ADCS Course Project

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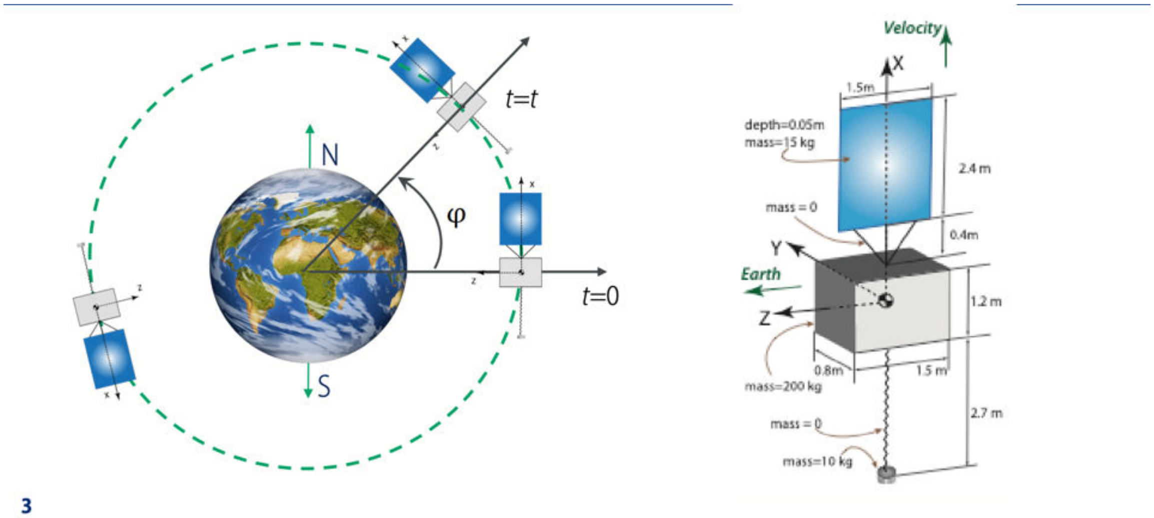
Scenario



750 km sun-synchronous circular orbit. Inclination is 98.4° and RAAN = 90°.

Principle Inertias, CM is geometric center, Sun is facing -Y. +X has the solar panel, instrument on -X. Instrument is considered a point mass.

Problem 1

Calculate the period of the orbit, given 750 km circular.

The formula we will use is:

$$P = 2\pi\sqrt{\frac{a^3}{\mu}}$$

```
mu = 3.986004356E5 % km^3/s^2
```

```
mu = 3.9860e+05
```

```
rE = 6378.14 % km
```

```
rE = 6.3781e+03
```

```
r_orbit = 750 % km
```

```
r_orbit = 750
```

```
a = rE + r_orbit
```

```
a = 7.1281e+03
```

```
P = 2*pi*sqrt(a^3/mu) % seconds
```

```
P = 5.9893e+03
```

```
P_min = P/60 % min
```

```
P_min = 99.8215
```

Problem 2

Calculate the Orbital Rate

The orbital rate, ω_0, is given by

$$\omega_0 = \frac{2\pi}{P}$$

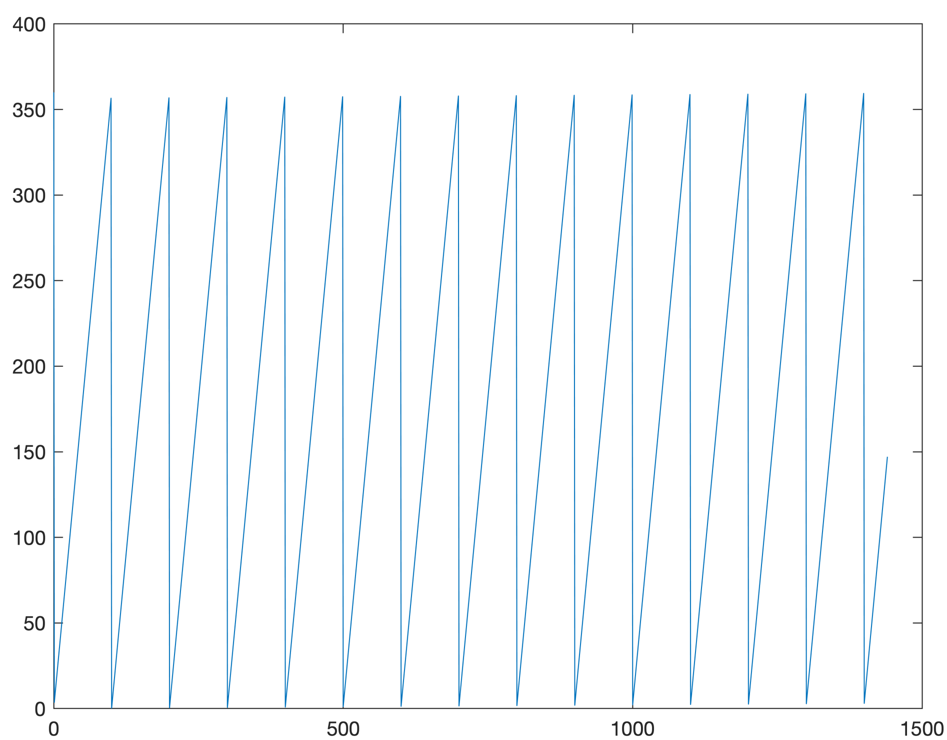
```
orbitalRate = 2 * pi / P % rad/s
```

```
orbitalRate = 0.0010
```

Problem 3-5

Use STK to generate latitude over time, plot

```
M = csvread("argOfLat_vs_time.csv",1,1);
timeCol = 0:1:length(M)-1;
plot(timeCol, M)
```



Problem 6

Show that the disturbance torque due to drag is zero

Consider the figure shown at the beginning of the document. The velocity facing side (X - axis) of the satellite can be considered symmetric due to both components lying in the X axis and the CM being the geometric center. With no protruding parts or anti-symmetric properties in the velocity face, all the drag force cross-products for the torque will cancel out.

Considering the CM as our origin, let

$$\mathbf{r}_{i,+} = (x_i, y_i, z_i)$$

and its symmetric counterpart

$$\mathbf{r}_{i,-} = (-x_i, -y_i, -z_i)$$

The velocity vector is

$$v = -v_0\hat{x}$$

The torque due to drag is

$$N_{\text{aero}} = -\frac{1}{2}C_D\rho\sum_i A_i[(r_{i,+} \times v) + (r_{i,-} \times v)]$$

Note that

$$r_{i,+} \times v = -(r_{i,-} \times v)$$

$$N_{\text{aero}} = 0$$

Problem 7

Calculate the Solar Pressure Disturbance Torque as a function of time for one orbit

$F_e = 1350$ **W/m^2,** $C_a = 0.7$, $C_d = 0.15$, $\theta = 0$, $\hat{S} = \hat{n}$

First, we will calculate the location of the center of mass, and make that our origin.

```
r_cm = [1.2/2, 0.8/2, 1.5/2] % m
```

```
r_cm = 1×3
    0.6000    0.4000    0.7500
```

Next, we will assume the rectangular plane surface of the solar panel is in line with the geometric center, so the only translation is the x coordinate

```
sp_surfaceCM = [2.4/2, 0.8/2, 1.5/2] % m
```

```
sp_surfaceCM = 1×3
    1.2000    0.4000    0.7500
```

```
r_cmToSP = (sp_surfaceCM + [0.4,0,0]) - r_cm % m
```

```
r_cmToSP = 1×3
     1     0     0
```

Now we will input our constants and calculate the solar force and torque

```
% Constants
Fe = 1350; Ca = 0.7; Cd = 0.15; c = 3.00E8; thetaSolar = 0; % m/s^2
Cs = 1 - Ca - Cd
```

```
Cs = 0.1500
```

```
A = 1.5 * 2.4 % m^2
```

```
A = 3.6000
```

The force for the solar pressure is

$$\vec{F}_{solar} = -\frac{F_e}{c} A\cos(\theta)\left[(1 - C_s)\hat{S} + 2\left(C_s\cos(\theta) + \frac{1}{3}C_d\right)\hat{N}\right]$$

```
solarForce = -(Fe/c)*A*cos(thetaSolar)*((1-Cs) + ...
    2*(Cs*cos(thetaSolar) + (1/3)*Cd)) .* [0 -1 0]
```

```
solarForce = 1×3
10^-4 ×
         0    0.2025         0
```

```
solarDisturbanceTorque = cross(r_cmToSP, solarForce)
```

```
solarDisturbanceTorque = 1×3
10^-4 ×
         0         0    0.2025
```

Since the geometry of the orbit is always in the $x - z$ plane, the torque will be constant throughout the orbit over time.

Problem 9

Calculate the disturbance due to residual magnetic torque

For here, we will use the simplified dipole equation

$$B_r = -\frac{2B_0}{R^3}\sin\lambda$$

$$B_\lambda = \frac{B_0}{R^3}\cos\lambda$$

where $B_0 = 31,200$nT.

The torque will be

$$\vec{N}_{mag} = \vec{m} \times \left(B_\lambda \hat{x} + 0\hat{y} + B_r \hat{z}\right)$$

This will vary over time, so $\lambda = \omega_0 t$, where $t \in [0, P]$

```
B0 = 31200E-9; % Teslas
m_dipole = 1; % A-T-m^2
m_dipole_vec = m_dipole .* [0 -1 0]
```

```
m_dipole_vec = 1×3
     0    -1     0
```

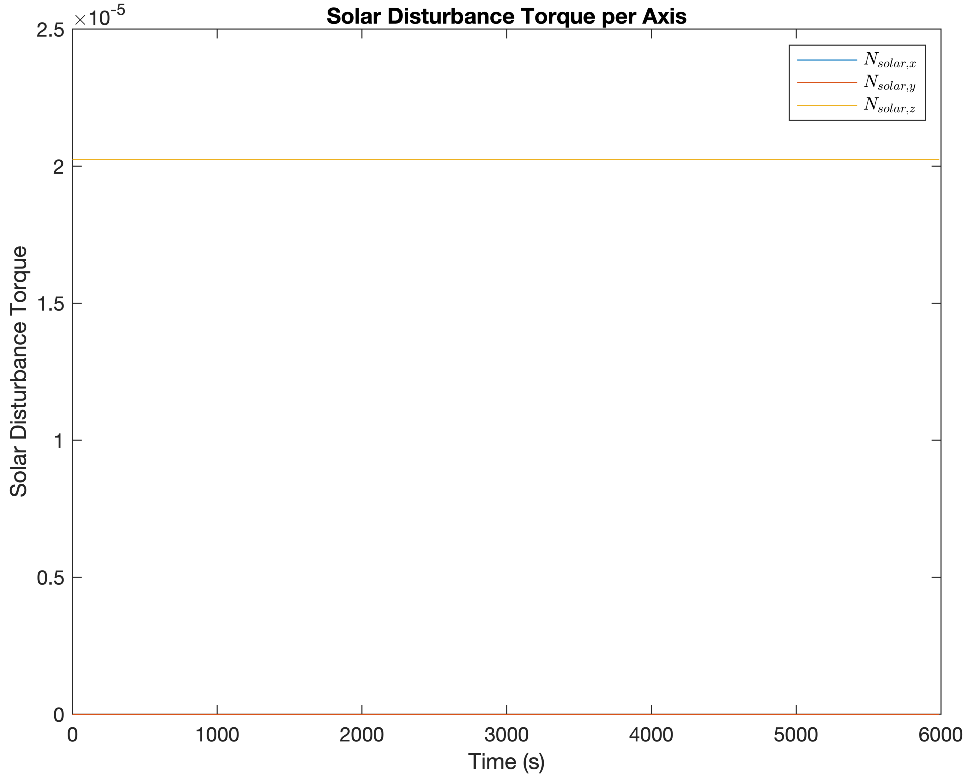
```
for t = 0:1:P
    idx = t+1;
    Bz(idx) = (-2*B0/a^3)*sin(orbitalRate*(t));
    By(idx) = 0;
    Bx(idx) = (B0/a^3)*cos(orbitalRate*(t));
    MagTorque(idx,:) = cross(m_dipole_vec, [Bx(idx), By(idx), Bz(idx)]);
end
```

Problem 10

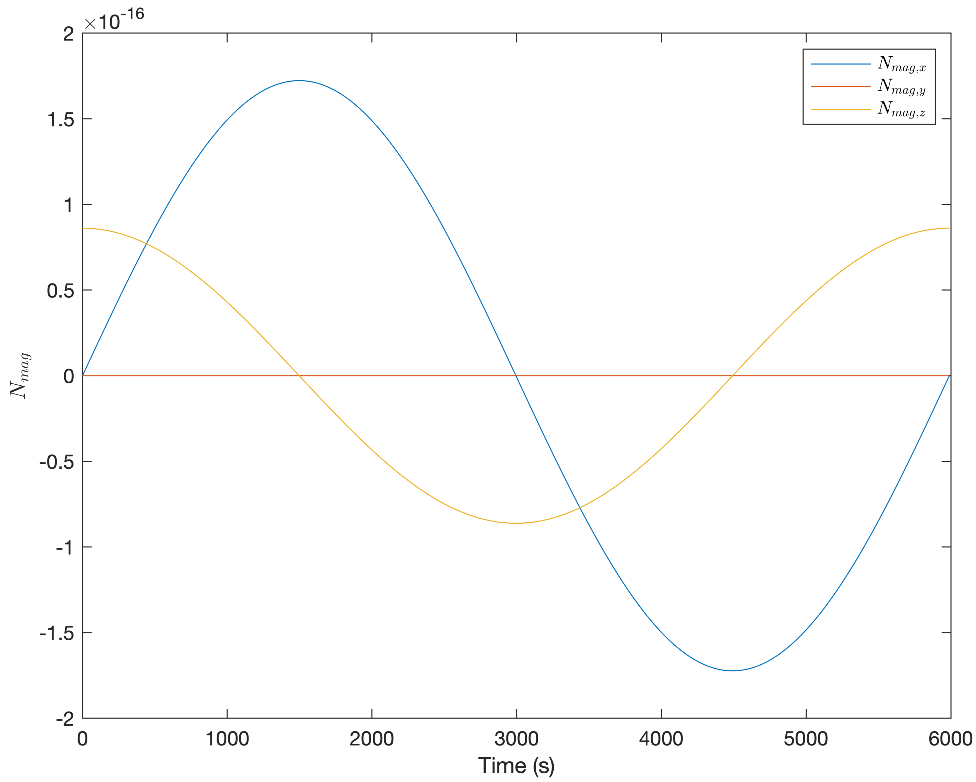
Plot the two disturbance torques and the total over the period of one orbit

```
% time vector
timeVec = 0:1:P;

% Solar Disturbance Torque
solarX = solarDisturbanceTorque(1)*ones(1,length(SolartimeVec));
solarY = solarDisturbanceTorque(2)*ones(1,length(SolartimeVec));
solarZ = solarDisturbanceTorque(3)*ones(1,length(SolartimeVec));
figure('Name', 'Solar Disturbance Torque')
plot(timeVec, solarX)
hold on
plot(timeVec, solarY)
plot(timeVec, solarZ)
xlabel("Time (s)")
ylabel("Solar Disturbance Torque")
title("Solar Disturbance Torque per Axis")
legend("$N_{solar, x}$", "$N_{solar,y}$", "$N_{solar,z}$", 'Interpreter', 'Latex')
```

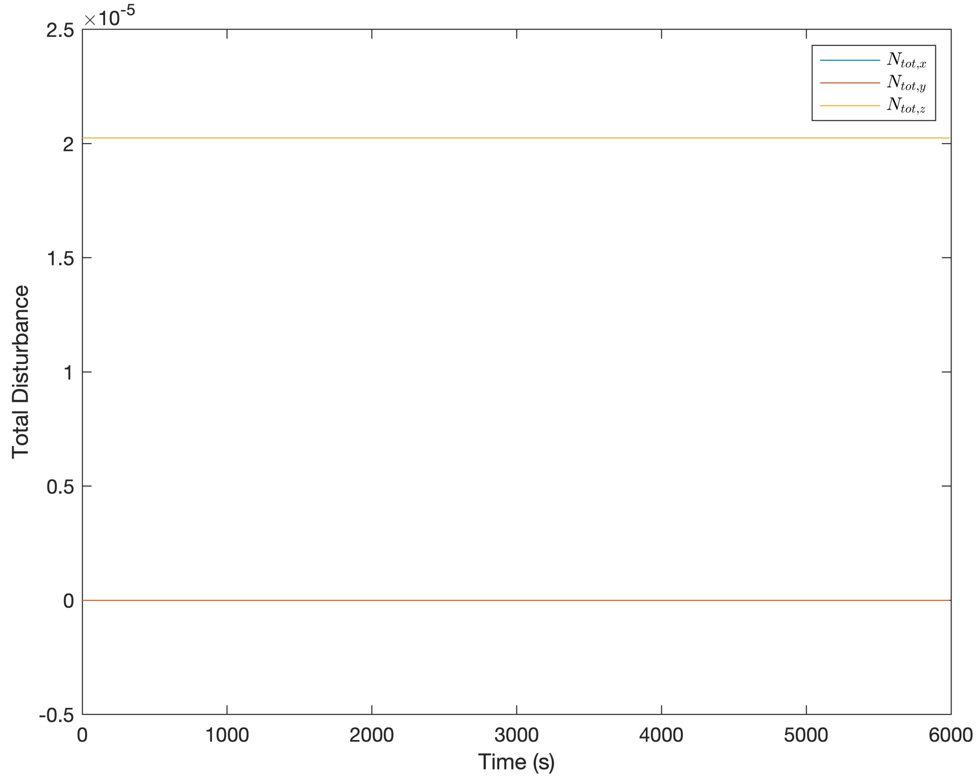



```
% Magnetic Disturbance Torque
magX = MagTorque(:,1);
magY = MagTorque(:,2);
magZ = MagTorque(:,3);
figure('Name', 'Magnetic Disturbance Torque')
plot(timeVec, magX)
hold on
plot(timeVec, magY)
plot(timeVec, magZ)
xlabel("Time (s)")
ylabel("$N_{mag}$", 'Interpreter', 'latex');
legend('$N_{mag,x}$', '$N_{mag,y}$', '$N_{mag,z}$', 'Interpreter', 'latex')
```




```
% total disturbance
xDisturbance = solarX + magX';
yDisturbance = solarY + magY';
zDisturbance = solarZ + magZ';

figure('Name', 'Total Disturbance Torque');
plot(timeVec, xDisturbance)
hold on
plot(timeVec, yDisturbance)
plot(zDisturbance)
xlabel("Time (s)")
ylabel("Total Disturbance")
legend("$N_{tot,x}$", "$N_{tot,y}$", "$N_{tot,z}$", 'Interpreter', 'latex')
```



Problem 11

What is the peak momentum over one orbit along the three axes?

For this, we can use the max value function for each disturbance. The $\Delta t = 1$ second, so the momentum is $N\Delta t = N \times 1$.

```
peakX = max(xDisturbance)
```

```
peakX = 1.7229e-16
```

```
peakY = max(yDisturbance)
```

```
peakY = 0
```

```
peakZ = max(zDisturbance)
```

```
peakZ = 2.0250e-05
```

Problem 13

What is the momentum change over the orbit along the three axes?

we can separate the first order differential equation to find momentum, i.e

$$\int_{p_0}^{p_f} dp = \int_{t_0}^{t_f} N \ \ dt$$

$$\Delta p = \int_{t_0}^{t_f} N \ dt$$

```
dp_x = trapz(xDisturbance)
```

```
dp_x = 7.5808e-21
```

```
dp_y = trapz(yDisturbance)
```

```
dp_y = 0
```

```
dp_z = trapz(zDisturbance)
```

```
dp_z = 0.1213
```