Mathematical Statistics Final

Dillon Allen

Problem 1

Number of Ants	Obs Num of Min	Expected Num of Min
0	10	13.66
1	29	27.20
2	25	27.06
3	27	17.95
4	6	8.93
5	3	3.554
6+	0	1.62

We combined the last entry to have 5+, meaning expected[5] = 3.554 + 1.62
Now, we calculate d by following case study 10.4.2, giving us $d = 7.67$. Our degrees of freedom is $df = 6 - 2 = 4$, so our chi-squared value is $\chi^2_{0.95,4} = 9.488$. The criteria to accept the distribution of data as Poisson is

$$d > \chi^2_{0.95,4}$$

Thus we cannot claim that this data follows the Poisson Distribution with $\lambda_{MLE} = mean(ants)$.

Problem 2

Using the formula

$$p_i = \binom{n}{n_i} p_{succ}^{n_i}(1 - p_{succ})^{n-n_i}$$

We can sum for p(A), p(B) and p(C) to get $p_{total} = 0.3482$

Problem 3

The wording of this problem confuses me but I feel like the answer to this, since the events are independent, is

$$P(type\ I) = 1 - P(three\ successes) = 1 - (0.75^3) = .5781$$

Problem 4

$$H_0: \mu_{CM} = \mu_C$$
$$H_A: \mu_{CM} > \mu_C$$

Using the formulas in section 9.2, we get $t = 1.469$

$$t_{\alpha=0.05,df=25} = 1.7081$$

In order to reject the null hypothesis in this case, $t > t_{\alpha=0.05,df=25}$. Since this is not the case, we cannot reject the null hypothesis.

Problem 5

I cannot figure out a better way of testing if these defects are independent from plant other than doing paired t-tests on each column to see if the mean defect counts are equal. If all of them come back equal, then that means there must be some sort of company-wide issue rather than a plant specific one.
Our Hypothesis setup is this:

$$H_0: \mu_i = \mu_j, \text{ j = plant}$$
$$H_a: \mu_i \neq \mu_j$$

Test	p-value	Reject Null?
A-B	0.859	No
A-C	0.03932	Yes
B-C	0.08691	No

Since the paired t-test for A-C has a p-value < 0.05, we can reject the null hypothesis. There is some sort of issue that needs to be investigated between plants A and C, and judging by the tabulated data provided on the final, it seems plant C has found a better way to produce their furniture.

Thus, failure mode is independent of plant.

Problem 6

The hypothesis setup for this problem was the following:

$$H_0: \sigma_B^2 = \sigma_C^2$$
$$H_a: \sigma_B^2 \neq \sigma_C^2$$
$$F_{\frac{0.05}{2},10-1,12-1} = .256$$
$$F_{1-\frac{0.05}{2},10-1,12-1} = 3.47$$

Following the procedure in case study 9.3.1, we calculate the following values:

$$s_B^2 = 0.5166$$
$$s_C^2 = 0.1643$$
$$F = \frac{s_C^2}{s_B^2} = .318$$

Since the calculated F value is in-between both F values from the table, we cannot reject the null hypothesis. Thus, we do not have statistically compelling evidence to conclude that the viscosity variances between plants B and C are different.

Problem 7

Using Theorem 9.4.1, we have the following setup

$$H_0: p_B = p_A$$
$$H_A: p_B > p_A$$
$$\alpha = 0.05$$

Now we have the following values

$$z_\alpha = 1.96$$
$$p_B = \frac{32}{180} = .175$$
$$p_A = \frac{20}{180} = .109$$
$$p_e = \frac{32 + 20}{360} = .1428$$
$$z_{Theorem\ 9.4.1} = 1.797$$

Thus, with $z_\alpha > z$, we cannot reject the null hypothesis. We do not have enough data to conclude that the traffic circle reduces accidents.

Problem 8

Using theorem 7.5.1, we get the following data from scores:

$$n = 15$$
$$s^2 = var(scores) = 20$$
$$\chi^2_{1-\frac{\alpha}{2},df-1} = 26.119$$
$$\chi^2_{\alpha,df-1} = 5.629$$
$$CI = (10.72, 49.74)$$