ADCS Homework 2

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Problem 2

A matrix is given by

```
A = [0.8138 0.4698 -0.3420;
     -0.5630 0.4915 -0.6645;
     -0.1441 0.7333 0.6645]
```

```
A = 3×3
    0.8138    0.4698   -0.3420
   -0.5630    0.4915   -0.6645
   -0.1441    0.7333    0.6645
```

Part A

Show that this matrix satisfies the six conditions for the a DCM (to numerical accuracy).

1. Unit Vector Columns

```
u1 = A(1,:)
```

```
u1 = 1×3
    0.8138    0.4698   -0.3420
```

```
u2 = A(2,:)
```

```
u2 = 1×3
   -0.5630    0.4915   -0.6645
```

```
u3 = A(3,:)
```

```
u3 = 1×3
   -0.1441    0.7333    0.6645
```

To show they are orthogonal, take the dot product between each unit vector

```
u1u2 = dot(u1,u2)
```

```
u1u2 = -3.7000e-06
```

```
u1u3 = dot(u1,u3)
```

```
u1u3 = -2.3240e-05
```

```
u2u3 = dot(u2,u3)
```

```
u2u3 = -1.5000e-05
```

```
u1u1 = dot(u1,u1)
```

```
u1u1 = 0.9999
```

```
u2u2 = dot(u2,u2)
```

```
u2u2 = 1.0001
```

```
u3u3 = dot(u3,u3)
```

```
u3u3 = 1.0001
```

2. Inversion is the same as the transpose

```
Ainv = inv(A)
```

```
Ainv = 3×3
    0.8138   -0.5629   -0.1441
    0.4698    0.4915    0.7333
   -0.3420   -0.6644    0.6644
```

```
Atranspose = transpose(A)
```

```
Atranspose = 3×3
    0.8138   -0.5630   -0.1441
    0.4698    0.4915    0.7333
   -0.3420   -0.6645    0.6645
```

```
abs(Ainv - Atranspose) < 5e-4 * ones(3,3)
```

```
ans = 3×3 logical array
   1   1   1
   1   1   1
   1   1   1
```

Both of these matrices are equal

3. The determinant is +1

```
detA = det(A)
```

```
detA = 1.0001
```

4. The only real eigenvalue is +1

```
[V,D] = eig(A)
```

```
V = 3×3 complex
  -0.7991 + 0.0000i   0.0644 - 0.4202i   0.0644 + 0.4202i
   0.1131 + 0.0000i   0.7026 + 0.0000i   0.7026 + 0.0000i
   0.5905 + 0.0000i  -0.0476 - 0.5687i  -0.0476 + 0.5687i
D = 3×3 complex
   1.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.4849 + 0.8746i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.4849 - 0.8746i
```

Where the only real eigenvalue is

```
D(1,1)
```

```
ans = 1.0000
```

Part B

1. If this represents a 3-2-1 sequence, what are the euler angles, yaw, pitch and roll?

Following the equations from lecture for a 3-2-1 rotation, we get the following

```
theta = -asin(A(1,3))*180/pi
```

```
theta = 19.9988
```

```
psi = atan(A(1,2)/A(1,1))*180/pi
```

```
psi = 29.9975
```

```
phi = atan(A(2,3)/A(3,3))*180/pi
```

```
phi = -45
```

So, the yaw (psi) was approximately 30 degrees, the pitch (theta) was approximately 20 degrees, and the roll (phi) was approximately -45 degrees.

2. If this represents a 3-1-3 DCM for Keplerian orbit elements, what are Ω, i, and f?

Using the equations I dervied in problem 1, the values are

```
Omega = -atan(A(3,1)/A(3,2))*180/pi
```

```
Omega = 11.1175
```

```
i = acos(A(3,3))*180/pi
```

```
i = 48.3560
```

```
f = atan(A(1,3)/A(2,3))*180/pi
```

```
f = 27.2336
```

3. What is the equivalent Quaternion for this DCM?

The formulas I will use are the following:

$$\mathbf{e} = [e_1, e_2, e_3]^T$$

Where $\mathbf{e}$ is the Euler axis of Rotation, the eigenvector that corresponds to $\lambda = 1$.

$$\alpha = \cos^{-1}\left(\frac{tr[A] - 1}{2}\right)$$

$$q_i = e_i \sin\left(\frac{\alpha}{2}\right),\ i = 1, 2, 3$$

$$q_4 = \cos\left(\frac{\alpha}{2}\right)$$

$$\mathbf{q} = [q_1, q_2, q_3]^T$$

$$[\mathbf{Q}] = \begin{pmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{pmatrix}$$

$$[\mathbf{A}(\mathbf{q})] = \left(q_4^2 - \mathbf{q}^T\mathbf{q}\right)\mathbf{1} + 2\mathbf{q}\mathbf{q}^T - 2q_4[\mathbf{Q}]$$

```
e = V(:,1)
```

```
e = 3×1
   -0.7991
    0.1131
    0.5905
```

```
e1 = e(1,1)
```

```
e1 = -0.7991
```

```
e2 = e(2,1)
```

```
e2 = 0.1131
```

```
e3 = e(3,1)
```

```
e3 = 0.5905
```

```

alphaAngle = acos((trace(A) -1)/2)
```

```
alphaAngle = 1.0645
```

```
q1 = e1 * sin(alphaAngle/2)
```

```
q1 = -0.4055
```

```
q2 = e2 * sin(alphaAngle/2)
```

```
q2 = 0.0574
```

```
q3 = e3 * sin(alphaAngle/2)
```

```
q3 = 0.2997
```

```
q4 = cos(alphaAngle/2)
```

```
q4 = 0.8617
```

```

qVec = [q1 q2 q3]'
```

```
qVec = 3×1
   -0.4055
    0.0574
    0.2997
```

```
Qmat = [0 -q3 q2; q3 0 -q1; -q2 q1 0]
```

```
Qmat = 3×3
         0   -0.2997    0.0574
    0.2997         0    0.4055
   -0.0574   -0.4055         0
```

```
A_q = ((q4)^2 - dot(qVec, qVec))*eye(3) + 2*dot(qVec,qVec) - 2*q4*Qmat
```

```
A_q = 3×3
    1.0000    1.0315    0.4162
   -0.0013    1.0000   -0.1838
    0.6140    1.2140    1.0000
```