ADCS Homework 5

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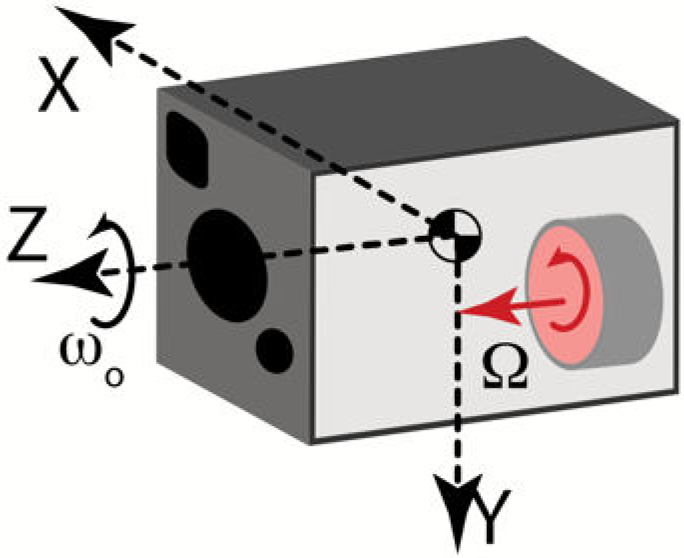

Problem 1

Write the Equations of Motion

The equations of motion are of the form

$$T_{ext} = \dot{H}_b + \omega \times H_b$$

Where

$$H_b = \begin{pmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \widetilde{\omega_z} \end{pmatrix} + \begin{pmatrix} I_w & 0 & 0 \\ 0 & I_w & 0 \\ 0 & 0 & I_w \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \Omega \end{pmatrix}$$

$$\widetilde{\omega_z} = \omega_z + \omega_0$$

Problem 2

Linearize about ω_0

To linearize about ω_0, we will make make the ω_i small such that any products $\omega_i \omega_j \approx 0$

The x-component of this equation of motion is

$$T_x = I_x \dot{\omega}_x + \omega_y (I_z(\omega_z + \omega_0) + I_w \Omega) - (\omega_z + \omega_0) I_y \omega_y$$

$$T_x = I_x\dot{\omega}_x + \omega_y[\omega_0(I_z - I_y) + I_w\Omega] = 0$$

$$\dot{\omega}_x = \omega_y\left(\omega_0\left(\frac{I_y - I_z}{I_x}\right) - \frac{I_w}{I_x}\Omega\right)$$

The y-component is

$$T_y = I_y\dot{\omega}_y + (\omega_z + \omega_0)(I_x\omega_x) - \omega_x(I_z(\omega_z + \omega_0) + I_w\Omega) = 0$$

$$T_y = I_y\dot{\omega}_y + \omega_x\omega_0 I_x - \omega_x\omega_0 I_z - \omega_x I_w\Omega = 0$$

$$\dot{\omega}_y = \omega_x\left(\omega_0\left(\frac{I_z - I_x}{I_y}\right) + \frac{I_w}{I_y}\Omega\right)$$

The z-component is

$$T_z = I_z\dot{\omega}_z + I_z\dot{\omega}_0 + I_w\dot{\Omega} + \omega_x(I_y\omega_y) - \omega_y(I_x\omega_x) = 0$$

$$I_z\dot{\omega}_z = 0$$

Problem 3

Show that the Z - Axis equation reduces to spin rate remaining constant

From the z-component equation for torques, we see that

$$I_z\dot{\omega}_z = 0$$

Which implies that $\omega_z = constant$.

Problem 4

Write the Equations of Motion for ω_x and ω_y, and write the characteristic equation

Taking the time derivative for the x-component

$$\ddot{\omega}_x = \dot{\omega}_y\left(\omega_0\left(\frac{I_y - I_z}{I_x}\right) - \frac{I_w}{I_x}\Omega\right)$$

Substituting $\dot{\omega}_y$, we get

$$\ddot{\omega}_x = \omega_x\left(\omega_0\left(\frac{I_z - I_x}{I_y}\right) + \frac{I_w}{I_y}\Omega\right)\left(\omega_0\left(\frac{I_y - I_z}{I_x}\right) - \frac{I_w}{I_x}\Omega\right)$$

$$\ddot{\omega}_x + \lambda^2\omega_x = 0$$

where

$$\lambda^2 = \left(\omega_0\left(\frac{I_z - I_x}{I_y}\right) + \frac{I_w}{I_y}\Omega\right)\left(\omega_0\left(\frac{I_z - I_y}{I_x}\right) + \frac{I_w}{I_x}\Omega\right)$$

WLOG, it is easy to see that this equation will be the same for $\ddot{\omega}_y$.

Problem 5

What is the characteristic equation?

For both ω_x and ω_y, we have the following equation:

$$\omega_{x,y} = exp(st)$$

$$s^2 + \lambda^2 = 0$$

$$s = \pm\sqrt{-\lambda^2}$$

Problem 6

What are the conditions for stability?

For our system to be stable, we need either both

$$\left(\omega_0\left(\frac{I_z - I_x}{I_y}\right) + \frac{I_w}{I_y}\Omega\right) > 0,\ \left(\omega_0\left(\frac{I_z - I_y}{I_x}\right) + \frac{I_w}{I_x}\Omega\right) > 0$$

or

$$\left(\omega_0\left(\frac{I_z - I_x}{I_y}\right) + \frac{I_w}{I_y}\Omega\right) < 0,\ \left(\omega_0\left(\frac{I_z - I_y}{I_x}\right) + \frac{I_w}{I_x}\Omega\right) < 0$$

Simplifying, our inequalities are:

$$(\omega_0(I_z - I_x) + I_w\Omega) > 0,\ (\omega_0(I_z - I_y) + I_w\Omega) > 0$$

and

$$(\omega_0(I_z - I_x) + I_w\Omega) < 0,\ (\omega_0(I_z - I_y) + I_w\Omega) < 0$$

Problem 7

Consider $I_z > I_x > I_y$. What are the conditions for stability on Ω?

$$(>) : \Omega > \omega_0\left(\frac{I_x - I_z}{I_w}\right),\ \Omega > \omega_0\left(\frac{I_y - I_z}{I_w}\right)$$

$$(<) : \Omega < \omega_0\left(\frac{I_x - I_z}{I_w}\right),\ \Omega < \omega_0\left(\frac{I_y - I_z}{I_w}\right)$$

Note that $\Omega = 0$ is also a valid solution as it returns our major axis inequality. Also note that these quotients are all negative.

Problem 8

Consider when Z is the minor axis spinner, $I_z < I_x < I_y$

The inequalities will be the same as derived in Problem 7 but this time the positive values. At $\Omega = 0$, we can divide by -1 to obtain the minor axis spinner inequality as well.

Problem 9

Consider the intermediate axis spinner, $I_x > I_z > I_y$

In this case, the Ω inequalities with (I_x, I_z) will be positive whereas Ω with (I_y, I_z) will be negative.

Problem 10

$I_x = 300\ \ kgm^2,\ I_y = 400\ \ kgm^2,\ I_z = 350\ \ kgm^2,\ I_w = 10\ \ kgm^2$

$\omega_0 = 60\ \ rpm$

```
% Put Variables into Code
Ix = 300
```

```
Ix = 300
```

```
Iy = 400
```

```
Iy = 400
```

```
Iz = 350
```

```
Iz = 350
```

```
Iw = 10
```

```
Iw = 10
```

```
omega0 = 60 % rpm
```

```
omega0 = 60
```

```

Omega_xz = omega0*((Ix - Iz)/Iw)
```

```
Omega_xz = -300
```

```
Omega_yz = omega0 * ((Iy - Iz)/Iw)
```

```
Omega_yz = 300
```

Ω_{yz} and Ω_{xz} reflect the interia quotients in the inequalities established in Problems 6-9. As we can see, the stability will be for $\Omega < -300$ and $\Omega > 300$.