

Lists

- A list is a finite sequence of elements
- Examples of lists in Prolog:

[mia, vincent, jules, yolanda]

[mia, robber(honeybunny), X, 2, mia]

[]

[mia, [vincent, jules], [butch, friend(butch)]]

[[], dead(z), [2, [b,c]], [], Z, [2, [b,c]]]

Important things about lists

- List elements are enclosed in square brackets
- The length of a list is the number of elements it has
- All sorts of Prolog terms can be elements of a list
- There is a special list:
the empty list `[]`

Head and Tail

- A non-empty list can be thought of as consisting of two parts
 - The head
 - The tail
- The head is the first item in the list
- The tail is everything else
 - The tail is the list that remains when we take the first element away
 - The tail of a list is always a list

Head and Tail example 1

- [mia, vincent, jules, yolanda]

Head:

Tail:

Head and Tail example 1

- [mia, vincent, jules, yolanda]

Head: mia

Tail:

Head and Tail example 1

- [mia, vincent, jules, yolanda]

Head: mia

Tail: [vincent, jules, yolanda]

Head and Tail example 2

- $[[], \text{dead}(z), [2, [b,c]], [], Z, [2, [b,c]]]$

Head:

Tail:

Head and Tail example 2

- `[[], dead(z), [2, [b,c]], [], Z, [2, [b,c]]]`

Head: `[]`

Tail:

Head and Tail example 2

- `[[], dead(z), [2, [b,c]], [], Z, [2, [b,c]]]`

Head: `[]`

Tail: `[dead(z), [2, [b,c]], [], Z, [2, [b,c]]]`

Head and Tail example 3

- [dead(z)]

Head:

Tail:

Head and Tail example 3

- $[\text{dead}(z)]$

Head: $\text{dead}(z)$

Tail:

Head and Tail example 3

- [dead(z)]

Head: dead(z)

Tail: []

Head and tail of empty list

- The empty list has neither a head nor a tail
- For Prolog, `[]` is a special simple list without any internal structure
- The empty list plays an important role in recursive predicates for list processing in Prolog

The built-in operator |

- Prolog has a special built-in operator | which can be used to decompose a list into its head and tail
- The | operator is a key tool for writing Prolog list manipulation predicates

The built-in operator |

?- [Head|Tail] = [mia, vincent, jules, yolanda].

Head = mia

Tail = [vincent,jules,yolanda]

yes

?-

The built-in operator |

?- [X|Y] = [mia, vincent, jules, yolanda].

X = mia

Y = [vincent,jules,yolanda]

yes

?-

The built-in operator |

?- [X|Y] = [].

no

?-

The built-in operator |

?- [X,Y|Tail] = [[], dead(z), [2, [b,c]], [], Z, [2, [b,c]]] .

X = []

Y = dead(z)

Tail = [[2, [b,c]], [], Z, [2, [b,c]]]

yes

?-

Anonymous variable

- Suppose we are interested in the second and fourth element of a list

```
?- [X1,X2,X3,X4|Tail] = [mia, vincent, marsellus, jody, yolanda].
```

```
X1 = mia
```

```
X2 = vincent
```

```
X3 = marsellus
```

```
X4 = jody
```

```
Tail = [yolanda]
```

```
yes
```

```
?-
```

Anonymous variables

- There is a simpler way of obtaining only the information we want:

```
?- [ _,X2, _,X4|_ ] = [mia, vincent, marsellus, jody, yolanda].
```

```
X2 = vincent
```

```
X4 = jody
```

```
yes
```

```
?-
```

- The underscore is an anonymous variable

The anonymous variable

- Is used when you need to use a variable, but you are not interested in what Prolog instantiates it to
- Each occurrence of the anonymous variable is independent, i.e. can be bound to something different

Member

- One of the most basic things we would like to know is whether something is an element of a list or not
- So let's write a predicate that when given a term X and a list L , tells us whether or not X belongs to L
- This predicate is usually called `member/2`

member/2

```
member(X,[X|T]).
```

```
member(X,[H|T]):- member(X,T).
```

?-

member/2

```
member(X,[X|T]).
```

```
member(X,[H|T]):- member(X,T).
```

```
?- member(yolanda,[yolanda,trudy,vincent,jules]).
```


member/2

```
member(X,[X|T]).  
member(X,[H|T]):- member(X,T).
```

```
?- member(yolanda,[yolanda,trudy,vincent,jules]).  
yes  
?-
```

member/2

```
member(X,[X|T]).
```

```
member(X,[H|T]):- member(X,T).
```

```
?- member(vincent,[yolanda,trudy,vincent,jules]).
```

member/2

```
member(X,[X|T]).  
member(X,[H|T]):- member(X,T).
```

```
?- member(vincent,[yolanda,trudy,vincent,jules]).  
yes  
?-
```

member/2

```
member(X,[X|T]).
```

```
member(X,[H|T]):- member(X,T).
```

```
?- member(zed,[yolanda,trudy,vincent,jules]).
```

member/2

```
member(X,[X|T]).
```

```
member(X,[H|T]):- member(X,T).
```

```
?- member(zed,[yolanda,trudy,vincent,jules]).
```

```
no
```

```
?-
```

member/2

```
member(X,[X|T]).
```

```
member(X,[H|T]):- member(X,T).
```

```
?- member(X,[yolanda,trudy,vincent,jules]).
```

member/2

```
member(X,[X|T]).
```

```
member(X,[H|T]):- member(X,T).
```

```
?- member(X,[yolanda,trudy,vincent,jules]).
```

```
X = yolanda;
```

```
X = trudy;
```

```
X = vincent;
```

```
X = jules;
```

```
no
```

Rewriting member/2

```
member(X,[X|_]).
```

```
member(X,[_|T]):- member(X,T).
```


Recurring down lists

- The member/2 predicate works by recursively working its way down a list
 - doing something to the head, and then
 - recursively doing the same thing to the tail
- This technique is very common in Prolog and therefore very important that you master it
- So let`s look at another example!

Example: a2b/2

- The predicate a2b/2 takes two lists as arguments and succeeds
 - if the first argument is a list of as, and
 - the second argument is a list of bs of exactly the same length

?- a2b([a,a,a,a],[b,b,b,b]).

yes

?- a2b([a,a,a,a],[b,b,b]).

no

?- a2b([a,c,a,a],[b,b,b,t]).

no

Defining a2b/2: step 1

```
a2b([],[]).
```

- Often the best way to solve such problems is to think about the simplest possible case
- Here it means: the empty list

Defining a2b/2: step 2

```
a2b([],[]).  
a2b([a|L1],[b|L2]):- a2b(L1,L2).
```

- Now think recursively!
- When should a2b/2 decide that two non-empty lists are a list of as and a list of bs of exactly the same length?

Testing a2b/2

```
a2b([],[]).
```

```
a2b([a|L1],[b|L2]):- a2b(L1,L2).
```

```
?- a2b([a,a,a],[b,b,b]).
```

```
yes
```

```
?-
```

Testing a2b/2

a2b([], []).

a2b([a|L1],[b|L2]):- a2b(L1,L2).

?- a2b([a,a,a,a],[b,b,b]).

no

?-

Testing a2b/2

```
a2b([],[]).
```

```
a2b([a|L1],[b|L2]):- a2b(L1,L2).
```

```
?- a2b([a,t,a,a],[b,b,b,c]).
```

```
no
```

```
?-
```

Further investigating a2b/2

```
a2b([],[]).
```

```
a2b([a|L1],[b|L2]):- a2b(L1,L2).
```

```
?- a2b([a,a,a,a,a], X).
```

```
X = [b,b,b,b,b]
```

```
yes
```

```
?-
```


Further investigating a2b/2

```
a2b([],[]).
```

```
a2b([a|L1],[b|L2]):- a2b(L1,L2).
```

```
?- a2b(X,[b,b,b,b,b,b,b]).
```

```
X = [a,a,a,a,a,a,a]
```

```
yes
```

```
?-
```

Append

- We will define an important predicate **append/3** whose arguments are all lists
- Declaratively, `append(L1,L2,L3)` is true if list `L3` is the result of concatenating the lists `L1` and `L2` together

```
?- append([a,b,c,d],[3,4,5],[a,b,c,d,3,4,5]).
```

```
yes
```

```
?- append([a,b,c],[3,4,5],[a,b,c,d,3,4,5]).
```

```
no
```

Append viewed procedurally

- From a procedural perspective, the most obvious use of append/3 is to concatenate two lists together
- We can do this simply by using a variable as third argument

```
?- append([a,b,c,d],[1,2,3,4,5], X).
```

```
X=[a,b,c,d,1,2,3,4,5]
```

```
yes
```

```
?-
```

Definition of append/3

```
append([], L, L).  
append([H|L1], L2, [H|L3]):-  
    append(L1, L2, L3).
```

- Recursive definition
 - Base clause: appending the empty list to any list produces that same list
 - The recursive step says that when concatenating a non-empty list $[H|T]$ with a list L , the result is a list with head H and the result of concatenating T and L

How append/3 works

- Two ways to find out:
 - Use trace/0 on some examples
 - Draw a search tree!
- Let us consider a simple example

```
?- append([a,b,c],[1,2,3], R).
```

Search tree example

?- append([a,b,c],[1,2,3], R).

```
append([], L, L).  
append([H|L1], L2, [H|L3]):-  
    append(L1, L2, L3).
```

Search tree example

?- append([a,b,c],[1,2,3], R).

/

\

append([], L, L).

append([H|L1], L2, [H|L3]):-
append(L1, L2, L3).

Search tree example

?- append([a,b,c],[1,2,3], R).

 /
 \
† R = [a|L0]
 ?- append([b,c],[1,2,3],L0)

append([], L, L).

append([H|L1], L2, [H|L3]):-
 append(L1, L2, L3).

Search tree example

?- append([a,b,c],[1,2,3], R).

/
†

\
R = [a|L0]
?- append([b,c],[1,2,3],L0)
/ \

append([], L, L).

append([H|L1], L2, [H|L3]):-
append(L1, L2, L3).

Search tree example

?- append([a,b,c],[1,2,3], R).

/ \
† R = [a|L0]
?- append([b,c],[1,2,3],L0)
/ \
† L0=[b|L1]
?- append([c],[1,2,3],L1)

append([], L, L).
append([H|L1], L2, [H|L3]):-
append(L1, L2, L3).

Search tree example

?- append([a,b,c],[1,2,3], R).

/ \
† R = [a|L0]
?- append([b,c],[1,2,3],L0)

/ \
† L0=[b|L1]
?- append([c],[1,2,3],L1)
/ \

append([], L, L).
append([H|L1], L2, [H|L3]):-
append(L1, L2, L3).

Search tree example

?- append([a,b,c],[1,2,3], R).

/ \
† R = [a|L0]
?- append([b,c],[1,2,3],L0)

/ \
† L0=[b|L1]
?- append([c],[1,2,3],L1)

/ \
† L1=[c|L2]
?- append([], [1,2,3], L2)

append([], L, L).
append([H|L1], L2, [H|L3]):-
append(L1, L2, L3).

Search tree example

?- append([a,b,c],[1,2,3], R).

/ \
† R = [a|L0]
?- append([b,c],[1,2,3],L0)

/ \
† L0=[b|L1]
?- append([c],[1,2,3],L1)

/ \
† L1=[c|L2]
?- append([], [1,2,3], L2)
/ \

append([], L, L).
append([H|L1], L2, [H|L3]):-
append(L1, L2, L3).

Search tree example

?- append([a,b,c],[1,2,3], R).

/ \
† R = [a|L0]
?- append([b,c],[1,2,3],L0)

/ \
† L0=[b|L1]
?- append([c],[1,2,3],L1)

/ \
† L1=[c|L2]
?- append([], [1,2,3], L2)

/ \
L2=[1,2,3] †

append([], L, L).
append([H|L1], L2, [H|L3]):-
append(L1, L2, L3).

Search tree example

?- append([a,b,c],[1,2,3], R).

/ \
† R = [a|L0]
?- append([b,c],[1,2,3],L0)

/ \
† L0=[b|L1]
?- append([c],[1,2,3],L1)

/ \
† L1=[c|L2]
?- append([], [1,2,3], L2)
/ \
L2=[1,2,3] †

append([], L, L).
append([H|L1], L2, [H|L3]):-
append(L1, L2, L3).

L2=[1,2,3]
L1=[c|L2]=[c,1,2,3]
L0=[b|L1]=[b,c,1,2,3]
R=[a|L0]=[a,b,c,1,2,3]

Using append/3

- Now that we understand how append/3 works, let's look at some applications
- Splitting up a list:

```
?- append(X,Y, [a,b,c,d]).
```

```
X=[ ]      Y=[a,b,c,d];
```

```
X=[a]      Y=[b,c,d];
```

```
X=[a,b]    Y=[c,d];
```

```
X=[a,b,c]  Y=[d];
```

```
X=[a,b,c,d] Y=[ ];
```

```
no
```


Prefix and suffix

- We can also use `append/3` to define other useful predicates
- A nice example is finding prefixes and suffixes of a list

Definition of prefix/2

```
prefix(P,L):-  
    append(P,_,L).
```

- A list P is a prefix of some list L when there is some list such that L is the result of concatenating P with that list.
- We use the anonymous variable because we don't care what that list is.

Use of prefix/2

```
prefix(P,L):-  
    append(P,_,L).
```

```
?- prefix(X, [a,b,c,d]).  
X=[ ];  
X=[a];  
X=[a,b];  
X=[a,b,c];  
X=[a,b,c,d];  
no
```

Definition of suffix/2

```
suffix(S,L):-  
    append(_,S,L).
```

- A list S is a suffix of some list L when there is some list such that L is the result of concatenating that list with S.
- Once again, we use the anonymous variable because we don't care what that list is.

Use of suffix/2

```
suffix(S,L):-  
    append(_,S,L).
```

```
?- suffix(X, [a,b,c,d]).  
X=[a,b,c,d];  
X=[b,c,d];  
X=[c,d];  
X=[d];  
X=[];  
no
```

Definition of sublist/2

- Now it is very easy to write a predicate that finds sub-lists of lists
- The sub-lists of a list L are simply the prefixes of suffixes of L

```
sublist(Sub,List):-  
    suffix(Suffix,List),  
    prefix(Sub,Suffix).
```

Reversing a List

- We will define a predicate that changes a list [a,b,c,d,e] into a list [e,d,c,b,a]
- This would be a useful tool to have, as Prolog only allows easy access to the front of the list

Reverse

- Recursive definition
 1. If we reverse the empty list, we obtain the empty list
 2. If we reverse the list $[H|T]$, we end up with the list obtained by reversing T and concatenating it with $[H]$
- To see that this definition is correct, consider the list $[a,b,c,d]$.
 - If we reverse the tail of this list we get $[d,c,b]$.
 - Concatenating this with $[a]$ yields $[d,c,b,a]$

Reverse

```
reverse([],[]).  
reverse([H|T],R):-  
    reverse(T,RT),  
    append(RT,[H],R).
```

- This definition is correct, but it does an awful lot of work
- It spends a lot of time carrying out appends
- But there is a better way...

Reverse using an accumulator

- A better way is using an accumulator
- The accumulator will be a list, and when we start reversing it will be empty
- We simply take the head of the list that we want to reverse and add it to the head of the accumulator list
- We continue this until we hit the empty list
- At this point the accumulator will contain the reversed list!

Reverse using an accumulator

```
accReverse([ ],L,L).  
accReverse([H|T],Acc,Rev):-  
    accReverse(T,[H|Acc],Rev).
```

Adding a wrapper predicate

```
accReverse([ ],L,L).  
accReverse([H|T],Acc,Rev):-  
    accReverse(T,[H|Acc],Rev).
```

```
reverse(L1,L2):-  
    accReverse(L1,[ ],L2).
```

Illustration of the accumulator

- List: [a,b,c,d] Accumulator: []

Illustration of the accumulator

- List: [a,b,c,d] Accumulator: []
- List: [b,c,d] Accumulator: [a]

Illustration of the accumulator

- List: [a,b,c,d] Accumulator: []
- List: [b,c,d] Accumulator: [a]
- List: [c,d] Accumulator: [b,a]

Illustration of the accumulator

- List: [a,b,c,d] Accumulator: []
- List: [b,c,d] Accumulator: [a]
- List: [c,d] Accumulator: [b,a]
- List: [d] Accumulator: [c,b,a]

Illustration of the accumulator

- List: [a,b,c,d] Accumulator: []
- List: [b,c,d] Accumulator: [a]
- List: [c,d] Accumulator: [b,a]
- List: [d] Accumulator: [c,b,a]
- List: [] Accumulator: [d,c,b,a]

A thick blue vertical bar is positioned on the left side of the slide, extending from the top to the bottom. A thick black horizontal line is positioned near the top of the slide, extending from the left edge to the right edge.

A few other things ...

Comparing Integers

Arithmetic

$x < y$

$x \leq y$

$x = y$

$x \neq y$

$x \geq y$

$x > y$

Prolog

$X < Y$

$X = < Y$

$X =: = Y$

$X = \backslash = Y$

$X > = Y$

$X > Y$

Comparison Operators

- Have the obvious meaning
- Force both left and right hand argument to be evaluated

?- 2 < 4+1.

yes

?- 4+3 > 5+5.

no

Comparison Operators

- The unification operator does not force evaluation but the numeric equality comparison operator does.

?- 4 = 4.

yes

?- 2+2 = 4.

no

?- 2+2 =:= 4.

yes

Similar looking symbols

$=$ The unification predicate. Succeeds if it can unify its arguments, fails otherwise.

$\backslash=$ The negation of the unification predicate. Succeeds if $=$ fails, and vice-versa.

$==$ The identity predicate. Succeeds if its arguments are identical, fails otherwise.

$\backslash==$ The negation of the identity predicate. Succeeds if $==$ fails, and vice-versa.

$:=$ The arithmetic equality predicate. Succeeds if its arguments evaluate to the same integer.

\neq The arithmetic inequality predicate. Succeeds if its arguments evaluate to different integers.

Negation as Failure

- The cut operator (!) suppresses backtracking.
- The fail predicate always fails.
- They can be combined to get **negation as failure** as follows:

```
neg(Goal):- Goal, !, fail.  
neg(Goal).
```

Vincent and burgers

```
enjoys(vincent,X):- burger(X),  
                    neg(bigKahunaBurger(X)).
```

```
burger(X):- bigMac(X).  
burger(X):- bigKahunaBurger(X).  
burger(X):- whopper(X).
```

```
bigMac(a).  
bigKahunaBurger(b).  
bigMac(c).  
whopper(d).
```


Vincent and burgers

```
enjoys(vincent,X):- burger(X),  
                    neg(bigKahunaBurger(X)).
```

```
burger(X):- bigMac(X).  
burger(X):- bigKahunaBurger(X).  
burger(X):- whopper(X).
```

```
bigMac(a).  
bigKahunaBurger(b).  
bigMac(c).  
whopper(d).
```

```
?- enjoys(vincent,X).  
    X=a  
    X=c  
    X=d
```

Another built-in predicate: \+

- Because negation as failure is so often used, there is no need to define it.
- In standard Prolog the prefix operator \+ means negation as failure
- We can define Vincent`s preferences as follows:

```
enjoys(vincent,X):- burger(X),  
                    \+ bigKahunaBurger(X).
```

```
?- enjoys(vincent,X).
```

```
X=a
```

```
X=c
```

```
X=d
```

Negation as failure and logic

- Negation as failure is not logical negation
- Changing the order of the goals in the vincent and burgers program gives a different behaviour:

```
enjoys(vincent,X):- \+ bigKahunaBurger(X),  
                    burger(X).
```

```
?- enjoys(vincent,X).  
    no
```

Complete negation example

```
enjoys(vincent,X):- burger(X),  
                    \+ bigKahunaBurger(X).
```

```
burger(X):- bigMac(X).  
burger(X):- bigKahunaBurger(X).  
burger(X):- whopper(X).
```

```
bigMac(a).  
bigKahunaBurger(b).  
bigMac(c).  
whopper(d).
```

```
?- enjoys(vincent,X).  
    X=a;  
    X=c;  
    X=d;  
    no
```



Miscellaneous Examples and Applications

Quick sort in Prolog

```
partition([], Pivot, [], []).
```

```
partition([H|T], Pivot, [H|Left], Right) :- H <= Pivot,  
                                             partition(T, Pivot, Left, Right).
```

```
partition([H|T], Pivot, Left, [H|Right]) :- H > Pivot,  
                                             partition(T, Pivot, Left, Right).
```

```
quicksort([], []).
```

```
quicksort([P|T], Sorted) :- partition(T, P, Left, Right),  
                             quicksort(Left, SortedLeft),  
                             quicksort(Right, SortedRight),  
                             append(SortedLeft, [P|SortedRight], Sorted).
```

Complex queries and Expert systems

person(ann).

person(bob).

personSkill(ann, software).

personSkill(bob, software).

personSkill(bob, floating).

job(lifeguard).

job(developer).

jobSkill(lifeguard, firstaid).

jobSkill(lifeguard, swimming).

hasSkills(P, Skills) :- person(P), findall(S, personSkill(P,S), Skills).

needsSkills(J, Skills) :- job(J), findall(S, jobSkill(J,S), Skills).

fit_for_job(P, J) :- hasSkills(P, SkillsHas),
 needsSkills(J, SkillsNeeded),
 forall(member(S, SkillsNeeded), member(S, SkillsHas)).

Natural language processing

- See:
 - Parsing and difference lists
 - Eliza chatbot
- IBM Watson:
 - Unstructured Information Management Architecture framework running on Hadoop

“We required a language in which we could conveniently express pattern matching rules over the parse trees and other annotations (such as named entity recognition results), and a technology that could execute these rules very efficiently. We found that Prolog was the ideal choice for the language due to its simplicity and expressiveness.” -- Watson development team