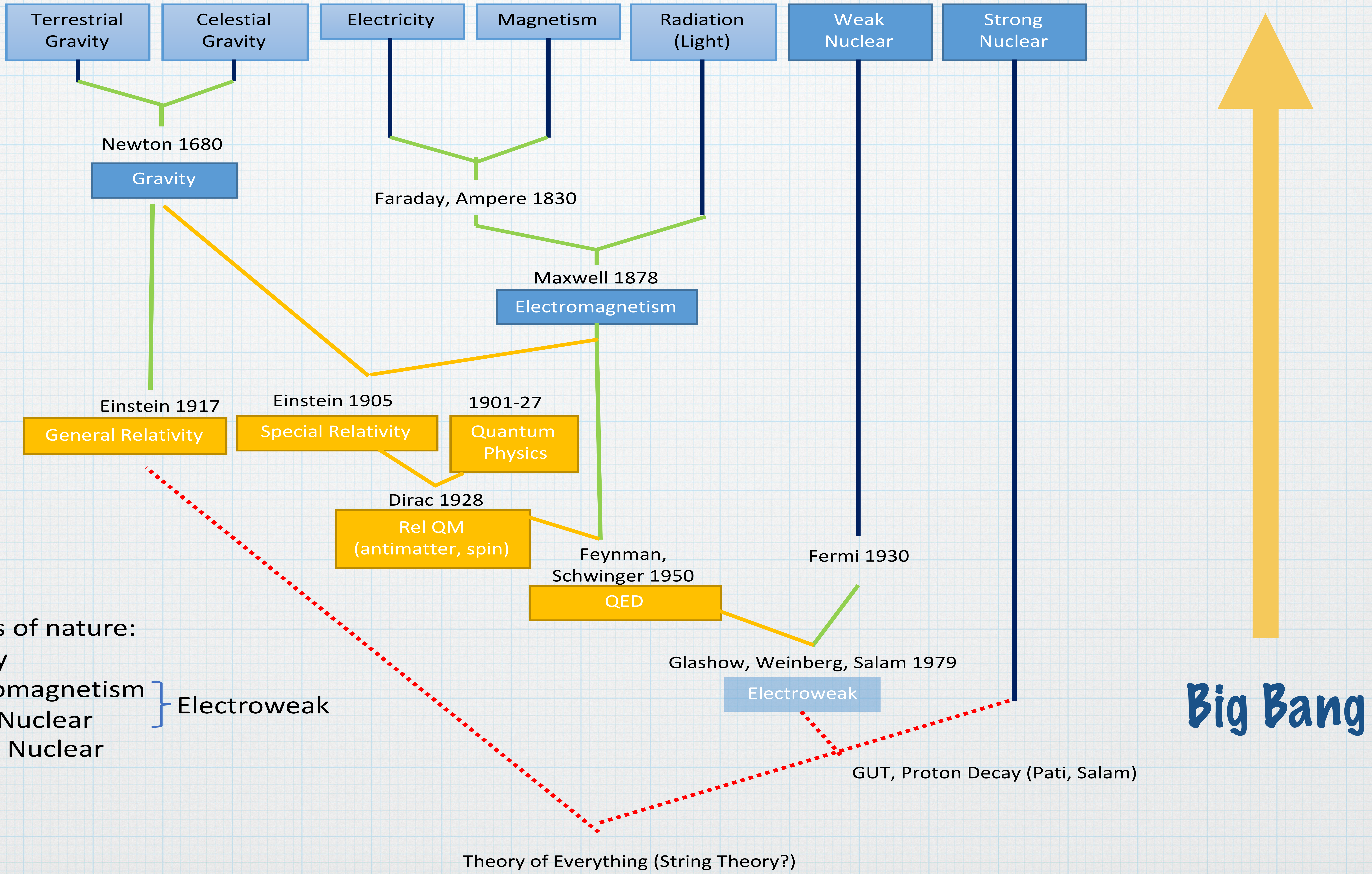
Phys 474 - Spring 2023

Lab 1 - Data Analysis
Fitting for the Z-boson Mass and Width

The Unification of Forces



"Four" forces of nature:

- Gravity
- Electromagnetism
- Weak Nuclear
- Strong Nuclear

Electromagnetism + Weak Nuclear: Electroweak

The Standard Model

FERMIONS
matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e electron	0.000511	–1
ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ muon	0.106	–1
ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

BOSONS
force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.39	–1
W^+ W bosons	80.39	+1
Z^0 Z boson	91.188	0

Strong (color) spin =1

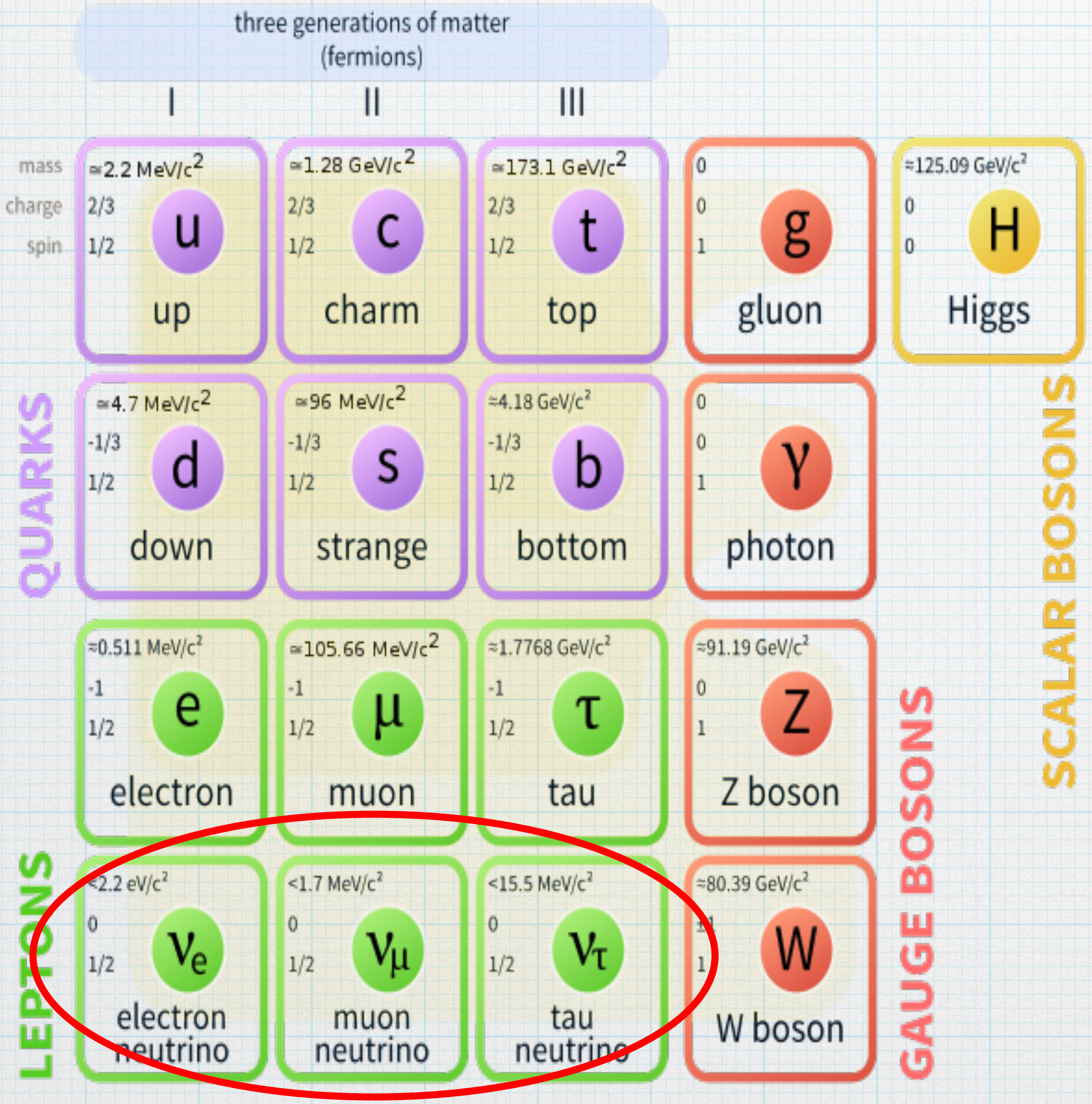
Name	Mass GeV/c^2	Electric charge
g gluon	0	0

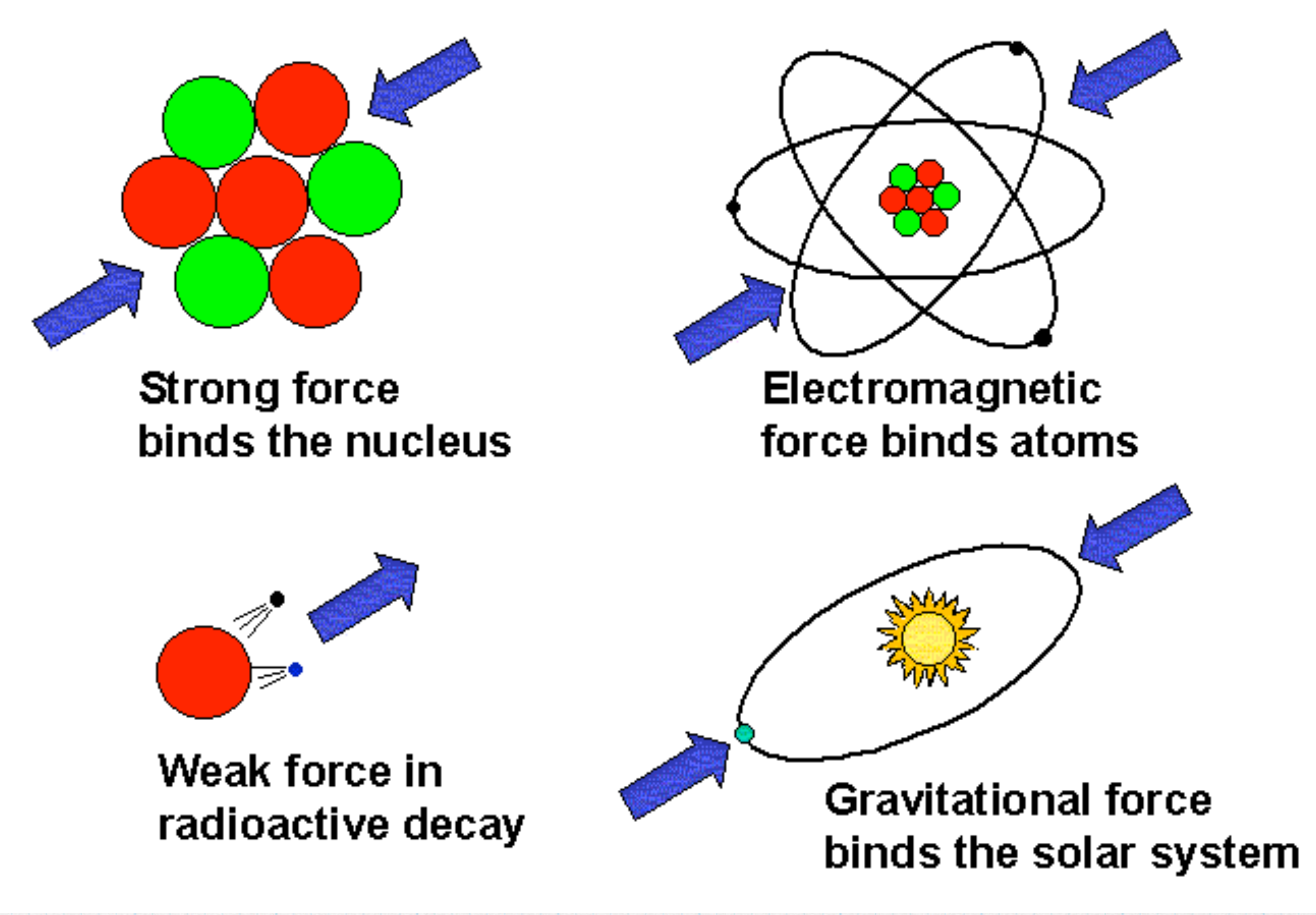
Properties of the Interactions

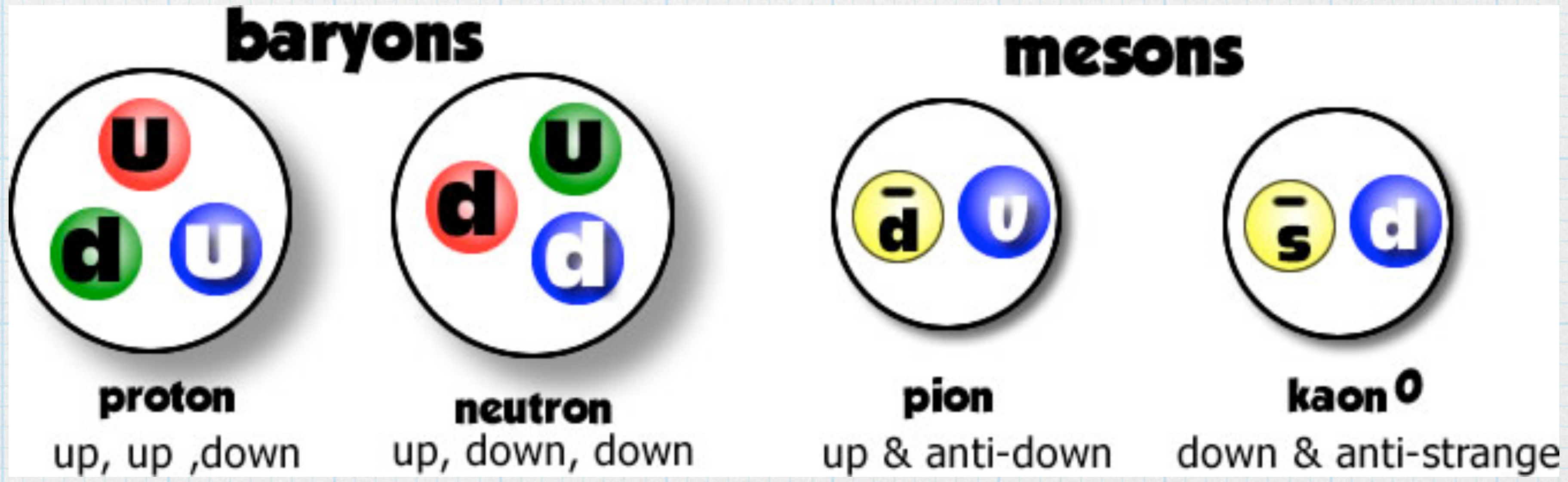
The strengths of the interactions (forces) are shown relative to the strength of the electromagnetic force for two u quarks separated by the specified distances.

Property	Gravitational Interaction	Weak Interaction (Electroweak)	Electromagnetic Interaction	Strong Interaction
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge
Particles experiencing:	All	Quarks, Leptons	Electrically Charged	Quarks, Gluons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons
Strength at 10^{-18} m	10^{-41}	0.8	1	25
Strength at 3×10^{-17} m	10^{-41}	10^{-4}	1	60

Standard Model of Elementary Particles





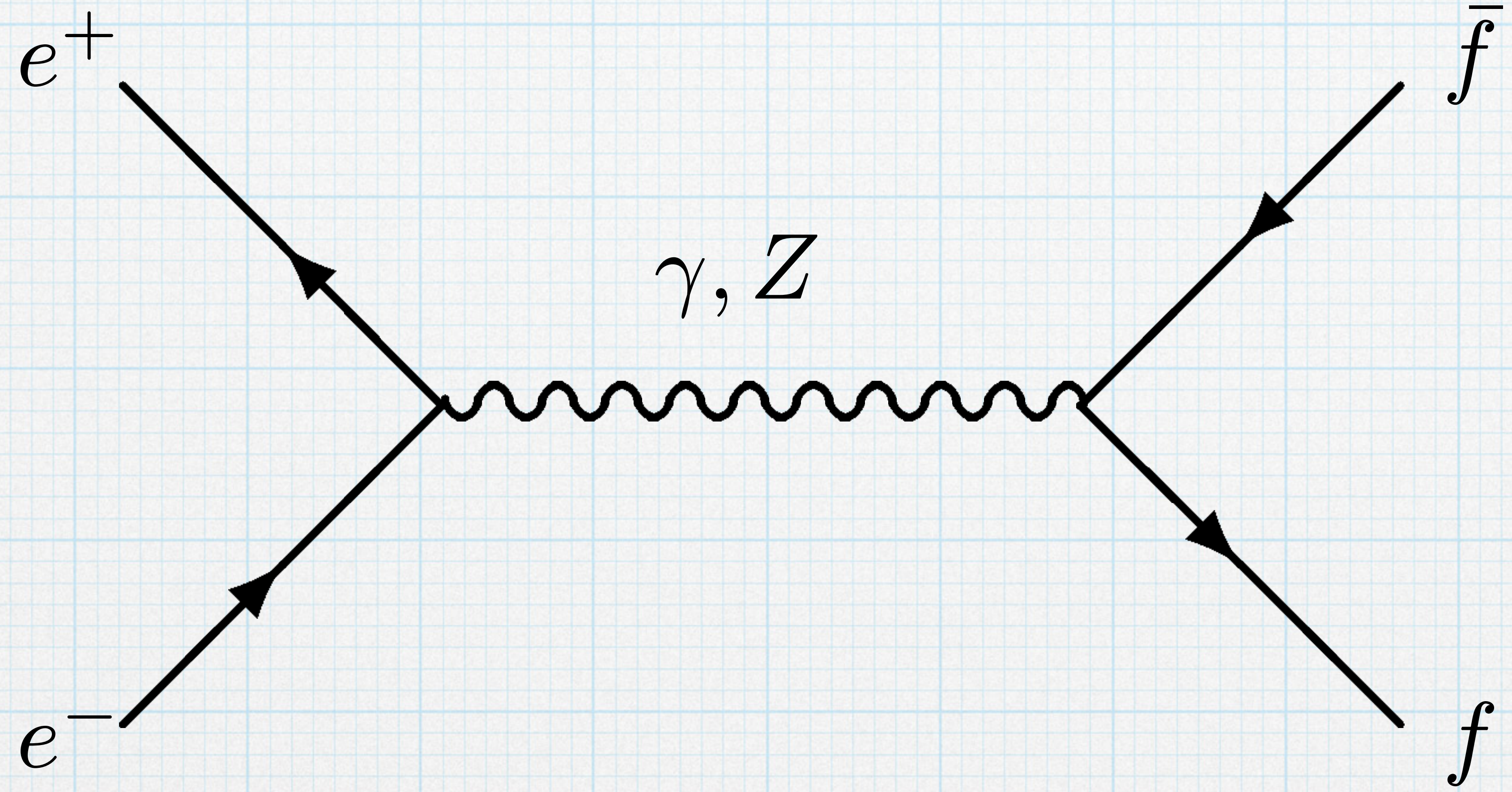


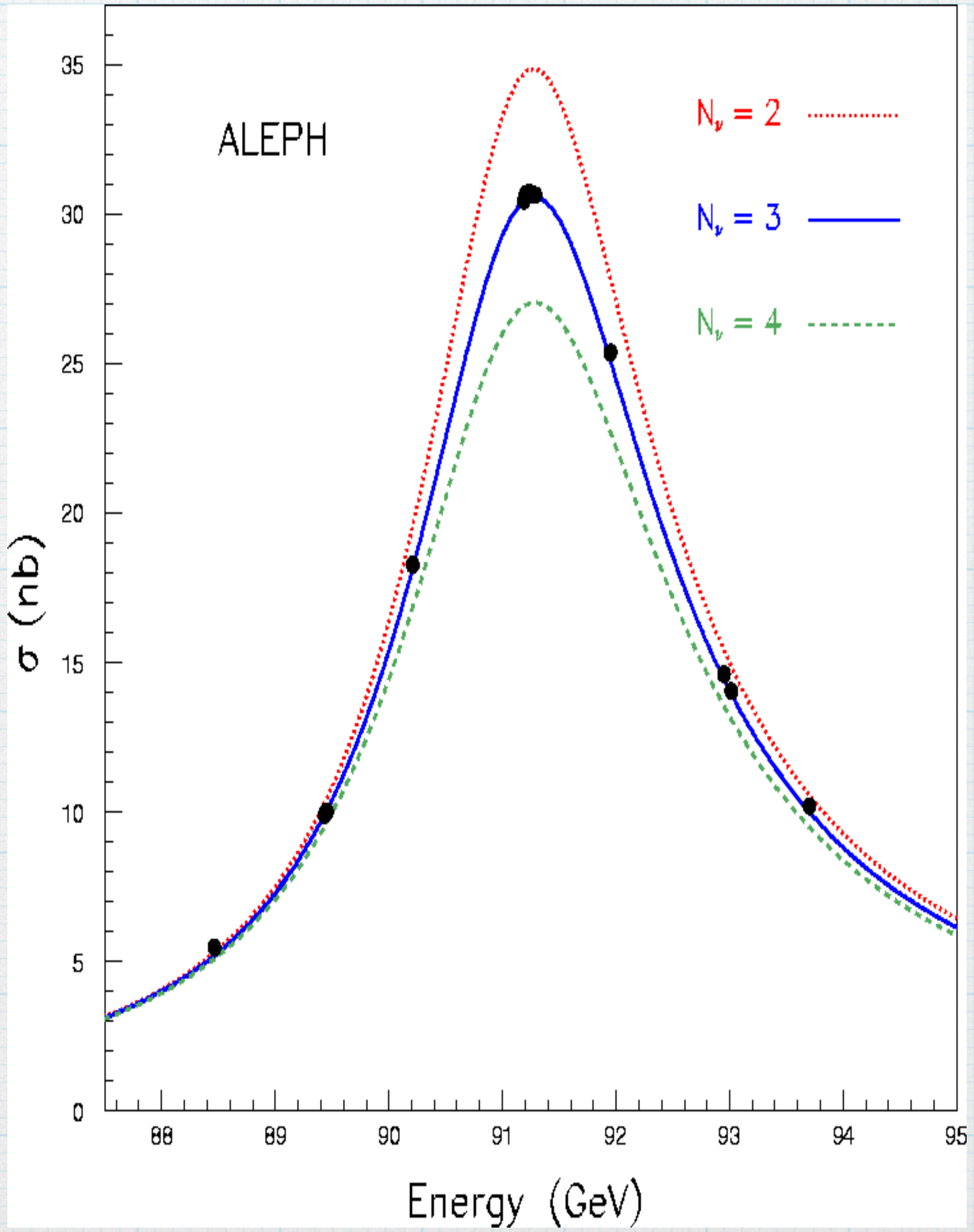
Z Production and Decay

Produce in $e^+ e^-$ collisions (annihilation)
When total $E_{com} = M_Z = 91$ GeV/c^2

Z then decays to fermion anti-fermion pairs. $Z \rightarrow f\ \bar{f}$

$$f = e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau, u, d, c, s, b$$



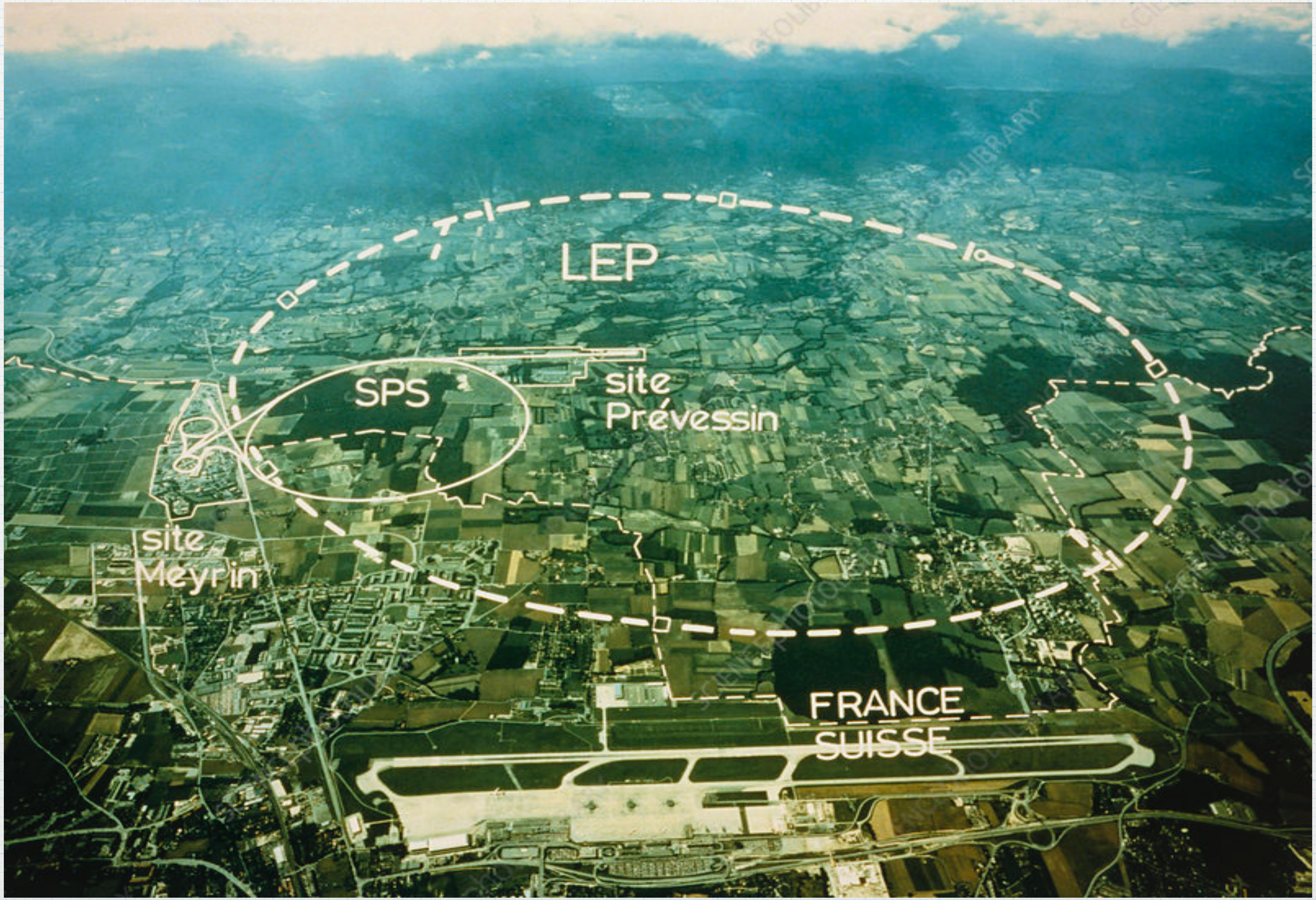


$$\Delta t\, \Delta E \geq \frac{\hbar}{2}$$

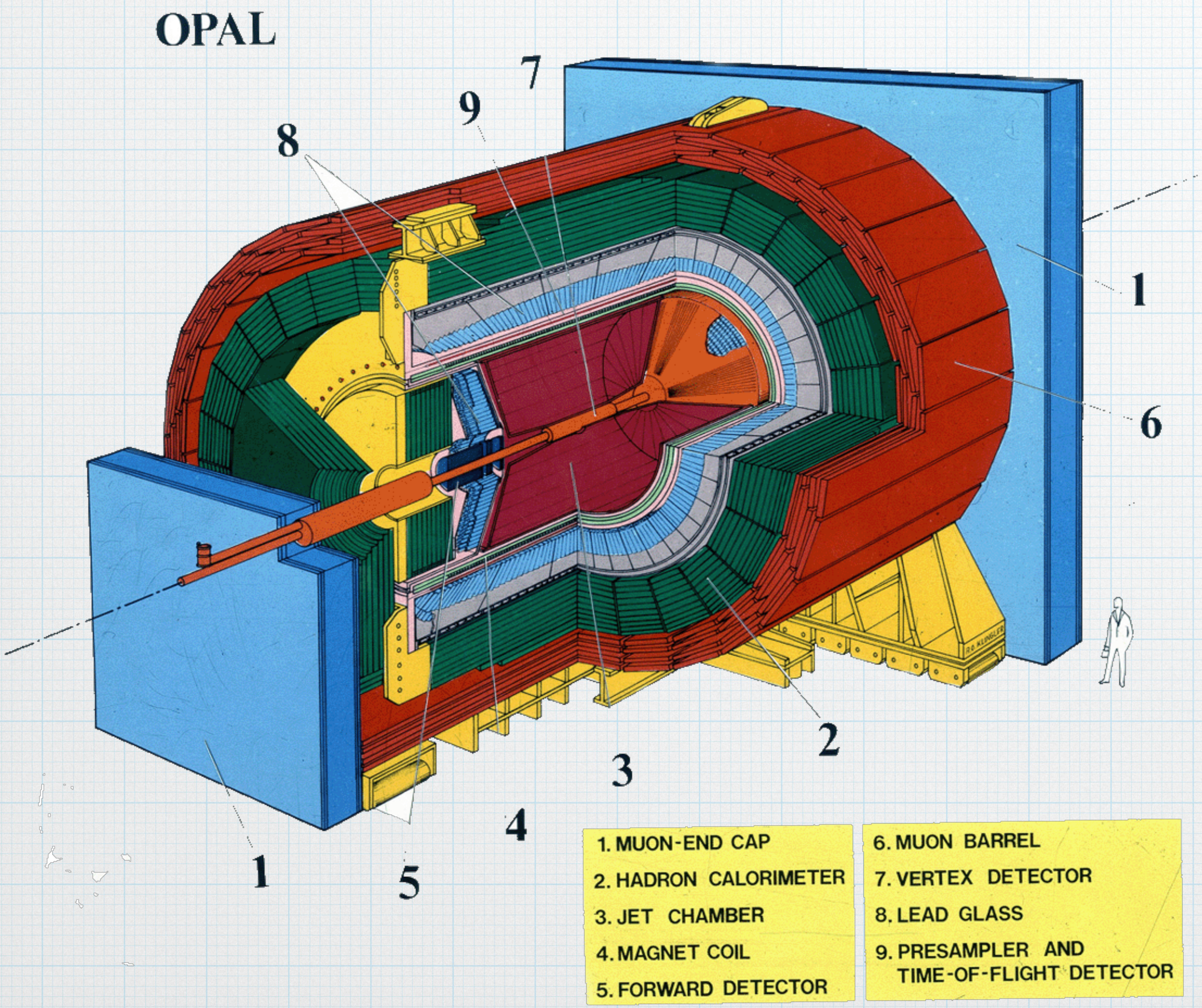
The more particles it can decay into, the faster it decays (shorter lifetime). And, therefore larger mass width.

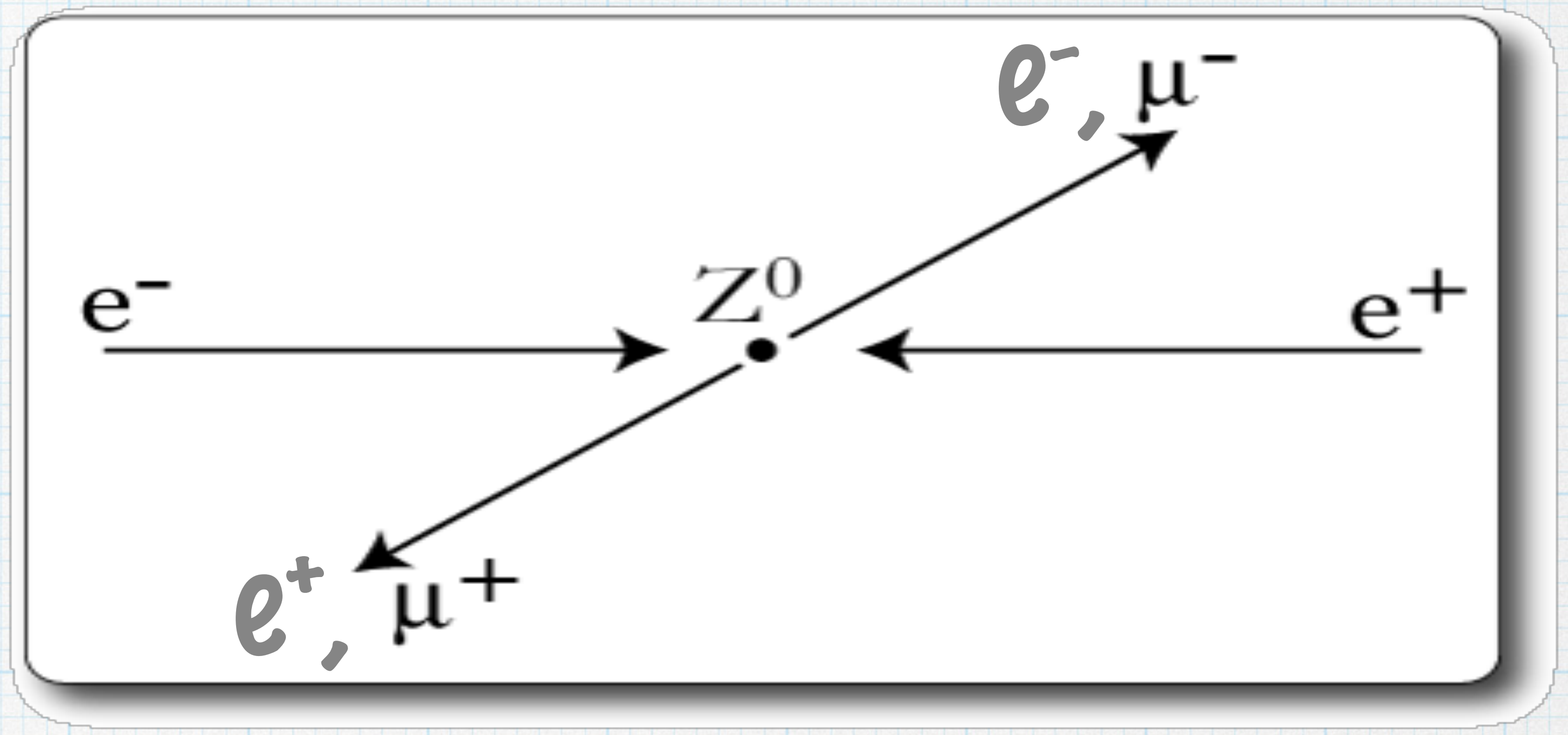
Measure Z boson mass width → # of neutrinos

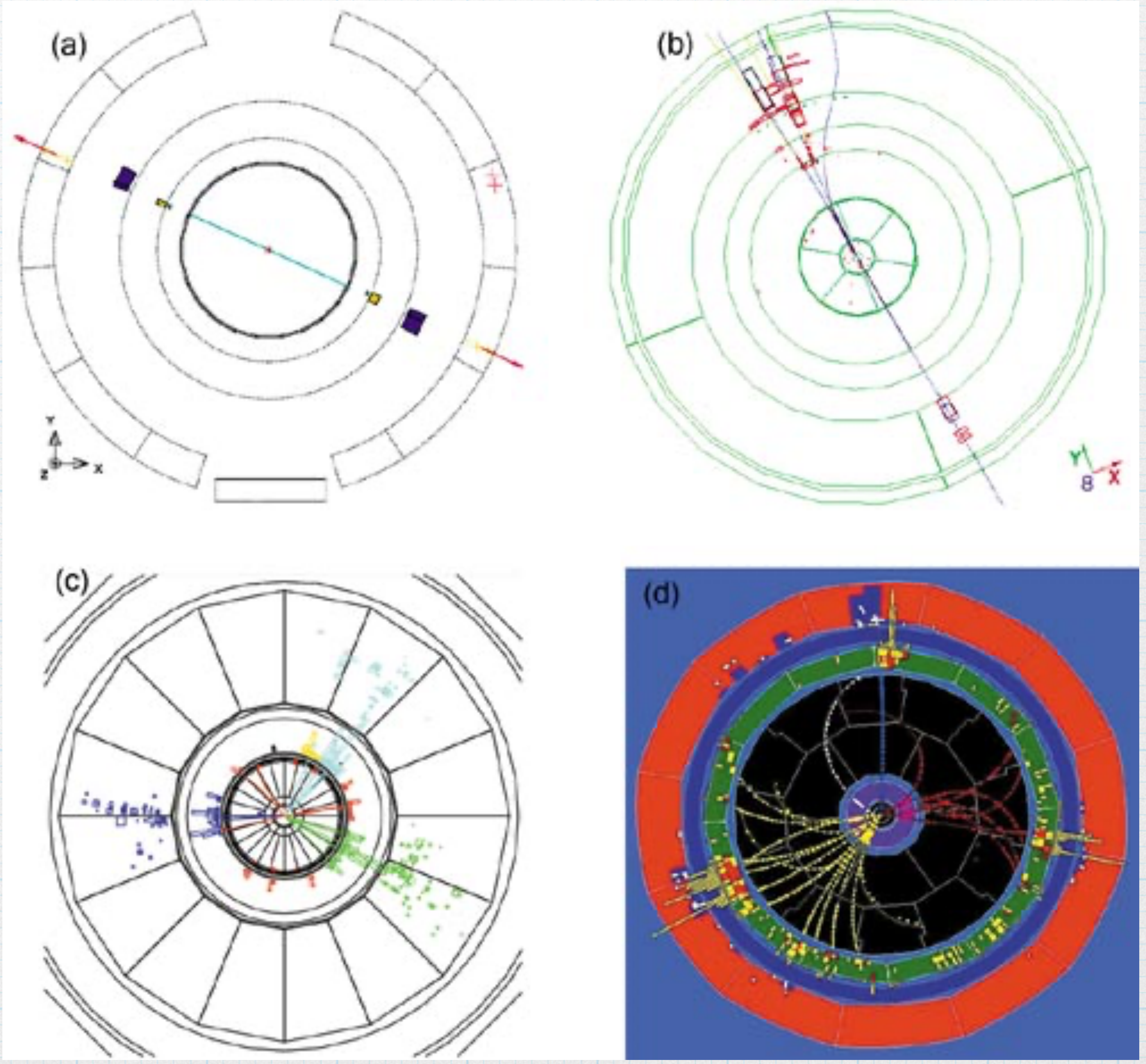
LEP collider at Cern (1990's)



Detector of collision products







Lab 1 - Fitting Data for Z mass and Z width

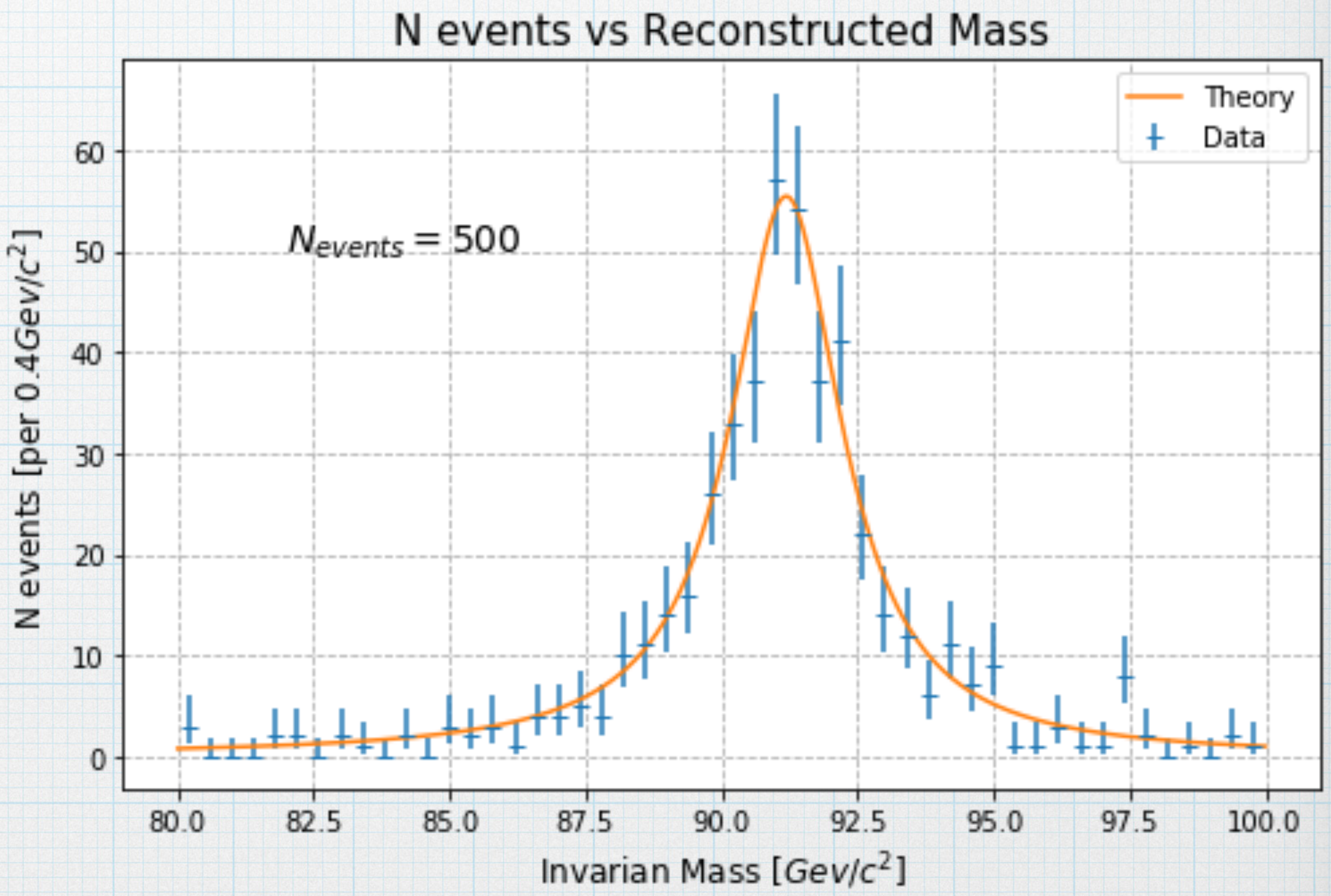
* Text file (.csv) with "data events"

  - Each row is 1 event consisting of $(E, \overrightarrow{p})$ for e^+, e^- pairs from data
  - n_event rows x 8 columns

* Reconstruct the invariant mass from pairs

* Fit the data to Breit-Wigner line shape in 2 ways

  1) binned data using χ^2 fit
  2) unbinned maximum likelihood



Some background on 4-vectors, 4-momentum and special relativity

Note: we are using particle physics units
- Energy (E) is GeV
- Momentum (P) is GeV/c
- Mass (m) is GeV/c^2

I will occasionally omit the c's in the formulas because I'm lazy ☺

$$E = \frac{m\,c^2}{\sqrt{1-\beta^2}} = \gamma m c^2 \qquad \beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\vec{p} = \frac{m\,\vec{v}}{\sqrt{1-\beta^2}} = \gamma m \vec{v} \qquad E^2 = p^2c^2 + m^2c^4$$

A 4-momentum, $\tilde{p}$, is a 4-vector with one element E and the other three $\vec{p} = (p_x, p_y, p_z)$
- $\tilde{p} = (E, p_x, p_y, p_z)$

A dot product of two 4-vectors $\tilde{a}, \tilde{b}$ in SR is defined as:
- $\tilde{a} \cdot \tilde{b} = (a_0, a_1, a_2, a_3) \cdot (b_0, b_1, b_2, b_3) = a_0 b_0 - (a_1 b_1 + a_2 b_2 + a_3 a_4)$
- The dot product is an invariant. Doesn't change with reference frame.
  - Dot products in normal 3-space are invariant too! The "space" in SR is a minkowski space , which defines the dot product as above.

Applying this to the 4-momentum $\tilde{p}$ gives :
- $\tilde{p}^2 = \tilde{p} \cdot \tilde{p} = E^2 - p^2 = m^2$ (see equation above!)
- → invariant mass squared! → This is independent of the reference frame!

Reconstructing the Z boson Mass

Z decay to $e^+ e^-$: $Z \rightarrow e^+ e^-$

Use 4-momenta: $\tilde{p} = (E, p_x, p_y, p_z)$ and energy-momentum conservation

Z 4-momentum = $\tilde{p}_z$
e^+ 4-momentum = $\tilde{p}_1$
e^- 4-momentum = $\tilde{p}_2$

$$\tilde{p}_z^2 = [\tilde{p}_1 + \tilde{p}_2]^2 = \tilde{p}_1^2 + \tilde{p}_2^2 + 2(\tilde{p}_1 \circ \tilde{p}_2)$$

Reconstructing the Z boson Mass

$$\tilde{p}_z^2 = [\tilde{p}_1 + \tilde{p}_2]^2 = \tilde{p}_1^2 + \tilde{p}_2^2 + 2(\tilde{p}_1 \circ \tilde{p}_2)$$

RECALL: Any 4-mom^2 is the particle mass^2, thus we get

$$m_z^2 = m_e^2 + m_e^2 + 2[\tilde{p}_1 \circ \tilde{p}_2]$$

$$m_z^2 = 2\, m_e^2 + 2[(E_1, \vec{p}_1) \circ (E_2, \vec{p}_2)]$$

Since Mass of electron is so small $m_e \ll E_e$ we make approximation $m_e \approx 0, E_{1,2} \approx |\vec{p}_{1,2}|$, then

$\boxed{m_z^2 \approx 2(E_1 E_2 - \vec{p}_1 \circ \vec{p}_2)\ \textit{use this eqn in Lab}}$

Breit-Wigner line shape

* The line shape probability distribution as a function of observed mass M of an unstable particle of central mass M_0 and FWHM width Γ is given by the Breit-Wigner distribution

$$\mathscr{P}(M; M_0, \Gamma) = \frac{1}{2\pi} \frac{\Gamma}{(M - M_0)^2 - (\Gamma/2)^2}$$

* This is the Fourier Transform of an exponential time decay of lifetime τ where $\Gamma = \hbar/\tau$

then for $\Delta E \approx \Gamma/2$ $\quad \Delta t \, \Delta E \approx \dfrac{\hbar}{2}$

Fit the data 2 ways

* Bin the reconstructed mass for the events and perform a χ^2 minimization to the Breit-Wigner distribution to determine the best fit mass and width (M_0, Γ)

* Use the un-binned data for the reconstructed mass and minimize the negative log likelihood to determine the best fit mass and width (M_0, Γ)

* Note that there are some background events that will need to be cut from the raw data sample before doing these fits!

See the Lab 1 notebook for some more details!