

$$\begin{aligned} -2x_1 - 6x_2 - x_3 &= -38 \\ -3x_1 - 1x_2 + 7x_3 &= -34 \\ -8x_1 + 1x_2 - 2x_3 &= -40 \end{aligned}$$

$$A = \begin{bmatrix} 2 & -6 & -1 \\ -3 & -1 & 7 \\ -8 & 1 & -2 \end{bmatrix}$$

$$R_2 + 3/2 R_1 \rightarrow \begin{bmatrix} 2 & -6 & -1 \\ 0 & -10 & 11/2 \\ -8 & 1 & -2 \end{bmatrix}$$

$$R_3 + 4 R_1 \rightarrow \begin{bmatrix} 2 & -6 & -1 \\ 0 & -10 & 11/2 \\ 0 & -23 & -6 \end{bmatrix}$$

$$R_3 \leftarrow 23/10 R_2 \rightarrow \begin{bmatrix} 2 & -6 & -1 \\ 0 & -10 & 11/2 \\ 0 & 0 & -373/20 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & -6 & -1 \\ 0 & -10 & 11/2 \\ 0 & 0 & -373/20 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -3/2 & 1 & 0 \\ -4 & 23/10 & 1 \end{bmatrix}$$

solve $L \vec{c} = \vec{b}$

$$\begin{bmatrix} 1 & 0 & 0 \\ -3/2 & 1 & 0 \\ -4 & 23/10 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} -38 \\ -34 \\ -40 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -38 \\ -3/2 & 1 & 0 & -34 \\ -4 & 23/10 & 1 & -40 \end{array} \right]$$

$$R_3 \leftrightarrow R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -38 \\ -4 & 23/10 & 1 & -40 \\ -3/2 & 1 & 0 & -34 \end{array} \right]$$

$$R_2 + 4R_1 \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -38 \\ 0 & 23/10 & 1 & -192 \\ -3/2 & 1 & 0 & -34 \end{array} \right]$$

$$R_2 \leftrightarrow R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -38 \\ -3/2 & 1 & 0 & -34 \\ 0 & 23/10 & 1 & -192 \end{array} \right]$$

$$R_2 + \frac{3}{2}R_1 \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -38 \\ 0 & 1 & 0 & -91 \\ 0 & 23/10 & 1 & -192 \end{array} \right]$$

$$R_3 - \frac{23}{10}R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -38 \\ 0 & 1 & 0 & -91 \\ 0 & 0 & 1 & 173/10 \end{array} \right]$$

$$\vec{c} = \begin{bmatrix} -38 \\ -91 \\ 173/10 \end{bmatrix}$$

$$\text{solve } U \vec{x} = \vec{c}$$

$$\begin{bmatrix} 2 & -6 & -1 \\ 0 & -10 & 11/2 \\ 0 & 0 & -373/20 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -38 \\ -91 \\ 173/10 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 2 & -6 & -1 & -38 \\ 0 & -10 & 11/2 & -91 \\ 0 & 0 & -373/20 & 173/10 \end{array} \right]$$

$$\begin{aligned} -373/20 z &= 173/10 \\ z &= -346/373 \end{aligned}$$

$$\begin{aligned} -10y + 11/2(-346/373) &= -91 \\ -10y &= -32040/373 \\ y &= 3204/373 \end{aligned}$$

$$2x - 6(3204/373) - (-346/373) = -38$$

$$\begin{aligned} 2x &= 4704/373 \\ x &= \frac{2352}{373} \end{aligned}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2352/373 \\ 3204/373 \\ -346/373 \end{bmatrix}$$