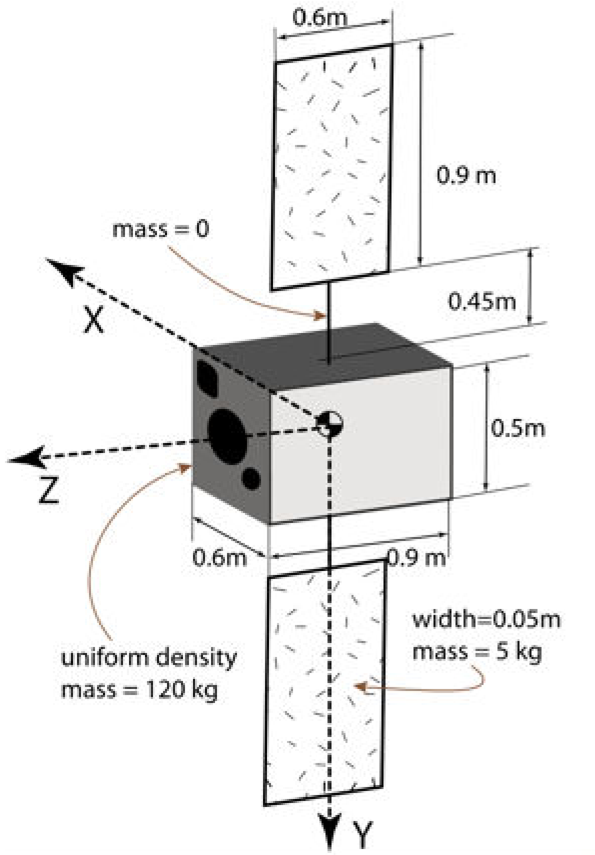
# ADCS Homework 4

## Problem 1




### Assumptions:

1. The thickness of the panels goes into the +x direction.

2. Origin is at (0,0,0) and each vector points to the geometric center of each rectangular prism.

3. Solar Panel 1 is in the +y direction

### (A) Find the Center of Mass Coordinates

```
% Vectors to each geometric center (GC)
origin_to_busGC = [0.5*0.6; 0.5*0.5; 0.5*0.9] % meters
```

```
origin_to_busGC = 3×1
    0.3000
    0.2500
    0.4500
```

```
origin_to_SP1GC = [0.5*0.005; 0.5*0.9 + 0.45; 0.5*0.6] % meters
```

```
origin_to_SP1GC = 3×1
    0.0025
    0.9000
    0.3000
```

```
origin_to_SP2GC = [0.5*0.005; -1*(0.5*0.9 + 0.45); 0.5*0.6] % meters
```

```
origin_to_SP2GC = 3×1
    0.0025
   -0.9000
    0.3000
```

```
busMass = 120 % kg
```

```
busMass = 120
```

```
SPMass = 5 % kg
```

```
SPMass = 5
```

```
totalMass = busMass + 2*SPMass
```

```
totalMass = 130
```

```
CM_Coordinates = (1/totalMass)*(busMass*origin_to_busGC + ...
                    SPMass*origin_to_SP1GC + SPMass*origin_to_SP2GC)
```

```
CM_Coordinates = 3×1
    0.2771
    0.2308
    0.4385
```

### (B) Find the Inertia Tensor relative to the CM

```
% CM to Geometric Centers
r_cmToBus = origin_to_busGC - CM_Coordinates;
r_cmToSP1 = origin_to_SP1GC - CM_Coordinates;
r_cmToSP2 = origin_to_SP2GC - CM_Coordinates;

% Diagonal Tensor Elements
Ixx = (busMass*(r_cmToBus(2)^2 + r_cmToBus(3)^2)) + ...
    (SPMass*(r_cmToSP1(2)^2+r_cmToSP1(3)^2)) + ...
    (SPMass*(r_cmToSP2(2)^2 + r_cmToSP2(3)^2))
```

```
Ixx = 8.8846
```

```
Iyy = (busMass*(r_cmToBus(1)^2 + r_cmToBus(3)^2)) + ...
    (SPMass*(r_cmToSP1(1)^2+r_cmToSP1(3)^2)) + ...
    (SPMass*(r_cmToSP2(1)^2 + r_cmToSP2(3)^2))
```

```
Iyy = 1.0247
```

```
Izz = (busMass*(r_cmToBus(2)^2 + r_cmToBus(1)^2)) + ...
    (SPMass*(r_cmToSP1(2)^2+r_cmToSP1(1)^2)) + ...
    (SPMass*(r_cmToSP2(2)^2 + r_cmToSP2(1)^2))
```

```
Izz = 9.4939
```

```
% Off Diagonals
Ixy = (-1)*((busMass*(r_cmToBus(1)*r_cmToBus(2))) + ...
```

```
    (SPMass*(r_cmToSP1(1)*r_cmToSP1(2))) + ...
    (SPMass*(r_cmToSP2(1)*r_cmToSP2(2))))
```

```
Ixy = -0.6865
```

```
Iyx = Ixy
```

```
Iyx = -0.6865
```

```
Iyz = (-1)*((busMass*(r_cmToBus(2)*r_cmToBus(3))) + ...
    (SPMass*(r_cmToSP1(2)*r_cmToSP1(3))) + ...
    (SPMass*(r_cmToSP2(2)*r_cmToSP2(3))))
```

```
Iyz = -0.3462
```

```
Izy = Iyz
```

```
Izy = -0.3462
```

```
Ixz = (-1)*((busMass*(r_cmToBus(1)*r_cmToBus(3))) + ...
    (SPMass*(r_cmToSP1(1)*r_cmToSP1(3))) + ...
    (SPMass*(r_cmToSP2(1)*r_cmToSP2(3))))
```

```
Ixz = -0.4119
```

```
Izx = Ixz
```

```
Izx = -0.4119
```

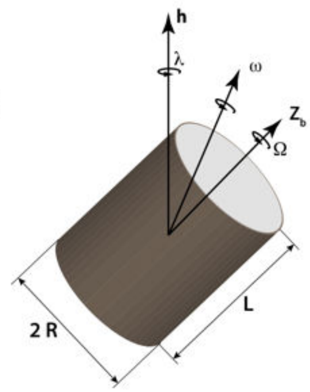
```

I = [Ixx Ixy Ixz; Iyx Iyy Iyz; Izx Izy Izz]
```

```
I = 3×3
    8.8846   -0.6865   -0.4119
   -0.6865    1.0247   -0.3462
   -0.4119   -0.3462    9.4939
```

## Problem 2

- Q2. (20). A spacecraft is a uniform cylinder of radius *R* and length *L*. It spins at a rate of Ω about the Z axis, which is also the axis of the cylinder. The spacecraft has a small damper which slowly seeps energy. What are the ranges of L which would make the spin about Z axis stable?
- Hint: $I_{zz} = \frac{1}{2}MR^2$ and $I_{xx} = I_{yy} = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$




Since we are spinning about the Z-axis, we need the following conditions to hold:

$$I_z > Ix \text{ and } I_z > I_y$$

or

$$I_z < I_x \text{ and } I_z < I_y$$

Since $I_{xx} = I_{yy}$, we have basically two inequalities instead of four.

1. $I_z > I_x$

$$\frac{1}{2}MR^2 > \frac{1}{4}MR^2 + \frac{1}{12}ML^2$$

$$R^2 > \frac{1}{3}L^2$$

$$|R| > \frac{1}{\sqrt{3}}L$$

2. $I_z < I_x$

$$\frac{1}{2}MR^2 < \frac{1}{4}MR^2 + \frac{1}{12}ML^2$$

$$3R^2 < L^2$$

$$\sqrt{3}R < |L|$$

## Problem 3

**The dimensions in the satellite above are $R = 1.0$m, $L = 2.0$m, $M = 150$kg, $\Omega = 5.0$rpm.**

## (a) What is the nutation Frequency?

The formula for the nutation frequency is

$$\lambda = \Omega \frac{I_z - Ix}{I_x}$$

```
R = 1.0 % m
```

```
R = 1
```

```
L = 2.0 % m
```

```
L = 2
```

```
M = 150 % kg
```

```
M = 150
```

```
Omega = 5.0 * (2*pi)*(1/60) % rad/s
```

```
Omega = 0.5236
```

```
Iz = (1/2)*M*R^2
```

```
Iz = 75
```

```
Ix = (1/4)*M*R^2 + (1/12)*M*L^2
```

```
Ix = 87.5000
```

```
Lambda = Omega*(Iz - Ix)/Ix
```

```
Lambda = -0.0748
```

## (b) If the nutation angle is $10^\circ$, what are the angular rates, the total momentum, and the total kinetic energy?

The formulas needed are

$$\tan(\theta) = \frac{I_x \omega_{xy}}{I_z \omega_z}$$

$$\tan(\gamma) = \frac{\omega_{xy}}{\omega_z}$$

$$\tan(\theta) = \frac{I_x}{I_z} \tan(\gamma)$$

$$h^2 = \omega_{xy}^2 I_x^2 + \omega_z^2 I_z^2$$

$$2T = \omega_{xy}^2 I_x + \omega_z^2 I_z$$

```
theta = 10 * (pi/180) % rad
```

```
theta = 0.1745
```

```
gamma = atan(Iz * tan(theta)/Ix)
```

```
gamma = 0.1500
```

If $\omega_z = \Omega$, we can find $\omega_{xy}$

```
omega_xy = Omega*tan(gamma)
```

```
omega_xy = 0.0791
```

```
h = sqrt(omega_xy^2 * Ix^2 + Omega^2*Iz^2)
```

```
h = 39.8757
```

```
T = (1/2)*(omega_xy^2 * Ix + Omega * Iz)
```

```
T = 19.9089
```

### (c) What is the lowest energy level? What is the nutation angle at this point?

If the kinetic energy has setlled into the lowest energy level, then $\dot{T} = 0$. From here, we can see that

$$\cos(\theta)\sin(\theta) = 0$$

$$\sin(2\theta) = 0$$

$$\theta = \frac{\pi}{4}$$

Using the nutation angle formula for $T$,

$$T = \frac{1}{2I_x}\left(h^2 - h^2\cos^2(\theta)\frac{I_z - I_x}{I_z}\right)$$

we get

```
theta_new = pi/4
```

```
theta_new = 0.7854
```

```
T = (1/(2*Ix))*(h^2 - h^2*(cos(theta_new))^2*((Iz - Ix)/Iz))
```

```
T = 9.8433
```