COSC122 (2018) Lab 8

Graphs

Goals

This lab will give you practice with graphs; in this lab you will:

- translate between the various ways of representing graphs;
- trace out Breadth First and Depth First searches (BFS and DFS);
- trace out the Single Shortest Path (SSP) for various graphs;
- generate minimum spanning trees using Prims and Kruskals's algorithms;
- have heaps of fun.

Representing Graphs

Draw up the adjaency matrix *and* adjacency list representations of the graphs in Figure 1.

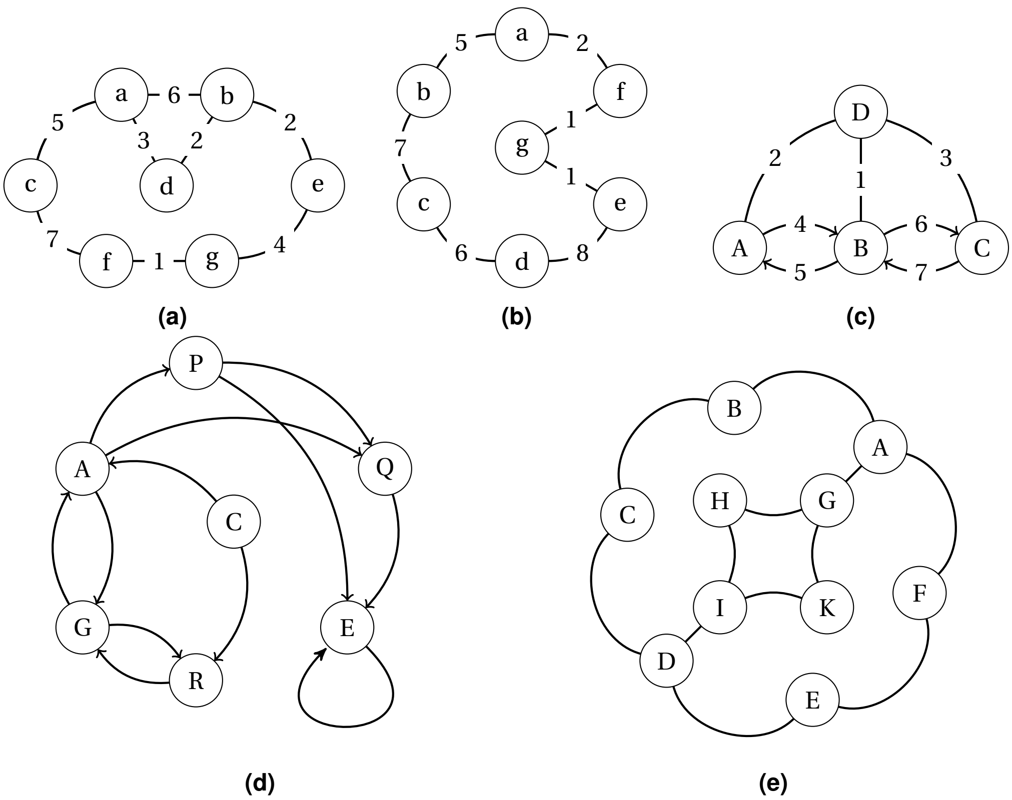
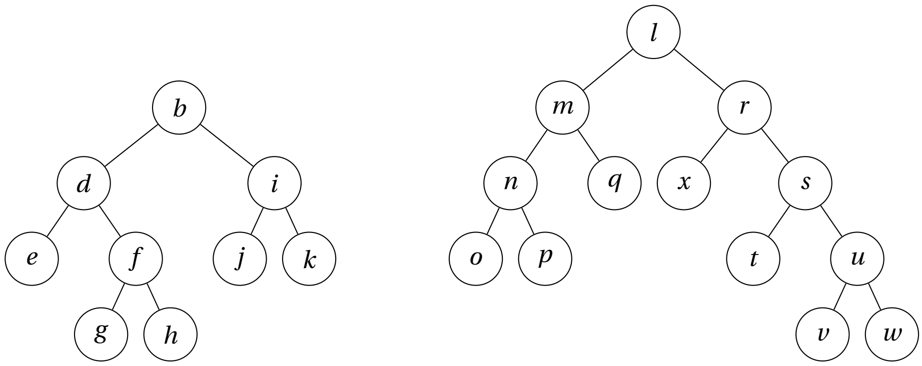


Figure 1: Reprazent!



(a) **(b)**

Figure 2: Tree Town

Searching/Traversing Graphs

Depth First Search

Trace out the state of the Stack and list the order that nodes are visited in when doing the following Depth First Searches on the graphs in Figure 2, starting from *b* and *l* respectively. You should put items onto the stack in alphabetical order.

How does this compare to a pre-order tree traversal?

Now do the following:

- DFS on Figure 2a, starting from *b*.
- DFS on Figure 2b, starting from *l*.
- DFS on Figure 3, starting from *a*.
- DFS on Figure 3, starting from *m*.

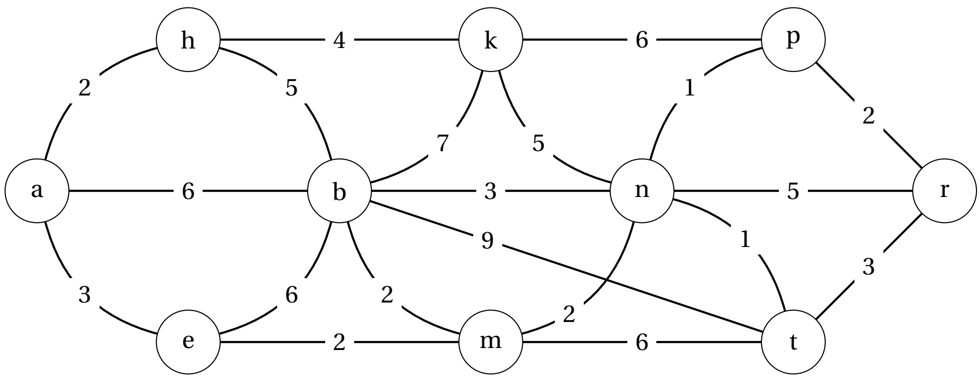


Figure 3: Pointy

Breadth First Search

Breadth first searches use a Queue to keep track of the nodes that are still to be visited. Trace out the state of the Queue and list the order that nodes are visited when doing the following Breadth First Searches.

- BFS on Figure 2b, starting from *l*.
- BFS on Figure 3, starting from *a*.
- BFS on Figure 3, starting from *m*.
- BFS on Figure 5, starting from *m*.

Paths and Cycles

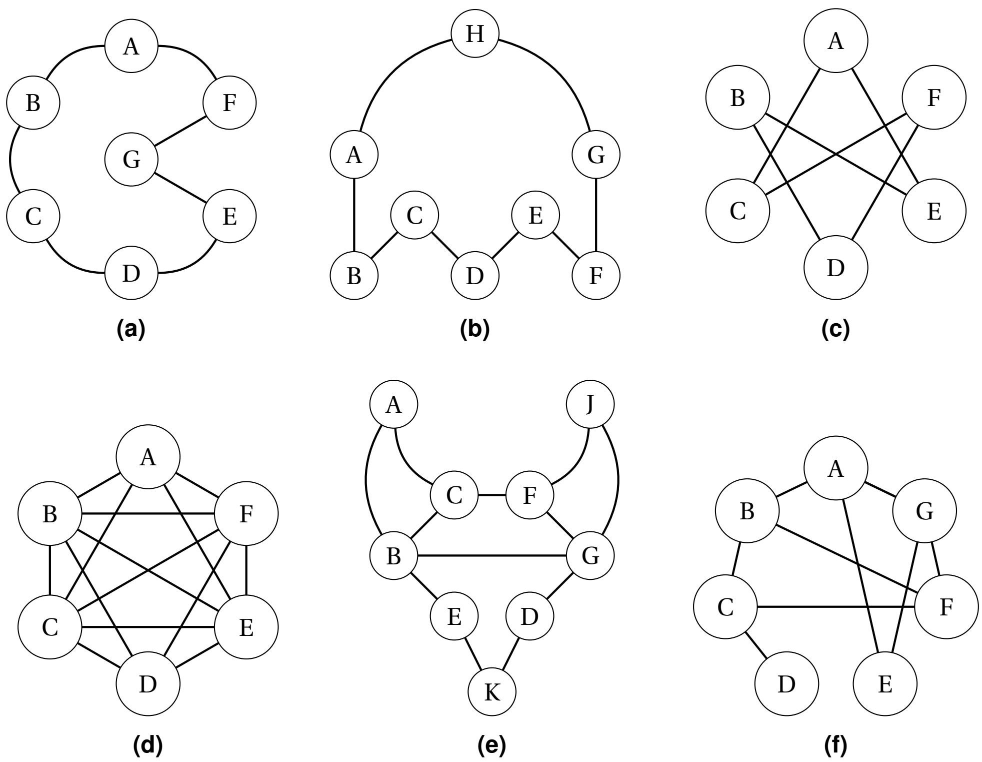


Figure 4: Graph Town

Identify a path from A to D in each of the graphs in Figure 4. Then given an example of a cycle in each of the graphs.

Eulerian Paths

Eulerian walk/path

This is a path that uses all the edges in a graph only once. For each graph in Figure 4 say whether it contains a Eulerian path and if so, write down a Eulerian path for the graph.

Write down the adjacency matrix for the graph in Figure 3 and use it to determine whether or not there is a Eulerian path in the graph.

Eulerian circuit/tour/cycle

A Eulerian Circuit/Tour is a Eulerian walk/path *and* starting and ending at the same node. Basically it's a cycle that is a Eulerian path.

For each graph in Figure 4 say whether it contains a Eulerian cycle and if so, give the cycle.

Write down the adjacency matrix for the graph in Figure 3 and use it to determine whether or not there is a Eulerian circuit in the graph.

Hamiltonian Paths

Hamiltonian walk/path

This is a path that visits all the nodes in a graph only once. For each graph in Figure 4 say whether it contains a Hamiltonian path and if so, write down a Hamiltonian path for the graph.

Hamiltonian circuit/tour/cycle

For each graph in Figure 4 say whether it contains a Hamiltonian cycle and if so, give the cycle.

Shortest Path between two nodes

Complete the following tasks for the graphs in Figure 3:

- Map out the shortest path from node *a* to node *r*.
- Starting from node *a* map out the shortest distance to each other node in the graph.
- Draw up a tree showing the shortest distance to all other nodes from node *m*.

Complete the following tasks for the graphs in Figure 5:

- Starting from node *h*, draw out a tree that maps out the shortest distance to each other node in the graph.
- Draw up another tree showing the shortest distance to all other nodes from node *e*.
- What is the shortest distance from *g* to *c* and what path does it follow.

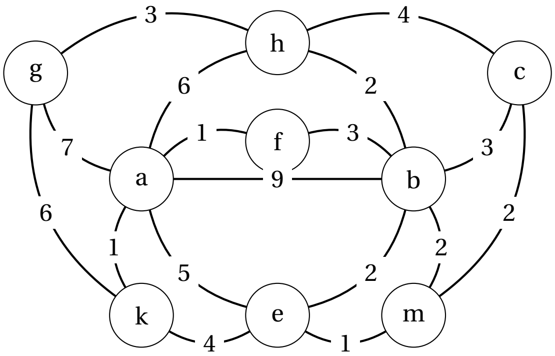


Figure 5: Around we go!

Minimum Spanning Trees

Prim's Algorithm

Do the following for the graphs in Figure 3 and Figure 5

- Use Prim's algorithm to find a minimum spanning tree.
- Find any other minimum spanning trees or say why there aren't any.

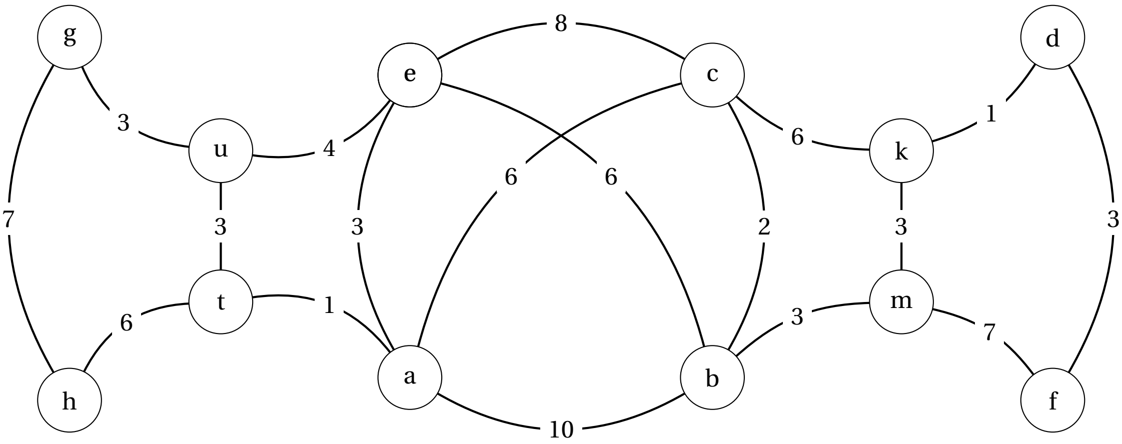


Figure 6: Bat graph

Kruskal's Algoritm

Use Kruskals's algorithm to find a minimum spanning tree for the graphs in Figure 3 and Figure 5. If the tree is different to that found using Prim's algorithm then make sure you confirm that both minimum spanning trees have the same length.

More fun with graphs

For more fun graph exercises have a look through a few old exams!

Extra blanks

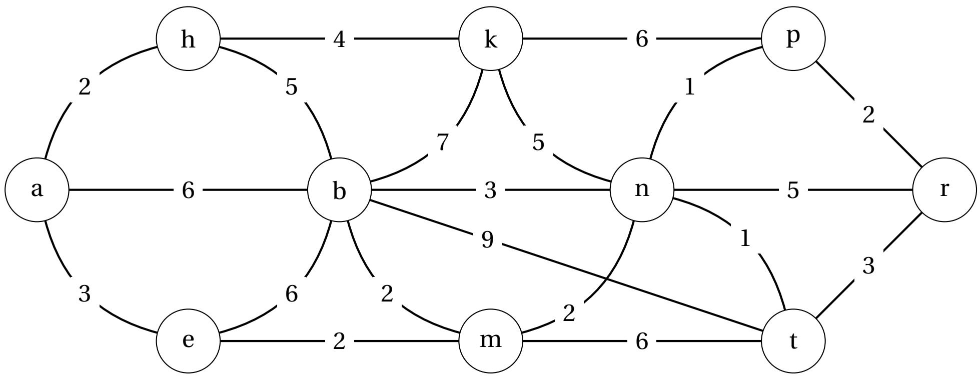


Figure 7: Another point

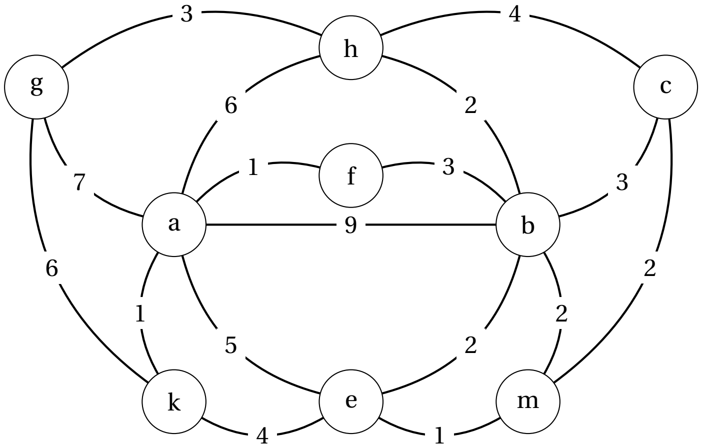


Figure 8: Around we go again

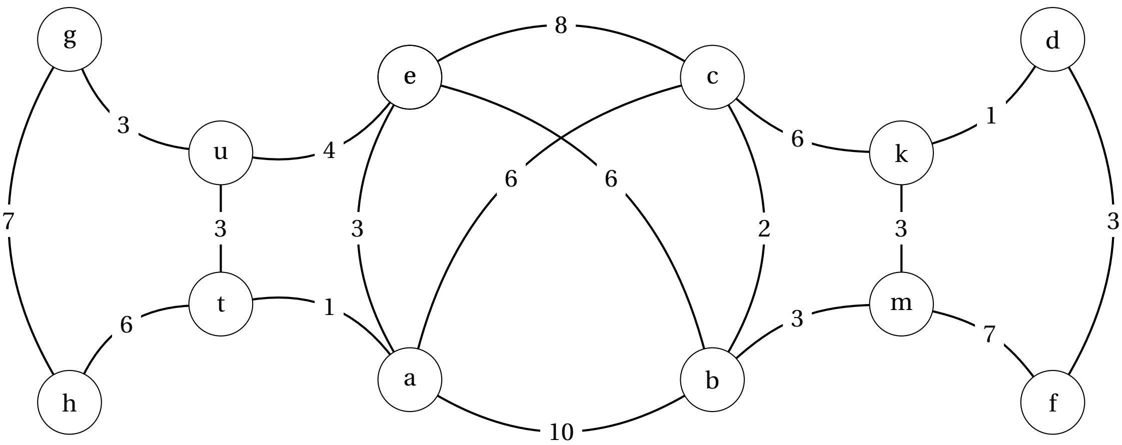


Figure 9: Bat graph again