Physics 410 - Homework 2

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1 Problem 1

1.1

$$\begin{aligned} v_{ter} &= \sqrt{\frac{mg}{c}} \\ m\frac{dv}{dt} &= -mg - cv^2 \\ \frac{m}{c}\frac{dv}{dt} &= -\frac{mg}{c} - v^2 \\ g(\frac{m}{c}\frac{dv}{dt}) &= (-v_{ter}^2 - v^2)g \\ v_{ter}^2\dot{v} = -g{v_{ter}}^2 - gv^2 \\ \dot{v} &= -g - g\frac{v^2}{v_{ter}^2} \\ \dot{v} &= -g(1 + \frac{v^2}{v_{ter}^2}) \end{aligned} \tag{1}$$

1.2

1.2.1 Solving differential equation

$$
\begin{aligned}
\frac{dv}{dt} &= -g(1+\frac{v^2}{v_{ter}^2}) \\
\frac{1}{(1+\frac{v^2}{v_{ter}^2})}\frac{dv}{dt} &= -g \\
\int_{v_0}^{v}\frac{dv}{(1+\frac{v^2}{v_{ter}^2})} &= \int_0^t -g \\
v_{ter}(\arctan(\frac{v}{v_{ter}}) - \arctan(\frac{v_0}{v_{ter}})) &= -gt \\
(\arctan(\frac{v}{v_{ter}}) - \arctan(\frac{v_0}{v_{ter}})) &= -\frac{gt}{v_{ter}} \\
\text{Multiply by tan} \\
\frac{v}{v_{ter}} &= \tan(\arctan(\frac{v_0}{v_{ter}}) - \frac{gt}{v_{ter}}) \\
v &= v_{ter}\tan(\arctan(\frac{v_0}{v_{ter}}) - \frac{gt}{v_{ter}})
\end{aligned} \tag{2}
$$

1.2.2 Case when $v(0) = v_0$

$$
\begin{aligned}
v(0) &= v_{ter}\tan(\arctan(\frac{v_0}{v_{ter}}) - \frac{g0}{v_{ter}}) \\
&= v_{ter}\tan(\arctan(\frac{v_0}{v_{ter}})) \\
&= v_{ter}\frac{v_0}{v_{ter}} \\
&= v_0
\end{aligned} \tag{3}
$$

1.3

$$
\begin{aligned}
&\text{At } t_{top},\ v = 0 \\
v = 0 &= v_{ter}\tan(\arctan(\frac{v_0}{v_{ter}}) - \frac{gt}{v_{ter}}) \\
&= \tan(\arctan(\frac{v_0}{v_{ter}}) - \frac{gt}{v_{ter}}) \\
\text{Multiply by arctan} \\
&= \arctan(\frac{v_0}{v_{ter}}) - \frac{gt}{v_{ter}} \\
\frac{gt}{v_{ter}} &= \arctan(\frac{v_0}{v_{ter}}) \\
t_{top} &= \frac{v_{ter}}{g}\arctan(\frac{v_0}{v_{ter}})
\end{aligned} \tag{4}
$$

1.4

$$t_{top} = \frac{35\frac{m}{s}}{9.8\frac{m}{s^2}} \arctan(\frac{5}{7}) \\ = 4.1193s \tag{5}$$

1.5

No air resistance time: $t = \frac{v_0}{g}$

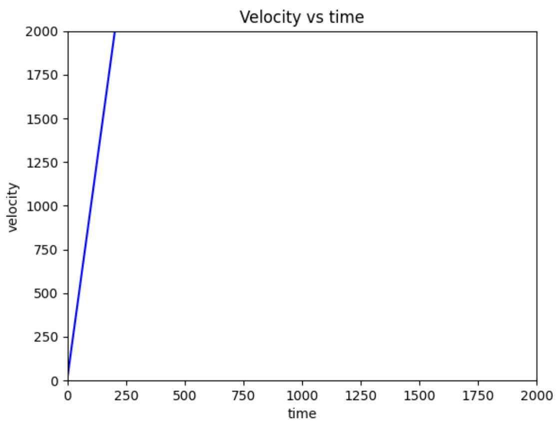
$$\tan^{-1}(x) \approx x - \frac{x^3}{3} + \frac{x^5}{5} + ... + \frac{x^n}{n}$$

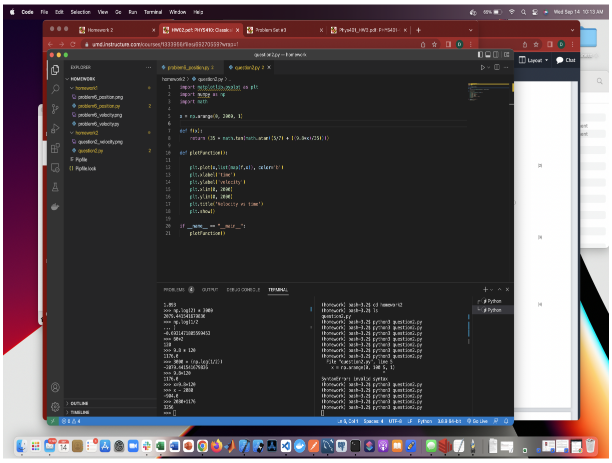
When x is very small, $\tan^{-1}(x) \approx x$

$$t = \frac{v_{ter}}{g} \frac{v_0}{v_{ter}} \\ = \frac{v_0}{g} \tag{6}$$

2 Problem 2

2.1





3 Problem 3

3.1

$$
\begin{aligned}
c &= \lambda\nu \\
1dm &= 10m \\
10MHz &= 30m
\end{aligned} \tag{7}
$$

This value is 300% larger than a decameter.

$$40MHz = 7.5m \tag{8}$$

This value is 75% of a decameter.

3.2

$$B = \frac{m_e \omega}{q_e}$$

3.3

3.4

4 Problem 4

4.1

$$
\begin{aligned}
\vec{E} &= (0, E, 0) \\
\vec{B} &= (0, 0, B) \\
\vec{v} &= (v_x, v_y, v_z) \\
\vec{F} &= q(\vec{E} + \vec{v} \times \vec{B}) \\
&= q(\vec{E} + (v_y B, -v_x B, 0))
\end{aligned} \tag{9}
$$

$$\begin{aligned} F_x &= qv_yB \\ m\dot{v}_x &= qv_yB \\ \dot{v}_x &= \omega v_y \end{aligned} \tag{10}$$

$$\begin{aligned} F_y &= qE - qv_xB \\ m\dot{v}_y &= qE - qv_xB \\ \dot{v}_y &= qE - \omega v_x \end{aligned} \tag{11}$$

$$\begin{aligned} F_z &= 0 \\ m\dot{v}_z &= 0 \\ \dot{v}_z &= 0 \end{aligned} \tag{12}$$

4.2

You want to show $\vec{F}_y = 0$

$$\begin{aligned} \vec{F}_y &= qE - v_xB \\ v_xB &= qE \\ v_x &= \frac{E}{B} \end{aligned} \tag{13}$$

Do not have to worry about q because it is a constant.

4.3

5 Problem 5

Still need my angles

6 Problem 6

6.1 Thrust

$$\begin{aligned} thrust &= -\dot{m}v_{ex} \\ &\approx -(-15000)(2500) \\ &\approx 37500000N \end{aligned} \tag{14}$$

6.2 Thrust expressed in Tons

$$\begin{aligned} 1Ton &\approx 9000N \\ xTons &\approx \frac{37500000}{9000} \\ &\approx 4166Tons \end{aligned} \tag{15}$$

This is $> 1000Tons(\approx 33\%)$ larger than the rocket's initial weight, which makes sense as a thrust of this magnitude is necessary to accelerate the rocket.

7 Problem 7

7.1

$$v_0 = 0$$

Using the derivation from Taylor equation 3.8 and subbing in our known values.

$$\begin{aligned} v &= v_{ex} \ln \frac{m_0}{m} \\ &= 3000 \ln 2 \\ &\approx 2080 \frac{m}{s} \end{aligned} \tag{16}$$

7.2

7.2.1 Thrust

$$\begin{aligned} thrust &= -\dot{m} v_{ex} \\ &= (1E6 \frac{kg}{s})(3000 \frac{m}{s}) \\ &= 3E9 \frac{kgm}{s^2} \end{aligned} \tag{17}$$

7.2.2 Weight on earth

$$\begin{aligned} weight &= mg \\ &= (2E6)kg * (9.8) \frac{m}{s^2} \\ &= 19.6E6 \frac{kgm}{s^2} \end{aligned} \tag{18}$$

This result shows the thrust to be larger than the weight on earth by several orders of magnitude, which makes sense, if this was not the case the rocket would not be able to leave earth.

8 Problem 8

8.1

$$\begin{aligned} P(t+dt) &= (m+dm)(v+dv) - dm(v - v_{ext}) \\ &= mv + mdv + dvdm + dmv_{ext} \\ &= mv + mdv + dmv_{ext} \end{aligned} \tag{19}$$

$$\begin{aligned} dP &= P(t+dt) - P(t) \\ &= (mv + mdv + dmv_{ext}) - mv \\ \frac{dP}{dt} &= m\frac{dv}{dt} + \frac{dm}{dt} v_{ext} \\ &= m\dot{v} + v_{ext}\dot{m} \\ \frac{dP}{dt} &= m\dot{v} + v_{ext}\dot{m} = F^{ext} \\ m\dot{v} &= F^{ext} - \dot{m} v_{ext} \end{aligned} \tag{20}$$

8.2

$$
\begin{aligned}
\dot{m} &= -k \\
m &= m_0 - kt \\
m\dot{v} &= -\dot{m}v_{ext} - mg \\
m\frac{dv}{dt} &= -\frac{dm}{dt}v_{ext} - mg \\
(m_0 - kt)\frac{dv}{dt} &= -(-k)v_{ext} - (m_0 - kt)g \\
\frac{dv}{dt} &= (\frac{k}{m_0 - kt}v_{ext} - g) \\
\int_0^v dv' &= \int_0^t (\frac{k}{m_0 - kt'}v_{ext} - g)dt' \\
v &= \int_0^t \frac{k}{m_0 - kt'}v_{ext}dt - \int_0^t gdt \\
&= kv_{ext}\int_0^t \frac{1}{m_0 - kt}dt - gt \\
&= v_{ext}[\ln|m_0 - kt| - \ln|m_0|] - gt \\
&= v_{ext}[\ln|\frac{m}{m_0}|] - gt
\end{aligned} \tag{21}
$$

8.3

$$
\begin{aligned}
v_{final} &= v_{ext}[\ln|\frac{m}{m_0}|] - gt \\
&= (3000)\frac{m}{s}(-0.693) - -(9.8\frac{m}{s^2}) * 120 \\
&= 2080 + 1176 \\
&= 3256\frac{m}{s}
\end{aligned} \tag{22}
$$

8.4

If $-\dot{m}v_{ter} << -mg$, the rocket will not overcome the gravitational force and therefore it will not take off.

9 Problem 9

9.1

$$v_0 = 0,\ m_{final} = m_0 - 0.6m_0 = 0.4m_0$$

$$
\begin{aligned}
v &= v_{ex}\ln\frac{m_0}{m} \\
&= v_{ex}\ln\frac{m_0}{0.4m_0} \\
&= 0.916v_{ex}
\end{aligned} \tag{23}
$$

9.2

9.2.1 Stage 1

$$
\begin{aligned}
v_1 &= v_{ex} \ln \frac{m_0}{0.3m_0} \\
&= 1.203 v_{ex}
\end{aligned} \tag{24}
$$

9.2.2 Stage 2

$$
\begin{aligned}
v - v_1 &= v_{ex} \ln \frac{0.6m_0}{0.3m_0} \\
&= 0.693 v_{ex} + 1.203 v_{ex} \\
&= 1.893 v_{ex}
\end{aligned} \tag{25}
$$

As shown above, the two stage is much better in terms of final velocity. It just about doubles the final velocity for the same fuel.