Homework Assignment 2

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I PROBLEM 1

A.

We can relate the energy of the system with the semi-major axis a using the following formula (since energy is conserved for the orbit):

$$E = \frac{1}{2}v^2 - \frac{1}{r} = -\frac{1}{2a}$$

Hence, we can find a with $\mathbf{v}$ and $\mathbf{r}$ as follows:

$$a = \frac{2}{r} - v^2 \tag{1}$$

Using the given vectors $\mathbf{v}$ and $\mathbf{r}$, we find that v^2 and r are the following:

$$v^2 = 0^2 + \left(\frac{1}{10}\right)^2 = \frac{1}{100} \tag{2}$$

$$r = \sqrt{10^2 + 0^2} = 10 \tag{3}$$

Therefore, we get the following value for the semi-major axis a:

$$\therefore \boxed{a = \frac{19}{100}} \tag{4}$$

To find $\mathscr{P}$, we simply find the angular momentum $\mathbf{L}$ and use the relation $\mathscr{P} = L^2$ to find $\mathscr{P}$ (since angular momentum is conserved, we can use the initial conditions).

$$\mathbf{L} = \mathbf{r} \times \mathbf{v} \tag{5}$$

$$= (r_1 v_2 - v_1 r_2)\hat{\mathbf{z}} \tag{6}$$

$$= \left(10 \cdot \frac{1}{10} - 0 \cdot 0\right)\hat{\mathbf{z}} \tag{7}$$

$$\therefore \mathbf{L} = \hat{\mathbf{z}} \tag{8}$$

$\mathscr{P}$ is then:

$$\therefore \boxed{\mathscr{P} = 1} \tag{9}$$

We can then use $\mathscr{P} = L^2$ to find the eccentricity (e) of the orbit as follows:

$$e = \sqrt{1 - \frac{L^2}{GMa}} \tag{10}$$

$$= \tag{11}$$

Finally, we can find the period (in reduced units) using the following a-dependent equation:

$$T^2 = 4\pi^2 a^3 \tag{12}$$

$$\implies T = 2\pi a^{3/2} \tag{13}$$

$$\therefore \boxed{T = 2\pi * \left(\frac{19}{100}\right)^{3/2}} \tag{14}$$

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