ADACS Homework 2

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September 2021

1 Problem 1

1.1 A

Problem: As we said, the 3-1-3 rotation is used in relating Kepler elements to the ECI frame, with the following:

- Rotation 1 is Ω about Z
- Rotation 2 is i about X (inclination)
- Rotation 3 is $f = \omega + \nu$ about Z

Answer:
Find the following:

- Derive the DCM for this 3-1-3 rotation (Write the individual rotations and then the full sequence)
- Derive the inverse relations: Expressions for Ω, i, f, given the DCM in the form $\{a_{ij}\}$
- Derive the DCM

  Solution:

  A 3-1-3 Rotation looks like the form

$$[A_{313}] = R_3\left(f\right) R_1\left(i\right) R_3\left(\Omega\right) \tag{1}$$

  Where

$$R_3\left(x\right) = \begin{pmatrix} \cos(x) & \sin(x) & 0 \\ -\sin(x) & \cos(x) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

  Where x can be either Ω or f.

  The X axis rotation is:

$$R_1(i) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(i) & \sin(i) \\ 0 & -\sin(i) & \cos(i) \end{pmatrix}$$

$$[A_{313}] = [A_f][A_i][A_\Omega]$$

$$A_{313} = \begin{pmatrix} \cos(f) & \sin(f) & 0 \\ -\sin(f) & \cos(f) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(i) & \sin(i) \\ 0 & -\sin(i) & \cos(i) \end{pmatrix} \begin{pmatrix} \cos(\Omega) & \sin(\Omega) & 0 \\ -\sin(\Omega) & \cos(\Omega) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A_{313} = \begin{pmatrix} \cos(f) & \sin(f) & 0 \\ -\sin(f) & \cos(f) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\Omega) & \sin(\Omega) & 0 \\ -\cos(i)\sin(\Omega) & \cos(i)\cos(\Omega) & \sin(i) \\ \sin(i)\sin(\Omega) & -\sin(i)\cos(\Omega) & \cos(i) \end{pmatrix}$$

$$A_{313} = \begin{pmatrix} \cos(f)\cos(\Omega) - \sin(f)\cos(i)\sin(\Omega) & \cos(f)\sin(\Omega) + \sin(f)\cos(i)\cos(\Omega) & \sin(f)\sin(i) \\ -\sin(f)\cos(\Omega) - \cos(f)\cos(i)\sin(\Omega) & -\sin(f)\sin(\Omega) + \cos(f)\cos(i)\cos(\Omega) & \cos(f)\sin(i) \\ \sin(i)\sin(\Omega) & -\sin(i)\cos(\Omega) & \cos(i) \end{pmatrix}$$

- Derive the Inverse Relations:

  Solution:

  We must find Ω, i, f, given the DCM that was derived above, using matrix component notation.

  $$a_{31} = \sin(i)\sin(\Omega), \;\; a_{32} = -\sin(i)\cos(\Omega)$$

  $$\Omega = -\arctan\left(\frac{a_{31}}{a_{32}}\right) \tag{2}$$

  i is pretty obvious, the final term, a_{33} is a singular term with just $\cos(i)$.

  $$i = \arccos(a_{33}) \tag{3}$$

  For f, we do more trigonometric ratios like for Ω.

  $$a_{13} = \sin(f)\sin(i), \;\; a_{23} = \cos(f)\sin(i)$$

  $$f = \arctan\left(\frac{a_{13}}{a_{23}}\right) \tag{4}$$