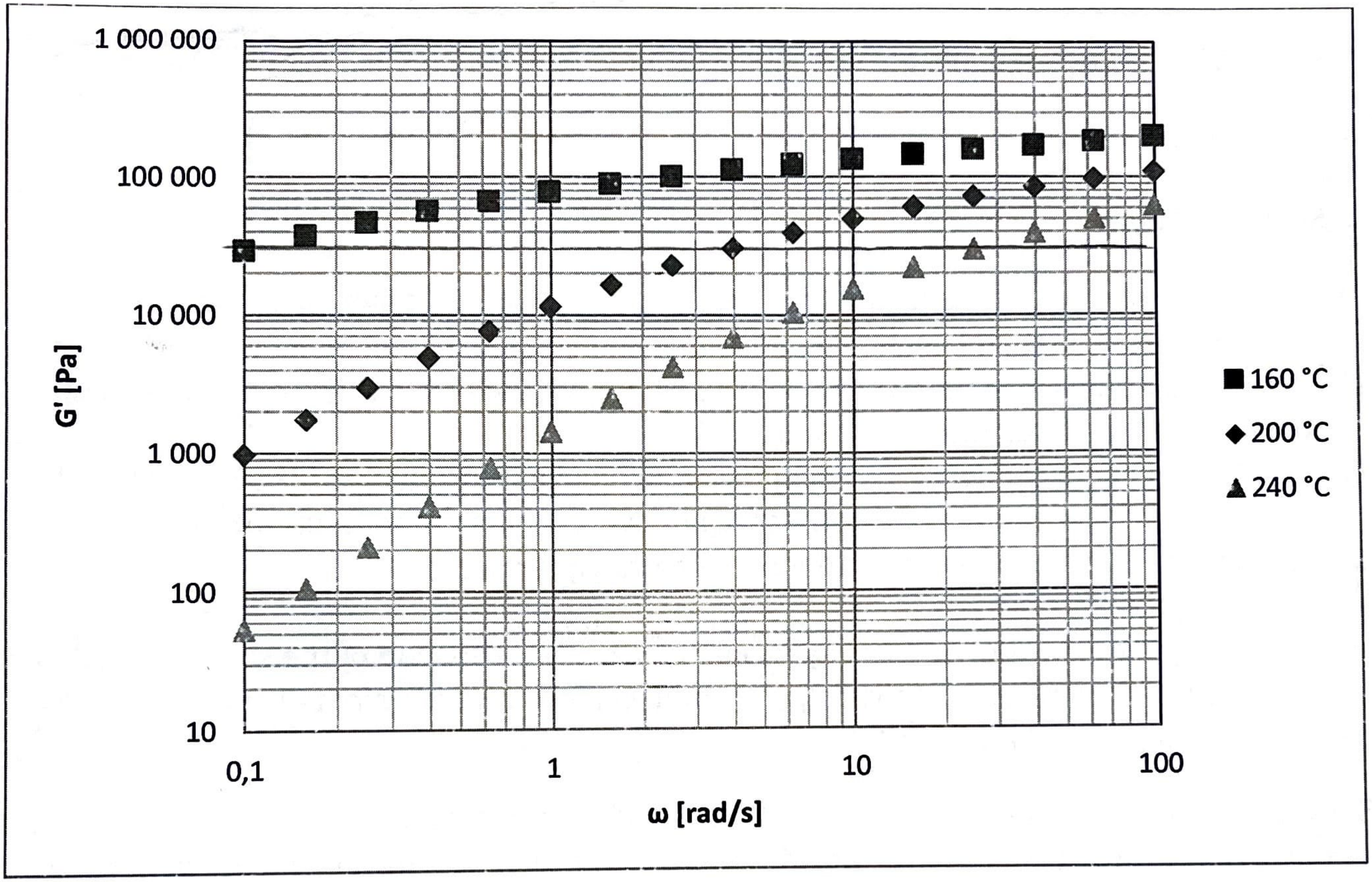
a) What type of measurements are the frequency and amplitude sweeps? Which one should be done first and why?
b) What conclusions may be made about the polymer structure based on the intersection of storage G' and loss moduli G'' and how?
c) What is the linear region when talking about viscoelastic properties? How are you able to make sure that you are in the linear region when doing the measurements?
d) What is the time temperature superposition principle?

**2. Essay.** Why the extensional viscosity should be measured instead of the shear viscosity in some cases? What it reveals about polymers molecular structure? What experimental challenges elongational flow has?

**3. Essay.** The figure below shows a relaxation modulus as function of time for four polymeric samples A-D. Name the zones that are visible from the curves and explain what molecular level relaxation mechanisms happen in different zones. How the four samples differ?



**4. Calculation.** The figure below shows the storage modulus G' (ω) for a polymer melt that follows time-temperature superposition principle at temperatures 160, 200 and 240 °C. Determine the temperature shift factor $\alpha_T$, at temperatures 160 and 240 °C, when the reference temperature is 200 °C. Does the shift factor follow the Arrhenius equation? The intersection of *G'* and *G"* at 200°C is at 3.5 rad/s. At which angular frequency the *G'* and *G"* are crossing at 240°C?



MSE.432 Rheology

18.10.2022

Ilari Jönkkäri

The formulary is enclosed, other literature is forbidden. Calculators are permitted.

Answer to all questions

**1. Short questions (2pts. each)**

a) How the concentration, molecular mass and solvent type effect on the polymer solutions viscosity?

*b)* What yield stress means with fluids, and what can cause it? Name a fluid that has a yield stress.

c) What is the major limitation of melt index device and Brookfield viscosimeter compared to capillary and rotational rheometers? How you can get a bit more information than usually about rheological behaviour of the measured fluids also with these devices?

**2. Essay (6pts.)** Draw a typical **shear** viscosity curve for a polymer melt. Explain what happens to the polymer chains at different regions of the curve. Comparison of viscosity curves can give a lot of qualitative information about the polymeric material. Explain how the following properties effect on the shape and location of the viscosity curve: Molecular weight, molecular weight distribution, branching

**3. Essay (6pts.)** What different factors effect on the viscosity of a suspension and how?

**4. Calculation (6pts.)** A polymer melt was measured with a capillary rheometer using two capillaries: L/D = 40 mm / 1mm and L/D = 10mm / 1mm. The table below shows the measured pressure drops at different flow rates. Calculate the true viscosities and shear rates for the data points.

| Q [$mm^3/s$] | Δp [bar] L/D = 40/1 | Δp [bar] L/D =10/1 |
|---|---|---|
| 4.909 | 73.2 | 21.6 |
| 19.64 | 137.0 | 44.3 |
| 49.09 | 200.0 | 65.0 |
| 98.18 | 259.8 | 84.6 |

**MSE.432 Rheology**

19.10.2021

Ilari Jönkkäri

The formulary is enclosed, other literature is forbidden. Calculators are permitted.

Answer to all questions

## 1. Short questions

a) Does the temperature have a stronger impact on the viscosity of polycarbonate or polyethylene? Why?

b) What is the melt flow index and how it is measured?

c) How the concentration, molecular weight and solvent type effect on the polymer solutions viscosity?

d) What is thixotrophy, and what can cause it? Name two applications where thixotrophy is important.

**2. Essay.** What different factors effect on the viscosity of a polymer melt and how?

**3. Essay.** Compare traditional rheometer and process equipment rheometer. Explain what they are and what are their strengths and weaknesses.

**4. Calculation** A polymer melt was measured with a capillary rheometer using two capillaries: L/D = 40 mm / 1mm and L/D = 10mm / 1mm. The table below shows the measured pressure drops at different flow rates. Make Bagley and Rabinowitch corrections for the data and calculate the true viscosities and shear rates for the measured data points.

| Q [$mm^3/s$] | Δp [bar] L/D = 40/1 | Δp [bar] L/D =10/1 |
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| 19.64 | 137.0 | 44.3 |
| 49.09 | 200.0 | 65.0 |
| 98.18 | 259.8 | 84.6 |

# MSE.432 RHEOLOGY – FORMULARY

**Viscosity models:**

Power-law: $\eta = K\dot{\gamma}^{n-1}$ $\quad (\tau = K\dot{\gamma}^{n})$

Carreau-Yasuda model: $\eta = \alpha_T \eta_o \left[1 + (\alpha_T \lambda \dot{\gamma})^a\right]^{(n-1)/a}$ $\quad$ (If $T$ = constant $\Rightarrow$ $\alpha_T = 1$)

Arrhenius: $\alpha_T = \exp\left[\frac{E}{R}\left(\frac{1}{T} - \frac{1}{T_o}\right)\right]$; $R = 8.314$ J/(mol·K) $\quad$ WLF: $\log(\alpha_T) = \frac{-C_1(T-T_o)}{C_2 + (T-T_o)}$

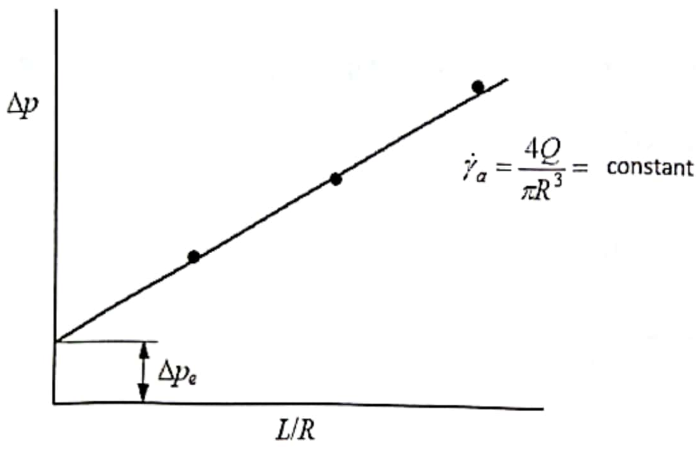
Relationship between WLF constants and reference temperature $T_o$:

$C_1C_2$ = constant, $T_o - C_2$ = constant

**Capillary rheometry:**

$$\tau_w = \frac{\Delta p - \Delta p_e}{2L/R} \qquad \dot{\gamma}_{aw} = \frac{4Q}{\pi R^3} \qquad \eta_a = \frac{\tau_w}{\dot{\gamma}_{aw}} \qquad \dot{\gamma}_w = \dot{\gamma}_{aw}\left(\frac{3n'+1}{4n'}\right) \qquad \eta = \frac{\tau_w}{\dot{\gamma}_w}$$

<u>Bagley correction:</u>




Bagley correction; 2 capillaries: $\Delta p_e = \frac{\Delta p_1 L_2 - \Delta p_2 L_1}{L_2 - L_1}$

Rabinowitsch correction: $n' = \frac{d(\log \tau_w)}{d(\log \dot{\gamma}_{aw})}$

Schümmerin approximation: $\eta(x^* \dot{\gamma}_{aw}) = \eta_a(\dot{\gamma}_{aw})$ $\quad x^* \approx 0.83$

**Rotation rheometry:**

Cone-plate geometry: $\dot{\gamma} = \frac{\Omega}{\theta}$ $\quad \tau = \frac{3M}{2\pi R^3}$ $\quad \eta = \frac{\tau}{\dot{\gamma}} = \frac{3M\theta}{2\pi R^3 \Omega}$

Cylinder geometry: $\dot{\gamma} = \frac{\Omega R_i}{R_o - R_i}$ $\quad \tau = \frac{M}{2\pi R_i^2 L}$ $\quad \eta = \frac{M(R_o - R_i)}{2\pi R_i^3 L \Omega}$

**Melt index ($MI$) $\rightarrow$ viscosity ($\eta$)**

$$\dot{\gamma} = \frac{1.85}{\rho} MI \qquad \eta = 4850 \cdot \rho \frac{m}{MI} \qquad [m] = \text{kg};\ [\rho] = \text{g/cm}^3;\ [\eta] = \text{Pa·s};\ [MI] = \text{g/10 min}$$

**Theory of viscoleasticity:**

Creep-recovery

$$J(t) = \frac{\gamma(t)}{\tau_o} \qquad J(t)_{stat} = J_s^0 + \frac{t}{\eta_o} \qquad J_s^0 = R_\infty^0$$

Maxwell: $\gamma = \frac{\tau_o}{G} + \frac{\tau_o}{\eta} t$ Kelvin: $\gamma = \frac{\tau_o}{G}\left[1 - \exp(-t/\lambda)\right]$ $\gamma = \frac{\tau_o}{G}\left[\exp(-t/\lambda)\right]$

Stress relaxation

$$G(t) = \frac{\tau(t)}{\gamma_o}$$

Maxwell: $\tau = \tau_o \exp(-t/\lambda) = G\gamma_o \exp(-t/\lambda)$

Oscillatory measurements:

$$\gamma = \gamma_o \sin(\omega t) \qquad \tau = \tau_o \sin(\omega t + \delta) = \gamma_o\left[G' \sin(\omega t) + G'' \cos(\omega t)\right] \qquad t_{cycle} = 1/f;\ \ f = \omega/2\pi$$

$$G' = \frac{\tau_o}{\gamma_o}\cos\delta \qquad G'' = \frac{\tau_o}{\gamma_o}\sin\delta \qquad \tan\delta = \frac{G''}{G'}$$

Plate-plate geometry: $\gamma_o = \frac{\varphi_o R}{h}$ $\tau_o = \frac{2M_o}{\pi R^3}$

$$\eta' = \frac{G''}{\omega} \qquad \eta'' = \frac{G'}{\omega} \qquad |\eta^*| = \sqrt{(\eta')^2 + (\eta'')^2} \qquad \eta'(\omega)\big|_{\omega\to 0} = \eta(\dot{\gamma})\big|_{\dot{\gamma}\to 0}$$

Cox-Merz:

$$\eta(\dot{\gamma}) \approx |\eta^*(\omega)| \qquad \text{kun}\ \dot{\gamma} = \omega$$

Oscillatory measurements: Maxwell model:

$$G' = \frac{G\lambda^2\omega^2}{1 + \omega^2\lambda^2} \qquad G'' = \frac{G\lambda\omega}{1 + \omega^2\lambda^2} \qquad \tan\delta = \frac{G''}{G'} = \frac{1}{\lambda\omega}$$

**Time-temperature superposition, master curves:**

$\alpha_T = t(T)/t(T_o) = \omega(T_o)/\omega(T)$

$G(t)$ vs $t/\alpha_T$ $\qquad$ $J(t)$ vs $t/\alpha_T$

$G'(\omega)$, $G''(\omega)$ vs $\omega\alpha_T$

$\eta'(\omega)/\alpha_T$, $\eta''(\omega)/\alpha_T$, $|\eta^*(\omega)|/\alpha_T$ vs $\omega\alpha_T$

$\eta(\dot{\gamma})/\alpha_T$ vs $\dot{\gamma}\alpha_T$

$\eta_o(T) = \alpha_T\eta_o(T_o)$

MSE.432 Rheology

2. Midterm exam 15.12.2021

Ilari Jönkkäri

The formulary is enclosed, other literature is forbidden. Calculators are permitted.

Answer to all questions

**1. Short questions**

a) What structural property of polymer chains is more visible from extensional than shear viscosity data? How it effects on the shape of the curves?

b) What conclusions may be made about the polymer structure based on the intersection of storage G' and loss moduli G'' and how?

c) What is the linear region when talking about viscoelastic properties? How are you able to make sure that you are in the linear region when doing the measurements?

d) What is the Cox-Merz rule?

**2. Essay.** What is the principle of oscillatory measurements (SAOS)? Name three quantities that you typically determine from the measured data. How the nature of the sample effects on the measured data?

**3. Calculation.** A polymer melt was measured with a rotational rheometer using a cone-plate geometry. The cone angle was 4°, radius of the plate 12.5mm and the measurement temperature 200 °C. Calculate shear rates, stresses and viscosities based on the data in table 1. What would be the lowest and highest shear rate of the master curve drawn to 200 °C, if you would have the same data measured also at 170 and 230°C (material follows Arrhenius equation: E/R=5000K)?

**Table 1.**

| Torque [mNm] | Angular Velocity [rad/s] |
|---|---|
| 0.11 | 0.000701 |
| 1.06 | 0.00701 |
| 9.25 | 0.0701 |
| 47.00 | 0.701 |
| 66.30 | 7.02 |

**4. Calculation.** The figure 1 shows results of a creep-recovery test (strain vs. time) for a polymer melt. The test was done in a linear region and the stress was 15 Pa. How are you able to determine if the creep test is done in a linear region? Determine from the figure: the zero viscosity $\eta_0$, the steady-state creep compliance $J_s^0$ and the steady-state recoverable compliance $R_\infty^0$. Evaluate what the strain would be at the end of the creep phase, if the stress was 5 Pa.

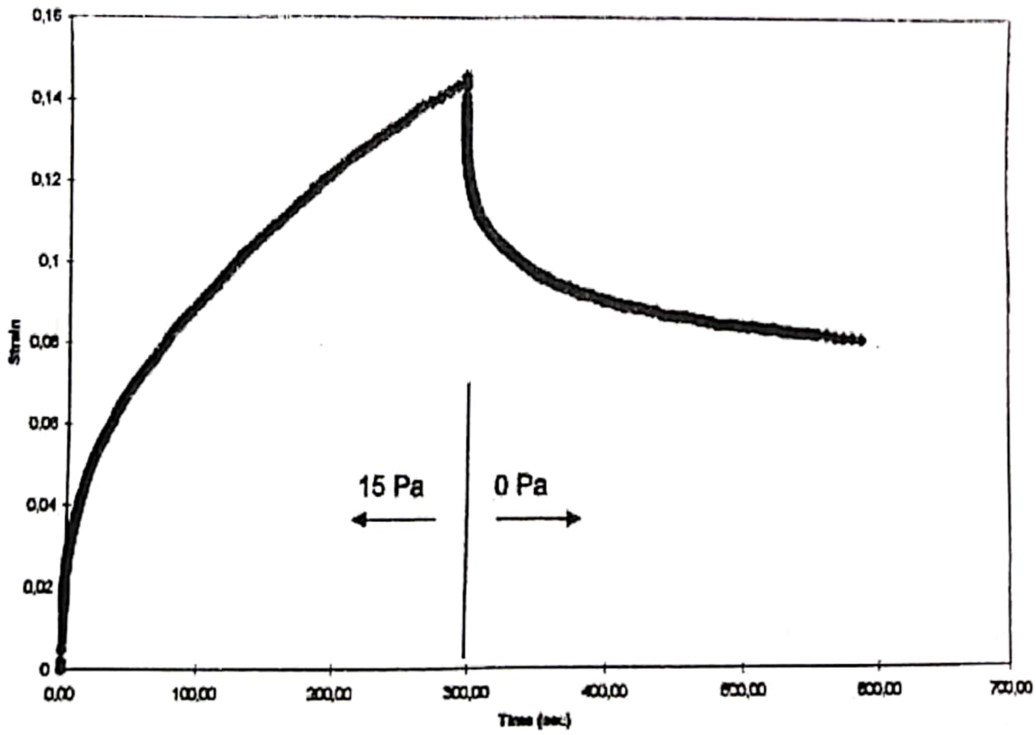



*Figure 1. The creep recovery test*

$\frac{1}{Pa} = \frac{1}{Pa} + \frac{s}{Pa \cdot s}$

# MSE.432 RHEOLOGY – FORMULARY

**Viscosity models:**

Power-law: $\eta = K\dot{\gamma}^{n-1}$ $\quad (\tau = K\dot{\gamma}^{n})$

Carreau-Yasuda model: $\eta = \alpha_T \eta_o \left[1 + (\alpha_T \lambda \dot{\gamma})^a\right]^{(n-1)/a}$ $\quad$ (If $T$ = constant $\Rightarrow \alpha_T = 1$)

Arrhenius: $\alpha_T = \exp\left[\frac{E}{R}\left(\frac{1}{T} - \frac{1}{T_o}\right)\right]$; $R = 8.314$ J/(mol·K) $\quad$ WLF: $\log(\alpha_T) = \frac{-C_1(T - T_o)}{C_2 + (T - T_o)}$

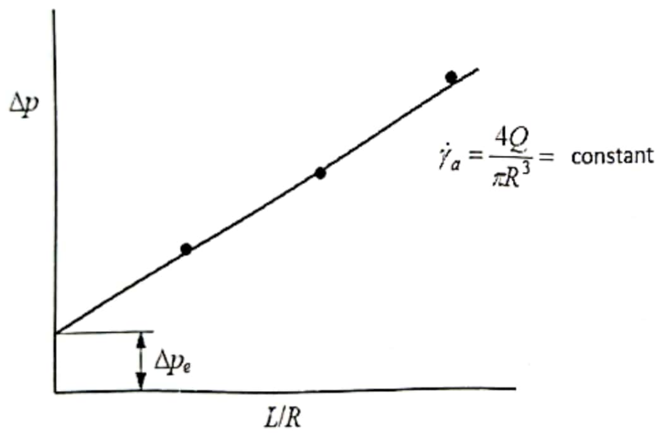
Relationship between WLF constants and reference temperature $T_o$:

$C_1 C_2 =$ constant, $T_o - C_2 =$ constant

**Capillary rheometry:**

$$\tau_w = \frac{\Delta p - \Delta p_e}{2L/R} \qquad \dot{\gamma}_{aw} = \frac{4Q}{\pi R^3} \qquad \eta_a = \frac{\tau_w}{\dot{\gamma}_{aw}} \qquad \dot{\gamma}_w = \dot{\gamma}_{aw}\left(\frac{3n'+1}{4n'}\right) \qquad \eta = \frac{\tau_w}{\dot{\gamma}_w}$$

Bagley correction:




Bagley correction; 2 capillaries: $\Delta p_e = \frac{\Delta p_1 L_2 - \Delta p_2 L_1}{L_2 - L_1}$

Rabinowitsch correction: $n' = \frac{d(\log \tau_w)}{d(\log \dot{\gamma}_{aw})}$

Schümmerin approximation: $\eta(x^* \dot{\gamma}_{aw}) = \eta_a(\dot{\gamma}_{aw})$ $\quad x^* \approx 0.83$

**Rotation rheometry:**

Cone-plate geometry: $\dot{\gamma} = \frac{\Omega}{\theta} \qquad \tau = \frac{3M}{2\pi R^3} \qquad \eta = \frac{\tau}{\dot{\gamma}} = \frac{3M\theta}{2\pi R^3 \Omega}$

Cylinder geometry: $\dot{\gamma} = \frac{\Omega R_i}{R_o - R_i} \qquad \tau = \frac{M}{2\pi R_i^2 L} \qquad \eta = \frac{M(R_o - R_i)}{2\pi R_i^3 L \Omega}$

**Melt index ($MI$) $\rightarrow$ viscosity ($\eta$)**

$$\dot{\gamma} = \frac{1.85}{\rho} MI \qquad \eta = 4850 \cdot \rho \frac{m}{MI} \qquad [m] = \text{kg};\ [\rho] = \text{g/cm}^3;\ [\eta] = \text{Pa·s};\ [MI] = \text{g/10 min}$$

**Theory of viscoleasticity:**

Creep-recovery

$$J(t) = \frac{\gamma(t)}{\tau_o} \qquad J(t)_{stat} = J_s^0 + \frac{t}{\eta_o} \qquad J_s^0 = R_\infty^0$$

Maxwell: $\gamma = \frac{\tau_o}{G} + \frac{\tau_o}{\eta} t$ Kelvin: $\gamma = \frac{\tau_o}{G}[1 - \exp(-t/\lambda)]$ $\gamma = \frac{\tau_o}{G}[\exp(-t/\lambda)]$

Stress relaxation

$$G(t) = \frac{\tau(t)}{\gamma_o}$$

Maxwell: $\tau = \tau_o \exp(-t/\lambda) = G\gamma_o \exp(-t/\lambda)$

Oscillatory measurements:

$$\gamma = \gamma_o \sin(\omega t) \qquad \tau = \tau_o \sin(\omega t + \delta) = \gamma_o [G' \sin(\omega t) + G'' \cos(\omega t)] \qquad t_{cycle} = 1/f;\ \ f = \omega / 2\pi$$

$$G' = \frac{\tau_o}{\gamma_o}\cos\delta \qquad G'' = \frac{\tau_o}{\gamma_o}\sin\delta \qquad \tan\delta = \frac{G''}{G'}$$

Plate-plate geometry: $\gamma_o = \frac{\varphi_o R}{h}$ $\tau_o = \frac{2M_o}{\pi R^3}$

$$\eta' = \frac{G''}{\omega} \qquad \eta'' = \frac{G'}{\omega} \qquad |\eta^*| = \sqrt{(\eta')^2 + (\eta'')^2} \qquad \eta'(\omega)\big|_{\omega \to 0} = \eta(\dot{\gamma})\big|_{\dot{\gamma} \to 0}$$

Cox-Merz:

$$\eta(\dot{\gamma}) \approx |\eta^*(\omega)| \qquad \text{kun } \dot{\gamma} = \omega$$

Oscillatory measurements: Maxwell model:

$$G' = \frac{G\lambda^2\omega^2}{1 + \omega^2\lambda^2} \qquad G'' = \frac{G\lambda\omega}{1 + \omega^2\lambda^2} \qquad \tan\delta = \frac{G''}{G'} = \frac{1}{\lambda\omega}$$

**Time-temperature superposition, master curves:**

$\alpha_T = t(T)/t(T_o) = \omega(T_o)/\omega(T)$

$G(t)$ vs $t/\alpha_T$ $\qquad$ $J(t)$ vs $t/\alpha_T$

$G'(\omega)$, $G''(\omega)$ vs $\omega\alpha_T$

$\eta'(\omega)/\alpha_T$, $\eta''(\omega)/\alpha_T$, $|\eta^*(\omega)|/\alpha_T$ vs $\omega\alpha_T$

$\eta(\dot{\gamma})/\alpha_T$ vs $\dot{\gamma}\alpha_T$

$\eta_o(T) = \alpha_T \eta_o(T_o)$