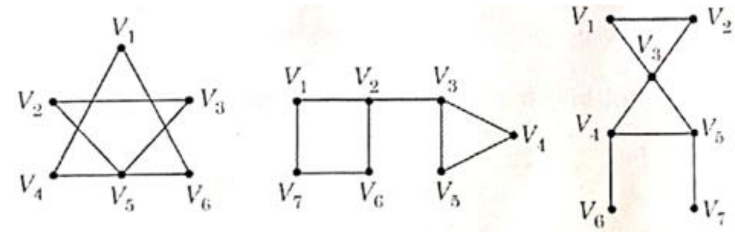
MA23303 - DISCRETE MATHEMATICS

IAT II – PRACTICE PROBLEMS

PART - A

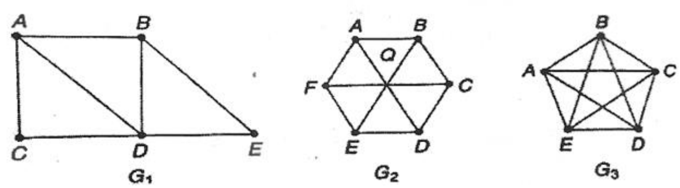
1. Define Connected graph. Test whether the following graph is connected.



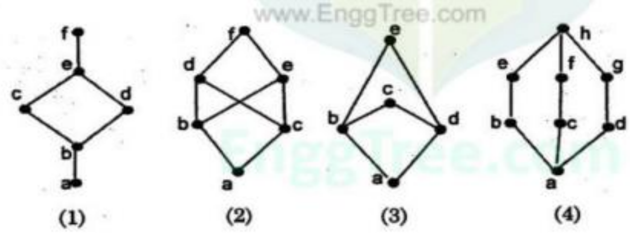
2. Give an example of a graph which is not Eulerian but Hamiltonian.
3. If G is a simple graph with $\delta(G) \geq \frac{|V(G)|}{2}$ then show that G is connected.
4. Let Z be the group of integers with binary operation $*$defined by $a * b = a + b - 2$, for all $a, b \in Z$. Find the identity element of the group $(Z, *)$.
5. Show that every cyclic group is abelian.
6. Let $f : (G, *) \to (G', \Delta)$ be a group homomorphism. Then prove that $[f(a)]^{-1} = f(a^{-1}) = [f(a)]^{-1} = f(a^{-1}), \forall a \in G$.
7. Draw the Hasse Diagram for $(D_{24}, /)$.
8. Prove that $a \vee (a \wedge b) = a$.
9. Prove that a lattice with five elements is not a Boolean algebra.
10. Define Modular lattice with an example.

PART - B

11. Find an Euler path or an Euler circuit, if it exists in each of the following three graphs. If it does not exist, explain why?



12. State and Prove Necessary and Sufficient condition for existence of Euler Graph.
13. Prove that, if G is a simple connected graph with $n \geq 3$ vertices and $\delta(u) \geq \frac{n}{2}$, for all $u \in V(G)$, then G is Hamiltonian graph.
14. Prove that a connected graph has an Euler path if and only if it has exactly two vertices of odd degree.
15. Prove that $(Z_5, +_5)$ is an abelian group.
16. State and Prove Lagrange's theorem.
17. Show that intersection two normal subgroups is a normal subgroup.
18. Prove that every finite group of order n is isomorphic to a permutation group of degree n.
19. Show that $(Z, \oplus, \odot)$ is a commutative ring with identity , where the operation $\oplus$ and $\odot$ are defined, for any $a, b \in Z$ as $a \oplus b = a + b - 1$ and $a \odot b = a + b - ab$.
20. Consider the Lattice D_{105} with the partial ordered relation, "divides" then (i) draw the Hasse diagram of D_{105} (ii) find the complement of each element of D_{105} (iii) find the set of atoms of D_{105} (iv) find the number of subalgebras of D_{105}.
21. Determine whether the following partial ordering sets with the Hasse diagrams are lattices.



22. Prove that Every chain is a distributive Lattice.
23. Prove that the De Morgan's laws are valid in a Boolean Algebra.
24. In a Boolean Algebra, prove the following: (i) $a \leq b$ (ii) $a * b' = 0$ (iii) $a' \oplus b = 1$ (iv) $b' \leq a'$.

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