$$10x_1 + 2x_2 - x_3 = 27$$
$$-3x_1 - 6x_2 + 2x_3 = -61.5$$
$$x_1 + x_2 + 5x_3 = -21.5$$

$$A = \begin{bmatrix} 10 & 2 & -1 \\ -3 & -6 & 2 \\ 1 & 1 & 5 \end{bmatrix}$$

$$R_2 + 3/10 R_1 \rightarrow \begin{bmatrix} 10 & 2 & -1 \\ 0 & -27/5 & 17/10 \\ 1 & 1 & 5 \end{bmatrix}$$

$$R_3 - \frac{1}{10} R_1 \rightarrow \begin{bmatrix} 10 & 2 & -1 \\ 0 & -27/5 & 17/10 \\ 0 & 4/5 & 51/10 \end{bmatrix}$$

$$R_3 + \frac{4}{27} R_2 \rightarrow \begin{bmatrix} 10 & 2 & -1 \\ 0 & -27/5 & 17/10 \\ 0 & 0 & 289/54 \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -3/10 & 1 & 0 \\ 1/10 & -4/27 & 1 \end{bmatrix}$$

solve $L\vec{c}_1 = \hat{i}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ -3/10 & 1 & 0 & 0 \\ 1/10 & -4/27 & 1 & 0 \end{array}\right]$$

$u = 1$

$v = 3/10$

$w = -[1/10 - (4/27 \cdot 3/10)] = -1/18$

$$\vec{c} = \begin{bmatrix} 1 \\ 3/10 \\ -1/18 \end{bmatrix}$$

solve $U\vec{x}_1 = \vec{c}$

$$\left[\begin{array}{ccc|c} 10 & 2 & -1 & 1 \\ 0 & -27/5 & 17/10 & 3/10 \\ 0 & 0 & 289/54 & -1/18 \end{array}\right]$$

$$\frac{289}{54} x_3 = -1/18 = \frac{-3}{289}$$

$$-27/5\, x_2 + \frac{17}{10}\left(\frac{-3}{289}\right) = 3/10$$

$$-\frac{27}{5} x_2 = \frac{27}{85} \rightarrow x_2 = -\frac{1}{17}$$

$$10 x_1 - \frac{2}{17} + \frac{3}{289} = 1 \rightarrow x_1 = \frac{32}{289}$$

$$\vec{x}_1 = \left\langle \frac{32}{289}, -\frac{1}{17}, -\frac{3}{289} \right\rangle$$

solve $L\vec{c}_2 = \hat{j}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ -3/10 & 1 & 0 & 1 \\ 1/10 & -4/17 & 1 & 0 \end{array}\right]$$

$u = 0$
$v = 1$
$w = 4/17$

solve $U\vec{x}_2 = c_2$

$$\left[\begin{array}{ccc|c} 10 & 2 & -1 & 0 \\ 0 & -27/5 & 17/10 & 1 \\ 0 & 0 & 289/54 & 4/17 \end{array}\right]$$

$$289/54\, x_3 = 4/17$$
$$x_3 = 8/289$$

$$-27/5\, x_2 + 17/10(8/289) = 1$$
$$-27/5\, x_2 = 4/85$$
$$x_2 = -3/17$$

$$10x_1 + 2(-3/17) - (8/289) = 0$$
$$10x_1 = 110/289$$
$$x_1 = 11/289$$

$$\vec{x}_2 = \left(\frac{11}{289}, -3/17, 8/289\right)$$

solve $L\vec{c}_3 = \hat{j}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ -3/10 & 1 & 0 & 0 \\ 1/10 & -47/27 & 1 & 1 \end{array}\right]$$

$x_1 = 0$
$x_2 = 0$
$x_3 = 1$

solve $U\vec{x} = \vec{c}_3$

$$\left[\begin{array}{ccc|c} 10 & 2 & -1 & 0 \\ 0 & -27/5 & 17/10 & 0 \\ 0 & 0 & 289/54 & 1 \end{array}\right]$$

$$x_3 = 54/289$$

$$-\frac{27}{5}x_2 + \frac{17}{10}\left(\frac{54}{289}\right) = 0$$

$$x_2 = \frac{1}{17}$$

$$10x_1 + \frac{2}{17} - \frac{54}{289} = 0$$

$$10x_1 = 20/289$$

$$x_1 = \frac{2}{289}$$

$$\vec{x}_3 = \left\langle \frac{2}{289}, \frac{1}{17}, \frac{54}{289} \right\rangle$$

$$A^{-1} = \begin{bmatrix} \frac{32}{289} & \frac{11}{289} & \frac{2}{289} \\ -\frac{1}{17} & -\frac{3}{17} & \frac{1}{17} \\ -\frac{3}{289} & \frac{8}{289} & \frac{54}{289} \end{bmatrix}$$