MATH 111, Course Notes Until Exam 1

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Lecture 1

In this lecture, we just learned the basics of LaTeX so nothing really happened.

Lecture 2: Statements and Hypotheses

1. Statements

   Definition 1. *A statement is a sentence that is either* ***true*** *or* ***false****.*

   Example 1. *The sky will is blue today.*

2. Definition of a Proof and Theorem

   Definition 2. *A* ***proof*** *is an argument that is logically sound. This argument is* ***thorough****,* ***logical****, and* ***complete****.*

   Definition 3. *A* ***theorem*** *is a true statement that has a proof.*

   Theorem structure: If HYPOTHESIS then CONCLUSION.
   This will later be written as: IF $\underline{P} \implies \underline{Q}$

Lecture 3: Implications and Mathematical Writing

This lecture was full of examples for writing mathematical statements into the $P \implies Q$ format.

Example 2. *If f is differentiable at $x = c$ then f is continuous at $x = c$.*

$P = f$ *is differentiable at* $x = c$

$Q = f$ *is continuous at* $x = c$.

Example 3. $x^2 \geq 0$, *for all* $x \in \mathbb{R}$

$P = x \in \mathbb{R}$

$Q = x^2 \geq 0$

Example 4. *If $x \in A \implies x \in B$ and $B \subseteq C$ then $x \in A \implies x \in C$.*

$P = x \in A \implies x \in B$ *and* $B \subseteq C$

$Q = x \in A \implies x \in C$

Lecture 4: AND, OR, Truth Tables

AND and OR Statements: The symbol for AND is $\wedge$ and the symbol for OR is $\vee$.
Truth Tables

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

Lecture 5: IFF and Negation

Definition 4. *If $P \implies Q$ and $Q \implies P$ are equivalent, then we can rewrite the expression as $P \iff Q$*

. This is known as the "If and Only If" expression.

Definition 5. *Reversing an implication is called the **converse**.*

Definition 6. *Given a statement P, the statement saying the exact opposite of P is called the **negation** of P, denoted as $\neg P$*

Lecture 6: DeMorgans Laws

Definition 7. *DeMorgan's Laws are*

1. $\neg(P \wedge Q) = \neg P \vee \neg Q$
2. $\neg(P \vee Q) = \neg P \wedge \neg Q$

Definition 8. *$P \implies Q$ is logically equivalent to $\neg P \vee Q$*

Examples: Negate the following sentences

1. It will be hot OR sunny tomorrow
   $P =$ It will be hot tomorrow
   $Q =$ It will be sunny tomorrow
   $\neg(P \vee Q) = \neg P \wedge \neg Q$
   It will not be hot AND it will not be sunny tomorrow.
2. All math professors are awesome
   This is tricky because it uses a quantifier $\forall$. $\neg\forall \implies \exists$ So the negation of this sentence is "Not all math professors are awesome" or "There exists a math professor that is not awesome".

Lecture 7: Quantifiers

There are three quantifiers that we need to know

1. For all
   The first quantifier to know is the "for all" symbol, denoted as $\forall$. An example of this symbol is
   $$\forall x \in \mathbb{N},\ x^2 + 1 > 0$$
2. There Exists
   The next symbol dictates the existence of some values within a set or collection of objects. This is denoted by the symbol $\exists$. An example of this is
   $$\exists x \in \mathbb{Z},\ \sqrt{x} - 3 < 0$$
3. Unique Existence
   The final quantifier is a more specific version of existence. This one states that there is only one value, a unique value, inside a set or collection. This is denoted as $\exists!$. An example of this symbol is
   $$\exists! n \in \mathbb{N}, \forall x \in \mathbb{R},\ x^n = x$$

Negation of Quantifiers

There are two main takeaways from negating quantifiers:

1. for all

   The negation of for all, $\forall$, is $\neg\forall = \exists$.
   An example would be

$$\neg\forall x \in \mathbb{N} \implies \exists x \in \mathbb{N}$$

2. There exists

   The negation of there exists, $\exists$, is $\neg\exists = \forall$.
   An example of this negation would be

$$\neg(\exists x \in \mathbb{R},\ x^2 - 7x > 0)$$

$$\implies \forall x \in \mathbb{R},\ x^2 - 7x \leq 0$$

A quick aside about inequalities. The negation of a strict inequality is the reverse, and includes said value. In the previous example, we had a strict inequality of being greater than 0, but the negation allowed us to be less than or equal to 0.